

Coordination Mechanisms for a Distribution System with One Supplier and Multiple Retailers

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We address a fundamental two-echelon distribution system in which the sales volumes of the retailers are endogenously determined on the basis of known demand functions. Specifically, this paper studies a distribution channel where a supplier distributes a single product to retailers, who in turn sell the product to consumers. The demand in each retail market arrives continuously at a constant rate that is a general decreasing function of the retail price in the market. We have characterized an optimal strategy, maximizing total systemwide profits in a centralized system. We have also shown that the same optimum level of channelwide profits can be achieved in a decentralized system, but only if coordination is achieved via periodically charged, fixed fees, and a nontraditional discount pricing scheme under which the discount given to a retailer is the sum of three discount components based on the retailer's (i) annual sales volume, (ii) order quantity, and (iii) order frequency, respectively. Moreover, we show that no (traditional) discount scheme, based on order quantities only, suffices to optimize channelwide profits when there are multiple nonidentical retailers. The paper also considers a scenario where the channel members fail to coordinate their decisions and provides numerical examples that illustrate the value of coordination. We extend our results to settings in which the retailers' holding cost rates depend on the wholesale price.

(Coordination; Pricing; Quantity Discounts; Supply Chain Management)

1. Introduction

A production and distribution channel often encompasses independent firms or decentralized divisions of the same firm. The channel members typically optimize their own performance based on locally available information. Driven by competitive pressures and enabled by modern information technology, many supply chains have come to realize that their overall performance can be improved dramatically by employing novel mechanisms to coordinate decisions.

Traditional mechanisms to optimize the overall channel performance include vertical or horizontal

integration, supply chain partnerships such as vendor managed replenishments, contracts specifying decision rules for all channel members, and profit-sharing schemes. See Jeuland and Shugan (1983), Johnston and Lawrence (1988), Tirole (1988), and Buzzell and Ortmeier (1995) for discussions of these mechanisms and their limitations. The preferred way to achieve coordination is often to maintain *decentralized* decision making but to structure the costs and rewards of all members so as to align their objectives with the systemwide objective, i.e., to identify a *coordination mechanism*. If the decentralized cost and reward structure

results in channelwide profits equal to those under a centralized system, the coordination mechanism is *perfect*.

We consider the following two-echelon system. A supplier distributes a single product to multiple retailers who in turn sell to consumers. The retailers serve geographically dispersed, heterogeneous markets, in which demands occur continuously at a rate that depends on the price charged by the retailer according to a general demand function. The supplier replenishes his inventory through orders (purchases, production runs) from a source with ample supply. If the system operates in a decentralized manner, the supplier charges a wholesale price (schedule) and makes its own replenishment decisions, each of the retailers determines its retail price as well as its replenishment policy from the supplier, and all parties maximize their own profit functions. (Our model applies equally to settings where the "supplier" and the "retailers" represent different divisions of the same firm, each operating as an independent profit center.)

The costs consist of holding costs for the inventories at the supplier and the retailers, and fixed and variable costs for their orders. For each retailer order, the supplier incurs a fixed order-processing cost and the retailer incurs a setup cost. We initially assume, as in most standard inventory models, that the holding-cost rates are exogenously specified parameters. (The results carry over to the case where the retailer holding-cost rates are functions of the wholesale price, see §8.) Our model permits an additional cost component: The supplier may incur a specific annual cost for managing each retailer's needs and transactions. In the consumer electronics industry, for example, some suppliers establish a "management team" of logistics managers and sales and production representatives for each of their major retailer accounts to monitor the retailers' needs, transactions, and forecasts, and to negotiate and implement sales and logistical terms, etc. The same management team may be devoted exclusively to a single retailer account or to multiple accounts. We model the "management costs" by a concave function of the retailer's annual sales volume, reflecting economies of scale.

The retailers are nonidentical, i.e., they can have different demand functions and cost parameters. All

demand functions and cost parameters are stationary and common knowledge among the channel members. Extensions to settings with asymmetric information, as in Corbett and de Groot (2000), are worthy of study.

We first characterize the solution to the centralized system with a central planner who makes all pricing and replenishment decisions so as to maximize the channelwide profits. We then show that the centralized solution can be realized in a decentralized system if the supplier offers a discount from a list price based on the sum of three discount components; each is based on a single retailer characteristic, i.e., (1) *annual sales volume*, (2) *order quantity*, and (3) *order frequency*. The first two discount components have been widely studied in the marketing, operations, and industrial organization literature, but this appears to be the first model in which order-frequency-based discounts arise as an essential component in the coordination mechanism. Under this pricing scheme, the centralized solution emerges as a strong type of equilibrium in the decentralized system.

We also show, with simple counterexamples, that traditional order-quantity discount schemes alone (pricing rules that determine discounts on an order-by-order basis as a function of the absolute order size, such that the average price per unit decreases with the order size) do not guarantee perfect coordination. While virtually all of the operations management literature has focused on traditional order-quantity discounts, in practice the vast majority of discount schemes are based on other criteria. In a recent field study Munson and Rosenblatt (1998) document that no less than 76% of the study participants experience discount plans that "aggregate over time," i.e., where the discount is based on the annual sales volume. They also document the prevalence of schemes where a discount is given "for every extra week's worth of sales that is added to an order" (p. 360), as well as schemes based on *multiple* criteria, e.g., annual sales volumes as well as individual order sizes. The authors conclude that new models are needed to "suggest schedules proposing business volume discounts aggregated over products and time to help suppliers appropriately parameterize schedules for these increasingly common quantity-discount

aggregations" (p. 365), and "to address the increasing practice of quantity-discount combinations, e.g., per-purchase, per-item discounts combined with time aggregation" (p. 365). The prevalence of discounts based on annual sales volumes has also been documented in the economics literature. See, e.g., Brown and Medoff (1990) and Lilien et al. (1992) for a general discussion of different types of discount schemes.

The discount scheme usually needs to be complemented with periodic fixed fees from each of the retailers to the supplier. The combined scheme is usually referred to as a block tariff, see, e.g., Oi (1971) and Schmalensee (1982). The fixed fees clearly do not affect the total profits of the supply chain or the parties' operational policies. These fees are, however, essential to achieve a proper allocation of the channel profits; see §5 below.

It is of interest to compare the performance of the supply chain under an optimal centralized strategy (or equivalently in a decentralized system with the above perfect coordination mechanism) with that of more typical decentralized chains without proper coordination. One important benchmark for the value of coordination arises when one of the parties in the chain, e.g., the supplier, is able to specify the terms (i.e., the wholesale price) unilaterally, so as to maximize his own profits. We consider two Stackelberg games with the supplier as the leader and the retailers as followers. In one, the supplier sets a constant wholesale price, and in the other, the supplier offers an order-quantity discount scheme with one breakpoint. The differences between the channelwide profits in these Stackelberg games and those in the coordinated system represent two possible measures of the value of coordination. Numerical examples suggest that the value of coordination can be significant.

The marketing literature on channel coordination focuses on pricing decisions. Jeuland and Shugan (1983) consider a channel with one supplier and one retailer, without inventory replenishment considerations. There, a simple quantity discount based on the annual sales volume results in perfect coordination. Moorthy (1987) points out that perfect coordination can also be achieved with a simple two-part tariff, i.e., the supplier sells the goods to the retailer at its marginal cost and charges a fixed franchise fee.

These mechanisms are designed to eliminate double marginalization (Spengler 1950). Ingene and Parry (1995) generalize the above model to multiretailer settings.

The operations literature on channel coordination, on the other hand, has until recently been confined to replenishment decisions, assuming that all demand processes are exogenously given. Except for Lal and Staelin (1984), this literature restricts itself to channels with a single retailer (or multiple identical retailers). Various mechanisms have been proposed to induce the retailer to adopt the globally optimal order quantity, effectively compensating the retailer for deviating from his locally optimal order quantity, see, e.g., Crowther (1964), Monahan (1984), and Lee and Rosenblatt (1986). Dealing with multiple nonidentical retailers, Lal and Staelin (1984) and Joglekar and Tharthare (1990) propose different coordination mechanisms that are based on order-quantity discounts and do not necessarily achieve the centralized optimum. See Dolan (1987), Boyaci and Gallego (1997), Cachon (1998), Lariviere (1998), Munson and Rosenblatt (1998), and Tsay et al. (1998) for further reviews.

Weng (1995) represents one of the first attempts to combine the above two streams of research. We generalize his model with a single retailer or multiple identical retailers to allow for an arbitrary number of nonidentical retailers. Although a scheme using an order-quantity discount and a periodic franchise fee suffices to achieve perfect coordination in the setting studied by Weng, we show that when the retailers are not identical, such a scheme is not guaranteed to coordinate the channel.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 considers the centralized system. Section 4 shows that order-quantity discounts alone are insufficient to guarantee perfect coordination when retailers are nonidentical. Section 5 describes our coordination mechanism. Section 6 analyzes the two Stackelberg games. Section 7 contains the numerical examples, and §8 considers an alternative model of retailer holding costs.

2. Model

A supplier distributes a single product to N retailers, who in turn serve geographically dispersed (thus independent) retail markets. The consumer demand in each retail market occurs continuously at a constant rate, which is determined by the retail price charged in the market in accordance with a *general* time-invariant (i.e., stationary) demand function. The retail price in each market, once determined, remains constant over time. All demands must be satisfied without backlogging. The supplier and the retailers are independent firms, and each firm has the objective to maximize its long-run average profits.

The supplier replenishes its inventory from a source with no capacity limit. The retailers place orders with the supplier. All replenishment orders incur fixed and variable costs. The fixed cost associated with a retailer order has two components: one incurred by the retailer and the other by the supplier (e.g., an order-processing cost). As with the consumer demand, retail orders cannot be backlogged. Each firm incurs inventory-carrying costs which at any point in time are proportional to its inventory level. In addition, the supplier incurs a specific annual cost for managing each retailer's account. All cost parameters are stationary. For $i = 1, \dots, N$, define

- p_i = retail price charged by retailer i
- $d_i(p_i)$ = annual consumer demand in the market served by retailer i , a decreasing function of p_i
- K_0 = fixed cost incurred by the supplier per order placed by the supplier
- K_i^s = fixed cost incurred by the supplier per order placed by retailer i
- K_i^r = fixed cost incurred by retailer i per order placed by retailer i
- $K_i = K_i^s + K_i^r$
- h_0 = annual holding cost per unit of inventory at the supplier
- \bar{h}_i = annual holding cost per unit of inventory at retailer i
- $h_i = \bar{h}_i - h_0$, incremental or echelon holding cost at retailer i
- c_0 = cost per unit ordered by the supplier

c_i = transportation cost per unit shipped from the supplier to retailer i

$\Psi(d_i)$ = annual cost incurred by the supplier for managing retailer i 's account, with $\Psi(\cdot)$ nondecreasing, concave and $\Psi(0) = 0$.

Let $p_i(d_i)$ denote the inverse demand function (because $d_i(\cdot)$ is decreasing). We assume that $p_i(d_i)d_i$ is concave in d_i . We assume that $h_i \geq 0$ for all i , which means the cost of carrying a unit at retailer i is at least as large as the cost of carrying it in the supplier's warehouse. Without loss of generality, we assume the transportation cost c_i is borne by the retailer. The above specified costs do not include any transfer payments between the supplier and the retailers. For notational convenience, we assume that all orders are received instantaneously upon placement. Positive but deterministic leadtimes can be handled by a simple shift in time of all desired replenishment epochs.

The supplier sets the wholesale price and determines its own replenishment policy. The retailers set the retail price (or alternatively the demand rate) in their markets and determine their own replenishment policy. The objective of each firm is to maximize its long-run average profits.

3. Centralized Solution

Assume that there is a central planner who makes all the pricing and replenishment decisions so as to maximize the systemwide profit, which is equal to the revenue ($\sum_{i=1}^N p_i(d_i)d_i$) minus costs. The cost components are: variable costs $\sum_{i=1}^N (c_0 + c_i)d_i$, account management costs $\sum_{i=1}^N \Psi(d_i)$, and inventory-carrying and setup costs.

Suppose, for a moment, that all the retail prices (thus demands) are given. Therefore, maximizing profit reduces to minimizing the inventory-carrying and setup costs. While a single-location model of this type is easily solved by the well-known EOQ formula, the two-stage distribution system is much more difficult and was not well understood until the seminal work of Roundy (1985). Roundy shows that, while the truly optimal replenishment strategy is intractable, a near-optimal solution can be found in the class of

integer-ratio policies. (The optimal integer-ratio policy is proven to come within 6% of a lower bound on the minimum achievable costs.) Let T_i be the replenishment interval for retailer i , $i = 1, \dots, N$, and T_0 the replenishment interval for the supplier. An integer-ratio policy requires that either T_0/T_i or T_i/T_0 be a positive integer. (This type of policy is widely used in practice to synchronize replenishment activities.) To synchronize, the system starts out empty (so that all locations place an order at Time 0), and it is easily verified that it is optimal for each facility to ensure that its inventory is down to zero at all subsequent replenishment epochs. This implies that each retailer i 's order is in the amount of $d_i T_i$ units.

Roundy also shows that the replenishment policies can be further restricted to the so-called power-of-two policies with virtually no loss of optimality. A power-of-two policy is one with

$$T_i = 2^{m_i} T_b, \quad m_i \text{ integer}, \quad i = 0, 1, \dots, N, \quad (1)$$

where T_b is a given base time period (e.g., a day). That is, the reorder interval at each location is restricted to be an integer power-of-two multiple of T_b . Obviously, a power-of-two policy is also an integer-ratio policy. We refer the reader to Roundy (1985) for an algorithm that determines an optimal power-of-two policy.

Throughout this paper, we assume that the order intervals at the supplier and any of the retailers are all chosen from the discrete set of power-of-two values $\{2^m T_b : m = -\infty, \dots, -1, 0, 1, \dots\}$. Although the power-of-two grid may appear to be sparse, it in fact limits the cost increase due to the grid restriction to a small percentage, while maximizing the benefits of coordination of replenishment intervals. For example, in the basic EOQ model, restriction to the set of power-of-two values results in an increase of the cost value by no more than 6%; see Brown (1959) and Roundy (1985). Moreover, power-of-two policies are appealing for a decentralized system because they require less coordination than integer-ratio policies. A power-of-two policy requires each firm to satisfy the power-of-two constraint independently; they do not have to coordinate their reorder intervals explicitly.

Having focused on the replenishment decisions given pricing/demand decisions, we now proceed to

consider the problem of maximizing the systemwide profit:

$$\begin{aligned} \Pi(d, T) &= \sum_{i=1}^N \left\{ (p_i(d_i) - c_0 - c_i) d_i - \Psi(d_i) - \frac{K_i}{T_i} \right. \\ &\quad \left. - \frac{1}{2} h_0 d_i \max\{T_0, T_i\} - \frac{1}{2} h_i d_i T_i \right\} - \frac{K_0}{T_0} \\ &= \sum_{i=1}^N G_i(d_i, T_i, T_0) - \frac{K_0}{T_0} \end{aligned} \quad (2)$$

where d is the demand vector, T the vector of replenishment intervals, and

$$\begin{aligned} G_i(d_i, T_i, T_0) &= (p_i(d_i) - c_0 - c_i) d_i - \Psi(d_i) - \frac{K_i}{T_i} \\ &\quad - \frac{1}{2} h_0 d_i \max\{T_0, T_i\} - \frac{1}{2} h_i d_i T_i. \end{aligned}$$

The problem is to determine a power-of-two vector T (satisfying (1)) and a demand vector d that maximize Π . Let $(T_0^*, T_1^*, \dots, T_N^*)$ and (d_1^*, \dots, d_N^*) be the optimal solution, which we assume to be unique. Let Π^* be the maximum channel profits. For an algorithm that determines the optimal solution, see Chen et al. (1998). (Multiple optimal solutions may arise if the functions $p_i(d) - \Psi(d)$ fail to be strictly concave or due to the rounding of replenishment intervals to power-of-two values. On the other hand, if the functions $p_i(d) - \Psi(d)$ are strictly concave, the optimal solution is unique almost surely when assuming that the cost parameters are drawn from general continuous distributions.)

4. Order-Quantity Discounts

The existing literature on channel coordination has repeatedly shown that a discount scheme based on the order quantity (i.e., an order-quantity discount scheme) coupled with fixed transfer payments achieves perfect coordination. We show, however, that an order-quantity discount scheme that is uniformly applied to all retailers does not guarantee perfect coordination for systems with nonidentical retailers. In the terminology employed by the economics literature on vertical contracting, uniform order-quantity discount schemes are an insufficient set of instruments; see Mathewson and Winter (1984, 1986). While

this is not surprising if the supplier's costs of serving the retailers are themselves nonidentical, it is more striking when this cost structure is uniform across all retailers. We therefore confine ourselves to the case with a uniform cost structure, i.e., $K_i^s = K^s$ for all $i = 1, \dots, N$.

THEOREM 1. *Even when the cost structure is uniform, a uniform order-quantity discount scheme cannot be guaranteed to coordinate the channel with multiple nonidentical retailers.*

PROOF. We prove this via a counterexample. Let $w(Q)$ be the average unit wholesale price when a retailer orders Q units from the supplier. (Thus, $Qw(Q)$ is the total price paid by the retailer for the lot.) For $w(\cdot)$ to be a discount scheme, we must have $w(Q_1) \geq w(Q_2)$ for any $Q_1 < Q_2$. Suppose $p_i(d_i) = a_i - b_i d_i$ for some positive constants a_i and b_i , $i = 1, \dots, N$. Let $Q_i^* = d_i^* T_i^*$, i.e., the (globally) optimal order quantity at retailer i .

Suppose $w(\cdot)$ induces the retailers to order the optimal quantity Q_i^* . (If not, it certainly does not achieve maximum channel profits.) Let $w_i^* = w(Q_i^*)$. The remaining problem for the retailers is to solve $\max\{(p_i(d_i) - c_i - w_i^*)d_i - d_i K_i^r / Q_i^* - \frac{1}{2} \bar{h}_i Q_i^* : d_i \geq 0\}$. It is easy to see that the optimal solution to the above problem is $d_i^0 = (a_i - c_i - w_i^* - K_i^r / Q_i^*) / 2b_i$. To achieve maximum channel profits, we must have $d_i^0 = d_i^*$ or $w_i^* = a_i - c_i - K_i^r / Q_i^* - 2b_i d_i^*$, $i = 1, \dots, N$. If there exists a pair i and j such that $Q_i^* < Q_j^*$ and $w_i^* < w_j^*$, then $w(\cdot)$ is not a discount scheme, i.e., there does not exist any traditional quantity-discount scheme that achieves maximum channel profits. This is the case when, for example, $N = 2$, $K_0 = 100$, $K_1^s = K_2^s = 0$, $K_1^r = K_2^r = 10$, $h_0 = h_1 = h_2 = 1$, $c_0 = 10$, $c_1 = c_2 = 1$, $\Psi(\cdot) \equiv 0$, $a_1 = a_2 = 100$, $b_1 = 10$, and $b_2 = 5$. In this example, the globally optimal solution is: $d_1^* = 4.3$, $d_2^* = 8.6$, $T_0^* = 4$, $T_1^* = T_2^* = 2$. Therefore, $Q_1^* = 8.6$ and $Q_2^* = 17.2$. However, $w_1^* = 11.8372$ and $w_2^* = 12.4186$. \square

Finally, is the insufficiency of (traditional) order-quantity discounts to achieve perfect coordination entirely due to the fact that demands are price sensitive and that retail prices are endogenously determined? In other words, would, under a uniform cost structure, an order-quantity discount scheme suffice

in Roundy's model with exogenously prespecified demand rates? The following proposition sheds light on this question.

THEOREM 2. *Assume all retailer demand rates $\{d_i^*, i = 1, \dots, N\}$ are exogenously given (or equivalently, all retail prices are fixed at their centralized-optimal values). Then, incremental, as well as all-unit, order-quantity discounts cannot be guaranteed to achieve perfect coordination even when the cost structure is uniform.*

PROOF. Consider a system with $N = 2$ retailers. Assume $K_i^s = 0$ for $i = 1, 2$, and K_0 is sufficiently large in relationship to the other parameters that in the centralized solution $T_0^* > \max\{T_1^*, T_2^*\}$. It is easily verified from Roundy (1985) that in this case, $T_i^* = \sqrt{2K_i^r / (h_i d_i^*)} = 2^{m_i}$ for $i = 1, 2$ if $K_i = 2^{2m_i - 1} h_i d_i^*$ for some integer m_i , $i = 1, 2$. (Assume the base period $T_b = 1$.) Let $Q_i^* = d_i^* T_i^*$ for $i = 1, 2$ denote the centralized optimal order quantities. Without loss of generality, assume $Q_1^* < Q_2^*$, or

$$\frac{d_1^* K_1}{h_1} < \frac{d_2^* K_2}{h_2}. \quad (3)$$

Let $t(Q)$ denote the total price the retailers pay for an order of size Q under an incremental order-quantity discount scheme. Thus, $t(Q) = \min_{i=1, \dots, L} \{k_i + w_i Q\}$, with $k_1 < k_2 < \dots < k_L$ and $w_1 > w_2 > \dots > w_L$. Because there are only two retailers, it suffices to have $L = 2$. Now consider the retailers' optimization problems (in the decentralized system with an incremental order-quantity discount scheme). Suppose retailer i chooses $k_i + w_i Q$ as its wholesale cost function. Then its annual cost can be written as $d_i^* w_i + (d_i^* (K_i + k_i)) / Q + \frac{1}{2} \bar{h}_i Q$, where Q is his order quantity. This is the standard EOQ cost function, and thus the optimal order quantity for retailer i is $Q_i^0 = \sqrt{2d_i^* (K_i + k_i) / \bar{h}_i}$. To induce retailer i to use his optimal order quantity in the centralized solution, i.e., $Q_i^0 = Q_i^*$, we must have $k_i / h_0 = K_i / h_i$. Assume $K_1 / h_1 \neq K_2 / h_2$. Thus the intercepts k_i that induce the retailers to choose their optimal order quantities are different. Because $Q_1^* < Q_2^*$, it must be the case that retailer i chooses $k_i + w_i Q$ as his wholesale cost function, $i = 1, 2$. Therefore,

$$k_1 = \frac{K_1 h_0}{h_1} < k_2 = \frac{K_2 h_0}{h_2}. \quad (4)$$

However, there exist examples where (3) and (4) cannot be satisfied simultaneously. Here is one such example: $d_1^* = 1$ and $d_2^* = 8$, $h_0 = h_1 = h_2 = 1$, $K_1 = 2$ and $K_2 = 1$.

We proceed to consider all-unit order-quantity discounts. Let $0 < q_1 < q_2 < \dots < q_{L-1}$ be the breakpoints, and $w_1 > w_2 > \dots > w_L$ the corresponding wholesale prices, for some positive integer L , i.e., if a retailer's order quantity Q is less than q_1 , he pays the wholesale price w_1 for every unit he buys, if Q is in $[q_1, q_2)$ then the wholesale price is w_2 , etc. Define $c_i(Q) = d_i^*K_i/Q + \frac{1}{2}h_iQ$, $i = 1, 2$, i.e., retailer i 's average holding and setup costs if Q is his order quantity (in the decentralized system). Let Q'_i be the (unconstrained) minimum point of $c_i(\cdot)$, i.e., $Q'_i = \sqrt{2d_i^*K_i/h_i}$. Clearly, $Q'_i < Q_i^*$. For $Q \in [q_{i-1}, q_i)$, retailer i 's total cost function is $w_id_i^* + c_i(Q)$. Retailer i 's objective is to choose an order quantity that minimizes his total cost. Because his objective function is piecewise convex, retailer i 's optimal order quantity can be either Q'_i or a breakpoint. To induce retailer i to choose the centralized optimal order quantity $Q_i^* (> Q'_i)$, we must have Q_i^* as a breakpoint. Because there are only two retailers, it suffices to consider an all-unit order-quantity discount scheme with $\{Q_1^*, Q_2^*\}$ as its only breakpoints. That is, $L = 3$, $q_1 = Q_1^*$, $q_2 = Q_2^*$, and (w_1, w_2, w_3) are the corresponding wholesale prices.

Assume $Q_1^* = Q_2^*$. (There exist examples that satisfy this condition.) For both retailers to choose their centralized optimal order quantities, the following conditions are necessary: $w_3d_2^* + c_2(Q_2^*) \leq w_2d_2^* + c_2(Q_2^*) = w_2d_2^* + c_2(Q_1^*)$ and $w_2d_1^* + c_1(Q_1^*) \leq w_3d_1^* + c_1(Q_2^*)$, where the first inequality ensures that retailer 2 does not choose the smaller breakpoint, and the second inequality prevents retailer 1 from choosing a lower wholesale price. Combining the two inequalities, we have

$$\frac{c_2(Q_2^*) - c_2(Q_1^*)}{d_2^*} \leq w_2 - w_3 \leq \frac{c_1(Q_2^*) - c_1(Q_1^*)}{d_1^*}. \quad (5)$$

However, in an example with $h_0 = 3$ and $h_1 = h_2 = 1$, $d_1^* = 2$ and $d_2^* = 1$, $K_1 = 1$ and $K_2 = 8$, it can be easily verified that $Q_1^* = Q_2^*$ and $(c_2(Q_2^*) - c_2(Q_1^*)) / d_2^* > (c_1(Q_2^*) - c_1(Q_1^*)) / d_1^*$, in which case (5) cannot be satisfied. Therefore, there exist examples where it is

impossible to find an all-unit order-quantity discount scheme that achieves perfect coordination. \square

Note that because any concave increasing function $t(Q)$ can be approximated arbitrarily closely by an incremental order-quantity discount scheme $\min_{i=1, \dots, L} \{k_i + w_iQ\}$, it is clear that any order-quantity discount scheme based on a concave $t(\cdot)$ function is still insufficient to achieve perfect coordination. Of course, Theorem 2 leaves open the possibility of the existence of an even more general order-quantity discount scheme that can achieve perfect coordination. However, such a scheme, even if it existed, is likely to have a most complex and unintuitive form, and might therefore be impractical.

REMARK. The single-retailer version of our model (without account management costs, i.e., $\Psi \equiv 0$) was first introduced by Weng (1995). While it is stated in his paper that an order-quantity discount scheme results in perfect coordination, Boyaci and Gallego (1997) question the validity of this result. Chen et al. (1997) formally establish that a scheme using an order-quantity discount and a periodic franchise fee suffices to achieve perfect coordination in the single retailer or multiple identical retailers settings.

5. Coordination Mechanisms

This section specifies a perfect coordination mechanism for the model described in §2. Every decentralized supply chain requires an upfront specification of a *contract*, i.e., a set of ground rules for the commercial interactions between the different parties involved. Such a contract may involve the specification of a pricing rule, the commitment to deliver in whole or in part (possibly within a specified lead time), return policies, restrictions on the times at which orders may be placed and delivered (e.g., daily, every Tuesday or Friday, etc.), among others. As explained in Tirole for systems with symmetric information (1988, p. 173) it is immaterial whether the contract is specified by one of the channel members, a consortium representing all, or a supply chain consultant, as long as all parties agree upon the terms. The channel members accept a contract only if it permits them to achieve a profit value at least equal to their outside opportunity or status quo.

In our model, the contract consists of the following provisions. The supplier commits himself to satisfy all retailer orders in their entirety and to deliver them with a fixed leadtime, which is without loss of generality normalized to be zero. All channel members place their replenishment orders at epochs taken from the discrete set $\{2^m T_b : m = \dots, -2, -1, 0, 1, 2, \dots\}$; see the discussion in §3. (We assume, without loss of generality, that the system is without inventory at Time 0.) All parties retain the revenues they collect, and incur their own costs. (Recall from §2 that the supplier incurs a fixed cost K_0 and a per unit purchase cost c_0 for every order it places, a fixed cost K_i^s for processing an order from retailer i , holding costs at its own site, and account management costs. Each retailer i incurs a fixed cost of K_i^r and a per unit transportation cost c_i for every order it places, and incurs holding costs at its own site. These costs do not include any transfer payments.) The following mechanism determines the wholesale price the retailers pay to the supplier: retailer $i, i = 1, \dots, N$

(i) is charged K_i^s by the supplier for every order it places;

(ii) is charged by the supplier a basic per unit cost equal to

$$c_0 + \frac{\Psi(d_i)}{d_i} + \frac{1}{2}h_0A; \text{ and} \quad (6)$$

(iii) is given a per unit discount equal to

$$\frac{1}{2}h_0 \min\{A, T_i\} \quad (7)$$

where $A = T_0^*$, a contract parameter from the centralized solution.

Finally, the contract specifies a *fixed* annual transfer payment F_i by facility i (in the forms of franchise fees or rebates). A large variety of fee vectors F may be chosen in the contract, subject to the restriction that it permits all channel members to achieve a profit value equal to or in excess of their status quo profits $\{\Pi_i^0, i = 0, 1, \dots, N\}$. Let Π_i denote facility i 's profit after any annual transfer payments, i.e., $\Pi_i = \Pi_i^* - F_i$. For the contract to be acceptable to all parties, we must have that $\Pi_i \geq \Pi_i^0$ for all $i = 0, 1, \dots, N$. One such possible profit allocation results in $\Pi_i = \Pi_i^0 + V/(N + 1)$,

$i = 0, 1, \dots, N$, where $V = \Pi^* - \sum_{i=0}^N \Pi_i^0 \geq 0$. (This corresponds with $F_i = \Pi_i^* - \Pi_i^0 - V/(N + 1), i = 0, 1, \dots, N$, and is in fact the Nash bargaining solution of an $(N + 1)$ -person bargaining game. See, e.g., Myerson (1991) for a systematic discussion of various allocation schemes and their properties.) Clearly, all fee vectors in $\mathcal{F} = \{F : \sum F_i = 0, F_i \leq \Pi_i^* - \Pi_i^0, i = 0, 1, \dots, N\}$ can be employed. The actual choice depends on the relative market and bargaining power of the channel members. Even though the fixed fees are essential in assuring that the contract is attractive to all channel members, they have no impact on the aggregate profits in the chain or on the pricing or replenishment strategies to be chosen by each of its members.

The above contract creates a game with the supplier and the retailers as its players. We will show that the retailers' demand rates and replenishment strategies that are part of the (unique) centralized solution surface as dominant strategies for the retailers in the game. (A player's strategy is dominant if it is the player's strictly best response to any strategies the other players might pick.) Moreover, given the retailers' dominant strategies, we show that it is optimal for the supplier to choose the same replenishment strategy, in particular the same constant reorder interval T_0^* , as in the centralized solution. In other words, the centralized solution is realized in the decentralized channel as an *iterated dominant* strategy equilibrium, all while maximizing channel profits. (An equilibrium is iterated dominant if the players can be sequenced in such a way that it has the following properties. Player 1's strategy is dominant. Knowing that Player 1 will therefore choose his dominant strategy, Player 2 has a strategy that is optimal regardless of the strategies adopted by the remaining players, and so forth. See Rasmusen (1990) for a definition of the equilibrium concept.) It will also become clear that the centralized solution prevails as a unique Nash equilibrium of the game. Finally, note that the contract is equally attractive when the "supplier" and "retailers" represent decentralized divisions of a single firm, which are organized as profit centers.

To analyze the above game, we begin with the problem facing retailer i . It is easily verified that it is optimal for the retailer to always place orders when his

inventory is down to zero. Adding Components (i)–(iii), the average wholesale price retailer i pays to the supplier is

$$w = \frac{K_i^s}{T_i d_i} + c_0 + \frac{\Psi(d_i)}{d_i} + \frac{1}{2} h_0 A - \frac{1}{2} h_0 \min\{A, T_i\}. \quad (8)$$

The long-run average profit for retailer i is given by

$$p_i(d_i)d_i - \left(c_0 + c_i + \frac{\Psi(d_i)}{d_i} + \frac{1}{2} h_0 A - \frac{1}{2} h_0 \min\{A, T_i\} \right) d_i - \frac{1}{2} h_i d_i T_i - \frac{K_i^s + K_i^r}{T_i}. \quad (9)$$

Notice that this profit function is independent of the other players' decisions. Therefore, any unique solution to the above problem represents a dominant strategy for retailer i . Because $A - \min\{A, T_i\} = \max\{A, T_i\} - T_i$ and $A = T_0^*$, (9) can be written as

$$(p_i(d_i) - c_0 - c_i)d_i - \Psi(d_i) - \frac{1}{2} h_0 d_i \max\{T_0^*, T_i\} - \frac{1}{2} h_i d_i T_i - \frac{K_i}{T_i} = G_i(d_i, T_i, T_0^*),$$

which is maximized at a unique point (d_i^*, T_i^*) (see (2)). Therefore, the dominant strategy for retailer i is to employ the demand rate d_i^* and the power-of-two replenishment strategy with interval T_i^* .

Now suppose the retailers all follow their dominant strategies. Under the above mechanism, the supplier receives the following average payments from the retailers:

$$\sum_{i=1}^N \frac{K_i^s}{T_i^*} + \left(c_0 + \frac{\Psi(d_i^*)}{d_i^*} + \frac{1}{2} h_0 A - \frac{1}{2} h_0 \min\{A, T_i^*\} \right) d_i^*. \quad (10)$$

It is again easily verified that the supplier is best off placing his orders when his inventory is down to zero. He then incurs the following average costs:

$$\frac{K_0}{T_0} + \sum_{i=1}^N \left\{ \left(\frac{1}{2} h_0 d_i^* \max\{T_0, T_i^*\} - \frac{1}{2} h_0 d_i^* T_i^* \right) + \frac{K_i^s}{T_i^*} + c_0 d_i^* + \Psi(d_i^*) \right\} \quad (11)$$

that include fixed costs, holding costs, order-processing costs, variable purchase costs, and account

management costs. Therefore, the supplier's profit function is

$$-\frac{K_0}{T_0} + \sum_{i=1}^N \frac{1}{2} h_0 d_i^* (A - \min\{A, T_i^*\} - \max\{T_0, T_i^*\} + T_i^*).$$

Collecting terms related to T_0 , the supplier maximizes $-K_0/T_0 - \sum_{i=1}^N \frac{1}{2} h_0 d_i^* \max\{T_0, T_i^*\}$ that is equal to $\Pi(d^*, T)$ with $T = (T_0, T_1^*, \dots, T_N^*)$ plus a constant, and it is maximized at a unique point $T_0 = T_0^*$. Hence, the supplier's optimal strategy is to use a power-of-two strategy with replenishment interval T_0^* . (The coordination mechanism has been designed to make each player's profit function equal to the total profit plus a constant, an application of the Groves mechanism; see Groves 1973.) In addition, observe that the centralized solution arises as an iterated dominant strategy equilibrium, considering any sequence of the players in which the supplier is last. Moreover, the above analysis indicates that the centralized solution is a unique Nash equilibrium of the game. (The uniqueness follows from the fact that (d^*, T^*) is a unique solution of the centralized problem.) Note that the supplier makes a negative profit, $-K_0/T_0^*$, under the contract. Therefore, transfer payments from the retailers are necessary for the supplier to accept the contract. We conclude:

THEOREM 3. *Under the above described coordination mechanism, in particular, the wholesale pricing mechanism (i)–(iii) and any fee vector in \mathcal{F} , the centralized solution (d^*, T^*) prevails as an iterated dominant strategy equilibrium (and also a unique Nash equilibrium) in the decentralized channel, and the total channel profit is maximized.*

Our coordination mechanism combines *three* kinds of discounts. Consider (8), the average wholesale price paid by retailer i . (1) Note that $d_i T_i$ is the retailer's order quantity. The higher the order quantity, the lower the unit price (*ceteris paribus*). This is an order-quantity discount. (2) The ratio $\Psi(d_i)/d_i$ decreases with d_i because $\Psi(\cdot)$ is concave and $\Psi(0) = 0$. Thus, the higher the annual sales volume d_i , the lower the wholesale price. This is a volume discount. (3) As the reorder interval T_i increases, the discount increases first and stays constant after it exceeds A , a contract parameter. This is an order-frequency discount. As mentioned in the introduction, all of the

above three types of quantity discounts have been used in practice, and many schemes use combinations of the three types of discounts (see Munson and Rosenblatt 1998).

The coordination mechanism is appealing because the differentials in the wholesale prices paid by different retailers are based on the underlying economics in the system. (1) The discount based on the order size is due to the order-processing costs K_i^s the supplier incurs. (2) The discount based on the annual sales volume reflects the economies of scale in managing a retailer account. (3) The discount based on the order frequency reflects the savings in holding costs the supplier enjoys when the retailer orders less frequently (i. e., the retailer keeps larger inventories). To see this, first suppose that the retailer's reorder interval is very small. In this case, the supplier has to hold all the inventory for the retailer. Because the supplier's replenishment cycle is T_0^* , the supplier incurs an average holding cost of $h_0 T_0^*/2$ for every unit sold by the retailer. This expense becomes part of the basic wholesale price in (6). Now suppose the retailer replenishes once every T_i units of time. If $T_i \leq T_0^*$, then the supplier on average saves $h_0 T_i/2$ of holding costs for every unit sold by retailer i . This explains the discount given in (7). Now if T_i exceeds T_0^* , it is easy to see that the supplier does not have to hold any inventory for the retailer, and therefore the discount in (7) is designed to cancel the holding-cost expense charged as part of the basic wholesale price.

Our coordination mechanism (including the fixed fees) ensures compliance with fair trade laws, in particular the Federal Robinson-Patman Act. Section 2(a) of this act specifies that it is unlawful "either directly or indirectly to discriminate in price between purchasers of commodities of like grade and quality." Quantity discounts are permitted under this section but only to the extent that they are fully justified by cost savings and, hence, identically applied to all retailers. This is precisely the case with our coordination mechanism because any discount awarded to a retailer is directly based on the costs incurred by the supplier to fill the retailer's order. McAfee and Schwartz (1994, p. 217) report that "courts and enforcement agencies (primarily the FTC) have generally been willing to consider as a defense the fact

that the challenged offer, while entailing a different marginal price, was made available to all competing buyers." Depending upon what allocation mechanism is used to allocate systemwide profits, the franchise fees paid by the retailers may either be guaranteed to be identical or not. It is our understanding that U.S. fair trade regulations do not require that identical franchise fee be charged to all retailers or that differences between them need to be justified as a sample function of one or more of the retailer characteristics. We refer to Stein and El-Ansary (1992) and Handler et al. (1990) for a more detailed discussion of the Robinson-Patman Act.

The above described coordination mechanism is by no means unique. For example, in lieu of the annual franchise fee F , one can use a profit-sharing plan under which the supplier receives a predetermined fraction θ_i of retailer i 's profit, $i = 1, \dots, N$. This modified coordination mechanism induces the same equilibrium strategies and is therefore perfect as well. (To verify the latter, note that the profit function for retailer i is now given by $(1 - \theta_i)G_i(d_i, T_i, A)$. The supplier maintains the same cost function as in (11) while receiving profit shares $\sum_{i=1}^N \theta_i G_i(d_i^*, T_i^*, A)$ in addition to revenues specified in (10), both of which are independent of the supplier's decision variable T_0 .) More generally, the coordination mechanism may use a combination of this type of profit sharing as well as annual franchise fee F or for that matter, any profit-sharing rule that is a fixed, predetermined and increasing function of the retailer's profits. In the remainder of this paper, we confine ourselves to the original coordination mechanism with franchise fees as the sole instrument to reallocate profits. Our choice is based on the simplicity and transparency of the scheme as well as the fact that most of the economics literature on vertical contracting confines itself to this instrument; see, e.g., Tirole (1988).

One possible limitation of the coordinating pricing scheme arises when the setup costs $\{K_i^s, i = 1, \dots, N\}$ are retailer-specific. In this case, the Pricing Scheme (8) has a component that fails to be uniform across all retailers, and it is therefore possible that a retailer i with a larger sales volume d_i and a longer replenishment interval T_i than some retailer j incurs a higher wholesale price, nevertheless.

REMARK. A critical assumption in this paper is that all demand functions are deterministic, i.e., demands are deterministically known once the retailer prices are selected. In practice, demands may be subject to a considerable amount of uncertainty, i.e., the demand functions are often stochastic. It is interesting to observe that the identified discount pricing scheme can be implemented in a stochastic setting. The supplier and retailers can continue to agree to place orders with replenishment intervals of constant length, possibly again chosen to be power-of-two multiples of a given base period. The three-part cost structure, specified in §5, can continue to be implemented, except for the term in (6). The latter may be replaced by its expectation over the relevant stochastic demand process; alternatively, the cost component may be paid periodically on the basis of the realized demand volume. Most importantly the three-part cost structure continues to adequately reflect the economics of the system even when demands are stochastic. While we do not surmise that the coordination mechanism is perfect in a stochastic setting, we conjecture that in expectation it is close to being perfect. This needs to be explored in future work.

We close this section with the following result, which shows how the coordination mechanism changes as new retailers enter the system.

THEOREM 4. *Assume that a new retailer, called retailer m ($m \neq 1, \dots, N$), joins the system with demand function $d_m(p_m)$ and parameters K_m, c_m and h_m . Assume the contract is modified to account for the new retailer, i.e., a new wholesale pricing scheme and a new set of franchise fees. The entry of this retailer results in a lower wholesale price schedule for all the existing retailers.*

PROOF. Note that (8) can be rewritten as $w = K_i^s/T_i d_i + c_0 + \Psi(d_i)/d_i + \frac{1}{2}h_0 \max\{A - T_i, 0\}$. Thus, the wholesale price schedule shifts down if A ($=T_0^*$) decreases. Let T_0^* (resp., T_0^{**}) denote the optimal reorder interval at the supplier obtained in the centralized solution before (respectively, after) retailer m enters the system. It thus suffices to show that $T_0^{**} \leq T_0^*$. Let $\pi(T_0)$ (respectively, $\bar{\pi}(T_0)$) denote the optimal channel profit in the absence (respectively, presence) of retailer m given the supplier's reorder interval T_0 . For any $T_0 > T_0^*$,

$\bar{\pi}(T_0) = \pi(T_0) + \max_{d_m, T_m} G_m(d_m, T_m, T_0) \leq \pi(T_0^*) + \max_{d_m, T_m} G_m(d_m, T_m, T_0^*) = \bar{\pi}(T_0^*)$, where the inequality follows because per definition $\pi(T_0) \leq \pi(T_0^*)$, and $G_m(d_m, T_m, T_0) \leq G_m(d_m, T_m, T_0^*)$ due to $\max\{T_0, T_m\} \geq \max\{T_0^*, T_m\}$. Therefore, the T_0 that maximizes $\bar{\pi}(T_0)$ must be less than or equal to T_0^* . \square

6. An Uncoordinated Channel

As mentioned, the franchise fees are necessary to ensure that the contract is attractive to all parties. To this end, all channel members must derive a profit value equal to or in excess of their status quo profit. This section considers a status quo, where the channel members fail to coordinate their decisions.

Consider the following scenario. The supplier charges a *constant* wholesale price and chooses a replenishment strategy with the objective of maximizing its own profits. The retailers take the wholesale price as given and maximize their individual profits by choosing an optimal retail price and replenishment strategy. In other words, the channel members play a Stackelberg game with the supplier as the leader and the retailers as followers. This modus operandi represents many traditional distribution channels in which the channel members fail to coordinate their decisions.

Once again, we assume that all replenishment intervals have to be chosen from the discrete set of power-of-two values $\{2^m T_b : m = \dots, -2, -1, 0, 1, 2, \dots\}$. It is then easily verified that all players are best off employing power-of-two strategies: Each player places an order when his inventory is down to zero and adopts a constant, player-specific reorder interval. Thus, in the Stackelberg game, the supplier's decision variables are the wholesale price, w , and its reorder interval T_0 . Given these decisions, the retailers then determine their own retail prices p_i (or demand rates d_i) and reorder intervals T_i to maximize their own profits. The supplier incurs a fixed cost (K_0) and a per unit variable cost (c_0) for every order he places, an order-processing cost (K_i^s) for each retailer order, holding costs at rate h_0 for his own inventories, as well as the account management costs ($\Psi(d_i)$). The retailers incur a fixed cost (K_i^r) and a per unit transportation cost (c_i) for each order they place with

the supplier, and holding costs at rate \bar{h}_i for the inventories they carry at their sites. Recall that the retailers' orders are always shipped immediately. Therefore, retailer i 's problem is

$$\max_{d_i, T_i} \pi_i(d_i, T_i | w) \stackrel{\text{def}}{=} (p_i(d_i) - c_i - w)d_i - \frac{K_i^r}{T_i} - \frac{1}{2} \bar{h}_i d_i T_i, \quad i = 1, \dots, N. \quad (12)$$

Note that the objective function depends only on a single parameter specified by the supplier, i.e., the wholesale price w . (It is independent of T_0 .) Let $d_i(w)$ and $T_i(w)$ be the optimal solution to (12), $i = 1, \dots, N$. On the other hand, the supplier's problem is

$$\max_{w, T_0} \pi_0(w, T_0) \stackrel{\text{def}}{=} \sum_{i=1}^N \left\{ (w - c_0)d_i(w) - \Psi(d_i(w)) - \frac{K_i^s}{T_i(w)} - \frac{1}{2} h_0 d_i(w) [T_0 - T_i(w)]^+ \right\} - \frac{K_0}{T_0}. \quad (13)$$

We have developed an efficient algorithm for solving the above Stackelberg game; see Chen et al. (1998) for details.

A more general Stackelberg game arises when the supplier offers a wholesale price schedule, i.e., the average unit wholesale price $w(Q)$ is a nonincreasing function of the order quantity Q . One can, for example, restrict $w(\cdot)$ to the class of incremental order-quantity discounts with a finite number of breakpoints. The resulting game, however, is much harder to solve. If there is only one breakpoint, it is possible to solve the resulting Stackelberg game via a complete search over all combinations of (i) the basic wholesale price, (ii) the discounted wholesale price, and (iii) the breakpoint.

7. Numerical Examples

In this section, we investigate how large the value of coordination, i.e., the increase in channel profits resulting from perfect coordination, can be. We use as our status quo scenario the Stackelberg games described in §6. Their channel profits are compared with the maximum channel profits to yield a value of coordination.

We consider two sets of examples, one with identical and one with nonidentical retailers. For

the former sets, $N = 10$ and the base case has the following parameters: $K_0 = 100$; $c_0 = 10$; $K_i^s = 0$, $K_i^r = 10$ and $c_i = 1$ for $i = 1, \dots, N$; $h_i = 1$ for $i = 0, 1, \dots, N$; $p_i(d_i) = a_i - b_i d_i$ with $a_i = 100$ and $b_i = 20$ for $i = 1, \dots, N$; $\Psi(d) = f + ed$ with $f = 10$ and $e = 1$. The other examples in this set are obtained by modifying the base case one parameter at a time. The set with nonidentical retailers has a base case with the same parameters as the previous base case except for c_i and b_i , now varying among the retailers as follows: $c_i = 1 + i\delta_c/N$ and $b_i = 10 + i\delta_b/N$, $i = 1, \dots, N$, with $\delta_c = 10$ and $\delta_b = 10$. The other examples in this set are again obtained by modifying the base case one parameter at a time.

The results are summarized in Figures 1 and 2. The channel profits under the two Stackelberg games are around 70% of the maximum, suggesting that the value of coordination can be significant. This suggests that a perfect coordination scheme can result in major performance improvements for the channel as a whole, thus encouraging companies and their supply chains to engage in coordination mechanisms of the type discussed here. Note that when comparing the two Stackelberg games, it is clear that the supplier is always better off with a quantity-discount scheme because the latter provides him with a larger set of feasible strategies, but the same cannot be said of the channel profits.

The double-marginalization phenomenon (Spengler 1950 and Tirole 1988) recurs in our model, i.e., coordination leads to lower (average) wholesale prices, lower retail prices, and larger demand rates. For the base case with identical retailers, the coordination mechanism requires retailer i to pay a wholesale price $12 + (10 + d_i)/d_i - 0.5 \min\{4, T_i\}$ per unit ordered. Because $d_i^* = 2.1$, and $T_i^* = 4$, each retailer pays a wholesale price of 15.76. This is much lower than the wholesale price of 53 in the Stackelberg game (with a constant wholesale price). With this Stackelberg game as the status quo, Table 1 shows for the base case a franchise fee of 36.1 based on equal division of the gains from coordination.

The same phenomenon is observed in the second set with nonidentical retailers. For its base case, the coordination mechanism uses the wholesale price

Figure 1 Uncoordinated Channel Profit As a Percentage of Maximum Channel Profit in Centralized Solution, when Retailers Are Identical

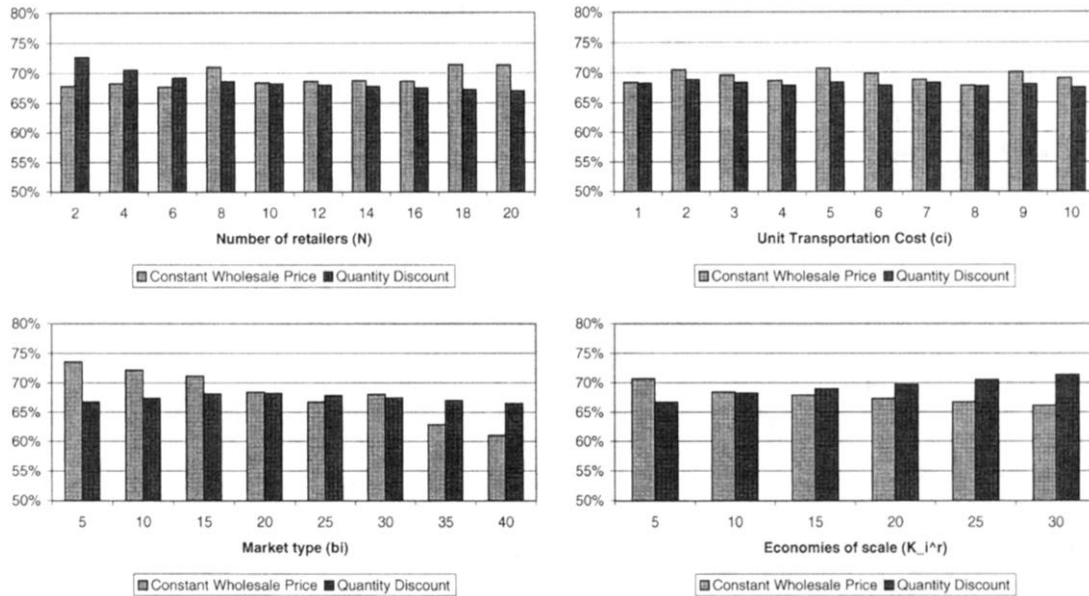


Figure 2 Uncoordinated Channel Profit As a Percentage of Maximum Channel Profit in Centralized Solution when Retailers Are Nonidentical

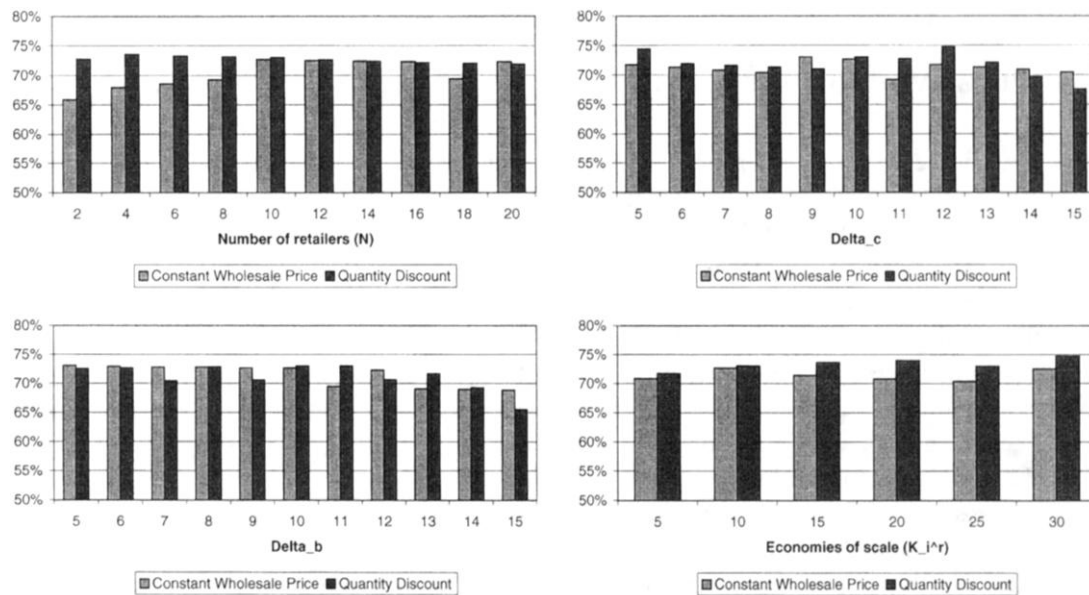


Table 1 Profit Table, Identical Retailers (Constant Wholesale Price)

	Stackelberg Game	Coordinated System	Bargaining Solution	Fixed Payments
Supplier	316.00	-25.00	336.05	-361.05
One Retailer	19.55	75.70	39.60	36.10
Systemwide	511.50	732.00	732.00	

$11 + (10 + d_i)/d_i - 0.5 \min\{2, T_i\}$. In the coordination system, we have, e.g., $(d_1, T_1) = (3.86, 2)$ and $(d_{10}, T_{10}) = (1.9, 2)$, resulting in a wholesale price of 13.59 for Retailer 1 and 16.26 for Retailer 10. Thus, Retailer 1 receives a substantial volume discount. In the Stackelberg game (with a constant wholesale

Table 2 Profit Table, Nonidentical Retailers (Constant Wholesale Price)

	Stackelberg Game	Coordinated System	Bargaining Solution	Fixed Payments
Supplier	405.00	-50.00	427.35	-477.35
Retailer 1	43.09	149.20	65.44	83.77
Retailer 2	37.19	132.00	59.53	72.47
Retailer 3	32.23	117.48	54.58	62.91
Retailer 4	28.02	105.07	50.36	54.71
Retailer 5	24.40	94.35	46.74	47.61
Retailer 6	21.27	85.00	43.61	41.39
Retailer 7	18.74	76.78	41.08	35.70
Retailer 8	16.51	69.50	38.86	30.64
Retailer 9	14.55	63.01	36.90	26.12
Retailer 10	12.81	57.20	35.16	22.04
Total	653.81	899.60	899.60	0.00

price), the retailers pay a price of 50. The franchise fees for this base case are in Table 2, using the Stackelberg game with a constant wholesale price as the status quo and equal division of gains from coordination.

8. An Alternative Holding-Cost Model

So far we have assumed that the inventory holding costs incurred by the retailers are independent of the wholesale price paid. Because the holding costs typically include both capital costs and physical holding such as warehousing costs, this assumption may be restrictive. In this section, we explicitly model holding costs as consisting of two components: physical holding costs and capital costs. We show that our coordination mechanism can easily be modified to accommodate this more general model.

Let I be the interest rate, which is identical across the firms. Redefine h_0 and \bar{h}_i to be the physical carrying-cost rates at the supplier and retailer i , respectively.

From the system's perspective, it costs $h'_0 = h_0 + Ic_0$ to hold a unit at the supplier, and $\bar{h}'_i = \bar{h}_i + Ic_0 + Ic_i$ at retailer i . (For more accurate models based on the net present value of cash flows, see Porteus 1985.) Hence, the incremental, holding-cost rate at retailer i is $h'_i = \text{def } \bar{h}'_i - h'_0 = \bar{h}_i - h_0 + Ic_i = h_i + Ic_i$, which is also

assumed to be nonnegative. The average systemwide inventory and setup costs can be expressed as

$$\frac{K_0}{T_0} + \sum_{i=1}^N \left(\frac{K_i}{T_i} + \frac{1}{2} h'_0 d_i \max\{T_0, T_i\} + \frac{1}{2} h'_i d_i T_i \right).$$

The channel profit is thus $\Pi(d, T) = \sum_{i=1}^N G'_i(d_i, T_i, T_0) - K_0/T_0$, where $G'_i(d_i, T_i, T_0) = (p_i(d_i) - c_0 - c_i)d_i - \Psi(d_i) - K_i/T_i - \frac{1}{2} h'_0 d_i \max\{T_0, T_i\} - \frac{1}{2} h'_i d_i T_i$. Again, let $T^* = (T_0^*, T_1^*, \dots, T_N^*)$ and $d^* = (d_1^*, \dots, d_N^*)$ be the optimal reorder intervals and the optimal demand rates, respectively. This optimal solution can be obtained by using the same algorithm outlined in §3. Our objective is to identify a coordination mechanism that induces the decentralized system to achieve the maximum channel profit $\Pi(d^*, T^*)$.

Consider the decentralized system and suppose we still use the wholesale price mechanism (i)–(iii) proposed in §5. Then retailer i incurs an interest cost of $I(c_i + \bar{w})$, where \bar{w} is the unit wholesale price. However, from the system's perspective, the interest cost at the retail level should be $I(c_i + c_0)$. Because $\bar{w} > c_0$, the price mechanism inflates the holding costs at the retail level. As a result, it can no longer induce the decentralized channel to achieve the maximum total profit. The system again suffers from double marginalization. There are two ways to fix this problem.

The first remedy is easy. Instead of paying the following full wholesale price at delivery,

$$c_0 + \frac{\Psi(d_i)}{d_i} + \frac{1}{2} h_0 A - \frac{1}{2} h_0 \min\{A, T_i\}, \quad (14)$$

the retailers pay only c_0 at delivery and the rest when the goods are sold. As a result, the retailers face an interest cost of $I(c_0 + c_i)$. It is easy to verify that with this partial consignment, the wholesale price mechanism (i)–(iii) still makes the retailer's profit function coincide with $G'_i(d_i, T_i, T_0^*)$, and therefore achieves perfect coordination.

The second approach does not use consignment but revises the wholesale price formula. Here we assume that the retailers pay the wholesale price upon delivery. (In practice, there are of course many other possible arrangements. One common arrangement is to pay the wholesale price after a fixed grace period, but this can easily be accommodated. To see this, let t be

the grace period. Set $w' = w(1 + It)$, where w is the desired unit wholesale price with zero grace period. Paying w' after the grace period is equivalent to paying w upon delivery. Interested readers are referred to Boyaci and Gallego 1997 for further discussions.) Change the wholesale price mechanism (i)–(iii) to the following. Retailer i , $i = 1, \dots, N$,

(i)' is charged K_i^s by the supplier for every order it places;

(ii)' is charged by the supplier a basic per unit cost equal to

$$c_0 + \frac{\Psi(d_i)/d_i + \frac{1}{2}h'_0A}{1 + \frac{1}{2}IT_i};$$

and

(iii)' is given a per unit discount equal to

$$\frac{\frac{1}{2}h'_0 \min\{A, T_i\}}{1 + \frac{1}{2}IT_i}$$

where $A = T_0^*$, the optimal reorder interval for the supplier in the (new) centralized system. Therefore, the unit wholesale price paid by retailer i is

$$w = c_0 + \frac{\Psi(d_i)/d_i + \frac{1}{2}h'_0A - \frac{1}{2}h'_0 \min\{A, T_i\}}{1 + \frac{1}{2}IT_i}.$$

Note that this new wholesale price is obtained by discounting part of (14). This is used to offset the additional inventory holding costs the retailer incurs due to the supplier's markup. Although the new formula is more complex, it preserves all the features of the original one: Discounts are based on the order quantity, the annual sales volume, and the order frequency.

Under the wholesale price mechanism (i)'–(iii)', retailer i 's problem is to select d_i and T_i to maximize its own profits: $(p_i(d_i) - c_i - w)d_i - (K_i^s + K_i^r)/T_i - \frac{1}{2}(\bar{h}_i + Ic_i + Iw)d_iT_i$, which can be shown to equal to $G'_i(d_i, T_i, T_0^*)$. Retailer i thus chooses d_i^* and T_i^* , $i = 1, \dots, N$. Given these decisions, now consider the supplier's problem. It is easy to see that the supplier's revenue stream is fixed; thus, maximizing profits reduces to minimizing costs. The costs that depend on the supplier's decision variable T_0 are the fixed costs and the inventory holding costs at the supplier site: $K_0/T_0 + \sum_{i=1}^N (\frac{1}{2}h'_0d_i^* \max\{T_0, T_i^*\} - \frac{1}{2}h'_0d_i^*T_i)$ that is equal to $-\Pi'(d^*, T)$ with $T = (T_0, T_1^*, \dots, T_N^*)$ plus a

constant. Thus, the supplier chooses T_0^* . The centralized solution again prevails as an iterated dominant strategy equilibrium in the decentralized channel, and the channel profits continue to be maximized.

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