# Core-polarization effects in doubly ionized cerium (Ce III) for transitions of astrophysical interest 

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#### Abstract

The importance of core-polarization effects on oscillator strength determination in Ce III is emphasized. The inclusion of these effects in relativistic Hartree-Fock (HFR) calculations leads to theoretical lifetime values for some 4 f 6 p levels of this ion in close agreement with recent time-resolved laser-induced fluorescence measurements. These new atomic data, the most accurate to date, have allowed the refinement of the stellar abundance determination of cerium in the chemically peculiar star HD 192913, but have not altered in a drastic way the conclusions derived in recent analyses regarding the overabundance of this element.


Key words: atomic data - atomic processes - techniques: spectroscopic - stars: chemically peculiar - stars: individual: HD 192913.

## 1 INTRODUCTION

The lanthanide elements ( $Z=57-71$ ) are important in astrophysics in relation to nucleosynthesis and star formation considerations. Spectroscopic investigations of the doubly ionized elements of this group are motivated by astrophysical interest because, in the hot chemically peculiar (CP) stars, there is an overwhelming presence of lines belonging to the third spectrum which corresponds to the dominant ionization stage. The present paper focuses on the particular case of Ce III, cerium being, of all the lanthanides, the most abundant one in the Ap stars (Cowley 1976).

Cerium has 19 isotopes and isomers among which only ${ }^{136} \mathrm{Ce}$, ${ }^{138} \mathrm{Ce},{ }^{140} \mathrm{Ce}$ and ${ }^{142} \mathrm{Ce}$ are stable. In the Solar system, most of the cerium ( 88 per cent) is in the form of ${ }^{140} \mathrm{Ce}$. In stars, ${ }^{136} \mathrm{Ce}$ and ${ }^{138} \mathrm{Ce}$ are produced by the p process, ${ }^{142} \mathrm{Ce}$ by the r process and ${ }^{140} \mathrm{Ce}$ by either the r or the s process (Jaschek \& Jaschek 1995). The first evidence of the presence of Ce III lines in stellar spectra is due to Struve \& Swings (1943) who, nearly sixty years ago, were able to identify lines of this ion in the spectrum of $\alpha^{2} \mathrm{CV}$. In the meantime, doubly ionized cerium has also been identified in spectra of other CP stars such as $\beta$ CrB (Adelman, Shore \& Tiernan 1973), HD 200311 (Adelman 1974b; Aikman, Cowley \& Crosswhite 1979), $\gamma$ Equulei (Adelman 1974a; Adelman, Bidelman \& Pyper 1979), HR 465 (Aikman et al. 1979), HD 51418 (Adelman \& Shore 1981) and HD 192913 (Ryabchikova et al. 1990). The first quantitative study of Ce III was performed by Bord, Cowley \& Norquist (1997) who considered the abundance of this ion in the spectrum of HD 200311

[^0]and, marginally, in the spectrum of two additional CP stars: HR 465 and HD 192913.

Despite their astrophysical interest, the radiative properties of Ce III have been the subject of rather rare experimental and theoretical investigations. More precisely, thirty years ago, natural lifetimes were determined for six 4f6p levels by Andersen \& Sorensen (1974) using the beam-foil method. More recently, Bord et al. (1997) calculated oscillator strengths for some Ce III lines with the atomic structure code of Cowan (1981). Using the same code, the spectroscopic analysis of Ce III was extended, with the support of detailed parametric studies of mixed configurations in both parities, by Wyart \& Palmeri (1998) who also gave a list of computed oscillator strengths for some selected transitions. Recently, radiative lifetimes of nine 4 f 6 p levels in this ion were accurately measured by Li et al. (2000) with the time-resolved laser-induced fluorescence technique. Transition probabilities in Ce III were also obtained by the same authors from the combination of theoretical branching fractions and the newly measured lifetime values.

It should be emphasized, however, that there are no direct measurements of transition probabilities and, in addition, in all the theoretical investigations carried out so far, the effects arising from the polarization of the ionic core (CPOL) have not been included in the calculations although it has been shown, in many recent papers, that these effects play an important role in the complex spectra of lowly charged heavy elements such as lanthanide ions (see e.g. Biémont et al. 2000a,b). This neglect of CPOL effects is reflected by the poor agreement observed between the theoretical lifetimes obtained so far and the accurate values recently measured by laser spectroscopy (Li et al. 2000). The main purpose of the present work is to investigate CPOL effects in Ce III and, as a direct consequence,
to provide for this ion an improved set of oscillator strengths for many transitions of astrophysical interest.

## 2 ATOMIC STRUCTURE CALCULATIONS

The computational procedure used for calculating the oscillator strengths and radiative lifetimes in $\mathrm{Ce}_{\mathrm{II}}$ is the relativistic HartreeFock (HFR) method modified in order to take into account the CPOL effects. The details of this approach have been described elsewhere (Cowan 1981; Quinet et al. 1999) and will not be repeated here.

The intravalence correlation was explicitly retained among the same configuration sets as those considered by Wyart \& Palmeri (1998), i.e. $4 \mathrm{f}^{2}+5 \mathrm{~d}^{2}+4 \mathrm{f} 6 \mathrm{p}+5 \mathrm{~d} 6 \mathrm{~s}+6 \mathrm{~s}^{2}+4 \mathrm{f} 7 \mathrm{p}+4 \mathrm{f} 5 \mathrm{f}+4 \mathrm{f} 6 \mathrm{f}$ $+5 \mathrm{~d} 7 \mathrm{~s}+5 \mathrm{~d} 6 \mathrm{~d}+4 \mathrm{f} 7 \mathrm{f}+6 \mathrm{p}^{2}$ for the even parity and $4 \mathrm{f} 5 \mathrm{~d}+4 \mathrm{f} 6 \mathrm{~s}$ $+4 \mathrm{f} 7 \mathrm{~s}+4 \mathrm{f} 6 \mathrm{~d}+5 \mathrm{~d} 6 \mathrm{p}+4 \mathrm{f} 5 \mathrm{~g}+6 \mathrm{~s} 6 \mathrm{p}+4 \mathrm{f} 7 \mathrm{~d}+4 \mathrm{f} 8 \mathrm{~s}+4 \mathrm{f} 6 \mathrm{~g}$ for the odd parity. They correspond to the experimentally known configurations (Martin, Zalubas \& Hagan 1978; Wyart \& Palmeri 1998) to which we have added $6 \mathrm{~s}^{2}, 5 \mathrm{~d} 7 \mathrm{~s}, 4 \mathrm{f} 7 \mathrm{f}, 6 \mathrm{~s} 6 \mathrm{p}, 4 \mathrm{f} 8 \mathrm{~s}$ and 4 f 6 g because interactions with the latter configurations could possibly affect the eingenvector compositions and, consequently, the oscillator strengths, particularly for the weakest transitions.

The estimation of core-polarization contributions requires knowledge of the dipole polarizability of the ionic core, $\alpha_{\mathrm{d}}$, and of the cutoff radius, $r_{\mathrm{c}}$. For the first parameter, we have used the value of the static dipole polarizability computed by Fraga, Karwowski \& Saxena (1976) for Ce v , i.e. $\alpha_{\mathrm{d}}=7.08 a_{0}^{3}$, while the cut-off radius has been chosen to be equal to $1.70 a_{0}$, which corresponds to the HFR average value $\langle r\rangle$ of the outermost core orbitals ( $5 \mathrm{p}^{6}$ ). However, as already mentioned previously (see e.g. Biémont et al. 2001; Li et al. 2001), the inadequacy of the analytical polarization corrections to the dipole operator as introduced in the model (see equation 6 of Quinet et al. 1999) for transitions involving the 4 f electrons imposes the consideration of a scaling factor to the $\langle 4 \mathrm{f}| r|\mathrm{nd}\rangle$ and $\langle 4 \mathrm{f}| \mathrm{r}|\mathrm{ng}\rangle$ dipole operators in order to compensate for the sudden collapse of the 4 f orbital inside the Xe -like core. For the matrix elements appearing in the present work, ranging from 0.08 to 1.80 a.u. according to the different types of transitions, a scaling factor equal to 0.80 has been derived directly from the curve of Fig. 1 showing the ratio between core-polarization corrected and uncorrected transition matrix elements for transitions not involving a 4 f electron as a function of the uncorrected matrix element. This factor has been applied to the $\langle 4 \mathrm{f}| \mathrm{r}|\mathrm{nd}\rangle$ and $\langle 4 \mathrm{f}| \mathrm{r}|\mathrm{ng}\rangle$ operators. A similar approach has been successfully considered previously, an excellent agreement between theoretical and experimental lifetimes having been found in Ce II (Zhang et al. 2001a), Pr III (Biémont et al. 2001), Tm III (Li et al. 2001) and Yb III (Zhang et al. 2001b). It will be seen hereafter that the procedure is entirely justified considering the excellent agreement found between HFR and experimental lifetime values obtained in the present work. It must be pointed out that our results are nearly insensitive to the choice of the scaling factor: adopting 0.85 instead of 0.80 would change the nine lifetime values by less than 0.1 per cent and, as a consequence, the change of the oscillator strengths, would be entirely negligible even for the weakest transitions considered in the present work (see Table 2, later).

Bord et al. (1997) have considered a smaller set of configurations in their calculation, arguing from the fact that the energies computed from the restricted set did agree with the corresponding energies found for the whole set to better than 0.1 per cent. The neglected configurations, however, could affect in a more substantial way the $f$ values of some weak transitions, and for this reason we have preferred to include all of them in the present calculation.


Figure 1. Absolute value of the ratio between transition matrix elements corrected by core-polarization effects ( $d_{\mathrm{pol}}$ ) and uncorrected matrix elements ( $d_{\text {nopol }}$ ) of transitions not involving a 4 f electron as a function of the uncorrected data (in atomic units). A smooth curve has been drawn showing the trend.

Using a least-squares fitting procedure (Cowan 1981), the average energies, the Slater and the spin-orbit integrals together with some 'forbidden' Slater integrals and effective interaction parameters ( $\alpha$, $\beta, \gamma)$ were adjusted to obtain the best agreement between the calculated and the experimental energy levels taken from the National Institute of Standards and Technology (NIST) compilation (Martin et al. 1978) and extracted from the recent extension of the level system by Wyart \& Palmeri (1998). The eight experimental even-parity levels above $130000 \mathrm{~cm}^{-1}$ were excluded from this optimization process because they were suspected to overlap levels belonging to unknown configurations such as 5 d 7 s and 4 f 7 f , these latter two being estimated to range between 130000 and $150000 \mathrm{~cm}^{-1}$ according to our HFR predictions. For the odd parity, the levels taken from the NIST table (Martin et al. 1978), which are all located below $130000 \mathrm{~cm}^{-1}$, were included in the fitting procedure. Starting with the adjusted parameter values obtained by Wyart \& Palmeri (1998), we have been able to refine the semi-empirical procedure. Indeed, at the end of the fitting process performed in the present work, the

Table 1. Radiative lifetimes (in ns) for selected 4f6p levels of Ce iII.

| Energy <br> $\left(\mathrm{cm}^{-1}\right)$ | $\mathrm{J}^{\prime}$ | $\mathrm{Bord}^{a}$ | $\mathrm{Li}^{b}$ | $\mathrm{HFR}^{c}$ | $\mathrm{HFR}+\mathrm{CPOL}^{c}$ | Andersen ${ }^{d}$ | $\mathrm{Li}^{b}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 48267.00 | 3 | 1.45 | 1.50 | 1.47 | 1.76 | $3.3 \pm 0.5$ | $1.80 \pm 0.20$ |
| 48404.86 | 2 | 1.49 | 1.74 | 1.60 | 1.93 | $3.3 \pm 0.4$ | $2.00 \pm 0.20$ |
| 50057.60 | 4 | 5.76 | 3.60 | 3.64 | 4.51 | $3.1 \pm 0.5$ | $4.2 \pm 0.4$ |
| 50375.00 | 3 | 1.48 | 1.57 | 1.54 | 1.85 | $3.2 \pm 0.4$ | $1.90 \pm 0.20$ |
| 51262.21 | 3 | 1.25 | 1.25 | 1.25 | 1.51 | $3.7 \pm 0.5$ | $1.55 \pm 0.20$ |
| 51640.68 | 2 | 1.46 | 1.46 | 1.39 | 1.67 |  | $1.70 \pm 0.20$ |
| 52440.96 | 4 | 1.19 | 1.29 | 1.30 | 1.57 | $3.5 \pm 0.4$ | $1.60 \pm 0.20$ |
| 53615.98 | 3 | 1.51 | 1.40 | 1.39 | 1.66 |  | $1.65 \pm 0.20$ |
| 54549.34 | 4 | 1.35 | 1.34 | 1.38 | 1.69 |  | $1.60 \pm 0.20$ |

Table 2. Oscillator strengths for Ce III lines. Only transitions for which $\log g f>-1.50$ are reported in the table. The lower and upper levels of each transition are represented by their experimental values (in $\mathrm{cm}^{-1}$ ), their parities [(e) for even and (o) for odd] and their $J$-values.

| $\lambda(\mathrm{nm})$ | Lower level |  |  | Upper level |  |  | $\log g f$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | A | B | C | D |
| 104.2737 | 3277 | (o) | 4.0 |  |  |  | 99178 | (e) | 5.0 | 0.33 |  | 0.44 |  |
| 107.2791 | 8350 | (o) | 6.0 | 101565 | (e) | 7.0 | 0.62 |  | 0.72 |  |
| 114.2546 | 16152 | (o) | 5.0 | 103676 | (e) | 6.0 | 0.38 |  | 0.46 |  |
| 179.6901 | 43517 | (e) | 4.0 | 99169 | (o) | 4.0 | 0.03 |  | 0.11 |  |
| 184.8984 | 41939 | (e) | 3.0 | 96022 | (o) | 3.0 | -0.06 |  | 0.02 |  |
| 195.0356 | 3277 | (o) | 4.0 | 54549 | (e) | 4.0 | -0.92 |  | -0.90 | -0.93 |
| 198.6519 | 3277 | (o) | 4.0 | 53616 | (e) | 3.0 | -1.31 |  |  | -1.12 |
| 202.8293 | 18444 | (o) | 1.0 | 67730 | (e) | 0.0 | -0.86 |  | -0.68 |  |
| 203.3342 | 3277 | (o) | 4.0 | 52441 | (e) | 4.0 | -0.83 |  | -0.79 | -0.90 |
| 206.1676 | 5127 | (o) | 4.0 | 53616 | (e) | 3.0 | -1.30 |  |  | -1.28 |
| 208.3316 | 6571 | (o) | 2.0 | 54556 | (e) | 2.0 | -0.45 |  | -0.38 |  |
| 210.9068 | 7150 | (o) | 4.0 | 54549 | (e) | 4.0 | -0.47 |  | -0.40 | -0.45 |
| 212.2546 | 3277 | (o) | 4.0 | 50375 | (e) | 3.0 | -1.10 |  |  | -1.10 |
| 212.4970 | 6571 | (o) | 2.0 | 53616 | (e) | 3.0 | -1.38 |  |  | -1.33 |
| 213.6949 | 3277 | (o) | 4.0 | 50058 | (e) | 4.0 | -0.45 |  | -0.34 | -0.36 |
| 215.1438 | 7150 | (o) | 4.0 | 53616 | (e) | 3.0 | -0.38 |  | -0.31 | -0.33 |
| 216.6716 | 5502 | (o) | 3.0 | 51641 | (e) | 2.0 | -0.66 |  |  | -0.59 |
| 216.6875 | 5127 | (o) | 4.0 | 51262 | (e) | 3.0 | 0.04 |  | 0.11 | -0.02 |
| 216.9473 | 6361 | (o) | 5.0 | 52441 | (e) | 4.0 | -0.06 |  | 0.01 | -0.07 |
| 218.0635 | 8350 | (o) | 6.0 | 54194 | (e) | 5.0 | 0.39 |  | 0.46 |  |
| 218.3712 | 7837 | (o) | 4.0 | 53616 | (e) | 3.0 | 0.59 |  |  | -0.75 |
| 218.4639 | 5502 | (o) | 3.0 | 51262 | (e) | 3.0 | -0.79 |  |  | -0.66 |
| 220.3146 | 6265 | (o) | 3.0 | 51641 | (e) | 2.0 | $-0.55$ |  |  | -0.60 |
| 220.7260 | 7150 | (o) | 4.0 | 52441 | (e) | 4.0 | $-0.53$ |  | -0.47 | -0.44 |
| 220.9367 | 5127 | (o) | 4.0 | 50375 | (e) | 3.0 | -1.33 |  | -1.06 | -1.26 |
| 221.0534 | 9326 | (o) | 5.0 | 54549 | (e) | 4.0 | -0.77 |  |  | -0.71 |
| 221.8114 | 6571 | (o) | 2.0 | 51641 | (e) | 2.0 | $-0.50$ |  | -0.43 | -0.53 |
| 222.1679 | 6265 | (o) | 3.0 | 51262 | (e) | 3.0 | -1.13 |  |  | -1.35 |
| 222.2008 | 3277 | (o) | 4.0 | 48267 | (e) | 3.0 | -0.01 |  | 0.07 | -0.08 |
| 222.4976 | 5127 | (o) | 4.0 | 50058 | (e) | 4.0 | -0.81 |  |  | -0.92 |
| 222.5086 | 6361 | (o) | 5.0 | 51289 | (e) | 4.0 | 0.01 |  | 0.07 |  |
| 222.7837 | 5502 | (o) | 3.0 | 50375 | (e) | 3.0 | $-0.32$ |  | -0.27 | -0.37 |
| 222.8051 | 9326 | (o) | 5.0 | 54194 | (e) | 5.0 | -0.13 |  | -0.06 |  |
| 223.6900 | 6571 | (o) | 2.0 | 51262 | (e) | 3.0 | -0.97 |  |  | -1.04 |
| 224.1244 | 7837 | (o) | 4.0 | 52441 | (e) | 4.0 | -0.70 |  |  | -0.89 |
| 224.2295 | 3822 | (o) | 2.0 | 48405 | (e) | 2.0 | -0.25 |  | -0.17 | -0.31 |
| 224.9251 | 3822 | (o) | 2.0 | 48267 | (e) | 3.0 | -0.78 |  |  | -0.98 |
| 226.6915 | 19236 | (o) | 2.0 | 63335 | (e) | 1.0 | -0.40 |  | -0.25 |  |
| 228.2211 | 6571 | (o) | 2.0 | 50375 | (e) | 3.0 | -1.23 |  |  | -1.51 |
| 228.7816 | 6361 | (o) | 5.0 | 50058 | (e) | 4.0 | -1.14 |  |  | -1.16 |
| 229.8700 | 10127 | (o) | 3.0 | 53616 | (e) | 3.0 | -0.90 |  |  | -1.92 |
| 230.0647 | 7837 | (o) | 4.0 | 51289 | (e) | 4.0 | -0.53 |  | -0.46 |  |
| 230.2086 | 7837 | (o) | 4.0 | 51262 | (e) | 3.0 | -0.52 |  | -0.41 | -0.43 |
| 231.2767 | 7150 | (o) | 4.0 | 50375 | (e) | 3.0 | -1.26 |  |  | -1.72 |
| 231.7337 | 5127 | (o) | 4.0 | 48267 | (e) | 3.0 | -0.57 |  | -0.54 | -0.46 |
| 231.8642 | 9326 | (o) | 5.0 | 52441 | (e) | 4.0 | -0.09 |  | -0.02 | -0.12 |
| 232.4311 | 8922 | (o) | 1.0 | 51932 | (e) | 1.0 | -0.66 |  | -0.60 |  |
| 233.7664 | 5502 | (o) | 3.0 | 48267 | (e) | 3.0 | -0.93 |  |  | -1.12 |
| 235.0104 | 7837 | (o) | 4.0 | 50375 | (e) | 3.0 | -0.25 |  | -0.20 | -0.29 |
| 236.2538 | 10127 | (o) | 3.0 | 52441 | (e) | 4.0 | -0.85 |  |  | -0.96 |
| 236.7773 | 7837 | (o) | 4.0 | 50058 | (e) | 4.0 | -1.07 |  |  | -1.03 |
| 237.2338 | 6265 | (o) | 3.0 | 48405 | (e) | 2.0 | -0.28 |  | -0.21 | -0.32 |
| 237.7070 | 12501 | (o) | 3.0 | 54556 | (e) | 2.0 | -0.66 |  | -0.62 |  |
| 237.7474 | 12501 | (o) | 3.0 | 54549 | (e) | 4.0 | -0.84 |  | -0.76 | -0.79 |
| 238.0125 | 6265 | (o) | 3.0 | 48267 | (e) | 3.0 | -0.40 |  | -0.34 | -0.37 |
| 238.2276 | 9326 | (o) | 5.0 | 51289 | (e) | 4.0 | -0.62 |  | $-0.53$ |  |
| 238.5057 | 12642 | (o) | 2.0 | 54556 | (e) | 2.0 | -0.93 |  | -0.85 |  |
| 239.5043 | 9900 | (o) | 2.0 | 51641 | (e) | 2.0 | -0.72 |  | -0.66 | -0.79 |
| 240.8085 | 10127 | (o) | 3.0 | 51641 | (e) | 2.0 | -0.89 |  | -0.82 | -0.92 |
| 242.8638 | 10127 | (o) | 3.0 | 51289 | (e) | 4.0 | -0.96 |  | -0.88 |  |
| 243.0242 | 10127 | (o) | 3.0 | 51262 | (e) | 3.0 | -0.58 |  | $-0.57$ | -0.57 |

Table 2 - continued

| $\frac{\lambda(\mathrm{nm})}{243.1449}$ | Lower level |  |  | Upper level |  |  | $\log g f$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | A | B | C | D |
|  | 12501 | (o) | 3.0 |  |  |  | 53616 | (e) | 3.0 | -0.21 |  | -0.16 | -0.13 |
| 243.9807 | 12642 | (o) | 2.0 | 53616 | (e) | 3.0 | -0.33 |  | -0.26 | -0.38 |
| 245.4324 | 9326 | (o) | 5.0 | 50058 | (e) | 4.0 | -0.71 |  | -0.65 | $-0.70$ |
| 246.9944 | 9900 | (o) | 2.0 | 50375 | (e) | 3.0 | -0.42 |  | -0.36 | -0.46 |
| 247.7248 | 11577 | (o) | 0.0 | 51932 | (e) | 1.0 | -0.79 |  | -0.73 |  |
| 247.9430 | 11613 | (o) | 1.0 | 51932 | (e) | 1.0 | -0.80 |  | -0.74 |  |
| 247.9506 | 6571 | (o) | 2.0 | 46890 | (e) | 2.0 | -1.09 |  | -0.89 |  |
| 248.3817 | 10127 | (o) | 3.0 | 50375 | (e) | 3.0 | -0.59 |  | $-0.50$ | $-0.50$ |
| 249.7498 | 11613 | (o) | 1.0 | 51641 | (e) | 2.0 | -0.61 |  | -0.55 | -0.63 |
| 250.3561 | 10127 | (o) | 3.0 | 50058 | (e) | 4.0 | -1.25 |  | -1.22 | -1.31 |
| 253.1987 | 8922 | (o) | 1.0 | 48405 | (e) | 2.0 | -0.62 |  | $-0.55$ | -0.65 |
| 256.3391 | 12642 | (o) | 2.0 | 51641 | (e) | 2.0 | -1.38 |  |  | -1.51 |
| 257.9108 | 12501 | (o) | 3.0 | 51262 | (e) | 3.0 | -1.22 |  |  | -2.00 |
| 260.3591 | 16152 | (o) | 5.0 | 54549 | (e) | 4.0 | 0.31 |  | 0.40 | 0.34 |
| 262.1105 | 10127 | (o) | 3.0 | 48267 | (e) | 3.0 | -1.12 |  |  | -0.97 |
| 264.9380 | 12642 | (o) | 2.0 | 50375 | (e) | 3.0 | -0.86 |  |  | -0.76 |
| 271.9301 | 10127 | (o) | 3.0 | 46890 | (e) | 2.0 | -0.77 |  | -0.58 |  |
| 274.3714 | 5502 | (o) | 3.0 | 41939 | (e) | 3.0 | -0.91 |  | -0.73 |  |
| 274.8902 | 7150 | (o) | 4.0 | 43517 | (e) | 4.0 | -0.85 |  | -0.68 |  |
| 275.4869 | 16152 | (o) | 5.0 | 52441 | (e) | 4.0 | -0.66 |  | -0.50 | -0.57 |
| 276.8280 | 18444 | (o) | 1.0 | 54556 | (e) | 2.0 | -0.43 |  | -0.38 |  |
| 279.5105 | 12501 | (o) | 3.0 | 48267 | (e) | 3.0 | -1.32 |  |  | -1.69 |
| 284.5162 | 16152 | (o) | 5.0 | 51289 | (e) | 4.0 | -1.06 |  | -0.84 |  |
| 284.9393 | 19464 | (o) | 3.0 | 54549 | (e) | 4.0 | -0.52 | -0.36 | -0.44 | $-0.57$ |
| 286.1387 | 5502 | (o) | 3.0 | 40440 | (e) | 2.0 | -1.45 |  | -1.25 |  |
| 290.7049 | 12501 | (o) | 3.0 | 46890 | (e) | 2.0 | -1.06 |  | -0.87 |  |
| 292.3809 | 9326 | (o) | 5.0 | 43517 | (e) | 4.0 | -0.55 |  | -0.33 |  |
| 292.5260 | 6265 | (o) | 3.0 | 40440 | (e) | 2.0 | -0.83 |  | -0.62 |  |
| 292.7271 | 19464 | (o) | 3.0 | 53616 | (e) | 3.0 | -1.23 | -1.11 |  | $-1.23$ |
| 293.1537 | 7837 | (o) | 4.0 | 41939 | (e) | 3.0 | -0.73 |  | $-0.51$ |  |
| 302.2745 | 21476 | (o) | 4.0 | 54549 | (e) | 4.0 | -0.22 | -0.18 | -0.10 | -0.18 |
| 303.1580 | 19464 | (o) | 3.0 | 52441 | (e) | 4.0 | 0.35 | 0.51 | 0.46 | 0.36 |
| 305.5591 | 21476 | (o) | 4.0 | 54194 | (e) | 5.0 | 0.63 | 0.74 | 0.73 |  |
| 305.6560 | 21849 | (o) | 3.0 | 54556 | (e) | 2.0 | 0.21 | 0.28 | 0.30 |  |
| 305.7227 | 21849 | (o) | 3.0 | 54549 | (e) | 4.0 | 0.30 | 0.44 | 0.40 | 0.33 |
| 305.7579 | 19236 | (o) | 2.0 | 51932 | (e) | 1.0 | -0.03 | 0.05 | 0.06 |  |
| 308.5100 | 19236 | (o) | 2.0 | 51641 | (e) | 2.0 | 0.01 | 0.06 | 0.12 | 0.05 |
| 310.6984 | 19464 | (o) | 3.0 | 51641 | (e) | 2.0 | -0.20 | -0.10 | -0.11 | -0.27 |
| 311.0532 | 21476 | (o) | 4.0 | 53616 | (e) | 3.0 | -0.04 | 0.08 | 0.05 | -0.11 |
| 312.1560 | 19236 | (o) | 2.0 | 51262 | (e) | 3.0 | 0.19 | 0.31 | 0.31 | 0.20 |
| 314.1282 | 19464 | (o) | 3.0 | 51289 | (e) | 4.0 | -0.13 | -0.24 | -0.04 |  |
| 314.3966 | 19464 | (o) | 3.0 | 51262 | (e) | 3.0 | -0.07 | -0.06 | 0.03 | -0.05 |
| 314.7058 | 21849 | (o) | 3.0 | 53616 | (e) | 3.0 | 0.14 | 0.20 | 0.23 | 0.22 |
| 321.0503 | 19236 | (o) | 2.0 | 50375 | (e) | 3.0 | -1.27 | -1.14 | -1.50 | -1.66 |
| 322.8573 | 21476 | (o) | 4.0 | 52441 | (e) | 4.0 | -0.18 | -0.23 | -0.09 | -0.17 |
| 323.4209 | 19464 | (o) | 3.0 | 50375 | (e) | 3.0 | -0.83 | $-0.73$ | -0.77 | -0.89 |
| 326.7765 | 19464 | (o) | 3.0 | 50058 | (e) | 4.0 | -0.81 | -0.81 | $-0.70$ | -0.80 |
| 326.7941 | 21849 | (o) | 3.0 | 52441 | (e) | 4.0 | -0.79 | -0.76 | $-0.73$ | $-0.83$ |
| 335.3287 | 21476 | (o) | 4.0 | 51289 | (e) | 4.0 | 0.18 | 0.35 | 0.28 |  |
| 339.5775 | 21849 | (o) | 3.0 | 51289 | (e) | 4.0 | -0.50 | -0.18 | -0.37 |  |
| 339.8912 | 21849 | (o) | 3.0 | 51262 | (e) | 3.0 | -0.80 | $-0.58$ | -0.76 | -0.94 |
| 342.7358 | 19236 | (o) | 2.0 | 48405 | (e) | 2.0 | -0.17 | -0.09 | -0.07 | -0.20 |
| 344.3634 | 19236 | (o) | 2.0 | 48267 | (e) | 3.0 | -0.03 | 0.05 | 0.05 | -0.03 |
| 345.4388 | 19464 | (o) | 3.0 | 48405 | (e) | 2.0 | -0.06 | 0.00 | 0.03 | -0.04 |
| 345.9391 | 21476 | (o) | 4.0 | 50375 | (e) | 3.0 | 0.08 | 0.14 | 0.18 | 0.13 |
| 347.0922 | 19464 | (o) | 3.0 | 48267 | (e) | 3.0 | 0.14 | 0.25 | 0.25 | 0.16 |
| 349.7810 | 21476 | (o) | 4.0 | 50058 | (e) | 4.0 | -0.39 | -0.69 | -0.31 | -0.35 |
| 350.4629 | 21849 | (o) | 3.0 | 50375 | (e) | 3.0 | -0.06 | 0.04 | 0.06 | -0.07 |
| 354.4064 | 21849 | (o) | 3.0 | 50058 | (e) | 4.0 | -0.09 | -0.25 | 0.00 | -0.03 |
| 378.4290 | 21849 | (o) | 3.0 | 48267 | (e) | 3.0 | -1.21 |  | -1.01 | -1.04 |
| 434.6355 | 100016 | (e) | 6.0 | 123017 | (o) | 7.0 | 1.08 | 1.15 |  |  |
| 452.1922 | 100814 | (e) | 4.0 | 122922 | (o) | 5.0 | 1.01 | 0.98 |  |  |
| 566.4198 | 100663 | (e) | 3.0 | 118312 | (o) | 4.0 | 0.74 | 0.82 |  |  |

Table 2 - continued

| $\lambda(\mathrm{nm})$ | Lower level |  |  | Upper level |  |  | $\log g f$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | A | B | C | D |
| 596.2213 | 104289 | (e) | 4.0 |  |  |  | 121057 | (o) | 5.0 | 0.83 | 0.75 |  |  |
| 598.3413 | 104293 | (e) | 3.0 | 121001 | (o) | 3.0 | 0.55 | 0.55 |  |  |
| 600.2623 | 101822 | (e) | 3.0 | 118477 | (o) | 4.0 | 0.79 | 0.76 |  |  |
| 603.2531 | 102222 | (e) | 4.0 | 118794 | (o) | 5.0 | 0.95 | 0.92 |  |  |
| 606.0901 | 104351 | (e) | 5.0 | 120846 | (o) | 6.0 | 1.03 | 1.04 |  |  |

A: present work; B: Bord et al. (1997); C: Wyart \& Palmeri (1998); D: Li et al. (2000).
average deviations were only $17 \mathrm{~cm}^{-1}$ for the 101 even-parity levels and $8 \mathrm{~cm}^{-1}$ for the 113 odd-parity ones, which is roughly a factor of 2 better than the results obtained by Wyart \& Palmeri (1998), i.e. 30 and $14 \mathrm{~cm}^{-1}$, respectively. As expected, the average deviations found by Bord et al. (1997) were substantially larger (i.e. $137 \mathrm{~cm}^{-1}$ for the 75 even levels considered and $89 \mathrm{~cm}^{-1}$ for the 71 odd levels) because smaller sets of configurations were considered.

## 3 RADIATIVE LIFETIMES AND OSCILLATOR STRENGTHS

Radiative lifetimes computed in the present work without and with the inclusion of CPOL effects are compared in Table 1 with other available theoretical results, and with experimental values for the nine 4f6p levels of Ce III considered by Li et al. (2000). As seen from this table, an excellent agreement (in any case within the experimental error bars) is observed when comparing the core-polarization corrected HFR values with the accurate laser lifetime measurements performed by Li et al. (2000). This very nice agreement and the fact that our HFR calculations, without core-polarization, agree well with the previous theoretical results of Bord et al. (1997) and Li et al. (2000) clearly demonstrate that, as expected, the neglect of CPOL contributions in the latter calculations was the main source of discrepancy between theory and experiment.

Calculated oscillator strengths $(\log g f)$ are reported in Table 2 for a sample of Ce iII lines of astrophysical interest between 100 and 1000 nm . This sample is limited to 129 intense transitions $(\log g f>-1.50)$ for which previous theoretical results are available for comparison. A more extensive table (about 3000 transitions) of $g f$ - and $g A$-values computed in the present work in doubly ionized cerium is available in the DREAM data base on the web at http://www.umh.ac.be/~astro/dream.shtml.

In Figs 2 and 3 we show the comparison between our oscillator strengths and those reported by Wyart \& Palmeri (1998) and Li et al. (2000), respectively. When looking at these figures, we can observe that our $g f$-values appear systematically smaller than those of Wyart \& Palmeri (Fig. 2), as expected, because these authors did not consider CPOL effects in their calculations. In fact, the mean ratio $g f$ (this work) $/ g f$ (Wyart \& Palmeri) was found to be equal to $0.796 \pm 0.135$ (where the uncertainty represents the standard deviation of the mean) when excluding from the mean the line at 294.8531 nm for which the ratio reaches the unexpected value of 4 . In the case of the results reported by Bord et al. (1997), the ratio was closer to unity, i.e. $0.951 \pm 0.347$ but with a somewhat larger dispersion. When comparing our calculated $f$-values with the scaled results derived by Li et al. (2000) from the combination of branching fractions obtained using the Cowan (1981) code and their experimental measurements of radiative lifetimes, a general good agreement is found for the strongest lines. Indeed, for the transitions charac-


Figure 2. Comparison of the oscillator strengths obtained in this work (HFR + CPOL values) with those of Wyart \& Palmeri (1998).
terized by $\log g f>-1.00$, the mean ratio $g f($ this work $) / g f(\mathrm{Li}$ et al.) is equal to $1.039 \pm 0.184$. However, for the weaker lines, a large scatter is observed in Fig. 3. This is probably due to the fact that Li et al. (2000) used the optimized radial parameters published by Wyart \& Palmeri (1998) only for the even configurations $4 \mathrm{f}^{2}$, $5 \mathrm{~d}^{2}, 4 \mathrm{f} 6 \mathrm{p}$ and 5 d 6 s and did not attempt to fit the energy levels belonging to all the other configurations for which they just applied a scaling factor of 0.80 to the corresponding $a b$ initio HFR parameters. Consequently, the results obtained by Li et al. (2000) for weak lines are expected to be affected by large uncertainties, the quality of the computed radiative data for these lines being generally very dependent upon a satisfactory representation of transition energies and of eigenvector compositions.

## 4 ASTROPHYSICAL IMPLICATIONS

The strongest Ce III lines lie in the near-ultraviolet region, and their observation with CCD detectors is not an easy task. Bord et al. (1997) made an analysis of cerium abundance based on Ce II and Ce III lines in three Ap stars: the hot star HD 200311, and the two cooler stars HD 192913 and HR 465. They used photographic spectra in their analysis. A reasonable agreement between abundances derived from


Figure 3. Comparison of the oscillator strengths obtained in this work (HFR + CPOL values) with those derived by Li et al. (2000) from their lifetime measurements. The open circles correspond to transitions affected by large cancellation effects.

Ce II and Ce III lines was obtained for the hotter star HD 200311, while for cooler stars a difference from 0.7 (HD 192913) to 1.0 dex (HR 465) was found. No details about their analysis for cooler stars were given. Here, we reanalyse observations of HD 192913 based on new transition probabilities both for $\mathrm{Ce}_{\text {II }}$ lines (Zhang et al. 2001a) and for $\mathrm{Ce}_{\text {III }}$ lines from the present work. Equivalent widths of 17 Ce II and four Ce III lines were measured in photographic spectra of HD 192913 described in the paper by Ryabchikova, Davidova \& Adelman (1990). Model atmosphere parameters $T_{\text {eff }}=11000 \mathrm{~K}$ and $\log g=3.5$ were taken from Ryabchikova \& Ptitsyn (1986). We used a value of $2 \mathrm{~km} \mathrm{~s}^{-1}$ for the microturbulence velocity, to take into account possible magnetic intensification. We obtained $\log \left(\mathrm{Ce} / N_{\text {tot }}\right)=-7.02 \pm 0.33$ from 17 Ce II lines and $\log \left(\mathrm{Ce} / N_{\text {tot }}\right)=$ $-7.37 \pm 0.21$ from four Ce III lines. These abundances agree within the uncertainties of their determination, these being due to errors of the equivalent width measurements in photographic spectra and to possible errors in atmospheric parameters, thus giving credit to the new atomic parameters for $\mathrm{Ce}_{\text {II }}$ and Ce III lines. Also non-local thermodynamic equilibrium (NLTE) effects may be significant and should be taken into account in abundance analysis, as was shown for other rare-earth ions, Eu II and Eu III (Mashonkina, Ryabtsev \& Ryabchikova 2002).

New reliable atomic data for $\mathrm{Ce}_{\text {II }}$ and Ce III are very important for abundance studies of cool oscillating Ap (roAp) stars, where Ryabchikova et al. (2001) found an anomaly in the behaviour of the other rare-earth elements Pr and Nd . Abundances obtained from the lines of the second ion appeared to be 1.5 dex higher than those derived from the lines of the first one. Such an anomaly is marginal or does not exist at all in the atmospheres of non-pulsating stars. The present data on Ce ions provide a possibility to extend the study of the rare-earth abundance anomalies in roAp stars.

## 5 CONCLUSIONS

The oscillator strengths and transition probabilities of Ce III calculated in the present work, for many transitions of astrophysical interest, clearly show the necessity of adequately introducing CPOL effects in the HFR calculations. The set of results obtained is expected to be the best one presently available. It has been tested and assessed through a comparison of the theoretical HFR lifetimes with recent laser measurements performed in Ce III, but its quality is, however, still dependent upon the theoretical branching fractions used in the calculations. The relevance of the new data is obvious for refining the chemical composition of some CP stars.

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