

## Coordinating Agile Systems through the Model-based Execution of Temporal Plans\*

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### Abstract

Agile autonomous systems are emerging, such as unmanned aerial vehicles (UAVs), that must robustly perform tightly coordinated time-critical missions; for example, military surveillance or search-and-rescue scenarios. In the space domain, execution of temporally flexible plans has provided an enabler for achieving the desired coordination and robustness. We address the challenge of extending plan execution to under-actuated systems that are controlled indirectly through the setting of continuous state variables.

Our solution is a novel model-based executive that takes as input a temporally flexible *state plan*, specifying intended state evolutions, and dynamically generates a near-optimal control sequence. To achieve optimality and safety, the executive plans into the future, framing planning as a disjunctive programming problem. To achieve robustness to disturbances and tractability, planning is folded within a receding horizon, continuous planning framework. Key to performance is a problem reduction method based on constraint pruning. We benchmark performance through a suite of UAV scenarios using a hardware-in-the-loop testbed.

### Introduction

Autonomous control of dynamic systems has application in a wide variety of fields, from managing a team of agile unmanned aerial vehicles (UAVs) for fire-fighting missions, to controlling a Mars life support system. The control of such systems is challenging for several reasons. First, they are under-actuated systems, which means they are described by models involving more state variables than input variables, so that not all state variables are directly controllable; second, their models involve continuous dynamics described by differential equations; third, controlling these systems usually requires tight synchronization; and fourth, the controller must be optimal and robust to disturbances.

To address these challenges, an autonomous controller for agile systems should provide three capabilities: 1) to handle tight coordination, the system should execute a temporal plan specifying time coordination constraints. 2) To deal

with the under-actuated nature of the system, it should elevate the interaction with the system under control (or *plant*) to the level at which the human operator is able to robustly program the plant in terms of desired state evolution, including state variables that are not directly controllable. 3) To deal with the under-actuated dynamics of the plant, the intended state evolution must be specified in a temporally flexible manner, allowing robust control over the system.

Previous work in model-based programming introduced a *model-based executive*, called *Titan* (Williams 2003), that elevates the level of interaction between human operators and hidden-state, under-actuated systems, by allowing the operator to specify the behavior to be executed in terms of intended plant state evolution, instead of specific command sequences. The executive uses models of the plant to map the desired state evolution to a sequence of commands driving the plant through the specified states. However, *Titan* focuses on reactive control of discrete-event systems, and does not handle temporally flexible constraints.

Work on dispatchable execution (Vidal & Ghallab 1996; Morris, Muscettola, & Tsamardinou 1998; Tsamardinou, Pollack, & Ramakrishnan 2003) provides a framework for robust scheduling and execution of temporally flexible plans. This framework uses distance graphs to tighten timing constraints in the plan, in order to guarantee dispatchability and to propagate the occurrence of events during plan execution. However, this work was applied to discrete, directly controllable, loosely coupled systems, and, therefore, must be extended to under-actuated plants.

Previous work on continuous planning and execution (Ambros-Ingerson & Steel 1988; Wilkins & Myers 1995; Chien *et al.* 2000) also provides methods to achieve robustness, by interleaving planning and execution, allowing on-the-fly replanning and adaptation to disturbances. These methods, inspired from model predictive control (*MPC*) (Propoi 1963; Richalet *et al.* 1976), involve planning and scheduling iteratively over short horizons, while revising the plan when necessary during execution. This work, however, needs to be extended to deal with temporally flexible plans and under-actuated systems with continuous dynamics.

We propose a model-based executive that unifies the three previous approaches and enables coordinated control of agile systems, through model-based execution of *temporally flexible state plans*. Our approach is novel with respect to three aspects. First, we provide a general method for encoding both the temporal state plan and the dynamics of the

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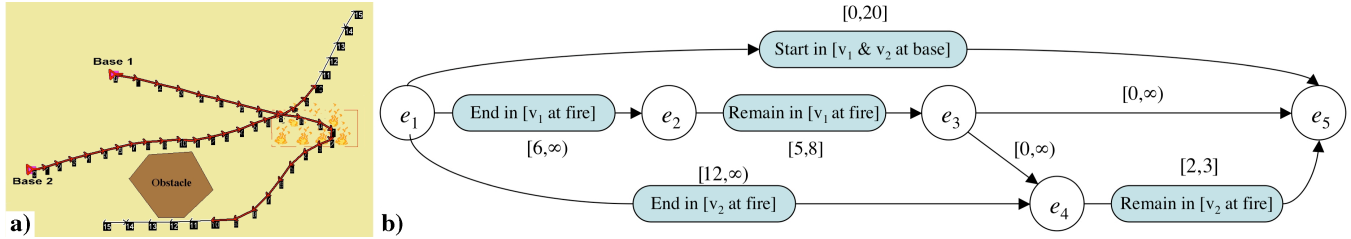


Figure 1: a) Map of the terrain for the fire-fighting example; b) Corresponding temporally flexible state plan.

system as a mixed discrete-continuous mathematical program. Solving this program provides near-optimal trajectories in the plant state space that satisfy the system dynamics and the state plan. Second, to achieve efficiency and robustness, we apply MPC for planning of control trajectories, in the context of continuous temporal plan execution for under-actuated dynamical systems. MPC allows us to achieve tractability, by reasoning over a limited receding horizon. Third, in order to further reduce the complexity of the program and solve it in real time, we introduce pruning policies that enable us to ignore some of the constraints in the state plan outside the current planning horizon.

### Problem Statement

Given a dynamic system (a *plant*) described by a *plant model*, and given a *temporally flexible state plan*, specifying the desired evolution of the plant state over time, the *continuous model-based execution (CMEx)* problem consists of designing a control sequence that produces a plant state evolution that is consistent with the state plan. In this section we present a formal definition of the CMEx problem.

### Multiple-UAV Fire-fighting Example

This paragraph introduces the multiple-UAV fire-fighting example used in this paper. In this example, the plant consists of two fixed-wing UAVs, whose state variables are their 2-D Cartesian positions and velocities. The vehicles evolve in an environment (Fig. 1a) involving a reported fire that the team is assigned to extinguish. To do so, they must navigate around unsafe regions (e.g. obstacles) and drop water on the fire. They must also take pictures after the fire has been extinguished, in order to assess the damage. An English description for the mission's state plan is:

*Vehicles  $v_1$  and  $v_2$  must start at their respective base stations.  $v_1$  (a water tanker UAV) must reach the fire region and remain there for 5 to 8 time units, while it drops water over the fire.  $v_2$  (a reconnaissance UAV) must reach the fire region after  $v_1$  is done dropping water and must remain there for 2 to 3 time units, in order to take pictures of the damage. The overall plan execution must last no longer than 20 time units.*

### Definition of a Plant Model

A *plant model*  $\mathcal{M} = \langle \mathbf{s}, S, \mathbf{u}, \Omega, \mathcal{SE} \rangle$  consists of a vector  $\mathbf{s}(t)$  of *state variables*, taking on values from the *state space*  $S \subset$

$\mathbb{R}^n$ , a vector  $\mathbf{u}(t)$  of *input variables*, taking on values from the *context*  $\Omega \subset \mathbb{R}^m$ , and a set  $\mathcal{SE}$  of *state equations* over  $\mathbf{u}$ ,  $\mathbf{s}$  and its time derivatives, describing the plant behavior with time.  $S$  and  $\Omega$  impose linear safety constraints on  $\mathbf{s}$  and  $\mathbf{u}$ .

In our multiple-UAV example,  $\mathbf{s}$  is the vector of 2-D coordinates of the UAV positions and velocities, and  $\mathbf{u}$  is the acceleration coordinates.  $\mathcal{SE}$  is the set of equations describing the kinematics of the UAVs. The unsafe regions in  $S$  correspond to obstacles and bounds on nominal velocities, and the unsafe regions in  $\Omega$  to bounds on accelerations.

### Definition of a Temporally Flexible State Plan

A *temporally flexible state plan*  $P = \langle \mathcal{E}, \mathcal{C}, \mathcal{A} \rangle$  specifies a desired evolution of the plant state, and is defined by a set  $\mathcal{E}$  of *events*, a set  $\mathcal{C}$  of *coordination constraints*, imposing temporal constraints between events, and a set  $\mathcal{A}$  of *activities*, imposing constraints on the plant state. A coordination constraint  $c = \langle e_1, e_2, \Delta T_{e_1 \rightarrow e_2}^{min}, \Delta T_{e_1 \rightarrow e_2}^{max} \rangle$  constrains the distance from event  $e_1$  to event  $e_2$  in  $[\Delta T_{e_1 \rightarrow e_2}^{min}, \Delta T_{e_1 \rightarrow e_2}^{max}] \subset [0, +\infty]$ . An *activity*  $a = \langle e_1, e_2, c_S \rangle$  has an associated *start event*  $e_1$  and an *end event*  $e_2$ . Given an assignment  $T : \mathcal{E} \mapsto \mathbb{R}$  of times to all events in  $P$  (a *schedule*),  $c_S$  is a *state constraint* that can take on one of the following forms, where  $D_S, D_E, D_{\forall}$  and  $D_{\exists}$  are domains of  $S$  described by linear constraints on the state variables:

1. *Start in state region  $D_S$* :  $\mathbf{s}(T(e_1)) \in D_S$ ;
2. *End in state region  $D_E$* :  $\mathbf{s}(T(e_2)) \in D_E$ ;
3. *Remain in state region  $D_{\forall}$* :  $\forall t \in [T(e_1), T(e_2)], \mathbf{s}(t) \in D_{\forall}$ ;
4. *Go by state region  $D_{\exists}$* :  $\exists t \in [T(e_1), T(e_2)], \mathbf{s}(t) \in D_{\exists}$ .

We illustrate a state plan diagrammatically by an acyclic directed graph in which events are represented by nodes, coordination constraints by arcs, labeled by their corresponding time bounds, and activities by arcs labeled with associated state constraints. The state plan for the multiple-UAV fire-fighting mission example is shown in Fig. 1b.

### Definition of the CMEx Problem

Schedule  $T$  for state plan  $P$  is *temporally consistent* if it satisfies all  $c \in \mathcal{C}$ . Given an activity  $a = \langle e_1, e_2, c_S \rangle$  and a schedule  $T$ , a state sequence  $\mathbf{S} = \langle \mathbf{s}_0 \dots \mathbf{s}_t \rangle$  *satisfies activity*  $a$  if it satisfies  $c_S$ .  $\mathbf{S}$  then *satisfies state plan*  $P$  if there exists a temporally consistent schedule such that  $\mathbf{S}$  satisfies every activity in  $\mathcal{A}$ . Similarly, given a plant model  $\mathcal{M}$  and

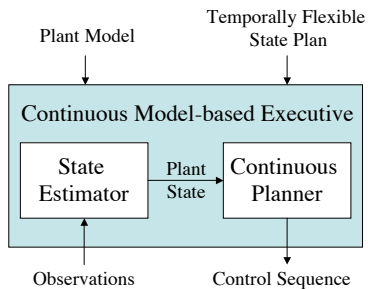


Figure 2: Continuous model-based executive architecture.

initial state  $\mathbf{s}_0$ , a control sequence  $\mathbf{U} = \langle \mathbf{u}_0 \dots \mathbf{u}_t \rangle$  satisfies  $P$  if it generates a state sequence that satisfies  $P$ .  $\mathbf{U}$  is *optimal* if it satisfies  $P$  while minimizing an *objective function*  $F(\mathbf{U}, \mathbf{S}, T)$ . A common objective is to minimize the scheduled time  $T(e_E)$  for the end event  $e_E$  of  $P$ .

Given an initial state  $\mathbf{s}_0$ , a plant model  $\mathcal{M}$  and state plan  $P$ , the CMEx problem consists of generating, for each time step  $t$ , a control action  $\mathbf{u}_t$  from a control sequence  $\langle \mathbf{u}_0 \dots \mathbf{u}_{t-1} \rangle$  and its corresponding state sequence  $\langle \mathbf{s}_0 \dots \mathbf{s}_t \rangle$ , such that  $\langle \mathbf{u}_0 \dots \mathbf{u}_t \rangle$  is optimal. A corresponding *continuous model-based executive* consists of a *state estimator* and a *continuous planner* (Fig. 2). The continuous planner takes in a state plan, and generates optimal control sequences, based on the plant model, and state sequences provided by the state estimator. The estimator reasons on sensor observations and on the plant model in order to continuously track the state of the plant. Previous work on hybrid estimation (Hofbauer & Williams 2004) provides a framework for this state estimator; in this paper, we focus on presenting an algorithm for the continuous planner.

## Overall Approach

Previous model-based executives, such as Titan, focus on reactively controlling discrete-event systems (Williams 2003). This approach is not applicable to temporal plan execution of systems with continuous dynamics; our continuous model-based executive uses a different approach that consists of planning into the future, in order to perform optimal, safe execution of temporal plans. However, solving the whole CMEx problem over an infinite horizon would present two major challenges. First, the problem is intractable in the case of long-duration missions. Second, it would require perfect knowledge of the state plan and the environment beforehand; this assumption does not always hold in real-life applications such as our fire-fighting scenario, in which the position of the fire might precisely be known only once the UAVs are close enough to the fire to localize it. Furthermore, the executive must be able to compensate possible drift due to approximations or errors in the plant model.

## Receding Horizon CMEx

Model Predictive Control (MPC), also called *Receding Horizon Control*, is a method introduced in (Propoi 1963; Richalet *et al.* 1976) that tackles these two challenges in

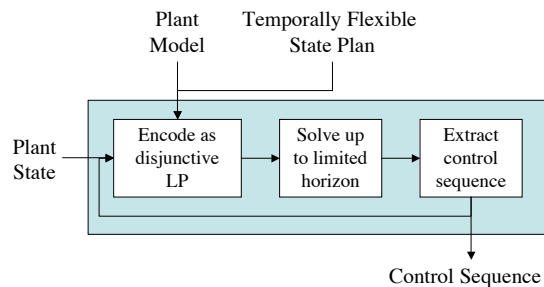


Figure 3: Receding horizon continuous planner.

the context of low-level control of systems with continuous dynamics. MPC solves the control problem up to a limited *planning horizon*, and re-solves it when it reaches a shorter *execution horizon*. This method makes the problem tractable by restricting it to a small planning window; it also allows for on-line, robust adaptation to disturbances, and generates control sequences that are optimal over the planning window, and globally near-optimal.

In this paper, we extend MPC to continuous model-based execution of temporal plans by introducing a *receding horizon continuous model-based executive*. We formally define receding horizon CMEx as follows. Given a state plan  $P$ , a plant model  $\mathcal{M}$ , and an initial state  $\mathbf{s}(t_0)$ , *single-stage, limited horizon CMEx* consists of generating an optimal control sequence  $\langle \mathbf{u}_{t_0} \dots \mathbf{u}_{t_0+N_t} \rangle$  for  $P$ , where  $N_t$  is the planning horizon. The *receding horizon CMEx* problem consists of iteratively solving single-stage, limited horizon CMEx for successive initial states  $\mathbf{s}(t_0 + i \cdot n_t)$  with  $i = 0, 1, \dots$ , where  $n_t \leq N_t$  is the execution horizon. The architecture for our model-based executive is presented in Fig. 3.

## Disjunctive Linear Programming Formulation

As introduced in Fig. 3, we solve each single-stage limited horizon CMEx problem by encoding it as a *disjunctive linear program (DLP)* (Balas 1979). A DLP is an optimization problem with respect to a linear cost function over decision variables, subject to constraints that can be written as logical combinations of linear inequalities (Eq. (1)). In this paper, we solve DLPs by reformulating them as Mixed-Integer Linear Programs. We are also developing an algorithm that solves DLPs directly (Krishnan 2004; Li & Williams 2005).

$$\begin{aligned} \text{Minimize :} & \quad f(\mathbf{x}) \\ \text{Subject to :} & \quad \bigwedge_i \bigvee_j g_{i,j}(\mathbf{x}) \leq 0 \end{aligned} \quad (1)$$

Any arbitrary propositional logic formula whose propositions are linear inequalities is reducible to a DLP. Hence, in this paper, we express formulae in propositional form as in Eq. (2), where  $\Phi(\mathbf{x})$  is defined in Eq. (3).

$$\begin{aligned} \text{Minimize :} & \quad f(\mathbf{x}) \\ \text{Subject to :} & \quad \Phi(\mathbf{x}) \end{aligned} \quad (2)$$

$$\Phi(\mathbf{x}) := \Phi(\mathbf{x}) \wedge \Phi(\mathbf{x}) \mid \Phi(\mathbf{x}) \vee \Phi(\mathbf{x}) \mid \neg \Phi(\mathbf{x}) \mid \Phi(\mathbf{x}) \Rightarrow \Phi(\mathbf{x}) \mid \Phi(\mathbf{x}) \Leftrightarrow \Phi(\mathbf{x}) \mid g(\mathbf{x}) \leq 0 \quad (3)$$

The rest of this paper is organized as follows: we first present our approach to encoding the constraints in the plant model and the state plan using the DLP formalism. We then introduce constraint pruning policies that enable us to simplify the DLP. Finally, we show results obtained on a multiple-UAV hardware-in-the-loop testbed.

### Single-stage Limited Horizon CMEx as a DLP

Recall that to solve the CMEx problem, we consider a smaller problem (single-stage limited horizon CMEx) by iteratively planning over limited planning windows. We now present how we encode single-stage limited horizon CMEx as a DLP. Our innovation is the encoding of the state plan as a goal specification for the plant.

### State Plan Encodings

In the following paragraphs, we present the encodings for the state plan. A range of objective functions are possible, the most common being to minimize completion time. To encode this, for every event  $e \in \mathcal{E}$ , we add to the DLP cost function, the time  $T(e)$  at which  $e$  is scheduled.

**Temporal Constraints Between Events:** Eq. (4) encodes a temporal constraint between two events  $e_S$  and  $e_E$ . For example, in Fig. 1b, events  $e_1$  and  $e_5$  must be distant from each other by a least 0 and at most 20 time units.

$$\Delta T_{e_S \rightarrow e_E}^{\min} \leq T(e_E) - T(e_S) \leq \Delta T_{e_S \rightarrow e_E}^{\max} \quad (4)$$

**State Activity Constraints:** Activities are of the following types: *start in*, *end in*, *remain in* and *go by*. *Start in* and *go by* are derivable easily from the primitives *remain in* and *end in*. We present the encodings for these two primitives below. In each case, we assume that the domains  $D_E$  and  $D_V$  are unions of polyhedra (Eq. (8)), so that  $\mathbf{s}_t \in D_E$  and  $\mathbf{s}_t \in D_V$  can be expressed as DLP constraints similar to (Eq. (9)).

**Remain in activity:** Eq. (5) presents the encoding for a *remain in state*  $D_V$  activity between events  $e_S$  and  $e_E$ . This imposes  $\mathbf{s} \in D_V$  for all time steps between  $T(e_S)$  and  $T(e_E)$ . Our example imposes the constraint “Remain in [ $v_1$  at fire]”, which means that  $v_1$  must be in the fire region between  $e_2$  and  $e_3$  while it is dropping water.  $T_0 \in \mathbb{R}$  denotes the initial time instant of index  $t = 0$ , and  $\Delta T$  is the granularity of the time discretization.

$$\bigwedge_{t=0 \dots N_t} \left\{ \begin{array}{l} T(e_S) \leq T_0 + t \cdot \Delta T \\ \wedge \\ T(e_E) \geq T_0 + t \cdot \Delta T \end{array} \right\} \Rightarrow \mathbf{s}_t \in D_V \quad (5)$$

**End in activity:** Consider an *end in* activity imposing  $\mathbf{s} \in D_E$  at the time  $T(e_E)$  when event  $e_E$  is scheduled. An example in our fire-fighting scenario is the “End in [ $v_2$  at fire]” constraint imposing  $v_2$  to be in the fire region at event  $e_4$ . The general encoding is presented in Eq. (6), which translates to the fact that, either there exists a time instant of

index  $t$  in the planning window that is  $\Delta T$ -close to  $T(e_E)$  and for which  $\mathbf{s}_t \in D_E$ , or event  $e_E$  must be scheduled outside of the current planning window .

$$\begin{aligned} & \bigvee_{t=0 \dots N_t} \left\{ \begin{array}{l} T(e_E) \geq T_0 + (t - \frac{1}{2})\Delta T \\ \wedge \\ T(e_E) \leq T_0 + (t + \frac{1}{2})\Delta T \\ \wedge \\ \mathbf{s}_t \in D_E \end{array} \right\} \\ & \vee \\ & T(e_E) \leq T_0 - \frac{\Delta T}{2} \\ & \vee \\ & T(e_E) \geq T_0 + (N_t + \frac{1}{2})\Delta T \end{aligned} \quad (6)$$

**Guidance heuristic for End in activities:** During execution of an “End in state region  $D_E$ ” activity, the end event may be scheduled beyond the current horizon. In this case, the model-based executive constructs a *heuristic*, in order to guide the trajectory towards  $D_E$ . For this purpose, an estimate of the “distance” to  $D_E$  from the end of the current partial state trajectory is added to the DLP cost function, so that the partial trajectory ends as “close” to  $D_E$  as possible. In the case of the multiple-UAV fire-fighting scenario, this “distance” is an estimate of the time needed to go from the end of the current trajectory to the goal region  $D_E$ .

The heuristic is formally defined as a function  $h_{D_E} : \mathcal{S} \mapsto \mathbb{R}$ , where  $\mathcal{S} = \{S_i \subset S\}$  is a finite partition of  $S$  such that  $D_E = S_{i_{D_E}} \in \mathcal{S}$ . Given  $S_i \in \mathcal{S}$ ,  $h_{D_E}(S_i)$  is an estimate of the cost to go from  $S_i$  to  $D_E$ . In the fire-fighting example,  $S$  is a grid map in which each grid cell  $S_i$  corresponds to a hypercube centered on a state vector  $\mathbf{s}_i$ , and  $h_{D_E}(S_i)$  is an estimate of the time necessary to go from state  $\mathbf{s}_i$  to the goal state  $\mathbf{s}_{i_{D_E}}$ . Similar to (Bellingham, Richards, & How 2002), we compute  $h_{D_E}$  by constructing a visibility graph based on the unsafe regions of the  $\langle x, y \rangle$  state space, and by computing, for every  $i$ , the cost to go from  $\langle x_i, y_i \rangle \in S_i$  to the goal state  $\langle x_{i_{D_E}}, y_{i_{D_E}} \rangle \in D_E$ .

Eq. (7) presents the constraint, for a given “End in state region  $D_E$ ” activity  $a$ , starting at event  $e_S$  and ending at event  $e_E$ . This encodes the fact that, if  $a$  is scheduled to start within the execution horizon but end beyond, then the executive must choose a region  $S_i \in \mathcal{S}$  so that the partial state trajectory ends in  $S_i$ , and the value  $h$  of the heuristic at  $S_i$  is minimized (by adding  $h$  to the DLP cost function).

$$\begin{aligned} & \left\{ \begin{array}{l} T(e_S) < T_0 + n_t \cdot \Delta T \\ \wedge \\ T(e_E) \geq T_0 + n_t \cdot \Delta T \end{array} \right\} \\ & \Rightarrow \bigvee_{S_i \in \mathcal{S}} \left\{ \begin{array}{l} h = h_{D_E}(S_i) \\ \wedge \\ \mathbf{s}_{n_t} \in S_i \end{array} \right\} \end{aligned} \quad (7)$$

Eq. (7) can be simplified by reducing  $\mathcal{S}$  to a subset  $\tilde{\mathcal{S}} \subset \mathcal{S}$  that excludes all the  $S_i$  unreachable within the horizon. For instance, in the multiple-UAV example, the maximum velocity constraints allow us to ignore the  $S_i$  that are not reachable by the UAVs within the execution horizon. We present in a later section how  $\mathcal{M}$  allows us to determine, in the general case, when a region of the state space is unreachable.

### Plant Model Encodings

Recall that a plant model  $\mathcal{M}$  consists of a state space  $S$  and a context  $\Omega$  imposing linear constraints on the variables, and a set of state equations  $\mathcal{SE}$ . We represent unsafe regions in

**Alg. 1** Pruning policy for the temporal constraint between events  $e_S$  and  $e_E$

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1: if  $T_{e_S}^{\max} < T_0$  then
2:   prune { $e_S$  has already been executed}
3: else if  $T_{e_S}^{\min} > T_0 + N_t \cdot \Delta T$  then
4:   prune { $e_S$  is out of reach within the current horizon}
5: else if  $T_{e_E}^{\max} < T_0$  then
6:   prune { $e_E$  has already been executed}
7: else if  $T_{e_E}^{\min} > T_0 + N_t \cdot \Delta T$  then
8:   prune { $e_E$  is out of reach within the current horizon}
9: end if

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$S$  and  $\Omega$  by unions of polyhedra, where a polyhedron  $\mathcal{P}_S$  of  $S$  is defined in Eq. (8). Polyhedra of  $\Omega$  are defined similarly.

$$\mathcal{P}_S = \{\mathbf{s} \in \mathbb{R}^n \mid \mathbf{a}_i^T \mathbf{s} \leq b_i, i = 1 \dots n_{\mathcal{P}_S}\} \quad (8)$$

The corresponding encoding (Eq. (9)) constrains  $\mathbf{s}_t$  to be outside of  $\mathcal{P}_S$  for all  $t$ . In the UAV example, this corresponds to the constraint encoding obstacle collision avoidance.

$$\bigwedge_{t=0 \dots N_t} \bigvee_{i=1 \dots n_{\mathcal{P}_S}} \mathbf{a}_i^T \mathbf{s}_t \geq b_i \quad (9)$$

The state equations in  $\mathcal{SE}$  are given in DLP form in Eq. (10), with the time increment  $\Delta T$  assumed small with respect to the plant dynamics. In our fixed-wing UAV example, we use the zero-order hold time discretization model from (Kuwata 2003).

$$\mathbf{s}_{t+1} = \mathbf{A}\mathbf{s}_t + \mathbf{B}\mathbf{u}_t \quad (10)$$

### Constraint Pruning Policies

Recall that our executive solves the CMEx problem by encoding it as a DLP and iteratively solving it over small planning windows. The ability of the executive to look into the future is limited by the number of variables and constraints in the DLP. In the next section, we introduce novel *pruning policies* that dramatically reduce the number of constraints.

#### Plant Model Constraint Pruning

Recall that  $\mathcal{M}$  defines unsafe regions in  $S$  using polyhedra (Eq. (8)). The DLP constraint for a polyhedron  $\mathcal{P}_S$  (Eq. (9)) can be pruned if  $\mathcal{P}_S$  is unreachable from the current plant state  $\mathbf{s}_0$ , within the horizon  $N_t$ . That is, if the region  $R$  of all states reachable from  $\mathbf{s}_0$  within  $N_t$  is disjoint from  $\mathcal{P}_S$ .  $R$  is formally defined in Eq. (11) and (12), with  $R_0 = \{\mathbf{s}_0\}$ .

$$\forall t = 0 \dots N_t - 1, \quad R_{t+1} = \left\{ \mathbf{s}_{t+1} \mid \begin{array}{l} \mathbf{s}_{t+1} = \mathbf{A}\mathbf{s}_t + \mathbf{B}\mathbf{u}_t, \\ \mathbf{s}_t \in R_t, \mathbf{u}_t \in \Omega \end{array} \right\} \quad (11)$$

$$R = \bigcup_{t=0 \dots N_t} R_t \quad (12)$$

Techniques have been developed in order to compute  $R$  (Tiwari 2003). In the UAV example, for a given vehicle, we use a simpler, sound but incomplete method: we approximate  $R$  by a circle centered on the vehicle and of radius  $N_t \cdot \Delta T \cdot v^{\max}$ , where  $v^{\max}$  is the maximum velocity.

**Alg. 2** Pruning policy for the absolute temporal constraint on an event  $e$

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1: if  $T_e^{\max} < T_0$  then
2:   prune { $e$  has already been executed}
3: else if  $T_e^{\min} > T_0 + N_t \cdot \Delta T$  then
4:   prune { $e$  is out of reach within the current horizon}
5:   POSTPONE( $e$ )
6: end if

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**Alg. 3** POSTPONE( $e$ ) routine to postpone an event  $e$

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1:  $T_e^{\min} \leftarrow T_0 + N_t \cdot \Delta T$ 
2: add  $T(e) \geq T_0 + N_t \cdot \Delta T$  to the DLP
3: for all events  $e'$  do {propagate to other events}
4:    $T_{e'}^{\min} \leftarrow \max(T_{e'}^{\min}, T_e^{\min} + \Delta T_{\langle e, e' \rangle}^{\min})$ 
5: end for

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### State Plan Constraint Pruning

State plan constraints can be either temporal constraints between events, *remain in* constraints, *end in* constraints, or heuristic guidance constraints for *end in* activities. For each type, we now show that the problem of finding a policy is equivalent to that of foreseeing if an event could possibly be scheduled within the current horizon. This is solved by computing bounds  $\langle T_e^{\min}, T_e^{\max} \rangle$  on  $T(e)$ , for every  $e \in \mathcal{E}$ . Given the execution times of past events, these bounds are computed from the bounds  $\langle \Delta T_{\langle e, e' \rangle}^{\min}, \Delta T_{\langle e, e' \rangle}^{\max} \rangle$  on the distance between any pair of events  $\langle e, e' \rangle$ , obtained using the method in (Dechter, Meiri, & Pearl 1991). This involves running an all-pairs shortest path algorithm on the distance graph corresponding to  $P$ , which can be done offline.

**Temporal Constraint Pruning:** A temporal constraint between a pair of events  $\langle e_S, e_E \rangle$  can be pruned if the time bounds on either event guarantee that the event will be scheduled outside of the current planning window (Alg. 1).

However, pruning some of the temporal constraints specified in the state plan can have two bad consequences. First, implicit temporal constraints between two events that can be scheduled within the current planning window might no longer be enforced. Implicit temporal constraints are constraints that do not appear explicitly in the state plan, but rather result from several explicit temporal constraints. Second, the schedule might violate temporal constraints between events that remain to be scheduled, and events that have already been executed.

To tackle the first issue aforementioned, rather than encoding only the temporal constraints that are mentioned in the state plan, we encode the temporal constraints between any pair of events  $\langle e, e' \rangle$ , using the temporal bounds  $\langle \Delta T_{\langle e, e' \rangle}^{\min}, \Delta T_{\langle e, e' \rangle}^{\max} \rangle$  computed by the method in (Dechter, Meiri, & Pearl 1991). This way, no implicit temporal constraint is ignored, because all temporal constraints between events are explicitly encoded.

To address the second issue, we also encode the absolute temporal constraints on every event  $e$ :  $T_e^{\min} \leq T(e) \leq$

**Alg. 4** Pruning policy for a “Remain in state region  $D_V$ ” activity starting at event  $e_S$  and ending at event  $e_E$

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1: if  $T_{e_E}^{\max} < T_0$  then
2:   prune {activity is completed}
3: else if  $T_{e_S}^{\max} < T_0$  then
4:   do not prune {activity is being executed}
5: else if  $T_{e_S}^{\min} > T_0 + N_t \cdot \Delta T$  then
6:   prune {activity will start beyond  $N_t$ }
7: else if  $T_{e_S}^{\max} \leq T_0 + N_t \cdot \Delta T$  then
8:   do not prune {activity will start within  $N_t$ }
9: else if  $R \cap D_V = \emptyset$  then
10:  prune; POSTPONE( $e_S$ )
11: end if

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**Alg. 5** Pruning policy for an “End in state region  $D_E$ ” activity ending at event  $e_E$

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1: if  $T_{e_E}^{\max} < T_0$  then
2:   prune { $e_E$  has already occurred}
3: else if  $T_{e_E}^{\max} \leq T_0 + N_t \cdot \Delta T$  then
4:   do not prune { $e_E$  will be scheduled within  $N_t$ }
5: else if  $T_{e_E}^{\min} > T_0 + N_t \cdot \Delta T$  then
6:   prune { $e_E$  will be scheduled beyond  $N_t$ }
7: else if  $R \cap D_E = \emptyset$  then
8:   prune; POSTPONE( $e_E$ )
9: end if

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$T_e^{\max}$ . The pruning policy for those constraints is presented in Alg. 2. The constraint can be pruned if  $e$  is guaranteed to be scheduled in the past (i.e. it has already been executed, line 1). It can also be pruned if  $e$  is guaranteed to be scheduled beyond the current planning horizon (line 3). In that case,  $e$  must be explicitly postponed (Alg. 3, lines 1 & 2) to make sure it will not be scheduled before  $T_0$  at the next iteration (which would then correspond to scheduling  $e$  in the past before it has been executed). The change in  $T_e^{\min}$  is then propagated to the other events (Alg. 3, line 3).

**Remain in Constraint Pruning (Alg. 4):** Consider the constraint  $c_S$  on a “Remain in state region  $D_V$ ” activity  $a$ , between events  $e_S$  and  $e_E$  (Eq. (5)). If  $e_E$  is guaranteed to be scheduled in the past, that is, it has already occurred (line 1), then  $a$  has been completed and  $c_S$  can be pruned. Otherwise, if  $e_S$  has already occurred (line 3), then  $a$  is being executed and  $c_S$  must not be pruned. Else, if  $a$  is guaranteed to start beyond the planning horizon (line 5), then  $c_S$  can be pruned. Conversely, if  $a$  is guaranteed to start within the planning horizon (line 7), then  $c_S$  must not be pruned. Otherwise, the time bounds on  $T(e_S)$  and  $T(e_E)$  provide no guarantee, but we can still use the plant model  $\mathcal{M}$  to try to prune the constraint: if  $\mathcal{M}$  guarantees that  $D_V$  is unreachable within the planning horizon, then  $c_S$  can be pruned (line 9; refer to Eq. (11) & (12) for the definition of  $R$ ). Similar to Alg. 2,  $e_S$  must then be explicitly postponed.

**End in Constraint Pruning (Alg. 5):** Consider a constraint  $c_S$  on an “End in state region  $D_E$ ” activity ending

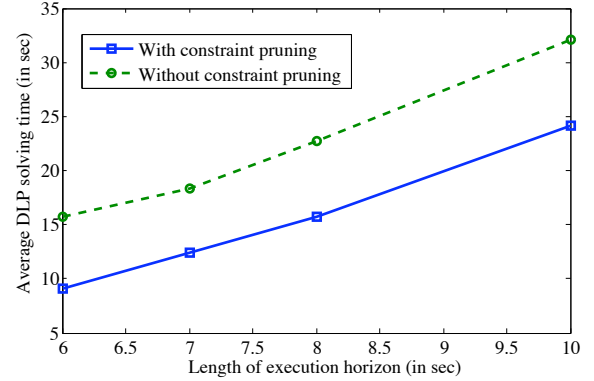


Figure 4: Performance gain by constraint pruning.

at event  $e_E$  (Eq. (6)). If  $e_E$  is guaranteed to be scheduled in the past (i.e., it has already occurred, line 1), then  $c_S$  can be pruned. Otherwise, if the value of  $T_{e_E}^{\max}$  guarantees that  $e_E$  will be scheduled within the planning horizon (line 3), then  $c_S$  must not be pruned. Conversely, it can be pruned if  $T_{e_E}^{\min}$  guarantees that  $e_E$  will be scheduled beyond the planning horizon (line 5). Finally,  $c_S$  can also be pruned if the plant model guarantees that  $D_E$  is unreachable within the planning horizon from the current plant state (line 7). Similarly to Alg. 2 and 4,  $e_E$  must then be explicitly postponed.

**Guidance Constraint Pruning:** The heuristic guidance constraint for an *end in* activity  $a$  between events  $e_S$  and  $e_E$  (Eq. (7)) can be pruned if  $a$  is guaranteed either to end within the execution horizon ( $T_{e_E}^{\max} \leq T_0 + n_t \cdot \Delta T$ ) or to start beyond ( $T_{e_S}^{\min} > T_0 + n_t \cdot \Delta T$ ).

In the next section, we show that our model-based executive is able to design optimal control sequences in real time.

## Results and Discussion

The model-based executive has been implemented in C++, using Ilog CPLEX to solve the DLPs. It has been demonstrated on fire-fighting scenarios on a hardware-in-the-loop testbed, comprised of CloudCap autopilots controlling a set of fixed-wing UAVs. This offers a precise assessment of real-time performance on UAVs. Fig. 1a was obtained from a snapshot of the operator interface, and illustrates the trajectories obtained for the state plan in Fig. 1b.

Fig. 4 and 5 present an analysis of the performance of the executive on a more complex example, comprised of two vehicles, two obstacles, and 26 activities, for a total mission duration of about 1,300 sec. These results were obtained on a 1.7GHz computer with 512MB RAM, by averaging over the whole plan execution, and over 5 different runs with random initial conditions. At each iteration, the computation was cut short if and when it passed 200 sec.

In both figures, the  $x$  axis corresponds to the length of the execution horizon,  $n_t \cdot \Delta T$ , in seconds. For these results, we maintained a planning buffer of  $N_t \cdot \Delta T - n_t \cdot \Delta T = 10$  sec

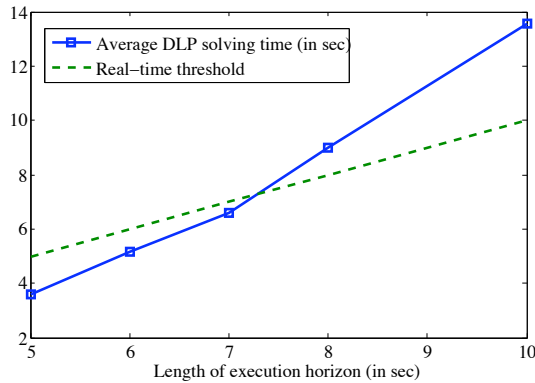


Figure 5: Performance of the model-based executive.

(where  $N_t \cdot \Delta T$  is the length of the planning horizon). The  $y$  axis corresponds to the average time in seconds required by CPLEX to solve the DLP at each iteration. As shown in Fig. 4, the use of pruning policies entails a significant gain in performance. Note that these results were obtained by disabling the *Presolve* function in CPLEX, which also internally prunes some of the constraints to simplify the DLP.

Fig. 5 presents an analysis of the model-based executive's capability to run in real time. The dotted line is the line  $y = x$ , corresponding to the real-time threshold. It shows that below the value  $x \simeq 7.3s$ , the model-based executive is able to compute optimal control sequences in real time (the average DLP solving time is below the length of the execution horizon). For longer horizons corresponding to values of  $x$  above  $7.3s$ , CPLEX is unable to find optimal solutions to the DLPs before the executive has to replan (the solving time is greater than the execution horizon). Note that in that case, since CPLEX runs as an anytime algorithm, we can still interrupt it and use the best solution found thus far to generate sub-optimal control sequences.

Note also that the number of the disjunctions in the DLP grows linearly with the length of the planning horizon; therefore, the complexity of the DLP is worst-case exponential in the length of the horizon. In Fig. 5, however, the relationship appears to be linear. This can be explained by the fact that the DLP is very sparse, since no disjunct in the DLP involves more than three or four variables.

## Conclusion

In this paper, we have presented a continuous model-based executive that is able to robustly execute temporal plans for agile, under-actuated systems with continuous dynamics. In order to deal with the under-actuated nature of the plant and to provide robustness, the model-based executive reasons on temporally flexible state plans, specifying the intended plant state evolution. The use of pruning policies enables the executive to design near-optimal control sequences in real time, which was demonstrated on a hardware-in-the-loop testbed in the context of multiple-UAV fire-fighting scenarios. Our approach is broadly applicable to other dynamic systems, such as chemical plants or Mars life support systems.

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