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Coordinated Stabilization of a Multimachine Power System

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COORDINATED STABILIZATION OF A MULTIMACHINE POWER SYSTEM

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Abstract - The paper presents a sequential procedure for coordinated stabilization of a multimachine power system with arbitrary complexity of the system model. The most effective machine to be equipped with a power system stabilizer is identified using eigenvalue analysis. The selection is based on the sensitivity of critical eigenvalues to increases in the coefficient of a damping term which is inserted in each equation of motion, in succession. The method is applied to a three-machine system and simulation studies show appreciable improvements in the small disturbance stability of the system.

INTRODUCTION

The coordinated design of power system stabilizers PSS in a multimachine power system has found a considerable attention in recent years [1] - [3] . The objective is to improve the stability of the power system. The first problem to be solved is to determine which machine is the most effective candidate for stabilization. A method based on eigenvalue - eigenvector analyses is described in [1] to guide selection of the machine which require stabilization the most.

The second problem is how to select the parameters of the stabilizers. A method has been proposed in [2] for selecting these parameters to achieve, approximately, predetermined improvements in the damping ratios of critical eigenvalues. A technique for optimal settings of the stabilizers has been described in [3] . The PSS of the assigned machine employes a signal from the shaft speed of that machine and feeds it back, through a lead-lag network, to the reference input of the machine voltage regulator.

This paper introduces an alternative approach for identifying the best candidate machine for stabilizer application in a multimachine power system. The selection procedure is based on the sensitivity of the eigenvalues, related to generator rotors, to changes in an intentionally inserted damping term coefficient in the equation of motion of each machine. This method is sequential, i.e. it takes into account the presence of stabilizers applied on previously selected machines, thus indicating whether new stabilizers are necessary. In contrast to existing methods [1] - [3] , this method is straightforward and can be applied to multimachine systems with arbitrary modelling complexity. The model is build from the linearized equations of the system components. Selection of PSS parameters is based on eigenvalue analysis to achieve satisfactory improvement in damping.

The proposed method is applied to a three-machine system comprising steam, hydro and nuclear power plants. Simulation results are presented to demonstrate the efficacy of this approach in improving the dynamic performance of the system. These include eigenvalue studies and time responses.

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COORDINATED STABILIZATION PROCEDURE

Multimachine System Model

A linearized model of the multimachine power system is obtained in the standard state-space form,

$$\dot{x} = A\,x + B\,u \qquad (1)$$

where x is the state vector, u is the input vector, A is the state coefficient matrix and B is the input coefficient matrix. The model can be derived by linearizing the nonlinear equations of the system(generators, exciters, governors, etc.) arround a nominal steady - state operating point. The components of x and u are the deviations of the variables from their corresponding steady-state values. A direct method for building the matrix A is given in the Appendix. Compared to the established method of [4] , the present method saves in computation time and effort by eliminating the need for the inversion of a matrix of the same order as the system model itself.

Eigenvalue Analysis

The eigenvalues of the matrix A contains the necessary information on the linearized stability of the multimachine system. The system is stable if all the eigenvalues lie on the left-hand side of the s-plane, i.e. the real parts are all negative. The real eigenvalues are related to exponential components in the time responses. The complex conjugate pairs of eigenvalues are associated with the oscillatory modes in the time responses. They have the form $\lambda = -\alpha + j\beta$, where β gives the frequency of oscillations in rad/sec and the value of $1/\alpha$ defines the time constant by which the magnitude of the oscillations is decayed. Study of the effects of changes in operating conditions and system parameters leads to the identification of the eigenvalues which may cause instability. The most important eigenvalues are those related to rotor oscillations.

Selection of the Best Candidate Machine For Stabilization

A damping term is added to the equation

of motion of each machine, i.e. Eq.11 becomes

$$\Delta\dot{\omega} = (\Delta T_m - \Delta T_e - K_d\Delta\omega)\ \omega_o/2H \qquad (2)$$

The damping coefficients of the machines are increased in turn, i.e. one at a time while the rest are set to zero. The eigenvalues of the system are calculated, and monitored for stability, in each case. The eigenvalues related to rotor oscillations are recorded and their normalized real parts are calculated as follows,

$$NRP = \frac{\text{Real part}\ \alpha \text{ with } K_d = \text{positive value}}{\text{Real part}\ \alpha \text{ with } K_d = 0} \qquad (3)$$

assuming the system is initially stable. This value facilitates a measure for relative improvement in the damping of oscillatory modes as K_d of each machine increases.

The damping coefficient which gives the maximum improvement in damping defines the best candidate machine for stabilizer application.

Selection of Stabilizer

The PSS thought here is a speed stabilizer applied to the voltage regulator loop. A signal from the shaft speed is applied through a lead-lag compensator to the reference of the voltage regulator. Analytical studies of [5] and field tests of [6] have shown that this is a very effective stabilizer for a single machine system. The same stabilizer structure was also suggested for multimachine power systems[1]-[3].

The parameters of the stabilizer should then be selected. The selection method adopted here is based on eigenvalue analysis. The time constants of the stabilizer are first specified and the gain is changed. The gain that gives satisfactory positions of the eigenvalues is selected. The stabilizer is then applied to the particular machine and simulation tests are made to assess its effectiveness.

Selection of the Next Machine to be Stabilized

If more damping is still required then we proceed to determine the next machine for stabilization, taking into account that machine one has already been equipped with a stabilizer. The procedure for determining this machine is the same as above but here the stabilizer of machine one is included. Then we study the effects of the damping coefficients of the remaining machines on the oscillatory modes. A stabilizer is then designed and applied to the second machine.

This process is continued to select other machines which require stabilization taking into account the effects of all stabilizers applied to the previously selected machines. If the improvement due to the damping coefficients is found to be insignificant, then the selection procedure is terminated. This indicates no more stabilizers are required.

THREE-MACHINE SYSTEM EXAMPLE

The multimachine power system considered here, and shown in Fig. 1 comprises three generating units. These are steam, hydro and nuclear plants. The rating of each unit is assumed to be the same.

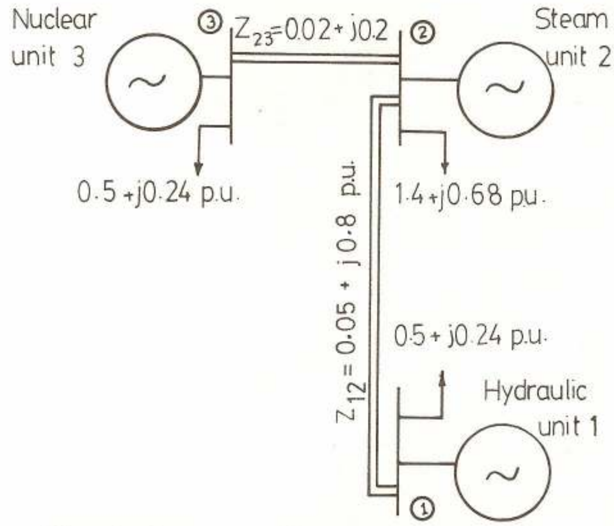


Fig. 1 Three-machine power system

generated powers (p.u.)		voltages (p.u.)
$P_1 = 0.81$	$Q_1 = 0.26$	$V_1 = 1.0\ \angle 0.0°$
$P_2 = 0.80$	$Q_2 = 0.77$	$V_2 = 1.0\ \angle 14.45°$
$P_3 = 0.80$	$Q_3 = 0.22$	$V_3 = 1.0\ \angle -2.31°$

The distance between the hydro unit and the steam unit is assumed to be much longer than the distance between the latter and the nuclear unit. Half of the load is concenterated on the busbar 2 of the steam plant. Busbar 1 has a quarter of the load and busbar 3 has the other quarter.

Each synchronous generator is represented by a seventh-order model based on Park's equations. The hydro and nuclear units are each equipped with a static exciter and the steam unit has a type 1 excitation system[7], Fig. 2 shows the block diagrams. Fig.3 shows the models of the turbine and speed governing systems [8]. The parameter values of the system are given in Table 1.

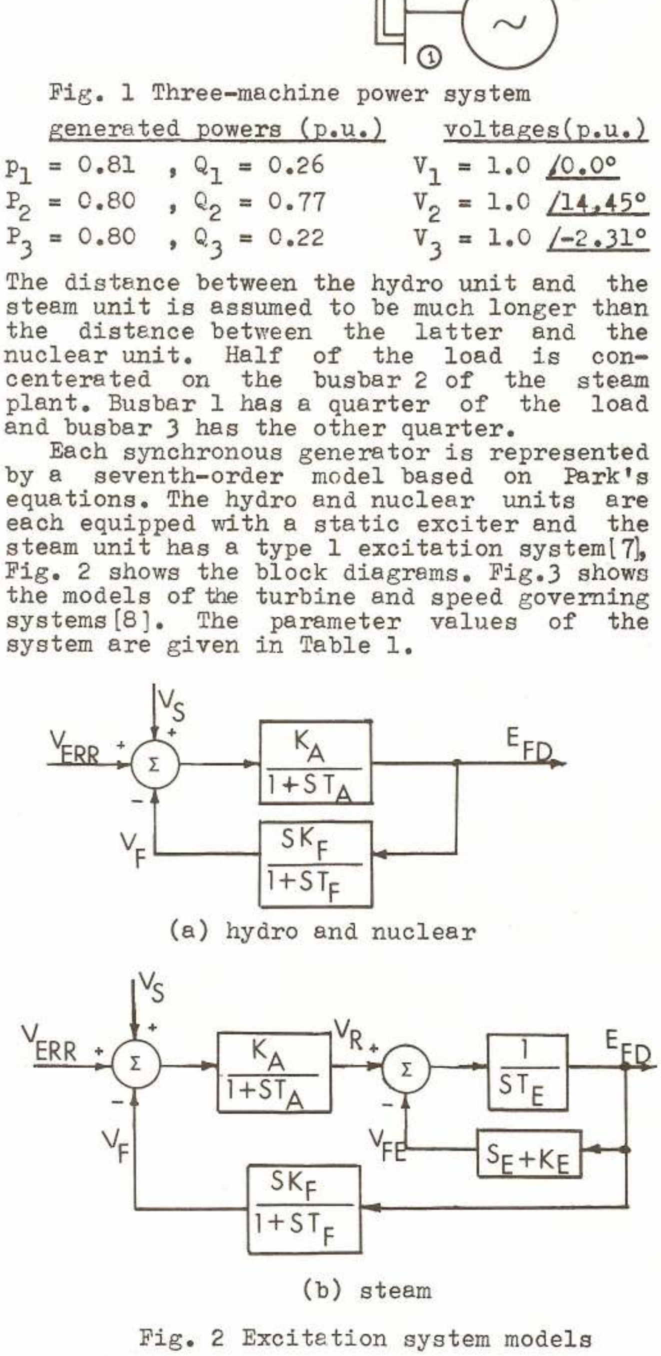
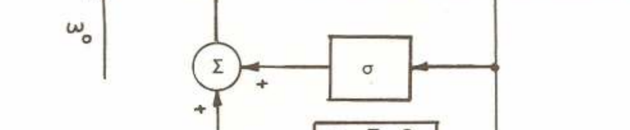


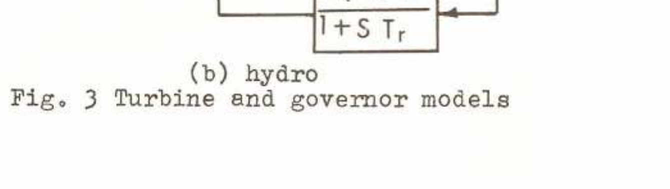
Fig. 2 Excitation system models

Table 1 Parameter values of the multimachine system . (all are in p.u., time constant sec.)

	Hydro unit	Steam unit	nuclear unit
x_{ffd}	1.1	1.65	1.97
x_{afd}	1.0	1.55	1.97
x_{fkd}	1.0	1.55	1.86
x_d	1.2	1.70	2.00
x_{akd}	1.0	1.55	1.86
x_{kkd}	1.1	1.605	1.936
x_q	1.8	1.64	1.91
x_{akq}	1.6	1.49	1.77
x_{kkq}	1.8	1.52	1.90
R	0.01	0.001	0.005
H	3.0	4.0	3.25
r_{kd}	0.02	0.0132	0.038
r_{kq}	0.04	0.0132	0.038
r_{fd}	0.0011	0.0007	0.0015
T_A	0.02	0.02	0.02
T_f	1.0	1.0	1.0
K_f	0.03	0.03	0.03
K_A	100	25.0	200
T_E	-	0.8	-
S_E	-	0.86	-
K_E	-	1.0	-
R_g	-	0.04	0.04
T_g	-	0.2	0.1
T_b	-	1.0	0.5
T_r	5.0	-	-
σ	0.04	-	-
δ_t	0.31	-	-
T_w	1.0	-	-
T_p	0.01	-	-
K_pK_h	0.2	-	-



(a) steam and nuclear



(b) hydro

Fig. 3 Turbine and governor models

RESULTS

The results of the load-flow solution of the three-machine system are summarized in Fig. 1. As the system has no infinite busbar, one arbitrary machine may be selected as a reference [4] . The calculated **eigenvalues** are not changed when any of the three machines is taken as a reference machine. Therefore, we selected the hydro unit 1 to be the reference machine. Thus the order of the system became thirty five. The system eigenvalues are listed in the first column of Table 2. These were calculated by using the QR standard algorithm. The eigenvalues related to rotor oscillations are $\lambda_{7,8}$ and $\lambda_{9,10}$. The mode with the lower frequency of oscillations, i.e. 7.27 rad/sec($\simeq$ 1.16 Hz), is the critical one. The real part of this eigenvalue pair may become positive for conditions such as operation with a leading power factor or operation with a single long transmission line, thus indicating instability. Other low frequency modes, shown in the Table, are related to control loops. The three highest frequency modes are related to the stator circuits of the **synchronous** generators. These are very rapidly damped as indicated by the large magnitude of the real parts. The real eigenvalue λ_{35} with the lowest magnitude is due to the slow-acting governor of the hydro unit.

Having calculated the eigenvalues and identified the modes of oscillations related to rotors, we then proceeded to determine which machine is the best candidate for stabilization. A damping term was added to each equation of motion as described before. The value of each damping coefficient K_{di} was changed, in turn, from 0 to a value of 10 and the normalized real parts (NRPs) of $\lambda_{7,8}$ and $\lambda_{9,10}$ are listed in Table 3.

As the value of K_{di} increases the value of NRP increases, the relation is almost linear as shown in Fig . 4. Clearly , the influence of K_{di} on the NRP of the **critical** eigenvalues $\lambda_{9,10}$ is more pronounced. The maximum increase in NRP of $\lambda_{9,10}$ is due to K_{d1}, therefore machine 1 (the hydro unit) is the most effective machine for stabilizer application. This machine is located at the far-end of the power system. It is interesting to notice that the proposed technique for selecting the best candidate machine for stabilization leads to the machine which has the maximum influence on damping of the lower frequency mode. This conclusion agrees with the results of [1] .

Fig. 5 shows the stabilizer block diagram. This structure was selected as it provided the most damping effect for the three-machine example. T_1 and T_2 are set to 0.15 and 0.015 sec., respectively and K_g is to be determined using eigenvalue analysis. As K_g increases the real part of $\lambda_{9,10}$ increases in the negative direction, thus showing improvement in damping of this mode. On the other hand increasing the value of K_g causes other stable eigenvalues to migrate to the right and then crossing the imaginary axes for high values of K_g, thus indicating

Table 2 System eigenvalues

3 m/c without stab.	3 m/c + stab. on hydro	3 m/c + stab. on hydro+nuclear
$\lambda(1,2) = -1586 \pm j\,1112$	$= -1445 \pm j\,1003$	$= -1445 \pm j\,1003$
$\lambda(3,4) = -261 \pm j\,956$	$= -197 \pm j\,832$	$= -197 \pm j\,832$
$\lambda(5,6) = -35.7 \pm j\,449$	$= -35.5 \pm j\,449$	$= -35.5 \pm j\,449$
$\lambda(7,8) = -1.51 \pm j\,10.14$	$= -1.55 \pm j\,10.189$	$= -2.37 \pm j\,10.68$
$\lambda(9,10) = -0.49 \pm j\,7.27$	$= -2.01 \pm j\,7.552$	$= -2.02 \pm j\,7.64$
$\lambda(11,12) = -5.49 \pm j\,1.69$	$= -3.99 \pm j\,2.67$	$= -3.79 \pm j\,2.77$
$\lambda(13,14) = -1.67 \pm j\,0.11$	$= -2.29 \pm j\,0.17$	$= -1.42 \pm j\,1.89$
$\lambda(15,16) = -1.06 \pm j\,0.77$	$= -1.43 \pm j\,0.79$	$= -1.26 \pm j\,0.63$
$\lambda(17,18) = -0.61 \pm j\,0.42$	$= -1.13 \pm j\,0.042$	$= -1.13 \pm j\,0.049$
$\lambda(19,20) = -0.41 \pm j\,1.67$	$= -0.93 \pm j\,1.47$	$= -0.59 \pm j\,0.35$
$\lambda(21) = -350 \pm j\,0.0$	$= -0.61 \pm j\,0.41$	$= -350 \pm j\,0.0$
$\lambda(22) = -200 + j\,0.0$	$= -350 + j\,0.0$	$= -200 + j\,0.0$
$\lambda(23) = -98.1 + j\,0.0$	$= -200 + j\,0.0$	$= -98.2 + j\,0.0$
$\lambda(24) = -74.6 + j\,0.0$	$= -98.2 + j\,0.0$	$= -80.9 + j\,0.0$
$\lambda(25) = -48.9 + j\,0.0$	$= -74.6 + j\,0.0$	$= -66.4 + j\,0.0$
$\lambda(26) = -34.3 + j\,0.0$	$= -66.4 + j\,0.0$	$= -58.1 + j\,0.0$
$\lambda(27) = -32.8 + j\,0.0$	$= -48.9 + j\,0.0$	$= -48.9 + j\,0.0$
$\lambda(28) = -25.8 + j\,0.0$	$= -33.3 + j\,0.0$	$= -33.2 + j\,0.0$
$\lambda(29) = -20.5 + j\,0.0$	$= -29.8 + j\,0.0$	$= -29.8 + j\,0.0$
$\lambda(30) = -10.52 + j\,0.0$	$= -25.5 + j\,0.0$	$= -25.7 + j\,0.0$
$\lambda(31) = -6.59 + j\,0.0$	$= -15.2 + j\,0.0$	$= -15.2 + j\,0.0$
$\lambda(32) = -5.31 + j\,0.0$	$= -10.53 + j\,0.0$	$= -10.6 + j\,0.0$
$\lambda(33) = -2.35 + j\,0.0$	$= -5.54 + j\,0.0$	$= -5.55 + j\,0.0$
$\lambda(34) = -1.09 + j\,0.0$	$= -4.88 + j\,0.0$	$= -4.91 + j\,0.0$
$\lambda(35) = -0.029 + j\,0.0$	$= -0.028 + j\,0.0$	$= -2.48 + j\,0.0$
$\lambda(36) =$ ——	——	$= -2.19 + j\,0.0$
$\lambda(37) =$ ——		$= -0.026 + j\,0.0$

Table 3 Normalized Real Parts of Oscillatory Modes

(a) $\lambda_{9,10} = -0.49 \pm j\,7.27$, for $K_{di} = 0$

Value of K_{di}	NRP of $\lambda_{9,10}$ for increase in K_{di}		
	K_{d1}	K_{d2}	K_{d3}
0	1	1	1
2	1.4	1.09	1.14
4	1.8	1.17	1.25
6	2.18	1.26	1.37
8	2.57	1.33	1.47
10	2.98	1.42	1.59

(b) $\lambda_{7,8} = -1.51 \pm j\,10.14$, for $K_{di} = 0$

Value of K_{di}	NRP of $\lambda_{7,8}$ for increase in K_{di}		
	K_{d1}	K_{d2}	K_{d3}
0	1	1	1
2	1.01	1.04	1.06
4	1.03	1.08	1.12
6	1.04	1.13	1.17
8	1.058	1.17	1.23
10	1.062	1.21	1.29

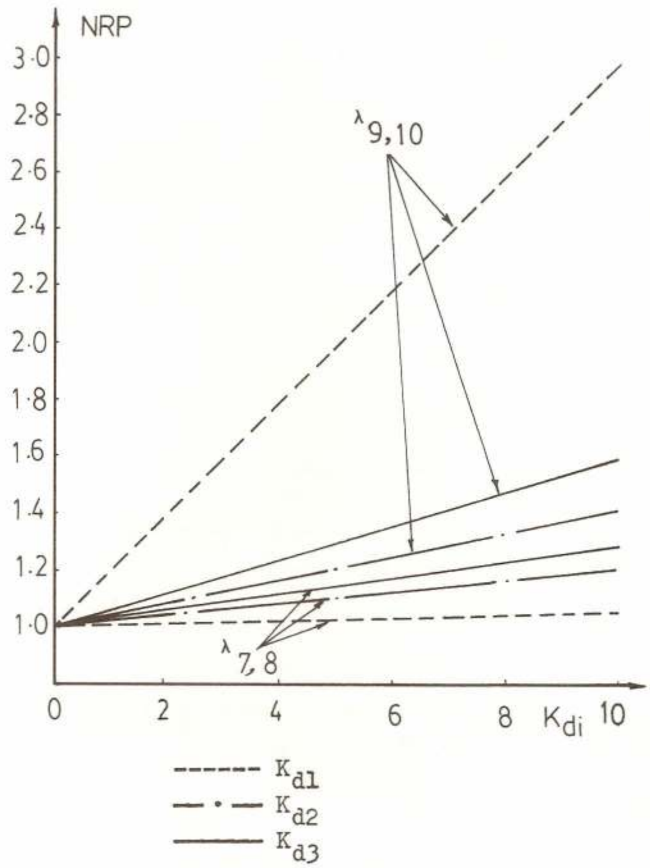


Fig. 4 Effect of K_{di} on NRP of $\lambda_{7,8}$ and $\lambda_{9,10}$.

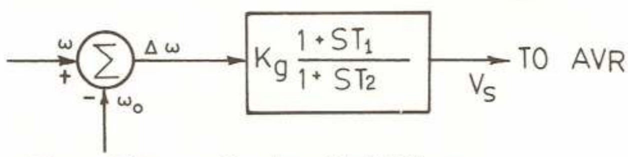


Fig. 5 Power System Stabilizer

instability. The choice of K_g is therefore a compromise between the improvement in the damping of $\lambda_{9,10}$ and the deterioration in the damping of other eigenvalues. A value of K_g = 0.04 p.u. was found to be a good choice.

Table 2, column 2, shows the system eigenvalues after the application of the stabilizer on the hydro unit 1. The stabilizer provides a significant improvement in the damping of the critical mode $\lambda_{9,10}$. A slight improvement in the damping of $\lambda_{7,8}$ is also indicated. Other eigenvalues are not significantly affected.

Fig. 6 shows the loci of the eigenvalues $\lambda_{7,8}$ and $\lambda_{9,10}$ with variations in the p.f. of the loads. The unstabilized system renders unstable for leading p.f. of 0.94 or less due to the critical eigenvalues $\lambda_{9,10}$. After the application of the stabilizer on the hydro unit 1, the loci of $\lambda_{9,10}$ is shifted a good distance to the left and the system is stable for the indicated range of the p.f. Although $\lambda_{7,8}$ do not lead to instability, as shown in Fig. 6 (b), their position is marginally improved due to the application of the stabilizer.

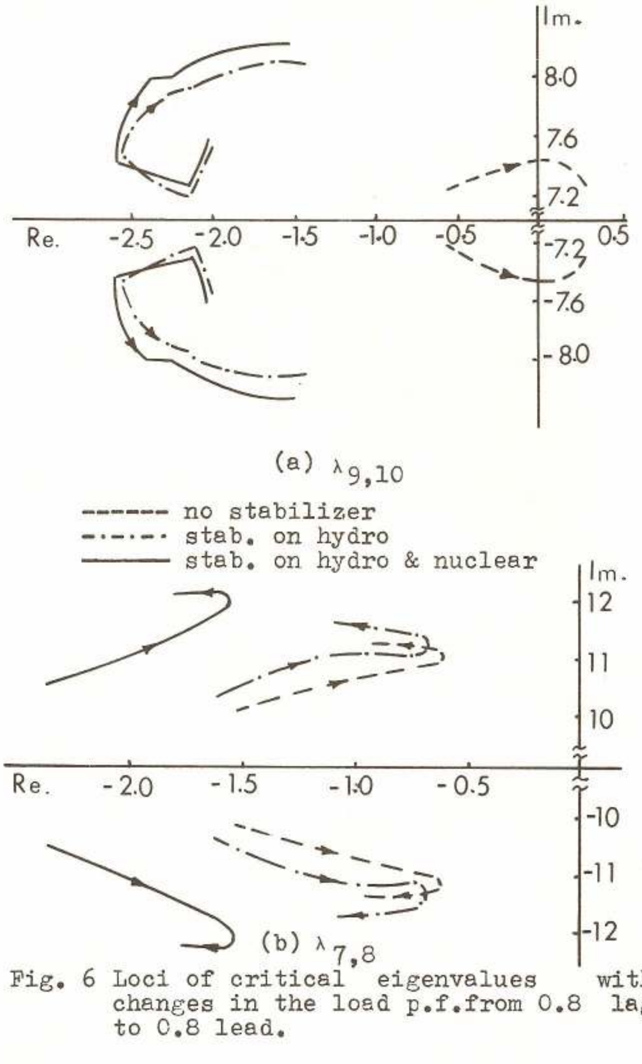


Fig. 6 Loci of critical eigenvalues with changes in the load p.f. from 0.8 lag to 0.8 lead.

Fig. 7 shows the system responses to a 0.05 p. u. pulse disturbances in the reference voltage of the AVR of the hydro unit. The response with the stabilizer on the hydro unit shows an appreciable improvement in the system damping when compared with the unstabilized response. The improvement is reflected in the entire system, thus confirming the effectiveness of the selection procedure. These responses were obtained by solving the linearized model of the system using the Runge-Kutta integration routine.

We now proceed to select the next machine to be equipped with a stabilizer. The stabilizer of the hydro unit is added to the system model, thus the order becomes thirty six. Table 4 shows the effect of changes in K_{d2} and K_{d3} on the NRPs of the eigenvalues $\lambda_{7,8}$ and $\lambda_{9,10}$, noting that the basic value of the real parts corresponding to $K_{di} = 0$ is increased due to the presence of the first stabilizer on the hydro unit.

Table 4 Normalized Real Parts of Oscillatory Modes

(a) $\lambda_{7,8} = -1.55 \pm j\,10.189$, for $K_{di} = 0$.

Value of K_d	NRP of $\lambda_{7,8}$ for increase in K_{di}	
	K_{d2}	K_{d3}
0	1.00	1.00
2	1.04	1.06
4	1.08	1.12
6	1.13	1.18
8	1.17	1.24
10	1.21	1.30

(b) $\lambda_{9,10} = -2.01 \pm j\,7.552$, for $K_{di} = 0$.

Value of K_d	NRP of $\lambda_{9,10}$ for increase in K_{di}	
	K_{d2}	K_{d3}
0	1.00	1.00
2	1.07	1.08
4	1.09	1.09
6	1.10	1.09
8	1.11	1.115
10	1.12	1.13

Analysis of these tables indicates that the best machine for stabilization is now machine no. 3, i.e. the nuclear unit. This is because the improvement in damping due to K_{d3} is greater than that due to K_{d2}. It is also observed that the improvement in the damping of the lower frequency mode $\lambda_{9,10}$ is less significance. This mode was well damped by the application of the first stabilizer on the hydro unit.

A similar stabilizer as that shown in Fig. 5 was applied to machine 3 and the resulting eigenvalues are listed in the last column of Table 2. The system has now two stabilizers one on the hydro unit 1 and the other on the nuclear unit 3. The addition of the second stabilizer leads to an appreciable improvement in the damping of the higher frequency mode $\lambda_{7,8}$. A slight improvement is

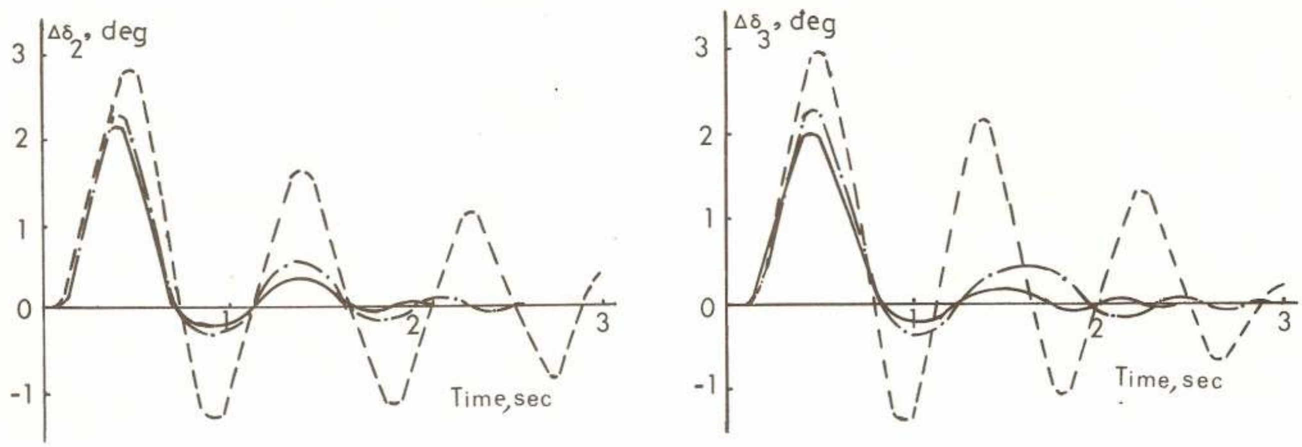


Fig. 7 System response to a small disturbance

also shown in the damping of $\lambda_{9,10}$. The loci of the eigenvalues shown in Fig. 6 supports the above two statements. The time response of Fig. 7 indicates the further improvement in the system damping after the addition of the second stabilizer.

We now investigate whether we can achieve further improvement in damping by the application of a stabilizer on the steam unit 2. Table 5 shows the effect of increasing K_{d2} on the NRPs of $\lambda_{7,8}$ and $\lambda_{9,10}$, noting that the basic values of the real parts are changed due to the presence of the two stabilizers. The improvement in the damping of both modes is evidently insignificant, thus indicating that the application of a stabilizer on the steam unit 2 is not necessary. The selection procedure is, therefore, terminated and the coordinated design of the multimachine stabilization is now completed.

Table 5 Normalized Real Parts of Oscillatory Modes.

$\lambda_{7,8} = -2.37 \pm j\,10.68$ and

$\lambda_{9,10} = -2.02 \pm j\,7.64$, for $K_{d2} = 0$

Value of K_{d2}	NRP of eigenvalues	
	$\lambda_{7,8}$	$\lambda_{9,10}$
0	1.00	1.00
2	1.02	1.01
4	1.033	1.02
6	1.04	1.025
8	1.046	1.03
10	1.051	1.032

CONCLUSIONS

A direct procedure for guiding selection of the most effective machines for stabilization, in a multimachine system, has been demonstrated. This procedure is sequential, i.e. it takes into account the influence of stabilizers applied on previously selected machines, thus indicating whether new stabilizers are necessary.

The procedure has been successfully applied to a three-machine power system and shown to provide an appreciable improvement in the dynamical performance of the entire system. The results have shown that the application of a power system stabilizer on each machine is not necessary provided that the best candidate machines have been properly selected and equipped with effective stabilizers.

Although a fairly detailed model has been used to represent the three-machine system, the procedure can be readily applied to systems with arbitrary modelling complexity. For large practical power systems, however, model reduction techniques may be employed if necessary.

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APPENDIX

Linearized Model of Multimachine Power System

The method of obtaining a linearized model for multimachine systems is explained here for a two-machine system in order to clarify the notations. Extension to systems with more machines can easily be made. The order of the submatrices given at the end of this appendix helps in the extension process.

Linearized Equations of a Synchronous Generator [4]

$$\begin{bmatrix} \Delta\psi_{fd} \\ \Delta\psi_{d} \\ \Delta\psi_{kd} \\ \Delta\psi_{q} \\ \Delta\psi_{kq} \end{bmatrix} = \begin{bmatrix} x_{ffd} & -x_{afd} & x_{fkd} & 0 & 0 \\ x_{afd} & -x_{d} & x_{akd} & 0 & 0 \\ x_{fkd} & -x_{akd} & x_{kkd} & 0 & 0 \\ 0 & 0 & 0 & -x_{q} & x_{akq} \\ 0 & 0 & 0 & -x_{akq} & x_{kkq} \end{bmatrix} \begin{bmatrix} \Delta i_{fd} \\ \Delta i_{d} \\ \Delta i_{kd} \\ \Delta i_{q} \\ \Delta i_{kq} \end{bmatrix} \quad (4)$$

$$\Delta\dot{\psi}_{fd} = -\omega_o r_{fd}\Delta i_{fd} + (\omega_o r_{fd}/x_{afd})\Delta E_{FD} \quad \ldots (5)$$

$$\Delta\dot{\psi}_{d} = \omega_o\,\Delta\psi_q + \psi_q\Delta\omega + \omega_o R\,\Delta i_d + \omega_o\Delta v_d \quad \ldots (6)$$

$$\Delta\dot{\psi}_{kd} = -\omega_o\, r_{kd}\,\Delta i_{kd} \quad \ldots\ \ldots\ \ldots \quad \ldots (7)$$

$$\Delta\dot{\psi}_{q} = -\omega_o\Delta\psi_d - \psi_d\Delta\omega + \omega_o R\,\Delta i_q + \omega_o\,\Delta v_q \quad \ldots (8)$$

$$\Delta\dot{\psi}_{kkq} = -\omega_o r_{kq}\,\Delta i_{kq} \quad \ldots\ \ldots\ \ldots \quad \ldots (9)$$

$$\Delta\dot{\delta} = \Delta\omega \quad \ldots\ \ldots\ \ldots\ \ldots \quad \ldots (10)$$

$$\Delta\dot{\omega} = (\Delta T_m - \Delta T_e)\,\omega_o/2H \quad \ldots\ \ldots \quad \ldots (11)$$

where

$$\Delta T_e = i_q\,\Delta\psi_d - i_d\,\Delta\psi_q - \psi_q\,\Delta i_d + \psi_d\,\Delta i_q$$

$$\Delta V_t = (V_d\,\Delta V_d + V_q\,\Delta V_q)/V_t \quad \ldots \quad \ldots (12)$$

Network Equations

A load flow solution of the power system is to be obtained first and the steady-state values of the system variables are then calculated. If the system has a non-synchronous load which can be represented by a constant admittance, the latter is added to the self admittance of the node at which the load is connected. All the node loads are then eliminated using the well known Kron reduction method. Thus node equations of the reduced network, which contains only machine nodes, may be written as :

$$I = Y\,V \quad \ldots\ \ldots\ \ldots\ \ldots (13)$$

Reference Frame

The node equations are referred to D and Q axes which rotate at the angular frequency of the network current. Eq. 13 can therefore be expressed in the double real form

$$\begin{bmatrix} I_{D1} \\ I_{Q1} \\ I_{D2} \\ I_{Q2} \end{bmatrix} = \begin{bmatrix} G_{11} & -B_{11} & G_{12} & -B_{12} \\ B_{11} & G_{11} & B_{12} & G_{12} \\ G_{21} & -B_{21} & G_{22} & -B_{22} \\ B_{21} & G_{21} & B_{22} & G_{22} \end{bmatrix} \begin{bmatrix} V_{D1} \\ V_{Q1} \\ V_{D2} \\ V_{Q2} \end{bmatrix}$$

or in the symbolic form

$$I_N = Y_N\,V_N \quad \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots (14)$$

The common reference frame may be chosen arbitrary. The system variables are referred to the common reference frame (D and Q) of the network, then we follow Undrill's [4] assumption considering the network frequency is always identical to that of one arbitrary selected machine.

System Transformation

The equations of each machine are usually referred to its rotor d and q axes. In steady-state the axes of the machines rotate at a constant speed with angular differences as shown in Fig. 8. Therefore the system equations referred to the common reference frame are

$$\begin{bmatrix} V_{D1} \\ V_{Q1} \\ V_{D2} \\ V_{Q2} \end{bmatrix} = \begin{bmatrix} \cos\delta_1 & -\sin\delta_1 & 0 & 0 \\ \sin\delta_1 & \cos\delta_1 & 0 & 0 \\ 0 & 0 & \cos\delta_2 & -\sin\delta_2 \\ 0 & 0 & \sin\delta_2 & \cos\delta_2 \end{bmatrix} \begin{bmatrix} v_{d1} \\ v_{q1} \\ v_{d2} \\ v_{q2} \end{bmatrix}$$

In symbolic form the above equation can be written as

$$V_N = T\,v_m \quad \ldots\ \ldots\ \ldots\ \ldots (15)$$

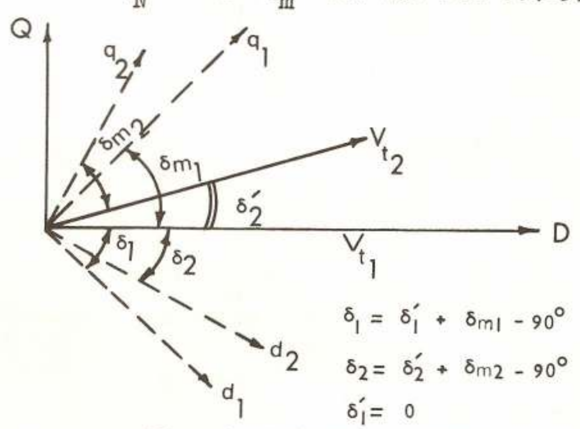

Fig. 8 Reference frame

For small perturbation we have :

$$\begin{bmatrix} \Delta V_{D1} \\ \Delta V_{Q1} \\ \Delta V_{D2} \\ \Delta V_{Q2} \end{bmatrix} = \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} \Delta v_{d1} \\ \Delta v_{q1} \\ \Delta v_{d2} \\ \Delta v_{q2} \end{bmatrix} + \begin{bmatrix} -v_{d1}\sin\delta_1 - v_{q1}\cos\delta_1 & 0 \\ v_{d1}\cos\delta_1 - v_{q1}\sin\delta_1 & 0 \\ 0 & -v_{d2}\sin\delta_2 - v_{q2}\cos\delta_2 \\ 0 & v_{d2}\cos\delta_2 - v_{q2}\sin\delta_2 \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \end{bmatrix}$$

which may be written in symbolic form as:

$$\Delta V_N = T\,\Delta v_m + D\,\underline{\Delta\delta} \quad \ldots\ \ldots\ \ldots\ \ldots (16)$$

The incremental form of the network node Eq. 14 is

$$\Delta I_N = Y_N \; \Delta V_N \quad \dots \; \dots \; \dots .(17)$$

From the power invariance theorem of Kron

$$i_m = T^t \; I_N \quad \dots \; \dots \; \dots .(18)$$

where i_m is the vector of the machines currents and T^t is the transpose of the transformation matrix.

For small perturbation, we have

$$\begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix} = [T]^t \begin{bmatrix} \Delta I_{D1} \\ \Delta I_{Q1} \\ \Delta I_{D2} \\ \Delta I_{Q2} \end{bmatrix} + \begin{bmatrix} i_{q1} & 0 \\ -i_{d1} & 0 \\ 0 & i_{q2} \\ 0 & -i_{d2} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \end{bmatrix}$$

or in compact form

$$\Delta i_m = T^t \, \Delta I_N + E \, \underline{\Delta\delta} \quad \dots \; \dots \; \dots \; \dots \quad (19)$$

Eliminating ΔI_N and ΔV_N from Eqs.16,17 and 19, and solving for Δv_m, we have :

$$\Delta v_m = G \, \Delta i_m + Q \, \underline{\Delta\delta} \quad \dots \; \dots \; \dots \; \dots \quad (20)$$

where

$$G = T^t \; Y_N^{-1} \; T$$

and $Q = - G \, E - T^t \, D$

Synchronous Machines

Eqs. 6 and 8 of each machine are now arranged in the matrix form :

$$\begin{bmatrix} \Delta\dot\psi_{d1} \\ \Delta\dot\psi_{q1} \\ \Delta\dot\psi_{d2} \\ \Delta\dot\psi_{q2} \end{bmatrix} = \begin{bmatrix} 0 & \omega_o & 0 & 0 \\ -\omega_o & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_o \\ 0 & 0 & -\omega_o & 0 \end{bmatrix} \begin{bmatrix} \Delta\psi_{d1} \\ \Delta\psi_{q1} \\ \Delta\psi_{d2} \\ \Delta\psi_{q2} \end{bmatrix} + \begin{bmatrix} \omega_o R_1 & 0 & 0 & 0 \\ 0 & \omega_o R_1 & 0 & 0 \\ 0 & 0 & \omega_o R_2 & 0 \\ 0 & 0 & 0 & \omega_o R_2 \end{bmatrix} \cdot$$

$$\begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d1} \\ \Delta i_{q2} \end{bmatrix} + \begin{bmatrix} \omega_o & 0 & 0 & 0 \\ 0 & \omega_o & 0 & 0 \\ 0 & 0 & \omega_o & 0 \\ 0 & 0 & 0 & \omega_o \end{bmatrix} \begin{bmatrix} \Delta v_{d1} \\ \Delta v_{q1} \\ \Delta v_{d2} \\ \Delta v_{q2} \end{bmatrix} + \begin{bmatrix} \psi_{q1} & 0 \\ -\psi_{d1} & 0 \\ 0 & +\psi_{q2} \\ 0 & -\psi_{d2} \end{bmatrix} \begin{bmatrix} \Delta\omega_1 \\ \Delta\omega_2 \end{bmatrix}$$

Compactly

$$\Delta\dot\psi = J \, \Delta\psi + K \, \Delta i_m + W \, \Delta v_m + P \, \underline{\Delta\omega} \quad \dots \; \dots (21)$$

Eliminating the vector Δv_m from Eq. 21 by using Eq. 20, we obtain

$$\Delta\dot\psi = J \, \Delta\psi + F \; \Delta i_m + C \, \underline{\Delta\delta} + P \, \underline{\Delta\omega} \quad \dots \; \dots (22)$$

where $F = W \, G + K$, $C = W \, Q$

The angular velocity equations of the machines, Eq. 10, can be combined to give

$$\begin{bmatrix} \Delta\dot\delta_1 \\ \Delta\dot\delta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta\omega_1 \\ \Delta\omega_2 \end{bmatrix} \quad \dots\dots .(23)$$

Also the swing equations of the machines, Eq. 11, are

$$\begin{bmatrix} \Delta\dot\omega_1 \\ \Delta\dot\omega_2 \end{bmatrix} = \begin{bmatrix} \frac{-\omega_o i_{q1}}{2\,H_1} & \frac{\omega_o i_{d1}}{2\,H_1} & 0 & 0 \\ 0 & 0 & \frac{-\omega_o i_{q2}}{2\,H_2} & \frac{\omega_o i_{d2}}{2\,H_2} \end{bmatrix} \begin{bmatrix} \Delta\psi_{d1} \\ \Delta\psi_{q1} \\ \Delta\psi_{d2} \\ \Delta\psi_{q2} \end{bmatrix} + \begin{bmatrix} \frac{\omega_o \psi_{q1}}{2\,H_1} & \frac{-\omega_o \psi_{d1}}{2\,H_1} & 0 & 0 \\ 0 & 0 & \frac{\omega_o \psi_{q2}}{2\,H_2} & \frac{-\omega_o \psi_{d2}}{2\,H_2} \end{bmatrix} \begin{bmatrix} \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta i_{d2} \\ \Delta i_{q2} \end{bmatrix} + \begin{bmatrix} \frac{\omega_o}{2H_1} & 0 \\ 0 & \frac{\omega_o}{2H_2} \end{bmatrix} \begin{bmatrix} \Delta T_{m1} \\ \Delta T_{m2} \end{bmatrix} \quad \dots \; \dots \; (24)$$

Now combining Eqs. 22, 23 and 24 and the field and amortisseur winding of each machine, Eqs.5,7 and 9, we obtain a model representing the two machine system . This model has the compact form

$$\dot x = L \, x + M \, \Delta I + B \; u \quad \dots \; \dots \; (25)$$

where the expanded form of this model is given at the end of the Appendix . The state-space form of the system can be obtained as follows : Multiply the matrix M by the inverse matrix of the reactances of the machines, Eqs.4 which will be 10 x 10 matrix for the two-machine system . Then adding the resultant matrix to L , we obtain the state-space Eq. 1 .

Selection of the Reference Axes

In the forgoing analysis, the network reference axes were assumed to be rotating at the constant speed ω_o. Since this assumption is not valid as explained by Undrill[4], we follow his approach which consider the network frequency is always identical to that of one arbitrary selected machine. Thus the (D-Q) axes rotate in synchronism with the axes (d_r, q_r) of that machine, i.e. $\Delta\delta_r$ of the r-th machine is zero. The system model of Eq. 25 should be modified as follows :

(i) deleting $\Delta\dot\delta_r$, $\Delta\delta_r$ and the rows and columns corresponding to $\Delta\delta_r$.

(ii) placing (-1) in the rows of the new L matrix corresponding to the remaining angle deviations and the column corresponding to $\Delta\omega_r$.

Excitation and Governing Systems

Excitation and governing systems of each machine are represented by state-space models and then added to the synchronous machine model Eq. 25 . For representing voltage regulator of generators , an expression for terminal voltages is required. It is obtained from Eq. 12 for each machine as follows :

$$\begin{bmatrix} \Delta V_{t1} \\ \Delta V_{t2} \end{bmatrix} = \begin{bmatrix} V_{d1}/V_{t1} & V_{q1}/V_{t1} & 0 & 0 \\ 0 & 0 & V_{d2}/V_{t2} & V_{q2}/V_{t2} \end{bmatrix} \begin{bmatrix} \Delta v_{d1} \\ \Delta v_{q1} \\ \Delta v_{d2} \\ \Delta v_{q2} \end{bmatrix}$$

or compactily

$$\Delta V_t = \Phi \, \Delta v_m \quad \dots \; \dots \; \dots \; \dots \; \dots \quad (26)$$

Substituting for Δv_m from Eq.20 , we have

$$\Delta V_t = Z \;\; \Delta i_m + S \, \underline{\Delta\delta} \quad \dots \; \dots \; \dots \quad (27)$$

where $Z = \Phi \, G$ and $S = \Phi \, Q$

Eq. 27 facilitates the inclusion of the voltage regulators to the system model of Eq. 25.

The Expanded Form

It is useful to record here the expansion of Eq. 25 as follows:

$$
\begin{bmatrix} \Delta\dot\psi_{fd1} \\ \Delta\dot\psi_{d1} \\ \Delta\dot\psi_{kd1} \\ \Delta\dot\psi_{q1} \\ \Delta\dot\psi_{kq1} \\ \Delta\dot\psi_{fd2} \\ \Delta\dot\psi_{d2} \\ \Delta\dot\psi_{kd2} \\ \Delta\dot\psi_{q2} \\ \Delta\dot\psi_{kq2} \\ \Delta\dot\delta_1 \\ \Delta\dot\delta_2 \\ \Delta\dot\omega_1 \\ \Delta\dot\omega_2 \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \omega_o & 0 & 0 & 0 & 0 & 0 & 0 & \omega_o C_{11} & \omega_o C_{12} & \psi_{q1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\omega_o & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega_o C_{21} & \omega_o C_{22} & -\psi_{d1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega_o & 0 & \omega_o C_{31} & \omega_o C_{32} & 0 & \psi_{q2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\omega_o & 0 & 0 & 0 & \omega_o C_{41} & \omega_o C_{42} & 0 & -\psi_{d2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & \frac{-\omega_o i_{q1}}{2H_1} & 0 & \frac{\omega_o i_{d1}}{2H_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{-\omega_o i_{q2}}{2H_2} & 0 & \frac{\omega_o i_{d2}}{2H_2} & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} \Delta\psi_{fd1} \\ \Delta\psi_{d1} \\ \Delta\psi_{kd1} \\ \Delta\psi_{q1} \\ \Delta\psi_{kq1} \\ \Delta\psi_{fd2} \\ \Delta\psi_{d2} \\ \Delta\psi_{kd2} \\ \Delta\psi_{q2} \\ \Delta\psi_{kq2} \\ \Delta\delta_1 \\ \Delta\delta_2 \\ \Delta\omega_1 \\ \Delta\omega_2 \end{bmatrix}
+
$$

$$
\begin{bmatrix}
-\omega_o r_{fd1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \omega_o F_{11} & 0 & \omega_o F_{12} & 0 & 0 & \omega_o F_{13} & 0 & \omega_o F_{14} & 0 \\
0 & 0 & -\omega_o r_{kd1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \omega_o F_{21} & 0 & \omega_o F_{22} & 0 & 0 & \omega_o F_{23} & 0 & \omega_o F_{24} & 0 \\
0 & 0 & 0 & 0 & -\omega_o r_{kq1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\omega_o r_{fd2} & 0 & 0 & 0 & 0 \\
0 & \omega_o F_{31} & 0 & \omega_o F_{32} & 0 & 0 & \omega_o F_{33} & 0 & \omega_o F_{34} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_o r_{kd2} & 0 & 0 \\
0 & \omega_o F_{41} & 0 & \omega_o F_{42} & 0 & 0 & \omega_o F_{43} & 0 & \omega_o F_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_o r_{kq2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\omega_o \psi_{q1}}{2H_1} & 0 & \frac{-\omega_o \psi_{d1}}{2H_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\omega_o \psi_{q2}}{2H_2} & 0 & \frac{-\omega_o \psi_{d2}}{2H_2} & 0
\end{bmatrix}
\begin{bmatrix} \Delta i_{fd1} \\ \Delta i_{d1} \\ \Delta i_{kd1} \\ \Delta i_{q1} \\ \Delta i_{kq1} \\ \Delta i_{fd2} \\ \Delta i_{d2} \\ \Delta i_{kd2} \\ \Delta i_{q2} \\ \Delta i_{kq2} \end{bmatrix}
+
\begin{bmatrix}
\frac{\omega_o r_{fd1}}{x_{afd1}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & \frac{\omega_o r_{fd2}}{x_{afd2}} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \frac{\omega_o}{2H_1} & 0 \\
0 & 0 & 0 & \frac{\omega_o}{2H_2}
\end{bmatrix}
\begin{bmatrix} \Delta E_1 \\ \Delta E_2 \\ \Delta T_{m1} \\ \Delta T_{m2} \end{bmatrix}
$$

Order of the Submatrices

Finally, we list the order of the submatrices to help in expanding the above method of building the matrix A of a multimachine power system. If the number of the machines is n, then Y is an (nxn) complex matrix, and I and V are both (nxl) complex vectors. The other submatrices are all real and have dimensions as follows:

Matrix	Dimension	Vector	Dimension
Y_N T G J K W	2nx2n	I_N i_m	2nxl
P D E Q	2nxn	V_N v_m	2nxl
Z Φ	nx2n	$\Delta\psi$	2nxl
S	nxn	$\Delta\delta$ $\Delta\omega$	nxl

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