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# CORRECTION

# A CENTRAL LIMIT THEOREM FOR STATIONARY PROCESSES AND THE PARAMETER ESTIMATION OF LINEAR PROCESSES

BY YUZO HOSOYA AND MASANOBU TANIGUCHI

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Assumptions (i) and (ii) of Theorem 2.1 are insufficient to guarantee the central limit theorem. For the theorem to be valid, a Lindeberg-type or a related condition is needed. Hosoya (1992) used a Lindeberg-type condition and Findley and Wei (1992) proposed a Liapounov-type condition as an additional assumption to the theorem [Findley communicated to us about the error in our assumptions].

Specifically, a version of correction according to the approach of Hosoya (1992) is given as follows.

**1.** Condition (i) of Lemma A2.4 should be

$$\lim_{m\to\infty} \frac{1}{n(m)} \sum_{k=1}^{n(m)} E\left[u_m(k)^2 I\{|u_m(k)| \geq \varepsilon n(m)^{1/2}\}\right] = 0.$$

**2.** Theorem 2.1 (page 134) should have the following additional assumption:

(iii) *For any* $\varepsilon > 0$, *there exists* $B_\varepsilon > 0$ *such that uniformly in* $N$ *and* $r$

$$E\left[S(N,r)^2 I\{S(N,r) > B_\varepsilon\}\right] < \varepsilon,$$

*where* $S(N,r) = [\sum_{\alpha=1}^{p}\{\sum_{t=1}^{N} x_\alpha(t+r)/N^{1/2}\}^2]^{1/2}$.

**3.** The line which contains (6.18) in the proof (pages 146 and 147) should be deleted and the following lines should be added after the last line of the proof:

It holds that

$$\begin{aligned} &E\left\{\left(\eta_k - E(\eta_k|\mathscr{F}_{k-1}^*)\right)^2 I\left(|\eta_k - E(\eta_k|\mathscr{F}_{k-1}^*)| > 2B_\varepsilon\right)\right\} \\ &\quad \leq 2E\left\{\eta_k^2 I\left(\left||\eta_k| - |E(\eta_k|\mathscr{F}_{k-1}^*)|\right| > 2B_\varepsilon\right)\right\} + 2E\left\{E(\eta_k|\mathscr{F}_{k-1}^*)^2\right\} \\ &\quad \leq 2E\left\{\eta_k^2 I(|\eta_k| > B_\varepsilon)\right\} + 2E\left\{\eta_k^2 I\left(|E(\eta_k|\mathscr{F}_{k-1}^*)| > B_\varepsilon\right)\right\} \\ &\qquad + 2E\left\{E(\eta_k|\mathscr{F}_{k-1}^*)^2\right\}, \end{aligned}$$

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where $2E\{\eta_k^2 I(|\eta_k| > B_\varepsilon)\} < 2\varepsilon$ in view of Assumption (iii). Since

$$\begin{aligned} E\{\eta_k^2 I(|E(\eta_k|\mathscr{F}_{k-1}^*)| > B_\varepsilon)\} &= E\big[E(\eta_k^2|\mathscr{F}_{k-1}^*) I(|E(\eta_k|\mathscr{F}_{k-1}^*)| > B_\varepsilon)\big] \\ &\le E\big|E(\eta_k^2|\mathscr{F}_{k-1}^*) - E(\eta_k^2)\big| \\ &\quad + E(\eta_k^2) Pr\{|E(\eta_k|\mathscr{F}_{k-1}^*)| > B_\varepsilon\}, \end{aligned}$$

the application of the Chebyshev inequality gives that

$$\begin{aligned} &\frac{1}{M}\sum_{k=1}^{M} E\{\eta_k^2 I(|E(\eta_k|\mathscr{F}_{k-1}^*)| > B_\varepsilon)\} \\ &\quad \le \frac{1}{M}\sum_{k=1}^{M} E\big|E(\eta_k^2|\mathscr{F}_{k-1}^*) - E(\eta_k^2)\big| + \frac{E(\eta_k^2)}{B_\varepsilon^2 M}\sum_{k=1}^{M} E\{E(\eta_k|\mathscr{F}_{k-1}^*)\}^2. \end{aligned}$$

As is seen in (6.17), the second member on the right-hand side tends to 0 as $N \to \infty$, and the first member tends to 0 in view of the inequality that comes after (6.19). Consequently, for sufficiently large $N$, it holds that

$$\frac{1}{M}\sum_{k=1}^{M} E\Big[\{\eta_k - E(\eta_k|\mathscr{F}_{k-1}^*)\}^2 I(|\eta_k - E(\eta_k|\mathscr{F}_{k-1}^*)| > B_\varepsilon)\Big] < \varepsilon,$$

from which the condition (i) of Lemma A2.4 follows by the line of argument given, for example, in Chow and Teicher [(1978), page 291].

**4.** Theorem 2.2 needs Assumption (v); namely:

(v) *For any* $\varepsilon > 0$ *and for any integer* $L \ge 0$, *there exists* $B_\varepsilon > 0$ *such that*

$$E\big[T(N,s)^2 E\{T(N,s) > B_\varepsilon\}\big] < \varepsilon$$

*uniformly in* $N$, $s$; *where*

$$T(N,s) = \left[\sum_{\alpha,\beta=1}^{p}\sum_{r=0}^{L}\left\{\sum_{t=1}^{N}\big(e_\alpha(t+s)e_\beta(t+s+r) - K_{\alpha\beta}\delta(0,r)\big)/N^{1/2}\right\}^2\right]^{1/2}.$$

**5.** The phrase "Assumptions (i) through (iv) of Theorem 2.2" in Theorem 3.1 and Proposition 4.1 is respectively changed to "Assumptions (i) through (v) of Theorem 2.2."

**6.** Hosoya (1989) also uses Theorem 2.2. Assumption (A) (page 403) of that paper should have the above Assumption (v) as additional Condition (iii).

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FACULTY OF ECONOMICS
TOHOKU UNIVERSITY
SEIDAI 980
JAPAN

DEPARTMENT OF MATHEMATICAL SCIENCE
FACULTY OF ENGINEERING SCIENCE
OSAKA UNIVERSITY
TOYONAKA 580
JAPAN