October 2001

COPING WITH CREDIT RISK*

Henri Loubergé, Department of Economics, University of Geneva, 40 Blvd. du Pont d'Arve, CH - 1211 Geneva 4 (Switzerland), Henri.Louberge@ecopo.unige.ch

Harris Schlesinger, Department of Finance, 200 Alston Hall, University of Alabama, Tuscaloosa AL 35487-0224 (USA), hschlesi@cba.ua.edu

Abstract

We consider a pool of bank loans subject to a credit risk and develop a method for decomposing the credit risk into idiosyncratic and systemic components. The systemic component accounts for the aggregate statistical difference between credit defaults in a given period and the long-run average of these defaults. We show how financial contracts might be redesigned to allow for banks to manage the idiosyncratic component for their own accounts, while allowing the systemic component to handled separately. The systemic component can be retained, passed off to the capital markets, or shared with the borrower. In the latter case, we introduce a type of floating rate interest, in which the rate is set in arrears, based on a composite index for the systemic risk. This is shown to increase the efficiency of risk sharing between borrowers, lenders and the capital market.

* Financial support from the International Center for Financial Asset Management and Engineering (FAME) is gratefully acknowledged. The authors thank Robert Brooks, Günter Franke, Frank Skinner, Richard Stapleton and Helmut Zink for helpful discussions and comments.

Credit risk is pervasive throughout financial markets.[1] Traditionally, various financial institutions have assumed the burden of credit risk. Banks have supported the credit risk attached to bank loans and forward contracts. Credit insurance companies have provided coverage for the commercial credit risk faced by suppliers of consumer and investment goods and services. Public insurers, such as the ECGD in the UK, have specialized in the coverage of credit risk attached to export trade and overseas investment. Specialized institutions, such as factoring companies, have offered credit risk coverage as one component in a basket of financial services. More recently, the proliferation of financial contracts that entail counter-party default risk -- such as swaps, back-to-back loans, and derivative products -- have focused attention on ways to deal with credit risk in the marketplace. New products, such as credit default swaps, credit spread options and total-rate-of-return swaps, have allowed firms and financial institutions to more effectively deal with credit risks. Indeed, a recent survey by the British Banker's Association estimates the current market for credit derivatives to be around $900 billion in notional principal, with that amount expected to jump to over $1.5 trillion in 2002.[2] In addition, insurance markets have reacted with an array of new products, many with the backing of larger capital markets, such as insurance-linked securities and finite-risk contracts (Shimpi 1999).

Much effort has gone into measuring the various parameters of credit risk.[3] For institutions dealing with credit risk, a particular concern arises from the fact that credit risk has both an idiosyncratic and a systemic component. Counter-party default may arise as a consequence of factors unique to the borrower, such as poor management or bad luck. However, it also may arise in the wider contexts of economic recessions, financial market crashes and political turmoil. This point is certainly underscored by the rate of defaults following the September 11 attacks on the World Trade Center in New York.

---

[1] A recent paper by Berrada, Gibson and Mougeot (2001) finds that credit risk is a significant factor in the pricing of US stocks, for example.
[2] See British Banker's Association (2000).
[3] See Longstaff and Schwartz (1995) for a good introduction to credit derivatives. Crouhy et al. (2000) provide a good critical summary of the main frameworks for measuring credit risk. An historical development of these models is examined by Altman and Saunders (1997).

The main point here is that defaults can be affected by various common factors, and that this results in default rates that are unstable over time. As an example, consider the market for corporate bonds that are rated by Standard & Poor's. There were approximately $4.3 billion in defaults for 1997, whereas in 1991 the value of defaults exceeded $20 billion. Thus, credit risk entails an important systemic component and it has, conceptually, much in common with other types of risks where an accumulation of losses may also arise as a consequence of market-wide phenomena.[4]

In this paper, we develop a method for coping with credit risk by decomposing this risk into idiosyncratic and systemic components that may be treated separately. We then show how some innovation in security design can aid in the treating of the systemic component. Since credit risk permeates many different types of financial (and nonfinancial) contracts, we limit our attention to a financial institution that has a pool of contracts exposed to a credit risk. To be more concrete, we refer to a bank with a pool of outstanding loans, although our main ideas are easily generalized to other settings. Our model is rather stylized in order that we may focus on the credit-risk component. Thus, for example, we ignore several important components of overall risk, such as market risk. Also, we focus on the probability of default, with less attention given to the level of partial payback under bankruptcy.

The systemic component of default risk is easy to quantify ex post: it basically consists of the difference between credit defaults in a given period and the statistical long-run average of these defaults. We show how this decomposition allows one to optimally manage the two risk components. In particular, we show how financial contracts might be redesigned to allow for a more efficient risk sharing between borrowers, lenders and the capital markets. Although, to the best of our knowledge, derivative contracts written exclusively on the systemic component of the credit risk do not currently exist, we show how existing methods can allow for the same result in risk sharing between the bank and the capital markets. We also introduce a type of floating-rate interest, in which the rate is set in arrears and is based on a composite index for systemic risk. This allows borrowers to retain a part of the systemic risk generated by their own loans. Our paper must be viewed as largely normative. Such floating-rate products do not currently exist in the form we suggest. However, they are essentially a type of structured note, so that our proposed contracts should be viewed more as "evolutionary" than "revolutionary."

[4] A good overview of the extensive literature in this area is provided by De Bandt and Hartmann (2000).

A method for isolating the systemic component of credit risk is introduced in the next section. Section 2 then develops a model of bank loans with three types of economic agents: firms, banks and capital-market investors. The banks have an intermediation role by contracting with firms on one side and with the capital market on the other side. Thus section also describes our structured loans. The welfare effects of these loans are analyzed in section 3, while section 4 derives the optimal retention of systemic risk by banks when firms demand such loans. The fifth section concludes by stressing the main features of our model and pointing out directions for future research along the line introduced in this paper.

1. RISK DECOMPOSITION

We consider a pool of bank loans subject to the risk of default. "Credit risk" in our model is essentially the probability that a loan defaults. Bank loans are assumed to be for one year, at which time principal is repaid and interest payments are made, assuming the borrower is solvent. All loans in the pool are assumed to receive the same default-risk classification from the bank. Such risk classification is done internally by the bank, although the bank may certainly access external information. For example, Carey and Hrycay (2001) recently examined methods of estimating one-year default probabilities using time series data, but point to problems with instability in the annual probabilities over time.

This instability is evidenced, for example, by observing a time series of default rates. As an illustration, we present one-year default rates on all corporate bonds that are rated by Moody's Investment Service in Table 1. As is easily observable, these rates are hardly stable from year to year.

Table 1

YEAR	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
DEFAULTS	3.28%	1.33%	0.96%	0.57%	1.06%	0.53%	0.67%	1.26%	2.20%	2.28%

Default rates on all corporate bonds rated by Moody's. Source: Hamilton (2001)

While this "instability" might be a problem in some empirical models, it turns out to be a key ingredient in our attempt to decompose the credit risk into idiosyncratic and systemic components. Moreover, by assuming that the pooled loans in our model are all in the same risk classification we mean more than just having the same long-run average one-year probability of default, as we explain below. The risk classification used by the bank is transparent in that we assume away any sources of asymmetric information. Borrower characteristics are observable by the bank and bank-rating production is observable by borrowers. We do, however, assume that the bank has a superior skill in producing the default-risk classifications. We do not mean to downplay the roles of moral hazard and adverse selection. However, by ignoring them here we can better focus on the risk decomposition and on risk-sharing efficiency.

The pool is assumed to consist of N loans. We will refer to the borrowers as "firms." Each loan has a long-run average one-year probability of default of p. However, in any given year, we might see a higher or lower relative frequency of defaults. For example, suppose the long-run probability of default in one year is p=0.01. Thus one percent of the pool's loans would be "expected" to default in a given year. In order to circumvent sampling error, we assume that N is large enough for the mean number of defaults to be realized with certainty.[5] However, suppose that we find 1.2 percent of the loans in default at year's end. Since we have assumed away any sampling errors, this would imply that the bank experienced 20 percent more defaults than "expected." Likewise, if 0.8 percent of the loans are in default at year's end, the bank experienced 20 percent fewer defaults than "expected."

To model this phenomenon, let $p(1+\tilde{\varepsilon})$ denote the relative frequency of defaults, where $\tilde{\varepsilon}$ is a random variable with support contained in the interval $[-1, \frac{1-p}{p}]$. In our illustration above, $\tilde{\varepsilon}$ took on realized values of $\varepsilon=0.2$ and $\varepsilon=-0.2$ respectively. For any realized value of $\tilde{\varepsilon}$, $p(1+\varepsilon)$ represents the relative frequency of defaults that has occurred. Taking some liberty with the terminology, we might say that $p(1+\varepsilon)$ represents the *ex-post probability* of default. If we randomly drew the name of a firm with an outstanding loan during the past year, the probability is $p(1+\varepsilon)$ that the loan was not fully repaid.

Using the examples above, note that the term "expected" appears in quotes. Letting E denote the expectation operator, if the mean value of $\tilde{\varepsilon}$ is zero, $E\tilde{\varepsilon} = 0$, then indeed the expected frequency of default is $E\,p(1+\tilde{\varepsilon}) = p$. However, this frequency is "expected" only in the

[5] In other words, the mean is realized almost surely. This is no more than a limiting case for the law of large numbers.

statistical sense. Indeed, we might think of a scenario in which only 0.1 percent of loans default in 4 out of 5 years, but 4.6 percent default in 1 out of 5 years. Setting p=0.01 and letting ε = -0.9 with an 80 percent probability and ε =3.6 with a 20 percent probability captures the above scenario. Thus, in a "typical" year, one might predict 0.1 percent of loan defaults, since .001 is both the median and modal level of relative frequency, even though the "expected" level of defaults is 1 percent.

Obviously, if we do not wish to require $E\tilde{\varepsilon}=0$, indefinitely many decompositions of the relative frequency of defaults are possible. For mathematical ease, we assume that $E\tilde{\varepsilon}=0$.

For loans that are in the same default-risk classification, we assume that the random loss probability, $p(1+\tilde{\varepsilon})$, is identical. It is important to emphasize that the random variable $\tilde{\varepsilon}$ is identical for every firm, not just identically distributed. The same realization of $\tilde{\varepsilon}$ affects all loans in this risk class. One might think of all the pool's loans as not just having the same p, but also being subject to the same type of macro shocks. Since we are assuming that N is large, the bank will experience a relative frequency of $p(1+\varepsilon)$ defaults. With respect to the annual risk of default, the random variable $\tilde{\varepsilon}$ is capturing what is often referred to as *parameter risk.* In our setting, there is assumed to be no parameter risk over the long run, but the instability of annual default frequencies from year to year creates parameter risk within a single year. [6]

Define $\tilde{\theta}(\varepsilon)$ as a Bernoulli random variable, where $\theta=1$ with probability $p(1+\varepsilon)$ and $\theta=0$ otherwise. Letting $\tilde{\theta}_i(\varepsilon)$ denote a value of θ for firm *i*, we interpret $\theta_i=1$ as indicating the loan of firm *i* is in default, *i*=1,…,N. We assume that $\tilde{\theta}_i(\varepsilon)$ is statistically independent from $\tilde{\theta}_j(\varepsilon)$ for all ε and for all $i \neq j$. Thus, given information on the relative defaults in a year, ε, defaults are independent among borrowers.

It is crucial to note that this does *not* imply that defaults are independent. If we define $\tilde{\theta}_i$ the same as $\tilde{\theta}_i(\varepsilon)$ but with $\tilde{\varepsilon}$ random, $\tilde{\theta}_i$ is a default indicator for firm *i* that is not conditional on the particular realization of ε. Note that $\tilde{\theta}_i$ and $\tilde{\theta}_j$ will be equal in distribution for all *i* and j, but note also that $\tilde{\theta}_i$ and $\tilde{\theta}_j$ are not independent. Observing one of these θ values gives us updated information on $\tilde{\varepsilon}$ in a Bayesian sense. This is easiest to illustrate with a stylized

[6] In the economic literature on behavior under uncertainty, this parameter risk is often labeled as "ambiguity." In a recent paper, Carey and Hrycay (2001) model a somewhat similar structure for the bank, but with $\tilde{\varepsilon}$ representing a distribution of classification errors among borrowers. In our model, the same realization of ε is applied to all loans.

example. Let p=0.5 and let $\varepsilon = \pm 1$, each with a 50 percent chance of occurrence. In this case, either all loans default or no loans default, with an equally likely chance of each. Thus, although we assume that the $\tilde{\theta}_i(\varepsilon)$ are mutually independent, the unconditional $\tilde{\theta}_i$ are not independent.

One key ingredient in our stylized model is that the $\tilde{\varepsilon}$ risk is identical for all loans in the pool. If we set up the model where $\tilde{\varepsilon}_i$ denotes the parameter uncertainty in the default probability for the ith loan, our assumption implies that $\tilde{\varepsilon}_i$ and $\tilde{\varepsilon}_j$ are perfectly correlated for all *i* and j. However, the default indicators $\tilde{\theta}_i$ and $\tilde{\theta}_j$, though positively correlated, may be only slightly so. A high correlation between defaults is not driving our model. Instead, it is the ability to separate out this systemic component $\tilde{\varepsilon}$. If the $\tilde{\varepsilon}_i$ are not perfectly correlated, but still highly correlated, our decomposition is still possible by using an aggregate or average $\tilde{\varepsilon}_i$ as an index. In this case, the same methods for hedging the $\tilde{\varepsilon}$ risk can be employed, but they would entail some basis risk if hedging is done by the firms themselves, for their individual $\tilde{\varepsilon}$ risk. It is only the aggregate or average value of ε that is directly observable in either case, and not the individual values of ε_i.[7]

2. A MODEL OF BANK LOANS

Firms are assumed to be identical with each firm assumed to have a one-year loan for a principal amount of F dollars. At the end of the year, firm *i* pays back F plus interest, if solvent. If firm *i* is insolvent, it pays back $(1-\gamma)F$, where we will assume that $0<\gamma\leq 1$.[8] We assume here that γ is constant and is the same for all firms.[9]

We allow the risk-free rate r_f to denote the bank's opportunity cost of funds. The bank's random profit from its pool of loans can be written as

[7] Modeling the correlation itself is no easy task; see for example Zhou (2001). For the bank, obtaining and processing the information to bundle credit risks is part of its "core business," for which it receives fair compensation.

[8] We could allow $\gamma \leq 0$. This would allow for a default in which principal is repaid in full, but not the interest. However, we wish to assume that small changes in the interest rate do not determine the insolvency of the company and, hence, we assume γ is positive. Moreover, such a minor technical default is not likely to lead to liquidation in the real world. For example, during the year 2000, senior unsecured bank loans in the US had a mean recovery rate, (1-γ), of $0.49 per dollar of loan value (Hamilton, 2001).

[9] If each firm has a different loan amount F_i, but with the default probability independent of the size of the loan, our assumption of a large N would allow us to use the mean value of the F_i in the ensuing analysis.

$$(1) \qquad \tilde{B} \equiv \sum_{i=1}^{N} F[(1-\tilde{\theta}_i)r_B-\tilde{\theta}_i\gamma-r_f] =_{a.s.} NF[(1-p)r_B-p\gamma-r_f-p\tilde{\varepsilon}(\gamma+r_B)].$$

Here r_B denotes the interest rate charged on the loan.

The large number of loans N is the key to eliminating all risk except the $\tilde{\varepsilon}$-risk in equation (1). The amount $NF[(1-p)r_B-p\gamma-r_f]$ represents the long-run average profit to the bank. If $\tilde{\varepsilon}$ were identically zero, so that default levels were stable from year to year, this profit would be realized almost surely.[10] However, the instability from year to year is precisely captured by the random quantity $NF[p\tilde{\varepsilon}(\gamma+r_B)]$. This amount is subtracted from the long-run average to adjust for the systemic component of the default risk. Note that for each dollar of loan value in default, the quantity $\gamma+r_B$ represents the revenue loss to the bank.

Since $\tilde{\varepsilon}$ is identical for each loan in the pool, it cannot be diversified away within the pool of N loans. We also assume that $\tilde{\varepsilon}$ cannot be effectively diversified within the bank via other pools of loans or via other banking activities. On the other hand, we assume that the distribution of $\tilde{\varepsilon}$ is observable, as is its realization. Thus, it is possible to write contracts based upon $\tilde{\varepsilon}$.

There are three relevant types of economic agents in our model: Banks (lenders), Firms (borrowers), and the Capital Markets (outside investors). Our focus in this paper is on the various types of contracts between these parties and their degree of efficiency with respect to risk sharing.

2.1. CONTRACTS BETWEEN BANKS AND THE CAPITAL MARKETS

Although the $\tilde{\varepsilon}$ risk is not fully diversifiable within the bank, it can be transferred to international capital markets. The $\tilde{\varepsilon}$ risk might or might not be fully diversifiable in the capital markets. If it were diversifiable, we might expect all of the risk to be transferred at a "fair" price of zero, absent other transaction costs. More than likely however, the $\tilde{\varepsilon}$ risk is not fully

[10] The "almost surely" (i.e. "with probability one") technicality is cumbersome, but we use it above to stress that we are assuming away any sampling risk, in order to focus on the systemic component. Hereafter, we will dispense with the "almost surely" notation.

diversifiable, but can be partly diversified in the capital markets. In this case, we would expect the market to exact a "price" for this risk, say $\lambda>0$. This would require swapping one dollar of $\tilde{\varepsilon}$ risk for the fixed value λ. That is, the bank could swap the systemic component of credit risk on one dollar's worth of loan, $-p\tilde{\varepsilon}(\gamma+r_B)$ in equation (1), for the fixed amount $-p\lambda(\gamma+r_B)$. This could be accomplished, for example, via a futures contract based on an ε index, where λ corresponds to a futures price.

Define $\tilde{R}$ as the bracketed term on the right-hand-side of equation (1):

$$\text{(2)} \qquad \tilde{R} \equiv (1-p(1+\tilde{\varepsilon}))r_B - p(1+\tilde{\varepsilon})\gamma - r_f.$$

Thus, $\tilde{R}$ is the excess return on the loan, which depends upon ε. In other words, using equations (1) and (2), it follows that $\tilde{B}=NF\tilde{R}$.

Here we assume that $\tilde{\varepsilon}$ can be exchanged for λ via either direct securitization of the $\tilde{\varepsilon}$ risk, or via derivative contracts with cash payouts contingent on an $\tilde{\varepsilon}$ index. We will refer to these procedures as hedging the $\tilde{\varepsilon}$ risk. Define

$$\text{(3)} \qquad \Delta \equiv (1-p(1+\lambda))r_B - p(1+\lambda)\gamma - r_f.$$

Thus, Δ is the "quasi-fixed" profit of the bank for each dollar of hedged loan value.[11]

By hedging a fraction of the loans in the pool, the bank can retain its own optimal share of the $\tilde{\varepsilon}$ risk. In particular, the bank can choose a level β^* to optimize the return on its hedged loan pool:

$$\text{(4)} \qquad \tilde{B}_H \equiv NF[\beta\Delta + (1-\beta)\tilde{R}].$$

In general, if $E\tilde{R}>\Delta$, it will follow that $\beta^*<1$; the bank always retains some of the $\tilde{\varepsilon}$ risk.[12]

[11] Note that Δ is a constant. This reflects our assumption of a large N. More realistically, even without $\tilde{\varepsilon}$ risk there would be some risk due to sampling error.
[12] This assumes "as if risk averse" behavior on the part of the bank, due to various types of contracting costs. See, e.g., Greenwald and Stiglitz (1990). We also acknowledge the possibility of writing option-like contracts on the

As an alternative to directly hedging the $\tilde{\varepsilon}$ risk, the bank can securitize the loans itself and then sell the loans in the capital markets, as is a common practice with home mortgages. Since the securitized loans include $\tilde{\varepsilon}$ risk, the bank can thus obtain its desired transfer/retention of aggregate $\tilde{\varepsilon}$ risk by securitizing a subset of its loan pool. Let δ denote the end-of-period future value of the bank's fee for passing through one dollar in loan value. By securitizing only a fraction β of its loan pool, the bank's return on its original pool of loans is

(5) $\tilde{B}_S \equiv NF[\beta\delta + (1-\beta)\tilde{R}]$.

Barring transactions costs, we expect $\delta = \Delta$. If $\Delta < \delta$, we would not expect any demand for the securitized loans in the capital market. If $\Delta > \delta$, there would be no supply of securitized loans by the bank, choosing instead to directly hedge the $\tilde{\varepsilon}$ risk. Of course, real-world transactions cost might lead to one method of mitigating the $\tilde{\varepsilon}$ risk being more preferred; or if the costs are nonlinear, perhaps a combination of techniques is more efficient. For our purpose in this paper, we maintain the stylized assumption of no transaction costs. Hence we will take Δ and δ as equal.

A more problematic issue is whether or not δ >0. If δ =0, banks as intermediaries are unnecessary and hence provide no value added that needs compensation. One basic argument is that banks add value via monitoring activity (Diamond 1984). Since we assume no asymmetry of information in our model, post-contractual monitoring is unnecessary. However, banks also can add value by initial screening of loan applicants and sorting them into appropriate risk classifications (see Rajan 1992). Absent hidden information by the bank, this information might be better acquired and analyzed via the bank. It is thus the bank's superior ability to package loans of similar risk in a transparent manner that is rewarded in the marketplace. Indeed, recent work has focused on internal credit rating as a "core business" of a bank (Brunner, et al.2000).

Since the value of financial intermediation is not the main focus of our paper, we simply note here that our model does not crucially depend on having δ >0. We do make this

---

$\tilde{\varepsilon}$ risk. We choose to ignore them here for simplicity, since our aim is simply to show that $\tilde{\varepsilon}$ -based contracts can improve risk-sharing efficiency. Also, $\beta^*<1$ follows in most common decision models, such as under expected utility. But even with $E\tilde{R} > \Delta$, we could have $\beta^*=1$ if risk aversion is of order 1. See Segal and Spivak (1990).

assumption, however, for the sake of exposition. We also assume that the banking market is competitive enough so that δ would be the same for all banks.[13]

2.2. CONTRACTS BETWEEN BANKS AND FIRMS

As a base case, consider first a case where $\tilde{\varepsilon}$ =0 with certainty. Given our assumption about a large N, bank profit is essentially nonrandom in this case. In particular, define

$$(6) \qquad B_0 \equiv NF[(1-p)r_0 - p\gamma - r_f],$$

where r_0 is the interest rate on the loan. Since there is (essentially) no risk bearing by the bank on the pool of loans, the bank will require a return of δ per dollar of loan value.

From (6) we can determine the interest rate charged on loans in this pool, namely[14]

$$(7) \qquad r_0 = \frac{r_f + \delta + p\gamma}{1-p} = (r_f + \delta) + \frac{p}{1-p}(r_f + \delta + \gamma).$$

If there were no possibility of default, the bank would simply charge $(r_f + \delta)$.[15] The term $\frac{p}{1-p}(r_f + \delta + \gamma)$ represents a type of fair insurance premium for the bank against default. Since a fraction (1-p) of the loans are performing, these loans are charged this insurance premium directly on an ex-post basis. Thus, the bank receives on average $p(r_f + \delta + \gamma)$ in excess of $(r_f + \delta)$ per dollar of loan value. This is exactly equal to the average loss per dollar of loan value due to default.

To see the insurance analogy more precisely, consider the following scenario. Suppose the bank charged an interest rate of $r_f + \delta$ for all loans, and required borrowers to post a bond (i.e. insurance) acquired via a third-party insurer. Let the net indemnification on the bond be the

---

[13] If bank information were imperfect, such an assumption might be problematic, as pointed out by Broecker (1990).

[14] Bierman and Hass (1975) derive this result for the case where $\delta = 1 - \gamma = 0$.

[15] We realize that δ could possibly vary with changes in p or in $\tilde{\varepsilon}$, but use this case only as a base case for illustration only.

shortfall, $r_f+\delta+\gamma$, in case of default. The gross indemnity is $\frac{1}{1-p}(r_f+\delta+\gamma)$ for a fair premium of $\frac{p}{1-p}(r_f+\delta+\gamma)$ per dollar of loan value. The payout per dollar of loan value for the insurer and for the bank is as follows:

State of Nature	**Insurer**	**Bank**
Firm solvent	$\frac{p}{1-p}(r_f+\delta+\gamma)$	$r_f+\delta$
Firm insolvent	$\frac{p}{1-p}(r_f+\delta+\gamma)-\frac{1}{1-p}(r_f+\delta+\gamma)$	$r_f+\delta$

In other words, the interest rate in (7) acts exactly "as if" the bank required the bond above. The only difference in (7) is that the bank itself also provides the insurance.

Now consider the above situation, but where the $\tilde{\varepsilon}$ risk is nondegenerate. Absent any hedging, the bank's profit is random as given in equation (1). Taking expectations in (1) we obtain

(8) $E\tilde{B}=NF[(1-p)r_B-p\gamma-r_f]$.

Let $k\geq 0$ denote the market compensation per dollar of loan value for bearing the embedded $\tilde{\varepsilon}$ risk. We expect the market compensation for this risk to be embedded in the loan rate r_B. In other words, the expected per dollar profit in (8) should equal $\delta+k$. Solving for r_B from (8), we obtain

(9) $r_B=\dfrac{(r_f+\delta+k)+p\gamma}{1-p}$

(9a) $\quad = r_0+\dfrac{k}{1-p}$

(9b) $\quad = (r_f+\delta+k)+\dfrac{p}{1-p}(r_f+\delta+k+\gamma)$.[16]

[16] Since $p\lambda(\gamma+r_B)$ replaces $p\tilde{\varepsilon}(\gamma+r_B)$ for each dollar of loan value, it is straightforward to solve for k in terms of r_0 and λ, namely

From (9a) we see that r_B equals r_0 plus a risk premium for bearing the $\tilde{\varepsilon}$ risk. Alternatively, we can interpret r_B in (9b) as guaranteeing a return of $r_f+\delta+k$ by charging a "fair insurance premium" of $\frac{p}{1-p}(r_f+\delta+k+\gamma)$. Important to note here, though, is that the insurance is "fair" only in an ex ante sense. Invoking the Law of Large Numbers to eliminate sampling error and assuming that $\varepsilon=0$, the "insurance" would earn a zero profit. However, when $\varepsilon\neq 0$ the bank profit in (1) will depend on the realized value of $\tilde{\varepsilon}$. In other words, with r_B as given in (9) the bank has an expected profit of $\delta+k$ per dollar of loan value, but the actual profit will be higher or lower as $\varepsilon \gtrless 0$. Thus, the "insurance" does not mitigate the systemic risk faced by the bank, which is inherent in the loan pool. The bank hedges this risk to the degree it desires as discussed in the previous section.

Since $k>0$, we see from (7) and (9) that $r_B>r_0$.[17] Thus, there exists some real number $\xi>0$ such that

$$(10)\qquad r_B=\frac{r_f+\delta+p(1+\xi)\gamma}{1-p(1+\xi)}=(r_f+\delta)+\frac{p(1+\xi)}{1-p(1+\xi)}(r_f+\delta+\gamma).$$

Assuming that market prices, such as k, are in equilibrium, the quantity $p(1+\xi)$ represents the so called risk-neutral probability of default.[18] That is, ξ represents an adjustment to a martingale probability measure. Replacing p with $p(1+\xi)$ in the expectation (8), we obtain the risk-neutral expectation of bank profit

$$(11)\qquad \hat{E}\,\tilde{B} \equiv \mathrm{NF}\{[1-p(1+\xi)]r_B-p(1+\xi)\gamma-r_f\}=\mathrm{NF}\delta,$$

where the last equality follows from (10).

$$k=\frac{p(1-p)}{1-(1+\lambda)p}(\gamma+r_0).$$

[17] Once again we wish to state the caveat that δ would not necessarily be the same with a degenerate ε. Our discussion of r_0 is only as a "base case" point of reference.

[18] Thinking of our simple model as partitioning states for each loan into default states and no-default state, $p(1+\xi)$ is simply the "state price" for receiving $1 contingent on the loan's default. The idea here is similar to that used by Litterman and Iben (1991) and Jarrow and Turnbull (1995) in an intertemporal model without ε risk.

2.3. STRUCTURED LOANS

The 1990's saw an increase in the types of debt instruments offered, whose payments were tied to some observable market phenomena. Such instruments generally fall under the rubric of "structured notes." In this section, we consider the possibility of setting up the loan from the bank to a particular firm as a type of structured note. In particular, we view a particular loan as a structured note issued by the firm and purchased by the bank. Here the loan's interest rate is tied to the observed value of ε at the end of the period. By allowing the interest rate to reflect the "ε index," we allow the firm to retain the $\tilde{\varepsilon}$ risk inherent in its own loan.

Let $r(\tilde{\varepsilon})$ represent a floating interest rate on the loan. The applicable value of $\tilde{\varepsilon}$ is the end-of-period realized value of ε.[19] If the firm retains all of the $\tilde{\varepsilon}$ risk, the bank still needs to evaluate the loan to categorize it properly with regards to the (long-run) probability of default p and the parameter risk $\tilde{\varepsilon}$. Thus, the bank would still require compensation of δ for processing this information. Bank profits, if all loans are floating-rate loans, is essentially nonrandom. In this case, we can replace (1) with the following, for all values of ε:

$$(12) \qquad B = NF\left\{[1-p(1+\varepsilon)]r(\varepsilon) - p(1+\varepsilon)\gamma - r_f\right\} = NF\delta.$$

From (12) we obtain the structured floating rate as follows.[20]

$$(13) \qquad r(\varepsilon) = \frac{r_f + \delta + p(1+\varepsilon)\gamma}{1-p(1+\varepsilon)} = (r_f + \delta) + \frac{p(1+\varepsilon)}{1-p(1+\varepsilon)}(r_f + \delta + \gamma)$$

Note that $r(0)=r_0$ and that $r(\varepsilon) \gtrless r_0$ as $\varepsilon \gtrless 0$. The rate can once again be interpreted as

extracting $r_f + \delta$ from all loans together with an "insurance." The only diversion here is that the "insurance premium" and the "indemnity" are both contingent on the realized value of $\tilde{\varepsilon}$.[21]

---

[19] Most floating rate loans are dependent upon market interest rates as realized at the beginning of the period. The floating rate considered here is based on the pool-related ε value, with the rate determined in arrears.
[20] We assume here that the level of interest payment $r(\varepsilon)$ does not itself induce default when ε is high.
[21] Real world insurance contracts with a premium determined in arrears do exist in the form of so-called "participating contracts."

From (13), it is easy to verify that r(ε) is increasing and convex in ε. This is illustrated in Figure 1. In Figure 1, we know that Er($\tilde{\varepsilon}$)>r_0 by Jensen's inequality. If ε=-1, there are no defaults and r(ε)=$r_f+\delta$.

<Insert Figure 1 about here>

What cannot be determined for the general case is whether or not Er($\tilde{\varepsilon}$) $\gtrless$ r_B. The comparison of Er($\tilde{\varepsilon}$) and r_B can be further illustrated by noting that $E\left[\frac{p(1+\tilde{\varepsilon})}{1-p(1+\tilde{\varepsilon})}\right] > \frac{p}{1-p}$ due to the convexity of r(ε) in ε. Thus, there exists a real number η>0 such that [22]

(14) $$E\left[\frac{p(1+\tilde{\varepsilon})}{1-p(1+\tilde{\varepsilon})}\right] = \frac{p(1+\eta)}{1-p(1+\eta)}.$$

Hence

(15) $$Er(\tilde{\varepsilon})=(r_f+\delta)+\frac{p(1+\eta)}{1-p(1+\eta)}(r_f+\delta+\gamma).$$

Thus, comparing (15) with (10), we see that Er($\tilde{\varepsilon}$) $\gtrless$ r_B as $\eta \gtrless \xi$.

When $\eta<\xi$, a firm can lower its average borrowing cost by retaining a fraction of the $\tilde{\varepsilon}$ risk. It accomplishes this by arranging with the bank for some of its debt to be fixed-rate debt and some of its debt to be floating-rate debt. Alternatively, it could arrange for equivalent payment streams via third parties. For example, the firm could take out a fixed-rate loan with the bank, and then arrange for a structured swap in which the firm pays r(ε) in arrears in exchange for receiving the fixed rate r_B.

What remains to be determined, however, is whether or not such structured loans would be demanded at all by borrowing firms and/or supplied by the bank.

3. WELFARE EFFECTS OF STRUCTURED LOANS

Consider first the payoff to the firm. Our goal here is to provide as simple a model as possible to capture the firm's situation. Thus, suppose for now that the firm has a constant amount of earnings before interest payments, in the event of solvency. If the firm is in default, it earns (1-γ)F, but this amount is paid to the bank as a partial recovery. The firm's share of earnings is zero when it defaults. Allowing the firm to chose some fraction α of its debt to be floating-rate debt, with the rest being fixed-rate debt, we can write firm *i*'s net earnings (after paying interest or bankruptcy claims) as a random variable:

$$(16) \qquad \tilde{\Pi}(\alpha) \equiv [1-\tilde{\theta}_i]\,[I - FR(\alpha,\tilde{\varepsilon})],$$

where $R(\alpha,\varepsilon) \equiv \alpha(1+r(\varepsilon)) + (1-\alpha)(1+r_B)$ and where we assume that $I - FR(\alpha,\varepsilon) > 0 \quad \forall\varepsilon.$[23]

We wish to know whether or not the optimal level of α by the firm is strictly positive; that is, whether the firm desires any floating-rate debt in its optimal loan structure. Recall that we are assuming that firms behave "as if" they are risk averse. Assuming second-order risk aversion, the optimal α will be positive if and only if $E\tilde{\Pi}(1) > E\tilde{\Pi}(0)$. That is, the firm will always choose to finance part of its debt via a floating rate if and only if expected profit is higher using 100 percent floating-rate debt than using 100 percent fixed-rate debt.[24]

It follows from (16) that $E\tilde{\Pi}(1) > E\tilde{\Pi}(0)$ whenever the following inequality holds:

$$(17) \qquad E[(1-p(1+\tilde{\varepsilon}))(r_B - r(\tilde{\varepsilon})] > 0.$$

Or equivalently,

$$(18) \qquad (1-p)(r_B+\gamma)] - E[(1-p(1+\tilde{\varepsilon}))(r(\tilde{\varepsilon})+\gamma)] > 0\,.$$

[22] Note, as a caveat, that η does *not* induce a martingale measure.
[23] Recall that we assume the interest payment itself is not a cause of firm default.
[24] This conclusion follows under any type of preference satisfying second-order risk aversion, such as in the standard expected-utility model (See Segal and Spivak, 1990).

Rearranging equations (10) and (13) we obtain

$$(19) \qquad [1-p(1+\xi)(r_B+\gamma)=[1-p(1+\varepsilon)](r(\varepsilon)+\gamma)=r_f+\delta+\gamma \qquad \forall\varepsilon\,.$$

Substituting, (19) into (18), we see that the inequality in (17) is equivalent to $p\xi(r_B+\gamma)>0$, which holds under risk aversion. Thus, the optimal α for the firm is strictly positive: the firm demands a positive level of floating-rate debt.

Now observe that $E[(1-p(1+\tilde{\varepsilon}))(r_B-r(\tilde{\varepsilon})]=(1-p)[r_B-Er(\tilde{\varepsilon})]+p\operatorname{cov}(\tilde{\varepsilon},r(\tilde{\varepsilon}))$. Since the covariance term is positive by the definition of r(ε), we see from the inequality in (17) that floating-rate debt will always be used whenever $r_B>Er(\tilde{\varepsilon})$. Moreover, and perhaps somewhat surprisingly at first, is that some floating-rate debt also might be optimal, even if r_B is slightly below $Er(\tilde{\varepsilon})$.

While this latter result seems a bit counter-intuitive, the reasoning becomes clear when one considers the probability of default $p(1+\tilde{\varepsilon})$. When ε is high, so is r(ε); but the probability of being in default, p(ε), also is higher. Thus, the odds of paying r(ε) are higher when ε is low. Since $Er(\tilde{\varepsilon})$ does not capture the probability of solvency and hence of paying the interest, it is not all too surprising that we might opt for some floating-rate debt, even if $Er(\tilde{\varepsilon})$ is slightly above r_B.

The above analysis assumes that the firm's earnings, contingent on solvency, are independent of the probability of solvency. Suppose more realistically that the level of earnings in the event of solvency is lower whenever ε is higher. In other words, suppose that a higher value of ε not only implies a higher chance of default, but also implies lower profit to the solvent firm. We capture this in the simplest manner, by making the pre-interest earnings, *I* in equation (16), a monotonic decreasing function of ε. Thus, (16) becomes

$$(20) \qquad \tilde{\Pi}(\alpha)\equiv[1-\tilde{\theta}_i]\,[I(\tilde{\varepsilon})-FR(\alpha,\tilde{\varepsilon})],$$

where *I*(ε) is a decreasing function. In this case, reducing the level of floating-rate debt α will act as a type of insurance against the falling earnings, when the firm is solvent. Thus, the firm will have two competing effects as it considers the use of floating-rate debt. One effect is the increase in the expected payoff. As before, the use of floating rate debt will decrease the firm's expected debt costs and hence increase its expected profit. However, now the use of floating-rate debt will increase the riskiness of the firm's net earnings, contingent upon solvency. This is due to the fact that the interest rate is high exactly in those solvent states of the world for which earnings $I(\varepsilon)$ are low, while the interest rate is low for solvent states in which the earnings $I(\varepsilon)$ are high. Thus, the decrease in expected debt repayment comes at the cost of more risk than in the case where earnings $I(\varepsilon)$ are constant for all ε. As a result, we cannot say a priori whether or not the firm would desire a positive level of floating-rate debt.[25]

Although our results do not show that firms must necessarily benefit by using some floating-rate debt when earnings in the solvent states are inversely related to ε, the point is that they might. If firms do not benefit, we expect that a market for the floating rate debt will not exist. On the other hand, if firms do benefit (and hence choose a positive α), we will at least have a demand for floating-rate debt.

4. SYSTEMIC-RISK RETENTION BY THE BANK

Having just established that firm's might have a demand for floating-rate loans, will the bank have any incentive to supply them? Since we have assumed that the bank is compensated the same for passing off the ε-risk to either the capital markets or back to the firm, the bank should be indifferent between the two, ceteris paribus. Suppose we have a market in which the firms demand some floating rate debt. Assuming that the bank does not wish to retain all of the ε-risk for its own account, we might expect that competitive pressures in the marketplace would force the bank to offer such floating-rate loans. If no other banks offered such loans, one bank could offer them at a slightly higher profit rate than δ. This would attract customers and earn an excess profit for the firm. Thus, in a long-run competitive setting, we would expect all banks to offer floating-rate loans to their clients who desire them.

[25] If the "insurance effect" is particular strong, the firm might even desire a negative α as a type of hedge against earnings risk. While a negative α might not be available from the bank loan per se, the firm could conceivably achieve the same affect by assuming some of the bank's ε-risk in the capital market.

Suppose that the bank allows for firms to choose their levels of fixed- versus floating-rate loans. Assume here that the firms each take out a fraction α of floating-rate debt, with $(1-\alpha)$ being fixed rate debt, $0 \le \alpha \le 1$. As previously, let β^* denote the optimal level of $\tilde{\varepsilon}$ risk to be transferred out of the bank, as in equation (5), with the fraction $(1-\beta^*)$ retained by the bank. The only difference in the previous setting is that the bank issued only fixed-rate debt and hedged only in the capital markets.

We now have part of the $\tilde{\varepsilon}$ risk retained by the borrowers themselves. The bank can achieve the same position by simply hedging the appropriate fraction t of the $\tilde{\varepsilon}$ risk remaining in its fixed-rate loans. Bank profit is given by

(21) $$\tilde{B}_S = NF\left\{\alpha\delta + (1-\alpha)[t\delta + (1-t)\tilde{R}]\right\}$$
$$= NF\{\beta\delta + (1-\beta)\tilde{R}\},$$

where $\beta = t(1-\alpha)+\alpha$.

If $\beta^* \ge \alpha$, the bank simply hedges a fraction t of the fixed-rate loans. If $\beta^* < \alpha$, the bank can only achieve β^* by setting $t<0$. In this case, several scenarios are possible. The bank might simply accept $t^*=0$ as a constrained optimum. The bank alternatively could impose a limit on α such that $\alpha \le \beta^*$. Obviously these remedies are sub-optimal without any further action.

Alternatively, the bank could effectively reach a desired level $t^*<0$ either by “buying” some of the securitized $\tilde{\varepsilon}$ risk of other banks or by arranging a float-for-fixed swap, where the “float” is based on the $\tilde{\varepsilon}$ index in arrears. The main point here is that there is no particularly severe impediment preventing the bank from achieving its own desired level of risk retention.

5. CONCLUDING REMARKS

We examined a pool of bank loans exposed to credit risk and showed how a particular risk decomposition, together with contracts structured on that decomposition, can increase risk-sharing efficiency in the marketplace. Although our model is couched in a setting of bank loans, the methodology employed is more general and would seem to be applicable to any pool of contracts in which promised payments are contingent partly on some type of macroeconomic risk.

Our results are largely normative. We use our risk decomposition to innovate on contract design. However, the types of contracts we consider certainly seem feasible, and would easily fit into the rubric of "structured notes," which became popular in financial markets during the 1990's.

The fact that ε can be "observed," if we know the expected frequency of defaults, would make our proposed contracts attainable. Of course the contracts examined here only consider proportional risk sharing. It would be easy to design other types of structured loans, for example loans with option-like features. For instance, we could put a cap on the firm's floating rate. However, our purpose here is mainly to point out how the risk decomposition allows for a new way to "carve up" the risk and, hence, share the risk. We make no claim that the types of instruments we propose are "optimal;" only that they improve risk-sharing efficiency.

The decomposition of the risk into idiosyncratic and systematic components is crucial to our analysis. We assumed no sampling error as well as a perfectly correlated systemic component, $\tilde{\varepsilon}$. More realistically, we might expect the $\tilde{\varepsilon}$ risk correlations to be less than perfect. Both the sampling error and the absence of a perfect correlation would induce a basis risk into our model. But this should hopefully be small if N is large (not too much sampling error) and if correlations are very high for the $\tilde{\varepsilon}$ components.

The same type of rationale also would apply to the bank's ability to correctly classify the credit risk. Misclassification error also could be a source of basis risk (see, for example, Carey and Hrycay 2000). In other words, although we model a perfect $\tilde{\varepsilon}$ -index for hedging purposes, even an imperfect index is capable of improving risk-sharing contracts. Note also that our decomposition does not require a high degree of overall correlation between defaults. If the distribution of $\tilde{\varepsilon}$ has a narrow support, overall default correlations may be only slightly positive. What is key is that the $\tilde{\varepsilon}$ component itself is somehow "observable" and is easy to separate out. We only require that ε component is highly correlated between firms, which might or might not show up as a high overall correlation between defaults.

If one believes the set-up in our model, grouping a large number of firms with identical ε-risk, we only need to observe total defaults to calculate ε. This avoids the complexities of modeling and measuring the correlations among the defaults themselves. See, for example, Zhou

(2001). Of course, it may be difficult to determine firms with identical, or nearly identical, ε-risk.

To isolate the credit risk in our model, we made several simplifications. For example, we ignored market risk, which is itself an important component of the overall risk faced by the economic agents in our model. We also realize that asymmetric information can play a large role in credit markets.[26] By ignoring adverse selection and moral hazard, we bypassed many potential pitfalls in structuring our proposed contracts. For instance, real-world cost considerations might force an $\tilde{\varepsilon}$ index to be constructed using data from a set of sample banks.[27] In such a situation, it is not clear whether or not an $\tilde{\varepsilon}$ index might be manipulable by the banks.

Too, we assumed that the level of default, γ, was constant and identical among borrowers. If $\tilde{\gamma}$ is random, but independent among insolvent firms, we can write contracts similar to those exposited here; simply replace γ with the mean of $\tilde{\gamma}$. The law of large numbers should take care of the sampling error as applied to the $\tilde{\gamma}$ realizations. More problematic would be a correlation between the $\tilde{\gamma}$ risk, especially if $\tilde{\gamma}$ risk is also statistically correlated with the $\tilde{\varepsilon}$ risk. At the same time, we note that banks and regulators might be more diligent in “pulling the plug” on delinquent loans during times with numerous defaults. Thus, it is not apparent that $\tilde{\gamma}$ and $\tilde{\varepsilon}$ must have a particular correlation.

We also realize that our model has a strong reliance on the existence of a market price for the $\tilde{\varepsilon}$ risk. How this $\tilde{\varepsilon}$ risk relates to other risks in the capital markets might be less than transparent. Thus, we might experience problems associated with a “thin” market. An understanding of how $\tilde{\varepsilon}$ risk is priced in a general-equilibrium setting would be helpful.

Still, we hope the ideas in this paper provide a foundation for dealing with the problem of credit risk. How robust our analysis is when pressed with real-world limitations, or when simplifying assumptions are relaxed, is a question left for further study.

[26] A recent paper by Duffee and Zhou (2001) points out the potential for conflict between efficient risk sharing, as is the focus within this paper, and achieving second-best optima in the face of informational asymmetries.
[27] Such a problem has long been a potential source of concern for setting property-damage indices in the market for catastrophe futures at the CBOT, for example.

REFERENCES

Altman, Edward I. and Anthony Saunders (1997), "Credit Risk Measurement: Development over the last 20 years." *Journal of Banking and Finance* 21, 1721-1742.

Berrada, Tony, Rajna Gibson and Nicolas Mougeot (2001), "Systematic Credit Risk and Asset Pricing: Empirical Study on the US Stock Market." SBI Working Paper, University of Zurich.

Bierman, Harold and Jerome E. Hass (1975), "An Analytic Model of Bond Risk Differentials." *Journal of Financial and Quantitative Analysis* 10, 757-773.

British Bankers Association (2000), "Global Credit Derivative Market Set to Hit $1581 Billion by Year 2002." Website URL: www.bba.org.uk/html/1601.html, July 20, 2000.

Broecker, Thorsten (1990), "Credit-Worthiness Tests and Interbank Competition." *Econometrica* 58, 429-452.

Brunner, Antje, Jan Pieter Krahnen and Martin Weber (2000), "Information Production in Credit Relationships: On the Role of Internal Ratings in Commercial Banking." Center for Financial Studies Working Paper, University of Frankfurt.

Carey, Mark and Mark Hrycay (2001), "Parameterizing Credit Risk Models With Rating Data," *Journal of Banking and Finance* 25, 197-270.

Crouhy, Michel, Dan Galai and Robert Mark (2000), "A Comparative Analysis of Current Credit Risk Models." *Journal of Banking and Finance* 24, 59-117.

DeBandt, Olivier and Phillipp Hartmann (2000), "Systemic Risk: A Survey." European Central Bank Working Paper.

Diamond, Douglas (1984), "Financial Intermediation and Delegated Monitoring." *Review of Economic Studies* 51, 393-414.

Duffee, Gregory R. and Chunsheng Zhou (2001), "Credit Derivatives in Banking: Useful Tools for Risk Management?" *Journal of Monetary Economics* 48, 25-54.

Greenwald, Bruce C. and Joseph E. Stiglitz (1990), "Asummetric Information and the New Theory of the Firm: Financial Constraints and Risk Behavior." *American Economic Review* 80, 160-165.

Hamilton, David T. (2001), "Default and Recovery Rates of Corporate Bond Issuers: 2000." *Moody's Special Comment*, Moody's Investors Service, February 2001, New York.

Jarrow, Robert A. and Stuart M. Turnbull, "Pricing Derivatives on Financial Securities Subject to Credit Risk." *Journal of Finance* 50, 53-85.

Litterman, Robert and Thomas Iben (1991), "Corporate Bond Valuation and the Term Structure of Credit Spreads." *Journal of Portfolio Management,* Spring 1991, 52-64

Longstaff, Francis and Eduardo Schwartz (1995), "Valuing Credit Derivatives." *Journal of Fixed Income*. June 1995, 6-12.

Rajan, Raghuram G. (1992), "Insiders and Outsiders: The Choice between Relationship and Armslength Debt." *Journal of Finance* 47, 1367-1400.

Segal, Uzi and Avia Spivak (1990), "First-order versus Second-order Risk Aversion." *Journal of Economic Theory* 51, 111-125.

Shimpi, Prakesh A. (1999), *Integrating Corporate Risk Management*. Swiss Re New Markets: New York.

Zhou, Chunsheng (2001), "An Analysis of Default Correlations and Multiple Defaults." *Review of Financial Studies* 14, 555-576.

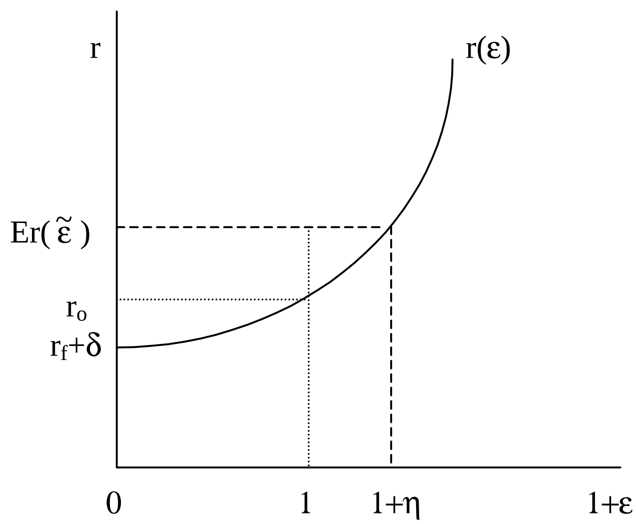

FIGURE 1