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Correcting Optimal Transmission Switching for AC Power Flows

Clayton Barrows
National Renewable Energy Laboratory
claytonpbarrows@gmail.com

Seth Blumsack
Penn State University
sethb@psu.edu

Paul Hines
University of Vermont
Paul.hines@uvm.edu

Abstract

Optimal Transmission Switching (OTS) has demonstrated significant savings potential on test systems when formulated in a linearized DC power flow framework. OTS solutions generated from DC models, however, are not guaranteed to produce a feasible AC dispatch. Additionally, whether AC-feasible OTS solutions will generate cost savings similar to those suggested in the DC model is not guaranteed. We present a method to correct OTS solutions obtained in the DC model to ensure feasible AC power flow solutions. When applied to the RTS-96 benchmark network, the method achieves results that are both AC feasible and generate significant system cost reductions – in some cases larger than the cost reductions suggested by the DC OTS.

1. Introduction

Thus far in the evolution of OTS literature and research, the transmission switching problem has been formulated and solved on a simplified and linearized model of a power grid, which neglects important variables such as variations in voltage magnitudes and reactive power [1]. The approximate effects of transmission switching on economic dispatch have been documented in previous literature using the linearized DC OPF model [1-6]. The voltage magnitude and reactive power flows, which are neglected in the linearized DC OPF, are crucial parameters when considering system reliability. Ignoring these parameters, the reliability of OTS results is uncertain on real systems. By analyzing the voltage magnitude response and reactive power flows resulting from topology reconfigurations under OTS, we can characterize the effects of OTS operations on AC power flows. While [7] focuses on a comparison between topology reconfigurations in the AC and DC power flow frameworks, this research develops a method to achieve OTS solutions that are demonstrably feasible in both the AC and linearized DC power flow models. The results suggest that the modified OTS problem can generate AC-feasible transmission topologies that can produce greater savings than originally suggested by the OTS solutions formulated using the linearized DC power flow framework.

2. The Accuracy of DC Power Flow Approximations When Considering AC Power Flow Constraints

The vast majority of electrical power systems components operate under an alternating current (AC) framework. The average active (P) and reactive (Q) power at each point in a power system is governed by the set of non-linear equations:

$$P_i = \sum_{k=1}^{N} |V_i||V_k| \left(G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)\right) \quad (1)$$

$$Q_i = \sum_{k=1}^{N} |V_i||V_k| \left(G_{ik} \sin(\theta_i - \theta_k) + B_{ik} \cos(\theta_i - \theta_k)\right) \quad (2)$$

Where $|V_i|$ is the root-mean-squared voltage magnitude at bus_i, G_{ik} and B_{ik} are the real and imaginary components of the bus admittance matrix (Y_{bus}) corresponding to the transmission line conductance and reactance of the transmission line connecting bus_i and bus_k, and θ_i is the bus voltage angle at bus_i. Many power flow analyses approximate active power flow using the linearized de-coupled (DC) system model.

Power system components are sensitive to both active and reactive power flow and reliable system operations must consider bus voltage magnitudes and voltage angles. The validity of the linearized DC power flow approximations is highly dependent on the system and load profile to which it is applied. For instance, imagine two simple systems: A single lossless line system connecting two generators each controlled to a voltage magnitude of 1p.u. would yield feasible operating points in both the AC and the linearized DC model. On the other hand, consider a system with a single generator at one bus and constant load at the other. If the load demands reactive power in addition to an active power quantity equal to the transmission line transfer capability, the linearized DC power flow would obtain a solution while an AC power flow would be infeasible [8]. Similarly, large perturbations in operating state may affect the validity of the DC power flow approximation. Joint dispatch of generation and transmission topology based on DC OTS may result in infeasible operating points in the more realistic AC model. These discrepancies in AC versus linearized DC power model results are the source of many arguments against the adoption of OTS [6].

In the linearized DC power flow model, transmission congestion is caused by binding transmission line flow constraints. However, in AC power flow models transmission losses and bus voltage magnitude constraints can also cause congestion. The linearized DC power flow model typically underestimates transmission line loadings while overestimating bus voltage angle differences [9]. Since optimal transmission switching problems have, thus far, been implemented using a linearized DC power flow model, it is useful to examine the results of OTS solutions in an AC power flow framework.

3. OTS for Feasible AC Power Flows

It is well known among power system engineers that it can often be “maddeningly difficult” to obtain solutions to the AC

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power flow problem [8]. Since AC solution algorithms attempt to iteratively converge on a solution from a known operating point, solutions are particularly difficult to obtain when the operating point is dramatically altered. Conventional practice typically avoids large deviations in operating point from contingencies and control actions such as transmission switching. While many power system models have demonstrably feasible AC optimal power flow (ACOPF) solutions prior to any transmission switching, OTS solutions obtained in the linearized DCOPF model may present infeasible ACOPF situations. The method presented here is designed to gain an understanding of the conditions where OTS solutions cause AC infeasibilities and the actions required to obtain AC feasible OTS solutions. The process diagram in Figure 2 describes the process of iteratively reducing the set of switchable lines used for topology optimization to achieve AC feasible solutions.

A sensitivity based transmission line screening method (the “Screen” step in Figure 2) is used to generate a switchable line set and improve the computation time [10]. The screened OTS problems are then solved using an OTS MIP solver developed in Comet Optimization Studio [11]. The resulting, topologically reconfigured, power system model is then fed into an ACOPF solver that neglects reactive power. The ACOPF is formulated in Equations (3)-(13); note that the transmission line flow constraints in Equations (8) and (9) consider only active power flows. This formulation is chosen for consistency to enable comparisons between the congestion in the linearized DC model and the AC model. Active power losses are accounted for in the power injection balance equations (10) and (11). ACOPF problems are solved using the Matpower version 4.1 toolbox in MATLAB [12], using the constrained nonlinear multivariate minimization tool in the MATLAB Optimization Toolbox. We focus on generating AC feasible OTS solutions, where AC feasibility is defined as an ACOPF solution that satisfies the constraints posed by (4)-(13).

Finally, if the ACOPF solver returns an infeasible solution, one of the “switched” lines is removed from the switchable set (by criteria described in section 4 and the screened OTS problem is re-applied to the original system. The process in Figure 2 is repeated until a feasible ACOPF solution is found.

$$min \sum_{g} c_g \, P_g \tag{3}$$

s.t.

$$\theta_n^{\min} \le \theta_n \le \theta_n^{\max}, \qquad \forall\, n \text{ busses} \tag{4}$$

$$v_n^{\min} \le v_n \le v_n^{\max}, \qquad \forall\, n \text{ busses} \tag{5}$$

$$P_g^{\min} \le P_g \le P_g^{\max}, \qquad \forall\, g \text{ generators} \tag{6}$$

$$Q_g^{\min} \le Q_g \le Q_g^{\max}, \qquad \forall\, g \text{ generators} \tag{7}$$

$$P_k^{\min} \le \Re\left\{S_{k_{ij}}\right\} \le P_k^{\max}, \qquad \forall\, k \text{ lines} \tag{8}$$

$$P_k^{\min} \le \Re\left\{S_{k_{ji}}\right\} \le P_k^{\max}, \qquad \forall\, k \text{ lines} \tag{9}$$

$$\sum_{i=n} P_{k_{ij}} - \sum_{j=n} P_{k_{ij}} - \sum_{g} P_{ng} - \sum_{d} P_{nd} = 0, \forall\, n \tag{10}$$

$$\sum_{i=n} Q_{k_{ij}} - \sum_{j=n} Q_{k_{ij}} - \sum_{g} Q_{ng} - \sum_{d} Q_{nd} = 0, \forall\, n \tag{11}$$

$$[V_i] Y_{ij}^* V^* - S_{k_{ij}} = 0, \qquad \forall\, k \text{ lines} \tag{12}$$

$$[V_j] Y_{ji}^* V^* - S_{k_{ji}} = 0, \qquad \forall\, k \text{ lines} \tag{13}$$

4. Selection Criteria for Switchable Set Reductions

The primary objective of OTS, as presented here, is to achieve cost reductions. However, the process in Figure 2 has the potential to degrade OTS savings if switchable set reductions are not chosen properly. This section outlines switchable set reduction criteria and the theory on which they are based.

In the linearized DC power flow model, transmission line reactances govern active power flows and bus voltage phase angles. A feasible DC power flow solution respects transmission line flow rating constraints and generator output constraints but assumes that all bus voltage magnitudes are normalized to 1p.u. In the ACOPF model, bus voltage magnitudes result from the solution of the AC power flow equations (12) and (13). Maintaining a feasible ACOPF solution requires that system generators be able to supply active and reactive power in such a way that none of the system constraints are violated. Removing transmission lines from service changes the impedance parameters of the network (represented by the Y_{bus} matrix) and affects the bus voltage magnitude, phase angles, and transmission line power flows. OTS can present scenarios where system generators cannot feasibly adjust to deliver load in the topologically reconfigured transmission network. Assuming transmission switching actions do not isolate network components (create islands), switching operations will only cause flow rating (8)-(9) or voltage magnitude (4)-(5) constraint violations. To understand the effect of transmission line impedance parameters on voltage phase angles (and thus active power flows) and voltage magnitudes, it is useful to construct an example.

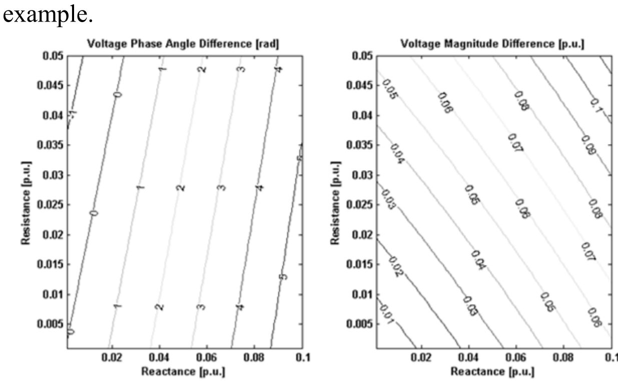

Figure 1: The resistance and reactance parameters of a single transmission line system are varied to generate voltage phase angle (left) and magnitude (right) differences between bus$_1$ and bus$_2$ where 100MW and 50MVAr are generated at bus$_1$ and consumed at bus$_2$.

Imagine a single line system where a generator located at bus_1 serves a 100MW active power and 50MVAr reactive power load at bus_2. To illustrate the effect of transmission line impedance parameters on the voltage at bus_2, the voltage at bus_1 is controlled to 1p.u. and 0° and the resistance and reactance parameters of the transmission line are varied. The contours of Figure 1 show the voltage phase angle (left pane) and magnitude differences (right pane) between bus_1 and bus_2 resulting from varying the transmission line impedance parameters. The left pane shows that phase angle differences are highly sensitive to transmission line reactance but not very sensitive to resistance. However, the right pane shows that voltage magnitude differences are sensitive to both resistance and reactance parameters. This analysis suggests that active power flow between two busses is primarily affected by the equivalent reactance while the bus voltage magnitudes are affected by the equivalent reactances and resistances. These results can be demonstrated analytically by examining equations governing active and reactive power flow for the two node example:

$$P_2 = |V_2||1|(G_{12}\cos(\theta_2 - 0) + B_{12}\sin(\theta_2 - 0)) \quad (14)$$

$$Q_2 = |V_2||1|(G_{12}\sin(\theta_2 - 0) + B_{12}\cos(\theta_2 - 0)) \quad (15)$$

Substituting $G = 1/R$ and $B = 1/X$ and calculating the partial derivatives of $|V_2|$ in (14) with respect to R and X:

$$\frac{\partial|V_2|}{\partial R_{12}} = -\frac{P_2}{\cos(\theta_2)} \quad (16)$$

$$\frac{\partial|V_2|}{\partial X_{12}} = -\frac{P_2}{\sin(\theta_2)} \quad (17)$$

Similarly, substituting for G and B and calculating the partial derivatives in (15) yields:

$$\frac{\partial|V_2|}{\partial R_{12}} = -\frac{Q_2}{\sin(\theta_2)} \quad (18)$$

$$\frac{\partial|V_2|}{\partial X_{12}} = -\frac{Q_2}{\cos(\theta_2)} \quad (19)$$

If we make the small angle assumption to assume that the value of θ_2 is relatively similar to the value of $\theta_1=0$ we can see that the most significant partial derivatives of $|V_2|$ are with respect to X for active power (17) and R for reactive power (18).

If we again make the small angle assumption in Equations (14) and (15), we can assume that $\sin(\theta_2-\theta_1)\approx\theta_2-\theta_1$ and $\cos(\theta_2-\theta_1)\approx 1$. Thus we can solve for θ_2:

$$\theta_2 = \frac{P_2 X_{12}}{|V_2|} - \frac{X_{12}}{R_{12}} \quad (20)$$

$$\theta_2 = \frac{Q_2 R_{12}}{|V_2|} - \frac{R_{12}}{X_{12}} \quad (21)$$

Taking the partial derivatives of θ_2 with respect to the impedance parameters in Equation (20) yields:

$$\frac{\partial\theta_2}{\partial R_{12}} = X_{12} \quad (22)$$

$$\frac{\partial\theta_2}{\partial X_{12}} = \frac{P_2}{|V_2|} - \frac{1}{R_{12}} \quad (23)$$

Similarly, the partial derivatives of the reactive power Equation (21) yields:

$$\frac{\partial\theta_2}{\partial R_{12}} = \frac{Q_2}{|V_2|} - \frac{1}{X_{12}} \quad (24)$$

$$\frac{\partial\theta_2}{\partial X_{12}} = R_{12} \quad (25)$$

If we make the realistic assumptions that the values of P_2 and X_{12} are significantly larger than the values of Q_2 and R_{12} respectively, the voltage angle at bus_2 is most sensitive to changes in line resistance.

In order to identify the switched transmission line properties that contribute to AC infeasible OTS solutions, the selection criteria for removing transmission lines that are optimally switched by OTS in the DC model from the set of switchable transmission lines is varied. Figure 1 and the subsequent analytical results demonstrate that ACOPF feasibility can be sensitive to both reactance and resistance, but the sensitivities of a particular system may depend on that system's parameters. So, it makes sense to consider corrective removal of lines from the switchable set based on both reactance and resistance criteria. This paper compares the following five criteria that could be used to identify which optimally switched line to remove from the switchable set:

1. Max resistance optimally switched line.
2. Min resistance optimally switched line.
3. Max reactance optimally switched line.
4. Min reactance optimally switched line.
5. Random optimally switched line.

For the analysis presented here, the OTS problem is solved without the *N-1* security constraints. Security constraints are ignored for two reasons: first, the un-secure OTS problem is significantly more tractable, enabling shorter computation times. Second, the lack of security constraints results in OTS solutions that present larger topology disruptions. That is, more transmission lines are switched in un-secure OTS than in *N-1* secure OTS solutions. Larger topology disruptions generally cause more constraint violations in the ACOPF and enable a more substantial analysis of the switched transmission lines that contribute to ACOPF infeasibility. Despite the lack of security constraints in the problem formulation, the effects of OTS on AC system security can be described using an ex-post security analysis. By checking the ability of the system to withstand the loss of any single element before and after OTS operations, we can assess the effect of topology reconfigurations on system security. Here, the ex-post security analysis is limited to transmission security. That is, a generation dispatch and transmission topology is said to be transmission secure if the system can withstand the loss of any single in-service transmission element without violating transmission line flow ratings.

It should be emphasized that the results obtained by the process in Figure 2 are not intended to produce a co-optimal generation dispatch and network topology in the AC power flow model. Rather, the method is intended to determine whether OTS solutions obtained through the DC framework are likely to cause constraint violations in real systems (modeled in the AC optimal power flow framework) and appropriate modifications to the OTS problem that enable feasible real system solutions.

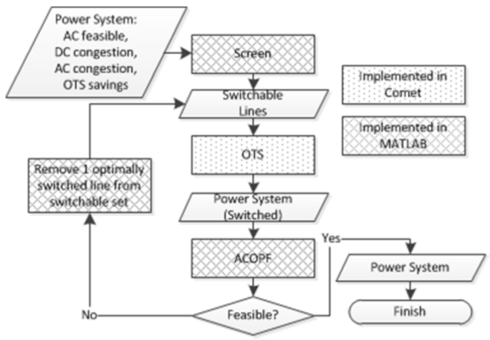

Figure 2: Process diagram for obtaining AC feasible transmission switching results.

5. DC Models may Underestimate Savings from Optimal Transmission Switching

The method described in 2 was applied to the 24 load periods of the IEEE RTS-96 network. Congestion is introduced into the RTS-96 system by removing line 11-13, shifting 480MW of load from buses 14,15,19 and 20 to bus 13, and decreasing the capacity of line 14-16 to 350MW [2, 13, 10]

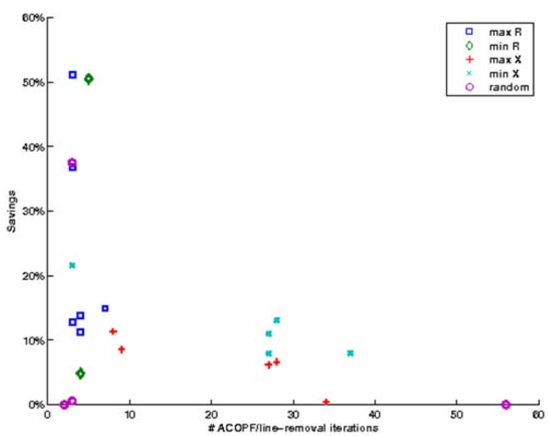

Figure 3: Optimally switched lines are removed from the switchable set until the OTS algorithm obtains a feasible ACOPF based on 5 ranking criteria. The horizontal axis represents the # lines that are removed from the switchable set (the # of iterations of the process in Figure 2), and the vertical axis shows the system cost savings obtained over the un-switched case in the AC model.

Table 1 and Figure 3 illustrate the results of the procedure presented in Figure 2 on the 24 summer peak weekday load periods of the RTS-96 network. The savings values presented on the vertical axis are the result of the following calculation:

$$Sav = \frac{f(ACOPF_{un-switched}) - f(ACOPF_{switched})}{f(ACOPF_{un-switched})} \qquad (26)$$

where *f(ACOPF)* is the value of the ACOPF objective function in Equations (3)-(13). $ACOPF_{switched}$ is the result of the first feasible ACOPF solution obtained in the process in Figure 2. Each process iteration is equivalent to removing a single line from the switchable set. The number of process iterations (switchable set reductions) required to obtain a feasible ACOPF solution is reported on the horizontal axis of Figure 3. The results show that feasible AC power flow solutions are achieved in the fewest iterations by successively removing high-resistance lines from the switchable set. Additionally, removing the maximum resistance line generally achieves greater system cost savings than removing lines by one of the other selection rules. This likely arises due to the effect of high resistance transmission lines on voltage magnitude differences. For a constant power flow between two busses, the bus voltage phase angle differences are driven primarily by the equivalent reactance between the two busses. Similarly bus voltage magnitude differences are driven primarily by the equivalent resistance between the two busses [9]. Removing a high resistance line from service can dramatically change the equivalent resistance between busses and cause significant changes bus voltage magnitudes at the line endpoints; causing voltage magnitude constraint violations if the induced changes are large enough. Thus, by removing the highest resistance optimally switched line from the switchable set of transmission lines, the next OTS solution will leave that line in service and improve the likelihood of obtaining feasible ACOPF solutions.

Another observation from Figure 3 is that the cost savings achieved in the ACOPF is significantly higher than the savings achieved by OTS in the DC model (see [10]). This can be explained by examining the system dispatch cost curve (i.e., the total dispatch cost as a function of system load) for the RTS-96 system, shown in Figure 4. The cost curve exhibits a substantial discontinuity in the ACOPF solution due to out-of-merit dispatch of high-cost generators. The DCOPF solution also exhibits such a discontinuity, but at a different (higher) level of demand than in the ACOPF solution (since the DC problem is lossless and constrained only by path ratings and voltage angle limits). Due to losses and additional binding constraints, the un-switched ACOPF system cost falls above the discontinuity, but AC-feasible transmission switching shifts the location of the discontinuity, resulting in large savings. OTS in the DC model does not shift the location of the discontinuity substantially. This result is sensitive to the system load profile and generator cost definitions as well as system constraints, but we would expect other systems to exhibit similar behavior under the right circumstances.

Table 1: Feasible ACOPF OTS Results (RTS-96)

	# Feasible ACOPF Periods	# Lines Removed from Switchable Set	24 hour OTS (DC) Savings	24 hour OTS (AC) Savings
Un-switched	13	--	--	--
Max *R*	15	75	0.539%	6.019%
Min *R*	18	429	0.350%	-13.300%
Max *X*	15	334	0.539%	5.810%
Min *X*	12	442	0.502%	10.500%
Random	10	395	0.315%	2.230%

Table 1 shows the ex-post security analysis. We assess the proportion of single transmission line outages that cause power flows in excess of 10% greater than transmission line flow limits. In the un-switched DCOPF and ACOPF models, 25% and 27% of transmission line outages cause security violations. By comparison, the security violation rates for the AC feasible OTS results are only slightly higher than for the un-switched results. When security constraints are left out of the optimization model, generation is dispatched so that transmission line security violations are common. These ex-post transmission security analysis results show that OTS reduces the occurrence of binding power flow constraints and thus reduces the proportion of line outages that cause overloads. Generalizing these results suggests that OTS (without security constraints) does not necessarily degrade transmission security.

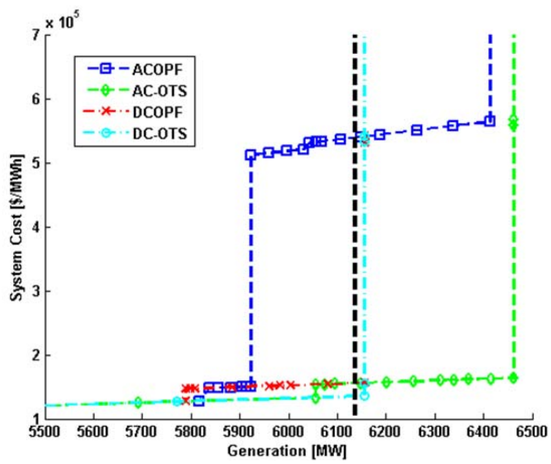

Figure 4: Total system cost (vertical axis) as a function of system load (horizontal axis) for IEEE RTS-96 hour 24 resulting from un-switched ACOPF ('□'), AC feasible OTS achieved by removing maximum resistance lines from the switchable set ('◊'), DCOPF ('X'), and DC OTS ('○'). The dotted vertical black line represents the total system load (6136 MW).

Table 1: Ex-Post Security Violations in the RTS-96

	Un-switched	Max *R*	Min *R*	Max *X*	Min *X*	Random
DC Security Violation	25%	27%	0%	9%	0%	34%
AC Security Violation	27%	32%	11%	10%	8%	40%

6. Conclusions

Although OTS has shown potential for cost savings in the linearized DC system model, there is still uncertainty about the behavior of DC-OTS solutions in the AC system environment. This research presents a step toward understanding AC system reliability under optimal topology reconfigurations made in the DC model.

The method developed here corrects OTS solutions generated in the DC model to generate feasible ACOPF solutions on topologically reconfigured networks by incrementally forcing switched lines to remain in service based on five selection criteria. The results show that removing the highest resistance lines from the switchable set (i.e. forcing high resistance lines to remain in service) generates feasible ACOPF solutions in fewer process iterations, relative to removing lines by other selection criteria. Additionally, when a feasible ACOPF solution is found, the cost savings over the un-switched ACOPF are significant when high resistance lines are removed from the switchable set. Removing lines from the switchable set by other selection criteria generates requires significantly more process iterations to achieve feasible ACOPF solutions and typically leads to lower savings. These results highlight the dependency of ACOPF feasibility on certain lines; particularly high resistance lines. Our results show that by screening for switchable lines, OTS solutions obtained in the linearized DC model can translate to system cost savings in the AC model. Under the right circumstances, the savings generated by topologically reconfigurations in the ACOPF framework can actually be larger than the savings highlighted in the linearized DC power flow framework.

Ultimately, we have demonstrated a voltage-corrected transmission switching method that is both feasible in terms of AC real power flows and still leads to system cost savings. The ex-post transmission security analysis shows that in both the AC and DC optimal power flow models, transmission line overloads can be less common in switched systems than in un-switched systems. Even with this relatively informal definition of transmission security, the results suggest that transmission topology reconfigurations do not necessarily degrade the security of the system.

Our findings are, at this point, suggestive. Further analysis includes incorporating reactive power considerations into the ACOPF, as well as conducting experiments to determine the source of voltage violations in the (real-power) AC OPF. We are also extending our analysis to incorporate test systems other than the RTS-96.

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