CORRECTION FACTORS FOR THE DETERMINATION OF OPTICAL-FIBRE REFRACTIVE-INDEX PROFILES BY THE NEAR-FIELD SCANNING TECHNIQUE

Indexing terms: Fibre optics, Optical waveguides, Refractive index

Numerical calculations are presented of the correction factor for use with the near-field scanning method of index profile determination. It is shown that a single curve can be applied to a range of possible profiles, and a numerically obtained average curve is given.

Introduction: In recent publications,[1, 2] we have described the determination of optical-fibre refractive-index profiles by a near-field scanning technique. The method provides a convenient means of obtaining detailed index profiles by observation of the light intensity distribution across the output face of a short length of fibre illuminated by a Lambertian source. However, it is necessary in most practical cases to correct the measured intensity profile to allow for the inevitable presence of leaky modes. This has led to the development[2] of a set of normalised correction curves enabling near-field measurements to be converted to refractive-index profiles.

To calculate a set of curves which would be applicable to a range of index profiles, and would not therefore require prior knowledge of the profile being measured, it was necessary to make a number of approximations. The object of the present contribution is to clarify these approximations, and to investigate their validity by presenting numerical calculations of the correction factors for several possible fibre profiles.

Leaky-mode attenuation coefficient: As shown in Reference 2, the near-field intensity $I(r)$ at radius r may be calculated by summing the power remaining in all propagating modes after a length z of fibre. When all bound modes are equally excited, this leads to a simple dependence[3] of near-field on refractive-index profile $n(r)$. However, in practice, leaky modes are also excited[4] and will contribute additional power to the near field. To determine the power remaining in leaky modes, and hence the magnitude of their contribution, the attenuation $\alpha(u, v)$ of each leaky mode (u, v) must be found. A correction factor $C(r, z)$ may then be formulated to account for the discrepancy between near field and refractive-index profile.

$\alpha(u, v)$ can be expressed in terms of the tunnelling coefficient[5] T which gives the probability of a photon at the central caustic r_2 of a leaky-ray path emerging at the outer caustic r_3 by a process analogous to quantum-mechanical tunnelling. α is normalised to the fibre core radius a and is given by the product of T with the number of 'reflections' $(1/\Delta z)$ per unit length at the central caustic r_2:

$$\alpha(u, v) = aT/\Delta z \qquad (1)$$

Δz may be interpreted as the ray half period for the parabolic fibre only. In general, Δz is given in the WKB approximation by*

$$\Delta z = 2\int_{r_1}^{r_2} \beta(k^2 n^2(r) - \beta^2 - (v^2/r^2))^{-1/2}\, dr \qquad (2)$$

where r_1 is the radius of the inner caustic, r_2 is that of the central caustic, β is the longitudinal propagation constant, k is the free-space wavenumber and v is the azimuthal wavenumber. Similarly, the WKB approximation yields, for the tunnelling coefficient,[2, 5-7]

$$T = \exp\left[-2\int_{r_2}^{r_3} ((v^2/r^2) + \beta^2 - k^2 n^2(r))^{1/2}\, dr\right] \qquad (3)$$

Eqns. 2 and 3 may be numerically integrated to generate a correction curve for a specific index profile. However, apart from the tedium of calculation, this is unsatisfactory when dealing with fibres having unknown index profiles, as in the near-field scanning technique. We therefore seek approximations which will allow a single average correction curve to cover a range of possible profiles.

Correction factors: In Reference 2 approximate forms of eqns. 2 and 3 were presented which were independent of the detailed form of the refractive-index profiles. The approximations used were:

(*a*) For the tunnelling coefficient of eqn. 3 it was assumed that $r_2 \simeq a$ in the integration, so that T no longer depends on $n(r)$ in the core region.

(*b*) Only the least leaky (near-helical) rays were considered, since these contribute most to the near field. Using again the approximation $r_2 \simeq a$, we obtain an average Δz for a wide range of profiles:

$$\Delta z \simeq a^2\beta/2v \qquad (4)$$

which is similarly independent of profile. The use of this approximation explains the apparent discrepancy between the generalised coefficient of Reference 2 and that developed for the specific case of the parabolic-index profile by Petermann,[6] and more recently by Snyder and Love.[7]

To investigate the validity of the above two approximations when applied to various fibres, we have numerically computed the correction factors for a range of power-law profiles. The tunnelling coefficient T for a given profile is obtained by using numerical quadrature for the integral of eqn. 3, while Δz is derived from Gloge's expression[8] for the number of propagating modes:

$$\Delta z \simeq \frac{a^2\beta\pi}{2u}\left(\frac{g+2}{2}\right)^{1/g}\left(\frac{g+2}{g}\right)^{1/2}\left(\frac{u}{v}\right)^{2/g} \qquad (5)$$

where g is the index exponent, and u and v have their conventional meanings. This approximation is exact for $g = 2$ and very good for other values, provided g is not too large.

The correction factors for index profiles represented by $g = 1{\cdot}5$, 2, 3 and 4 are given in Fig. 1, together with the step-index curves for comparison. The X values shown refer to the normalisation parameter[2] $X = (1/v)\ln(z/a)$, which renders the curves applicable to fibres of any radius a, length $z > 10^3\, a$ and normalised frequency v. It may be seen that the correction factors are remarkably similar up to a normalised radius

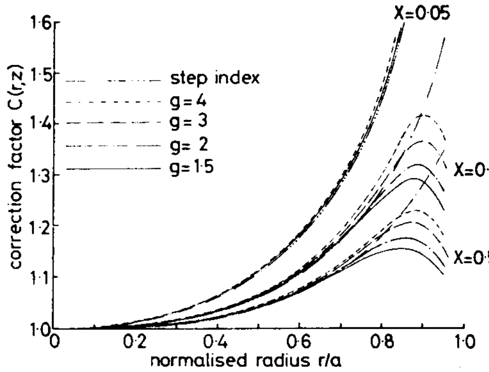

Fig. 1 *Correction factors C(r, z) plotted as function of normalised fibre radius r/a for fibres having a range of index profiles and X = 1/vln(z/a) values of 0·05, 0·25 and 0·5*

Near-field intensity profiles must be divided by $C(r, z)$ to give the refractive-index distribution.

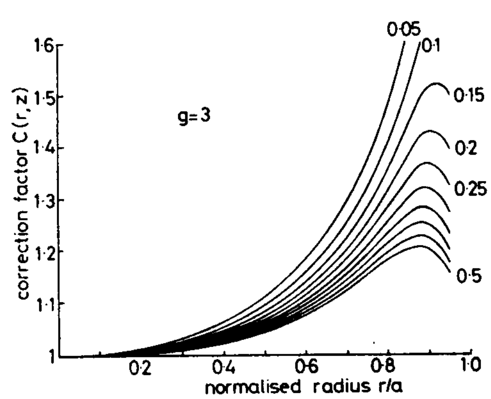

Fig. 2 *Near-field correction factors for fibres having an index exponent of 3 and X values from 0·05 to 0·5, in increments of 0·05*

The curves represent a useful median value for application when the approximate profile is unknown

* Note that Petermann's derivation[6] of $\alpha(u, v)$ differs by a factor of $\frac{1}{2}$

of 0·8 for a wide range of index profiles, after which a spread of ±6% may occur. Thus our initial assumption that the correction factor would be approximately independent of profile is verified. Moreover, the errors incurred by applying an inappropriate correction factor are only significant at a normalised radius of greater than 0·8. Fortunately, since the magnitude of most graded-index profiles at this radius is small, the error will be hardly noticeable.

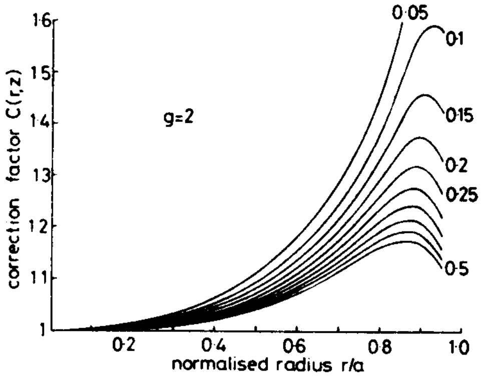

Fig. 3 *Correction factors for fibres having an index exponent of 2 and X values from 0·05 to 0·5, in increments of 0·05*

Comparison of Fig. 1 with the curves presented in Reference 2 shows the latter to be slightly low; this is a consequence of approximations (*a*) and (*b*) both producing an error of similar sign. It would therefore appear that a numerically obtained average correction factor would be preferable to that obtained previously by analytic approximation, and, in particular, the curves for $g = 3$ represent a median value for the range of profiles normally encountered. A set of such curves for X values from 0·05 to 0·5 is given in Fig. 2. An error of 8% at a normalised radius of 0·9 would result if these curves were used to correct a $g = 1{\cdot}5$ profile for a typical fibre having $X = 0{\cdot}25$. This represents a deviation of about 1% of the index difference at the core centre, an insignificant error.

Since it is likely that most interest will be centred on near-parabolic-index fibres, for completeness we present curves for $g = 2$ in Fig. 3.

Conclusion: normalisation parameters: In conclusion, it should be noted that the normalisation parameter $X = (1/v)\ln(z/a)$ is unchanged by the improved correction factors given here and remains valid for $z/a > 10^3$. This is further borne out by eqn. 5, which may be used to generate profile-dependent normalisation parameters for any power-law index distribution. The approximate parameter for each g value is given by

$$D(g) = \frac{1}{v}\left\{\ln\left(\frac{v}{\pi a k n(0)}\right) + \ln\left[\frac{1}{2}\left(\frac{g+2}{2}\right)^{1/g}\left(\frac{g+2}{g}\right)^{\frac{1}{2}}\right] + \ln\left(\frac{z}{a}\right)\right\} \quad (6)$$

which reduces exactly to the expression derived by Love and Pask[9] for the parabolic case ($g = 2$). The normalisation parameter introduced in Reference 2 retains only the third term of eqn. 6, on the basis that, in practice, the variation in this term dominates. For very short lengths of fibre, less than 3 cm for a fibre having a 60 μm core diameter, it is possible that a more accurate normalisation based on eqn. 6 may be useful. However, in a practical case the small improvements in accuracy thus obtained would hardly merit the added complexity.

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