# CORRECTION FACTOR TECHNIQUES FOR IMPROVING AERODYNAMIC PREDICTION METHODS 

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MAY 1976

Prepared under Contract No. NASI-13835 By MCDONNELL DOUGLAS CORPORATION DOUGLAS AIRCRAFT COMPANY LONG BEACH, CALIFORNIA

## For

National Aeronautics And Space Administration

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CORRECTION FACTOR TECHNIQUES FOR IMPROVING AERODYNAMIC PREDICTION METHODS*<br>By Joseph P. Giesing, Terez P. Kalman<br>and William P. Rodden<br>Douglas Aircraft Company

## SUMMARY

This report describes a method for correcting lifting surface theory so that it reflects known experimental data. Specifically the theoretical pressure distribution is modified such that imposed constraints are satisfied (e.g., lift, moment, etc.) while minimizing the change to the theoretical pressure distribution. It is assumed that a finite element or discretized lifting surface method is used, such as either the Doublet or Vortex Lattice Methods.

There are several ways in which the theoretical pressures are modified. One is a direct application of a set of correction factors to the pressures. This is accomplished by premultiplying the pressures with a diagonal matrix of correction factors. A second approach to correcting the theory is to modify the downwash. This modification can be accomplished by either multiplying the downwash by a diagonal matrix of correction factors or by adding an incremental downwash (which is proportioned to the pressure) to the theoretical downwash. In any case the correction factors are adjusted so that the imposed experimental constraints are satisfied by the corrected pressure distribution while the changes in the pressure distribution are minimized.

There are several features that have been built into the basic method and these include: (1) the ability to consider together experimental data

* The authors wish to acknowledge Dr. Edward Albano for interesting discussions of alternate formats (non-diagonal) of correction matrices and their potential derivations.
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Prepared under Contract No. NASI-13835
from more than one mode (e.g., control surface rotation, pitch, camber, etc.), (2) the ability to limit the excursions of the correction factors (i.e., establish minimum and maximum values for them) and (3) the ability to use correction factor mode shapes (i.e., construct correction factors from known distributions or functions).

The methods developed have been implemented on the computer and many correlations and calculations made. Specifically cases involving all three Mach Number ranges are considered. For instance in the subsonic speed range a swept wing with an oscillating partial span flap and a swept wing with a leading edge droop are discussed. In the transonic speed range a two-dimensional symmetric airfoil with an oscillating flap is treated in detail. An arrow wing with and without camber is used in the supersonic analysis.

The computer program used to generate the correction factors for these cases is also fully described in this report and test cases are provided.

Finally, a new, simple method for accounting for transonic effects in the lifting surface theory is described and correlated for the two-dimensional case. Basically, a transformed distance between the sending element and receiving point is employed. The transformation depends on the time delay encountered by a signal traveling from the sending point to the receiving point.

## INTRODUCTION

Wind tunnel data have provided the basis for semi-empirical methods of aeroelastic analysis for many years, whether in the estimation of stability and control characteristics, the calculation of structural loads, or in flutter analysis by modified strip methods. These semi-empirical methods have been tailored to aerodynamic lifting-line theory or to strip theory and not to the more general (and more accurate) lifting-surface methods. The use of a diagonal correction matrix to be applied as a premultiplying factor to matrices of aerodynamic influence coefficients obtained from lifting surface theory has been considered by a number of authors. A premultiplier may be regarded as a correction to the pressure distribution; as an alternative, a postmultiplier would be regarded as a correction to the downwash to account for thickness effects and for camber induced by boundary-layer displacement effects. Rodden and Revell (refs. 1-4) considered a real correction matrix derived from static wind tunnel measurements and theoretical load predictions. Bergh and Zwaan (refs. 5 and 6) investigated a complex correction matrix derived from oscillatory wind-tunnel pressure measurements and theoretical predictions. These authors assumed measurements were available only for a single mode, a steady angle of attack or an oscillatory pitching (or yawing) mode.

Current interest in using actively controlled aerodynamic surfaces to minimize aeroelastic response requires an improvement in accuracy in predicting unsteady aerodynamic characteristics of lifting surfaces equipped with control surfaces. The correction matrix provides one means of improving the accuracy but it requires experimental data on control surface characteristics in addition to the angle of attack characteristics of the surface. Hence, an extension of references $1-4$ is necessary to obtain the correction matrix for more than one aerodynamic mode. Furthermore, the discrepancies between theory and experiment in predicting trailing-edge control surface loads are most likely caused by boundary-layer displacement effects on the effective downwash. Hence, another extension is necessary to obtain a postmultiplying correction matrix. These two extensions are considered in the present
development. The diagonal format has been retained* and complex pre- and. postmultiplying correction matrices have been derived which satisfy the constraints of matching experimental data from multiple aerodynamic downwash modes.

The use of correction matrices is in the time-honored engineering tradition of empirical correction factors. It retains the generality of the theory while approaching the limiting values of the test results. Such a posteriori adjustment obviously cannot be regarded as addressing any of the fundamental causes of the discrepancies. Other possibilities exist whereby empirical corrections can be introduced directly into the theoretical solution. Ashley (ref. 7) has discussed two such "irrational correction methods". The first of these is of interest here and concerns the calculation of the downwash boundary condition and the pressure distribution by "local linearization" in terms of the local velocity $V_{L}$ rather than the free stream velocity $U_{\infty}$. The dimensionless downwash then becomes

$$
\frac{W}{U_{\infty}}(x, y, 0, t)=\frac{1}{U_{\infty}}\left[\frac{\partial h}{\partial t}+V_{L}(x, y) \frac{\partial h}{\partial x}\right]
$$

where $h(x, y, t)$ is the deflection of the mean surface. Applications of this "local linearization" to the downwash boundary condition (but not to the kernel function nor pressure coefficient) have been made for control surfaces by Ashley and Rowe (ref. 8) and by Rowe, Winther, and Redman (ref. 9), and improved correlations have been obtained. Tijdeman and Zwaan (ref. 10) have also employed the local linearization of the downwash boundary condition but have suggested another modification for use in the Doublet-Lattice Method for high subsonic flows, viz., that the free stream Mach number be replaced by a mean Mach number $M_{j}$ for each panel (lifting element or box). The downwash induced at box $i$ by the lifting pressure on box $j$ then becomes

* The format of a full matrix was briefly investigated but it was found to destroy the distributional character of the theoretical aerodynamic influence ccefficients, and was not considered further.

$$
\frac{w_{i}}{U_{\infty}}=D_{i j}\left(M_{j 1}, k_{r}\right) \Delta C_{p j}
$$

where $D_{i j}\left(M_{j l}, k_{r}\right)$ is the downwash influence coefficient between the $j$ th lifting element and the ith downwash collocation point and its functional dependence on $M_{j 1}$ and the reduced frequency $k_{r}$ is indicated. Tijdeman and Zwaan denote the freestream Mach number by $M_{\infty}$ and the local Mach number at the surface of box $\mathfrak{i}$ by $\mathrm{M}_{\mathrm{i} 2}$, so that the locally linearized downwash for harmonic motion becomes

$$
\frac{w_{i}}{U_{\infty}}=\frac{M_{i 2}}{M_{\infty}} \frac{\partial h_{i}}{\partial x}+i \frac{\omega}{U_{\infty}} h_{i}
$$

The values of $M_{1}$ and $M_{2}$ for a certain box are not equal, in general, because $M_{1}$ has to reflect the influence of the Mach number distribution normal to the surface ranging from $M_{2}$ at the surface to the freestream Mach number far away from the wing. Preliminary results from NLR calculations have shown that $M_{1}$ can be chosen simply to be the average value of $M_{2}$ and $M_{\infty}$.

Matrix that gives pressures in terms of downwash. Inverse of $D$

## Speed of sound in the free stream

Constraining power of a constraint. If $\tilde{a}=1$ constraint is $100 \%$ effective. If $\tilde{a}=0$ constraint has no effect at all

Wing bending moment (about $x$-axis)
Semi span of wing
Aerodynamic coefficient (e.g. lift or moment coefficient). $\mathrm{C}_{e}$ is used as an experimental constraint

Lift coefficient
Moment coefficient
Wing root bending moment coefficient
Reference chord length
Section lift coefficient
Section moment coefficient
Section hinge moment coefficient
Matrix that gives downwash (normalwash) in terms of pressures
Deflection normal to lifting surface
Unit vector in direction of axis
Reduced frequency $\frac{\omega \bar{C}}{2 U_{\infty}}$
An average Mach Number between $M_{2}$ and $M_{\infty}$

| $M_{2}$ | Local Mach Number at wing surface |
| :---: | :---: |
| $M_{\infty}$ | Free stream Mach Number |
| $\bar{M}_{\infty}$ | Average Mach Number between sending and receiving point |
| R | Compressible radius (see Eqn. 73) |
| S | Matrix that integrates pressures into aerodynamic parameters (e.g., $C_{L}, C_{M}, C_{\ell}$, etc.) |
| $S_{p}$ | $[S]\left[\Delta C_{p_{t}}\right]$ see Equation (9) |
| $S_{w}$ | [S][A][W] see Equation (30) |
| $\bar{s}$ | See Equation (45) |
| $\overline{\bar{S}}$ | [S] $\left.\mathrm{E}_{0}^{\phi_{1}^{+}} \mathrm{I}\right]$ ] See Equation (54) |
| $S_{\text {p }}^{*}$ | See Equation (67.) |
| T | Weights given to the correction factors $\bar{\varepsilon}$ for the minimization process, $\sum T \bar{\varepsilon}^{-2}=\min$ |
| t | Time |
| $\mathrm{U}_{\infty}$ | Free stream velocity |
| $V_{L}$ | Local surface speed |
| W | Correction factor $=1+\varepsilon$ (Called CF in computer program) |
| w | Downwash (or normalwash) (Called W in computer program) |
| wT | Weights in the minimization process for estimates |
| $x, y, z$ | Cartesian coordinates right handed system $x$ aft, y lateral (starboard), z vertical |
| $\alpha$ | Angle-of-attack, also direction cosine for force or moment axis |
| B | $\sqrt{1-M_{\infty}^{2}}$ |
| $\gamma$ | Direction cosine for force or moment axis |


| $\bar{\gamma}$ | Dihedral of lifting surface |
| :---: | :---: |
| $\Delta C_{p}$ | Lower surface minus upper surface pressure coefficient |
| $\Delta A$ | Box area |
| $\varepsilon$ | Incremental correction factors $=W-1$ |
| $\bar{\varepsilon}$ | Generalized incremental correction factors $\varepsilon=\phi \bar{\varepsilon}$ |
| $\tilde{\varepsilon}$ | $\varepsilon \sqrt{T}$ |
| ¢ | Correction factor mode shapes |
| $\phi_{d}$ | Doublet potential function |
| $\omega$ | Circular Frequency |
|  | Subscripts and Superscripts |
| a | Stands for either p or w |
| d | Designated or known correction factors |
| e | Experimental |
| q | Identifies estimates as opposed to constraints |
| H | Hermetian transpose |
| mod. | Modified values |
| $p$ | Identifies pressure modifying terms in the correction factor procedure |
| t | Theoretical values |
| u | Undesignated or unknown correction factors |
| w | Identifies downwash modifying terms |
| 1 | Deflection mode 1 |
| 2 | Deflection mode 2 |
| 3/4 | Three quarter chord point |
| 1/4 | One quarter chord point |

Matrix Notation
\{ \} Column Matrix
[ ] Rectangular
「」 Diagonal

# THEORETICAL DEVELOPMENT 

## Basic Method

Premultiplying Correction Factor Matrix. - A derivation of a real premultiplying correction matrix constrained to match aerodynamic data from a single downwash mode is presented in references 1-4. The use of Lagrange multipliers considerably simplifies the derivation so we present this alternative derivation here. As an introduction we will rederive the same case first, i.e., the premultiplier for a single mode; then we will consider multiple modes and the postmultiplier. Whether the correction matrix is real or complex depends only on the experimental data: static data lead to a real matrix and oscillatory data lead to a complex matrix

Assume that we have a matrix [A] of theoretical aerodynamic influence coefficients (AIC's) that relates the theoretical pressures $\left\{C_{p_{f}}\right\}$ on a set of aerodynamic finite elements to the dimensionless downwashes $\left\{_{W}\right\}$ at the same aerodynamic elements by

$$
\begin{equation*}
\left\{\Delta C_{P_{t}}\right\}=[A]\{W\} \tag{1}
\end{equation*}
$$

The AIC's correspond to the reduced frequency of the experimental data and, hence, are real for static data and complex for oscillatory data. The premultiplying correction matrix $\left\lceil W_{p}\right\rfloor$ is used to obtain an estimate of the experimental pressure distribution $\left\{\Delta C_{p_{e}}\right\}$ from the theoretical distribution from

$$
\begin{equation*}
\left\{\Delta C_{p_{e}}\right\}=\left\lceil W_{p}\right\rfloor\left\{\Delta C_{p_{t}}\right\} \tag{2}
\end{equation*}
$$

The subscript $p$ refers to modification of the pressure distribution. The experimental force distribution is usually not known from the test data but only the integrated force and moment coefficients $\left\{C_{e}\right\}$ are measured. An integration matrix [S] relates the experimental force distribution to the measured force coefficients through;

$$
\begin{equation*}
\left\{C_{e}\right\}=[S]\left\{\Delta C_{p_{e}}\right\} \tag{3}
\end{equation*}
$$

Combining equations (1) - (3) yields

$$
\begin{equation*}
\left\{C_{e}\right\} \cdot=[S]\left\ulcorner W_{p}\right\rfloor[A]\{w\} \tag{4}
\end{equation*}
$$

which is the equation to be solved for the correction matrix $\left\lceil W_{p}\right\rfloor$ given all the remaining terms in the equation. The remaining terms are all known: $\left\{C_{e}\right\}$ and $\{w\}$ are obtained from the test data, and [S] and [A] are known from the mathematical model and the theoretical aerodynamic analysis of the configuration. In general, equation (4) is underdetermined, i.e., there are many more unknowns than equations. The method of least squares provides a solution. We require that changes in the theoretical load distribution shall be as uniform as possible or, in least-squares terminology, the weighted sum of the squares of the deviations shall be a minimum, where the deviation $\left\{\varepsilon_{p}\right\}$ is defined as the difference between the correction factors and unity.

$$
\begin{equation*}
\left\{\varepsilon_{p}\right\}=\left\{W_{p}-I\right\} \tag{5}
\end{equation*}
$$

We denote the weighting function by $T_{p}$; it will be discussed below. The weighted least-squares condition then becomes

$$
\begin{align*}
\sum T_{p} \varepsilon_{p}^{2} & \left.=\left\{\varepsilon_{p}\right\}^{H} \Gamma T_{p}\right\rfloor\left\{\varepsilon_{p}\right\}  \tag{6}\\
& =\text { a minimum }
\end{align*}
$$

where $H$ denotes a Hermitian (complex conjugate) transpose. The Lagrange multipliers may be introduced by defining the error functional

$$
\begin{equation*}
f_{p}=(1 / 2)\left\{\varepsilon_{p}\right\}^{H}\left\lceil T_{p} j\left\{\varepsilon_{p}\right\}\right. \tag{7}
\end{equation*}
$$

and rewriting the measured generalized force coefficients (the constraints) as

$$
\begin{aligned}
\left\{C_{e}\right\} & =[s]\left[1+\varepsilon_{p}\right]\left\{\Delta C_{p_{t}}\right\} \\
& \left.=[s]\left\{\Delta C_{p_{t}}\right\}+[s] \Gamma \Delta C_{p_{t}}\right\}\left\{\varepsilon_{p}\right\}
\end{aligned}
$$

The term $[s]\left\{\Delta C_{p_{t}}\right\}$, is just the theoretical integrated pressures which are the theoretical cdefficients, $\left\{C_{t}\right\}$. Thus

$$
\begin{equation*}
\left\{c_{t}\right\}=[s]\left\{\Delta C_{p_{t}}\right\} \tag{8}
\end{equation*}
$$

Introducing also the following

$$
\begin{align*}
& {\left[S_{p}\right]=[S]\left\lceil\Delta C_{P_{t}}\right\}}  \tag{9}\\
& \left\{\Delta C_{e}\right\}=\left\{C_{e}\right\}-\left\{C_{t}\right\}
\end{align*}
$$

gives finally

$$
\begin{equation*}
\left\{\Delta C_{e}\right\}=\left[S_{p}\right]\left\{\varepsilon_{p}\right\} \tag{10}
\end{equation*}
$$

The variation of the error functional $f_{p}$ is

$$
\begin{equation*}
\delta f_{p}=\left\{\varepsilon_{p}\right\}^{H}\left[T_{p}\right]\left\{\delta \varepsilon_{p}\right\} \tag{11}
\end{equation*}
$$

and the variations of the incremental constraints, $\Delta C_{e}$, given by equation (10) are

$$
\begin{aligned}
\left\{\delta \Delta C_{e}\right\} & =\left[S_{p}\right]\left\{\delta \varepsilon_{p}\right\} \\
& =0
\end{aligned}
$$

The condition for the minimum subject to the constraints is then a linear combination of equations (11) and (12) set to zero in which the linear factors
are the Lagrange multipliers $\lambda_{p}$,

$$
\begin{equation*}
\delta f_{p}+\left\{\lambda_{p}\right\}^{H}\left\{\delta \Delta C_{e}\right\}=0 \tag{13}
\end{equation*}
$$

Substituting equations (11) and (12) into equation (13) yields

$$
\left(\left\{\varepsilon_{p}\right\}^{H}\left[T_{p}\right]+\left\{\lambda_{p}\right\}^{H}\left[S_{p}\right]\right)\left\{\delta \varepsilon_{p}\right\}=0
$$

Since the variation $\left\{\delta \varepsilon_{p}\right\}$ is arbitrary,

$$
\left\{\varepsilon_{p}\right\}^{H}\left[T_{p}\right]+\left\{\lambda_{p}\right\}^{H}\left[S_{p}\right]=0
$$

or, after Hermitian transposition.

$$
\begin{equation*}
\left[T_{p} j\left\{\varepsilon_{p}\right\}+\left[S_{p}\right]^{H}\left\{\lambda_{p}\right\}=0\right. \tag{14}
\end{equation*}
$$

The simultaneous solution of equations (5), (10) and (14) yields the desired solution. The simultaneous solution leads first to the Lagrange‘multipliers and then to $\varepsilon_{p}$ as follows:

$$
\begin{gather*}
\left\{\lambda_{p}\right\}=-\left(\left[S_{p}\right]\left\lceil T_{p}\right]^{-1}\left[S_{p}\right]^{H}\right)^{-1}\left\{\Delta C_{e}\right\}  \tag{15}\\
\left\{\varepsilon_{p}\right\}=-\left\lceil T_{p}\right]^{-1}\left[S_{p}\right]^{H}\left\{\lambda_{p}\right\} \tag{16}
\end{gather*}
$$

and the correction factors are then

$$
\begin{equation*}
\left\{W_{p}\right\}=\{I\}+\left\{\varepsilon_{p}\right\} \tag{17}
\end{equation*}
$$

The premultiplying correction factors are written in a diagonal format for use in subsequent aeroelastic analyses as in equation (2).

The above results can be restated in summary form as follows:

$$
\begin{equation*}
\text { Solution of } \rightarrow\left\{\Delta C_{e}\right\}=\left[S_{p}\right]\left\{\varepsilon_{p}\right\} \tag{18}
\end{equation*}
$$

$$
\begin{align*}
\text { subject to } & \rightarrow \sum \varepsilon_{p}^{2} T_{p}=\min .  \tag{19}\\
& \rightarrow\left\{\tilde{\varepsilon}^{2}\right\}=\left[\tilde{S}_{p}\right]^{H}\left(\left[\tilde{S}_{p}\right]\left[\tilde{S}_{p}\right]^{H}\right)^{-1}\left\{\Delta C_{e}\right\}  \tag{20}\\
\text { where }\left[\tilde{S}_{p}\right] & =\left[S_{p}\right]\left[\sqrt{T_{p}}\right]^{-1}  \tag{21}\\
\text { and }\left\{\varepsilon_{p}\right\} & =\{\tilde{\varepsilon}\} / \sqrt{T_{p}} \tag{22}
\end{align*}
$$

The weighting function $T_{p}$ is arbitrary; the only requirement on it is that it should be positive. However, engineering judgment provides some guidance: if only a single constraint, e.g., the lift curve slope, is available, one would prefer all the correction factors to be simply the ratio of its experimental value to its theoretical estimate. Accordingly, the recommended choice for the weighting function for the premultiplier is

$$
\begin{equation*}
\left\{T_{p}\right\}=|[A]\{I\}| \tag{23}
\end{equation*}
$$

However, other choices may be deserving of further investigation:
Multiple Modes. - We next consider multiple experimental downwash modes. The derivation will be presented using two modes $\left\{w_{1}\right\}$ and $\left\{w_{2}\right\}$. For these two modes, the theoretical pressure distributions are

$$
\begin{aligned}
& \left\{\Delta C_{\left.p_{t}\right]}\right\}=[A]\left\{w_{1}\right\} \\
& \left\{\Delta C_{p_{t 2}}\right\}=[A]\left\{w_{2}\right\}
\end{aligned}
$$

so the incremental experimental force coefficients $\Delta C_{e_{1}}$ and $\Delta C_{e_{2}}$ become

$$
\begin{align*}
\left\{\Delta C_{e l}\right\} & =\left\{C_{e}\right\}-[S]\left\lceil\Delta C_{p_{t 1}}\right\rfloor\left\{W_{p}\right\} \\
& \equiv\left\{C_{e}\right\}-\left[S_{p l}\right]\left\{W_{p}\right\} \tag{24}
\end{align*}
$$

$$
\begin{align*}
\left\{\Delta C_{e 2}\right\} & =\left\{C_{e}\right\}-[S]\left\lceil\Delta C_{p_{t}}\right\rfloor\left\{W_{p}\right\} \\
& \equiv\left\{C_{e}\right\}-\left[S_{p 2}\right]\left\{W_{p}\right\} \tag{Continued}
\end{align*}
$$

These two equations may be combined into one set as follows:

$$
\begin{equation*}
\left\{\Delta C_{e}\right\}=\left[S_{p}\right]\left\{\varepsilon_{p}\right\} \tag{25}
\end{equation*}
$$

where

$$
\left\{\Delta C_{e}\right\}=\left\{\begin{array}{c}
\Delta C_{e_{1}} \\
\bar{\Delta} C_{e_{2}}^{-}
\end{array}\right\}
$$

and

$$
\begin{equation*}
\left[S_{p}\right]=\left[s_{p_{1}}: s_{p_{2}}\right] \tag{26}
\end{equation*}
$$

If again we impose the minimization condition

$$
\begin{equation*}
\sum \varepsilon_{p}^{2} T_{p}=\min \tag{27}
\end{equation*}
$$

then the solution is identical to equation (20) since equations (25) and (27) are identical to (18) and (19).

Postmultiplying Correction Factor Matrix. - Now we consider the postmultiplying correction matrix. It is only necessary to consider a single downwash mode; the multiple mode case can be generalized by reference to equations (25) and (26). Since the postmultiplier modifies the downwash mode it defines an effective experimental downwash given by

$$
\left\{w_{e}\right\}=\left\lceil W_{w}\right\rfloor\{w\}
$$

in which the subscript $w$ refers to modification of the downwash. Our new estimate of the experimental pressure distribution becomes:

$$
\begin{align*}
\left\{\Delta C_{p_{e}}\right\} & =[A]\left\{W_{e}\right\} \\
& =[A]\left[W_{w}\right]\{w\} \tag{28}
\end{align*}
$$

The experimental generalized forces or force coefficients are again given by equation (3) which with equation (28) becomes

$$
\begin{align*}
\left\{C_{e}\right\} & =[S][A]\left[1+\varepsilon_{w}\right\rfloor\{w\} \\
& =[S]\left\{\Delta C_{P_{t}}\right\}+[S][A][w]\left\{\varepsilon_{w}\right\} \tag{29}
\end{align*}
$$

And again noting that $[S]\left\{\Delta \mathrm{C}_{\mathrm{p}}\right\}=\left\{\mathrm{C}_{\mathrm{t}}\right\}$ and introducing

$$
\begin{equation*}
\left[S_{w}\right]=[S][A][w] \tag{30}
\end{equation*}
$$

gives

$$
\begin{align*}
\left\{C_{e}\right\}-\left\{C_{t}\right\} & =\left[S_{w}\right]\left\{\varepsilon_{w}\right\} \\
\left\{C_{e}\right\}-\left\{C_{t}\right\} & =\left\{\Delta C_{e}\right\} \\
\left\{\Delta C_{e}\right\} & =\left[S_{w}\right]\left\{\varepsilon_{w}\right\} \tag{31}
\end{align*}
$$

or since

Again the minimization condition is imposed,

$$
\begin{equation*}
\sum \varepsilon_{w}^{2} T_{w}=\min . \tag{32}
\end{equation*}
$$

The solution for $\varepsilon_{w}$ is then identical to equation (20) since equations (31) and (32) are identical to equations (18) and (19). Again the correction factors are calculated from $\varepsilon_{\mathrm{w}}$ as follows:

$$
\begin{equation*}
\left\{W_{w}\right\}=\{I\}+\left\{\varepsilon_{w}\right\} \tag{33}
\end{equation*}
$$

The weighting function $T_{W}$ is also arbitrary. However, the considerations that led to the recommendation of equation (23), in the premultiplying case, also lead to

$$
\begin{equation*}
\left\{T_{W}\right\}=\left|\{I\}^{T}[A]\right| \tag{34}
\end{equation*}
$$

which is to say that the weighting function is the lift coefficient induced by a unit downwash at each lifting element. Equation (34) is the recommended choice for the weighting function in the postmultiplying case, although other choices may still warrant further investigation.

For multiple modes, say two, equations (31) and (32) provide the following:

$$
\begin{align*}
& \left\{\Delta C_{e_{1}}\right\}=\left[S_{W_{1}}\right]\left\{\varepsilon_{w}\right\}  \tag{35}\\
& \left\{\Delta C_{e_{2}}\right\}=\left[S_{w_{2}}\right]\left\{\varepsilon_{w}\right\}  \tag{36}\\
& \sum \varepsilon_{w}^{2} T_{w}=\min \tag{37}
\end{align*}
$$

Again equations (35) and (36) can be combined into one as follows:

$$
\begin{equation*}
\left\{\Delta C_{e}\right\}=\left[S_{W}\right]\left\{\varepsilon_{w}\right\} \tag{38}
\end{equation*}
$$

where

$$
\left\{\Delta C_{e}\right\}=\left\{\begin{array}{c}
\Delta C_{e_{1}} \\
--C_{e_{2}}
\end{array}\right\}
$$

and where

$$
\begin{equation*}
\left[S_{W}\right]=\left[S_{W_{1}}: S_{W_{2}}\right] \tag{39}
\end{equation*}
$$

Equations (38) and (37) are now identical to equations (18) and (19) respectively and thus the solution is the same as before, i.e., equation (20).

## Modifications to the Basic Method

In some instances correction factors become unrealistic. In order to correct this situation when it occurs or to minimize the probability of its occurrence initially, various modifications can be introduced. Three such modifications are discussed here, i.e., estimates, correction factor modes and limits. Estimates are like constraints except that the "constraining power" can be varied. Correction factor modes constrain the distribution of correction factors such that the final distribution is a superposition of a limited set of well behaved, user input mode shapes. The "limit" feature constrains the correction factors (or any subset of them) to be above a given minimum and below a given maximum.

Estimates. - In some instances data will be available in the form of estimates. These estimates can be based on past data, data from related configurations, two-dimensional data, empirical methods, or just past experfence. In any case they do not have equal weight with the experimental data considered so far. Consider the case where some experimental data are available, leading to $\left\{\Delta C_{e}\right\}$, then the usual equation applies to these data:

$$
\left\{\Delta C_{e}\right\}=\left[S_{a}\right]\left\{\varepsilon_{a}\right\}
$$

where the subscript "a" stands for either p or w. If estimates exist, leading to $\left\{\Delta C_{g}\right\}$, then it is desirable to minimize the difference between these astimates and the modified theoretical values. Let this difference be termed $\left\{\varepsilon_{\mathrm{g}}\right\}$, then:

$$
\left\{\Delta C_{g}\right\}=\left[S_{g}\right]\left\{\varepsilon_{a}\right\}+\left\{\varepsilon_{g}\right\}
$$

where $\left\{\Delta C_{g}\right\}=\left\{C_{g}-C_{t}\right\}$ and $S_{g}$ is analogous to $S_{a}$ with the exception that it refers to the estimates $C_{g}$ and not the constraints $C_{e}$. Thus we wish to minimize both $\left\{\varepsilon_{a}\right\}$ and $\left\{\varepsilon_{g}\right\}$ together and this is done as follows:

$$
\left\{\begin{array}{c}
\Delta c_{e} \\
\hdashline \Delta C_{g}
\end{array}\right\}=\left[\begin{array}{c:c}
S_{a} & 0 \\
\hdashline S_{g} & I
\end{array}\right]\left\{\begin{array}{c}
\varepsilon_{a} \\
\hdashline \varepsilon_{g}
\end{array}\right\}
$$

This equation can then be solved in the usual manner producing the following result:

$$
\sum T \varepsilon_{\mathrm{a}}^{2}+\sum \varepsilon_{\mathrm{g}}^{2}=\min
$$

If it is desired to give the $\varepsilon_{g}$ values more or less weight in the minimization scheme, then the $\varepsilon_{g}$ values must be weighted.

$$
\begin{equation*}
\sum T \varepsilon_{a}^{2}+\sum\left(w T \varepsilon_{g}\right)^{2}=\min \tag{40}
\end{equation*}
$$

The equations for the constraints then become

$$
\left\{\begin{array}{c}
\Delta c_{e}  \tag{41}\\
\hdashline \Delta c_{g}
\end{array}\right\}=\left[\begin{array}{c:c}
s_{a} & 0 \\
\hdashline s_{g} & \frac{1}{w T}
\end{array}\right]\left\{\begin{array}{c}
\varepsilon_{a} \\
-\varepsilon_{-} \\
\varepsilon_{w T}
\end{array}\right\}
$$

where

$$
\begin{equation*}
\left\{\varepsilon_{W T}\right\}=[w T]\left\{\varepsilon_{g}\right\} \tag{42}
\end{equation*}
$$

and where the values $w T$ are the weights assigned to the errors $\varepsilon_{g}$. If the estimates are of high quality then the weights will be large. In the limit as $W T \rightarrow \infty, \Delta C_{g}$ becomes a constraint instead of an estimate and equation (41) reduces to the form of equation (18). Equation (41) can also be cast into the same form as equation (18) for the general case, i.e., wT finite, as follows:

$$
\begin{equation*}
\left\{\Delta C_{e}\right\}=[\bar{S}]\{\varepsilon\} \tag{43}
\end{equation*}
$$

with

$$
\begin{align*}
& \left\{\Delta C_{e}\right\}=\left\{\begin{array}{l}
\Delta C_{e} \\
\hdashline \Delta C_{g}
\end{array}\right\}  \tag{44}\\
& {[\bar{S}]=\left[\begin{array}{l:l}
S_{a} & 0 \\
\hdashline S_{g} & \frac{1}{W T}
\end{array}\right\}}  \tag{45}\\
& \{\varepsilon\}=\left\{\begin{array}{l}
\varepsilon_{a} \\
-\varepsilon_{W T}
\end{array}\right\} \tag{46}
\end{align*}
$$

Equation (40) can be written in terms of $\varepsilon$ as:

$$
\begin{equation*}
\sum T^{*} \varepsilon^{2}=\min \tag{47}
\end{equation*}
$$

where

$$
T^{\star}=\left\{\begin{array}{l}
T \text { for constraints } \\
\frac{1}{(W T)^{2}} \text { for estimates }
\end{array}\right.
$$

Thus equation (43) and (47) are formally identical to equation (18) and (19) and thus have the same solution, i.e., equation (20).

Currently the term $\frac{1}{W T}$ is obtained from a term a where

$$
\begin{gather*}
\frac{1}{w^{T}}=\frac{1-\frac{\tilde{a}}{\tilde{a}}}{a}  \tag{48}\\
10^{-4} \leq \tilde{a} \leq 1.0
\end{gather*}
$$

where $\tilde{a}$ is called the constraining power of the estimate $C_{g}$.

Correction Factor Modes. - The correction factors can be expressed in terms of a set of modes $\phi$ as follows:

$$
\begin{equation*}
\{\varepsilon\}=[\phi]\{\bar{\varepsilon}\} \tag{49}
\end{equation*}
$$

Placing equation (49) into (43) gives

$$
\begin{align*}
\left\{\Delta C_{e}\right\} & =[\bar{S}][\phi]\{\bar{\varepsilon}\} \\
& =[\overline{\bar{S}}]\{\bar{\varepsilon}\} \tag{50}
\end{align*}
$$

where

$$
\begin{equation*}
[\overline{\bar{s}}]=[\bar{s}][\phi] \tag{51}
\end{equation*}
$$

If the minimization process is applied to $\bar{\varepsilon}$ as usual

$$
\begin{equation*}
\sum \varepsilon^{-2}=\min \tag{52}
\end{equation*}
$$

Here the weight $T$ is missing since it is usually not used with modes. Equations (50) and (52) are then identical to equations (18) and (19) and thus have the same solution, i.e., equation (20). A similar expression exists for the postmultiplying correction factors. This approach allows a bias based on experience and past tests and physical reasoning, to be built into the correction factors. When estimates are considered equation (50) must be altered since the $\varepsilon_{W T}$ are not fitted with correction factor modes. Thus

$$
\{\varepsilon\}=\left\{\begin{array}{c}
\varepsilon_{a}  \tag{53}\\
\varepsilon_{-} \\
\varepsilon_{w T}
\end{array}\right\}=\left[\begin{array}{c:c}
\Phi & 0_{-} \\
0 & I
\end{array}\right]\left\{\begin{array}{c}
\bar{\varepsilon}_{a} \\
-\varepsilon_{-} \\
\varepsilon_{w T}
\end{array}\right\}
$$

and thus

$$
[\overline{\bar{s}}]=[\bar{s}]\left[\begin{array}{c:c}
\Phi & 0  \tag{54}\\
0 & I
\end{array}\right]
$$

and then the solution proceeds as before.
Limits. - If certain basic properties of the weight factors are known, they could be limited to fall within a given set of bounds. If for instance the sign of an incremental weight factor is known to be positive, then it could be constrained to be positive. Also, for practical reasons, the maximum value of the weight factors should be limited and thus the incremental weight factors are constrained to lie below this maximum. In general, the weight factors can be constrained to lie between a maximum and a minimum.

$$
\begin{equation*}
\bar{\varepsilon}_{\min } \leq \bar{\varepsilon} \leq \bar{\varepsilon}_{\max } \tag{55}
\end{equation*}
$$

Notice that the generalized incremental correction factors, $\bar{\varepsilon}$, are the ones limited in the solution and not the actual ones, $\varepsilon$. The values of $\bar{\varepsilon}$ are the coefficients of the correction factor modes, $\phi$, and not the incremental correction factors themselves.

The basic procedure would be to set any generalized incremental weight factor to its maximum or minimum value if it exceeded these limits. This would require a multistep operation: (1) solving for the factors, (2) checking and setting those that exceeded the limits to the limit values, and (3) resolving. Before this can be done a capability must exist for assigning weight factors special values. This is easily accomplished as follows:

$$
\left\{\Delta C_{e}\right\}=\left[\overline{\bar{S}}_{u} \mid \overline{\bar{S}}_{d}\right]\left\{\begin{array}{l}
\bar{\varepsilon}_{u}  \tag{56}\\
\bar{\varepsilon}_{d}
\end{array}\right\}
$$

where $[\overline{\bar{S}}]$ is defined in equation (51). The subscript $u$ indicates those factors that are undesignated and d indicates those that are designated. This equation can be solved for $\left\{\varepsilon_{u}\right\}$ in terms of the known quantities:

$$
\begin{equation*}
\left\{\Delta C_{e}\right\}-\left[\overline{\bar{S}}_{d}\right]\left\{\bar{\varepsilon}_{d}\right\}=\left[\overline{\bar{S}}_{u}\right]\left\{\bar{\varepsilon}_{u}\right\} \tag{57}
\end{equation*}
$$

Equation (57) effectively eliminates the designated factors from the minimi-
zation process. This equation can then be solved in the usual manner for $\left\{\bar{\varepsilon}_{u}\right\}$ since $\left\{\bar{\varepsilon}_{d}\right\}$ is given. Specifically

$$
\begin{equation*}
\left\{\Delta C_{e_{\text {mod }}}\right\}=\left[\overline{\bar{S}}_{u}\right]\left\{\bar{\varepsilon}_{u}\right\} \tag{58}
\end{equation*}
$$

where

$$
\begin{equation*}
\left\{\Delta C_{e_{\text {mod }}}\right\}=\left\{\Delta C_{e}\right\}-\left[\overline{\bar{S}}_{d}\right]\left\{\bar{\varepsilon}_{d}\right\} \tag{59}
\end{equation*}
$$

The minimization scheme is then

$$
\begin{equation*}
\sum T^{*} \bar{\varepsilon}_{u}^{2}=\min \tag{60}
\end{equation*}
$$

Equations (58) and (60) are now formally identical to equations (18) and (19) and thus the solution is identical to equation (20). In the computer program the final $\bar{\varepsilon}$ array that are modified or have reached their limits is called $\overline{\bar{\varepsilon}}$.

## A New Postmultiplying Correction Factor Matrix

The postmultiplying correction factor matrix developed in a previous section has been applied successfully to wings operating in pitch. Problems arise however when control surface modes are used. The discussion to be presented in the "Correlation Studies" section describes some of these problems. As a result of these, a new postmultiplying correction factor matrix was developed and it is derived here.

Viscous effects on airfoils can be thought of in terms of a displacement thickness added to the airfoil. The difference between the upper and lower surface displacement thicknesses produces a "decambering" of the airfoil or a change in the downwash $w$.

$$
\begin{equation*}
w_{e}=w+\delta w \tag{61}
\end{equation*}
$$

The changes in downwash, $\delta w$, exist over the entire airfoil or wing and not just in the region where $w$ is non zero. These changes are a function of the pressure distribution on the airfoil. Usually the displacement thickness at a point is an integral function of the pressure distribution upstream of that point. If the general case of correction factor mode shapes is assumed then the downwash correction $\delta \mathrm{w}$ can be expressed as:

$$
\begin{equation*}
\{\delta w\}=[\phi]\{\delta \overline{\mathrm{e}}\} \tag{62}
\end{equation*}
$$

where $\{\delta \overline{\mathrm{e}}\}$ is proportional to the integrated pressires, $\lceil\ell\rfloor$.

$$
\begin{equation*}
\{\delta \overline{\mathrm{e}}\}=\lceil\ell]\{\bar{\varepsilon}\} \tag{63}
\end{equation*}
$$

where「 $\ell]$ is given in terms of an integration matrix [ $N$ ] and the pressures $\left\{\Delta C_{p}\right\}$.

$$
\begin{equation*}
\{\ell\}=[N]\left\{\Delta C_{p}\right\} \tag{64}
\end{equation*}
$$

Combining equations (63) with (62) and using the result to obtain the corrected pressures leads to

$$
\begin{align*}
\left\{\Delta C_{p_{e}}\right\} & =[A]\left\{w_{e}\right\} \\
& =[A]\{w+[\phi]\lceil\ell]\{\bar{\varepsilon}\}\} \tag{65}
\end{align*}
$$

The constraints $\left\{C_{e}\right\}$ are obtained by integrating $\left\{\Delta C_{p_{e}}\right\}$ as follows:

$$
\begin{align*}
\left\{C_{e}\right\} & =[S]\left\{\Delta C_{p_{e}}\right\} \\
& =[S]\left\{\Delta C_{p_{t}}\right\}+\left[S_{p}^{*}\right]\left\{\varepsilon_{w}\right\} \tag{66}
\end{align*}
$$

where

$$
\begin{equation*}
\left[S_{p}^{*}\right]=[S][A][\phi][\ell] \tag{67}
\end{equation*}
$$

Noting that $\left\{\Delta C_{e}\right\}=\left\{C_{e}\right\}-\left\{C_{t}\right\}$ equation (65) can be written as:

$$
\begin{equation*}
\left\{\Delta C_{e}\right\}=\left[S_{p}^{\star}\right]\left\{\varepsilon_{w}\right\} \tag{68}
\end{equation*}
$$

Equation (68) has a form identical to that of equation (18) except $\left[S_{p}\right]$ is replaced by $\left[S_{p}^{*}\right]$ and thus has the same solution, i.e., equation (20). Once found, $\left\{\varepsilon_{W}\right\}$ can be placed into the expression for $\left\{\Delta C_{p_{e}}\right\}$, in equation (65), and the desired modified pressure found.

Currently in the computer program the matrix [N] is simply either the identity matrix or the matrix $[\phi]^{\top}$. The identity matrix implies that the correction to the downwash is proportional to the local lifting pressure. In addition the above derivation is good for only one mode and thus the multiple mode option can not be used with the new postmultiplier. The new postmultiplier can be extended to multiple modes by simply replacing

$$
\left\{\Delta C_{e}\right\} \quad \text { with }\left\{\begin{array}{c}
\Delta C_{e_{1}}  \tag{69}\\
\Delta C_{e_{2}}
\end{array}\right\}
$$

and

$$
\left[S_{p}^{*}\right] \text { with }\left[\begin{array}{c}
S_{p_{1}^{*}}^{*}  \tag{70}\\
\hdashline S_{P_{2}^{*}}^{*}
\end{array}\right]
$$

but this has not yet been tried.

Empirical modification of theory is most meaningful if the theory qualitatively matches experimental data. If the theory misses an important feature of the data the modified theory will also usually miss it. Transonic effects fall in this category. The classic lifting surface theory makes no provision for transonic effects and it is the purpose of this section to investigate some simple modifications to help remedy this situation.

Direct Application of Local Mach Number. - Several methods based on the steady local Mach Number distribution have been tried and the results are discussed in later sections.

One of these methods, discussed in the Introduction, consists of making a simple substitution of a local Mach Number distribution in place of its free stream value both in the kernel function and in the boundary conditions and pressure equation (see refs. 7 and 10). The local Mach Number distribution is taken from steady flow results. For the kernel function, application of a Mach Number distribution that lies somewhere in between the surface values and the free stream value was used. Tijdeman and Zwaan (ref. 10) suggest a local Mach Number distribution that lies half way in between the actual local and the free stream values. The reason for this is that acoustic signals propagate to the surface along various paths out in the fluid and thus propagate at some average between the surface value and free stream value. For the kernel, the local receiving point value of Mach Number and the free stream values were averaged and used in place of the free stream value.

For the normalwash boundary condition and pressure evaluation the local Mach Number on the surface was used. Specifically, if $M_{2}(x)$ is the local surface steady Mach Number distribution then the normalwash boundary condition $w$ is (see ref. 10):

$$
\begin{equation*}
\frac{w}{U_{\infty}}=\frac{M_{2}(x)}{M_{\infty}} \frac{\partial h}{\partial x}+i \frac{\omega}{U_{\infty}} h \tag{71}
\end{equation*}
$$

The second order Bernoulli Equation for steady flow is (see ref. 7):

$$
\begin{equation*}
\Delta C_{P_{s o}}=\Delta C_{p}\left[1+\beta^{2}\left(\frac{M_{2}(x)}{M_{\infty}}-1\right)\right] \tag{72}
\end{equation*}
$$

where $\Delta C_{p}$ and $\Delta C_{P_{\text {so }}}$ are the first and second order pressures respectively. This method did not prove to be all that was hoped for and thus a second method was investigated.

A New Transonic Effects Method. - In this section a derivation of the newly developed Douglas transonic effects method is presented. The basic method was conceived under the McDonnell-Douglas IRAD program however its implementation in two-dimensions and its application to the airfoil-control surface problem was done under the current contract.

The lifting surface method is based on the following expression for the potential of a doublet, $\phi_{d}$ :

$$
\begin{equation*}
\phi_{d}=\frac{\partial}{\partial n}\left\{\frac{e^{i \omega t} e^{i \lambda\left[M_{\infty}(x-\xi)-R\right]}}{R}\right\} \tag{73}
\end{equation*}
$$

where

$$
\lambda=\frac{\omega M_{\infty}}{\beta^{2} U_{\infty}}, \quad R=\sqrt{(x-\xi)^{2}+\beta^{2} r^{2}}
$$

and $n$ is the direction normal to the lifting surface. This expression can be rewritten as:

$$
\begin{equation*}
\phi_{d}=\frac{\partial}{\partial n}\left\{\frac{e^{i \omega(t-\tau)}}{R}\right\} \tag{74}
\end{equation*}
$$

where

$$
\begin{equation*}
\tau=\frac{M_{\infty}}{\beta^{2} U_{\infty}}\left[R-M_{\infty}(x-\xi)\right] \tag{75}
\end{equation*}
$$

It can be shown that $\tau$ has a physical significance. The term $\tau$ is the acoustic time delay between the sending and receiving points. That is, $\tau$ is the time it takes an acoustic signal, originating at the point $\xi, n, \zeta$ to reach the point $x, y, z$ as the acoustic pulse washes downstream.

This statement can be illustrated by the example of figure 1.


Figure 1

A signal is emitted at $(\xi, \eta, \zeta)$ at time $\tau=0$. At time $\tau$ the wave front, traveling at the speed of sound, $a_{\infty}$, has reached the receiving point at $(x, y, z)$. During this time the wave center has travelled a distance $U_{\infty} \tau$. Using the right triangle relations gives:

$$
\begin{equation*}
r^{2}+\left(\xi-x+U_{\infty} \tau\right)^{2}=a_{\infty}^{2} \tau^{2} \tag{76}
\end{equation*}
$$

Solving for $\tau$ using the solution for a quadratic equation gives:

$$
\tau=\frac{M_{\infty}}{U_{\infty} \beta^{2}}\left(M_{\infty}(\xi-x)+R\right)
$$

which is exactly what is given in equation 75 .

Thus $\tau$ in the expression for $\phi_{d}$ has physical significance and it is the acoustic time delay between the sending and receiving points in a fluid moving with uniform velocity. This physical insight can form the basis of a correction factor for the theory. For instance if the wave is in a flow field whose velocity varies in the longitudinal direction then the distance $d$ travelled by the wave center is not $U_{\infty} \tau$ but is:

$$
\begin{equation*}
d=\int^{\tau} U(t) d t \tag{77}
\end{equation*}
$$

If we consider $U(t)$ to be made up of $U_{\infty}+\delta U(t)$ then $d$ also can be so split.

$$
\begin{align*}
d & =U_{\infty} \tau+\delta d  \tag{78}\\
\delta d & =\int_{0}^{\tau} \delta U(t) d t \tag{79}
\end{align*}
$$

The wave center velocity $U$ is being discussed, however this is not the velocity of the fluid particle located at the wave center as is the case for a uniform flow. The velocity $U$ actually reflects the wave front speed and location and is the speed of an imagined wave center for the wave that strikes the receiving point. The wave front speed varies around the circumference of the wave, but the most important part of the wave is that part that strikes the receiving point. Thus the velocity $U(t)$ is the time history of the wave center corresponding to that part of the wave that strikes the receiving point ( $x, y, z$ ). As an approximation to the location of this part of the wave, it is assumed that it lies along a line connecting the receiving and sending points (shown dotted in figure 2). Thus $\delta U$ is the difference between the local velocity and the free stream velocity along the dotted line. If a


Figure 2
coordinate $\bar{R}$ is defined lying along this line then the integral in time of equation (79) can be converted into a space integral in $\bar{R}$ as follows:

$$
\delta d=\int_{0}^{R} \delta U(\bar{R}) \frac{d t}{d \bar{R}} d \bar{R}
$$

where $\bar{R}$ is defined below equation (73) and where $\frac{d \bar{R}}{d t}$ is the speed with which the wave front moves along the radial coordinate.

$$
\frac{d \bar{R}}{d t}=\left(a_{\infty} \stackrel{\rightharpoonup}{i_{a}}+U(\bar{R}) \vec{i}\right) \cdot \stackrel{\rightharpoonup}{i}_{R}
$$

The unit vectors $\vec{i}_{a}$, $\vec{i}$ and $\vec{i}_{R}$ are defined in figure 2 .
For example, in the two-dimensional analysis for coplanar surfaces:

$$
\begin{align*}
& \delta d=\int_{x}^{\xi} \Delta U \frac{d t}{d \bar{x}} d \bar{x} \\
& \frac{d \bar{x}}{d t}=a-U(\bar{x}) \tag{80}
\end{align*}
$$

Thus

$$
\begin{equation*}
\delta d=\int_{\xi}^{X} \frac{M(\bar{x})-U_{\infty} / a}{1-M(\bar{x})} d \bar{x} \tag{81}
\end{equation*}
$$

where $M(\bar{x})$ is the local Mach Number distribution. As an approximation set $U_{\infty} / a=U_{\infty} / a_{\infty}=M_{\infty}$.

The time $\tilde{\tau}$ can now be calculated using the right-triangle relations and the quadratic formula solution.

$$
\left[U_{\infty} \tilde{\tau}+\delta d+\xi-x\right]^{2}+r^{2}=a_{\infty}^{2} \tilde{\tau}^{2}
$$

Solving for $\tilde{\tau}$ gives:

$$
\begin{equation*}
\tilde{\tau}=\frac{M_{\infty}}{U_{\infty} \beta^{2}}\left\{M_{\infty}(\xi-\tilde{x})+\tilde{R}\right\} \tag{82}
\end{equation*}
$$

where

$$
\begin{gather*}
\tilde{R}^{2}=(\xi-\tilde{x})^{2}+\beta^{2} r^{2}  \tag{83}\\
\tilde{x}=x-\delta d \tag{84}
\end{gather*}
$$

It is immediately evident that $\tilde{\tau}$ has exactly the same form as $\tau$ (see equation (75)) except that $x$ is replaced, in the expression for $\tau$, by $x$ - $\delta d$ in the expression for $\tilde{\tau}$. In essence then the receiving and sending points
have increased their separation (in the x-direction) as far as the acoustic time delay $\tau$ is concerned. Is there any reason to carry this increase in distance to other parts of the potential function, specifically, to the radius term in the denominator? It seems so. First, this radial distance is already modified in the expression for $\tilde{\tau}$, see equation (82). Second it is known that transonic effects exist in steady flow ( $\omega=0$ ) where $\tau$ has no effect; i.e., $\phi_{d}=\frac{\partial}{\partial n}(1 / R)$. Thus it seems appropriate to add $-\delta d$ to all $x-\xi$ terms. Thus

$$
\begin{equation*}
\tilde{\phi}_{d}\left(x-\xi, y-n, z-\zeta, \omega, M_{\infty}\right)=\phi_{d}\left(x-\xi-\delta d, y-n, z-\zeta, \omega, M_{\infty}\right) \tag{85}
\end{equation*}
$$

where $\tilde{\phi}_{d}$ is the potential modified for transonic effects.
This method has been implemented for the two-dimensional case and the results are discussed subsequently.

One variation of this method that is possible is to use an average Mach Number between sending and receiving points and define $\delta U$ as the difference between the local value of velocity and this average. Thus the term $M_{\infty}$ is replaced with $\bar{M}_{\infty}$ where

$$
\begin{equation*}
\bar{M}_{\infty}=\frac{1}{(x-\bar{\xi})} \int_{\xi}^{x} M(\bar{x}) d x \tag{86}
\end{equation*}
$$

This method has also been tried and results using this variation are also discussed subsequently.

A final consideration is the determination of the local Mach Number distribution, $M(\bar{x})$. Tijdeman and Zwaan (ref. 10) note that the local surface Mach Number distribution should not be used but that some average between it and the free stream Mach Number should be used. This is because the signal arriving at a point has traveled both in the vicinity of the airfoil and out in the flow field. The recommendation of Tijdeman and Zwaan has been adopted in the present method.

Other work by Tijdeman and Bergh (ref. 11) can also be brought to bear on this work. Specifically a fully two-dimensional acoustic solution of a source pulse located at the control surface hinge was calculated for the case of a nonuniform flow field. This solution produced the exact time lag $\tilde{\tau}$ from the hinge line to all other points on the airfoil. The equivalent distance $\tilde{x}$, and also $\tilde{R}$, from the hinge line to the receiving point can then be calculated using this information and the equation relating $\tilde{\tau}$ to $\tilde{x}$. Thus

$$
\xi-\tilde{x}=\sqrt{\tau^{2} U_{\infty}^{2} \beta^{2}-r^{2}}
$$

If acoustic solutions were obtained for all other sending points then all the necesary $\tilde{x}$ for this theory would be available. This method would be accurate, however it would require many expensive acoustic solutions. It appears that each of these solutions requires a computing effort comparable to a direct solution, by finite difference, of the original problem. This conclusion however, remains to be seen and further study is required.

## Local Mach Number Studies

Several methods of accounting for local steady Mach Number variations in the oscillatory lifting surface theory have been studied for the twodimensional case. A mathematical description of these methods has been presented in the Theoretical Development section. The cases considered here are for a two-dimensional symmetric airfoil (NACA 65A006) with an oscillating $25 \%$ chord flap. The local Mach Number variations over the airfoil at zero angle of attack are given in reference 11.

The first and simplest of the methods studied involves simply making a direct substitution of the steady local Mach Number in place of its free stream value. In general, this approach does not produce substantial changes in the pressures from their classical values.

In figure 3 the symbols marked by triangles indicate the pnessures calculated using the local Mach Number in the downwash boundary condition. The pressures are reduced from their classical values (indicated by dots and a dashed line) as expected, but not by very much.

The circles indicate the pressures calculated using the local Mach Number in the downwash as well as in the kernel function. In the kernel function the average between the local receiving point Mach Number and the free stream value is used. The pressures again are generally reduced but not by any substantial amount.

The use of local Mach Number in the second order Bernoulli equation (Ashley, ref. 7) also produces very little change. In figure 5 this change is observed as the difference between the circles and triangles. This change is about the same order of magnitude as the other changes except it is generally in the opposite direction.

The second method studied is new and is described in the Theoretical

Development section. The basic idea of this method is to transform the longitudinal distance between sending and receiving points depending on the time it takes an acoustic signal to travel that distance. A variation of this method is simply to replace the free stream Mach Number, $M_{\infty}$, by $\bar{M}_{\infty}$, an integrated average of the Mach Number distribution between sending element and receiving point defined in equation (86).

On the face of it the new method gives the best correlation when $M_{\infty}$ and not $\bar{M}_{\infty}$ is used. Figure 4 presents a comparison between the two methods at a Mach Number of 0.875 . Near the leading edge the basic method designated "Present Method $\left(M_{\infty}\right)$ " and indicated by triangles produces the best agreement between experiment and calculation. Near the position of the steady shock wave however the peak pressure is better predicted by the variation of the basic method designated "Present Method ( $\bar{M}_{\infty}$ ) and indicated by circles. The location of the calculated peak is slightly forward of the experimentally observed peak. Either method, however, is better than the classic theory (indicated by dots) for predicting pressures as comparisons with experimental data shown.

Two features of the experimental pressure distribution illustrate transonic effects. One of these is the reduced leading edge pressure levels, and a second is the bump or peak in pressure near the location of the steady shock wave location. The new method qualitatively reproduces these features. However there is reason to suspect that the basic version of the new method (triangle) under-predicts the leading edge pressure. The reason for this lies in the fact that, even though the calculated and experimental pressures agree near the leading edge, viscous effects have not yet been accounted for and these effects reduce the calculated loads even further. A drop in the leading edge loading caused by application of viscous effects to the ( $\bar{M}_{\infty}$ ) variation of the basic method (circles), renders this method more acceptable than before. However these effects are not large enough to bring the calculated pressures in line with the experimental values (see fig. 38). Further study is required in this area to decide which method is best or to discover other more accurate variations of the basic method.

The application of the new transonic method to a lower Mach Number, ( 0.85 ), is shown in figure 5. The agreement is good near the leading edge but only a slight indication of the shock bump is given by the theory. Also shown in this figure is the effect of local Mach Number on the Bernoulli equation (see equation (72)). The difference between the circles and triangles indicates this effect.

All applications thus far have been for the steady case. Figure 6 presents a comparison of the Present Method (new transonic theory) for the case of the control surface oscillating at a reduced frequency $k_{r}=\frac{\omega C}{2 U}=0.059$. Also shown in this figure is a calculation done using the Traci et al method (ref. 12) and a calculation done using the classic theory. The finite element theory of Traci et al predicts the bump at the shock wave fairly accurately however is not as good as the Present Method elsewhere. One part of the pressure distribution that does not seem to be predicted by any of the theories is the depth of the dip in pressure behind the shock.

Figure 7 presents a comparison similar to that in Figure 6 but at a lower Mach Number (0.85). Also instead of in phase (Real) and out of phase (Imaginary) parts given, amplitude $\Delta C_{p}=\sqrt{\left(\operatorname{Re} \Delta C_{p}\right)^{2}+\left(\operatorname{Im} \Delta C_{p}\right)^{2}}$ and phase angle $=\tan ^{-1}\left(\operatorname{Im} \Delta C_{p} / \operatorname{Re} \Delta C_{p}\right)$ are presented. Again, as in the steady case the bump at the shock is barely noticeable in the Present Method and of course absent altogether in the classic theory.

Figure 8 presents a comparison of the Present Method, classic theory and experimental data for a case similar to that presented in figure 7 except that the Mach Number is 0.875 and the reduced frequency, $k_{r}=0.176$. Again pressure amplitude and phase angle are shown. The two variations of the Present Method are in better agreement with the experimentally obtained pressure amplitudes than is the classic theory. However the same can not be said of the phase angles. The Present Method follows the experimental phase angle curve from about the $40 \%$ chord on to the trailing edge. However none of the theories follows the curve forward of that point. Tijdeman and Bergh (ref. 11) present a modified phase angle curve based on a full two-dimensional
acoustic solution of a pulse located at the control surface leading edge (see section on New Transonic Effects Method). This approach gives very good agreement with the experimental phase angle data (see figure 31 of reference 11). The correction was applied only to the phase angles and not the pressure amplitudes. The calculated phase angle was simply corrected using the additional time lag over and above that experienced in uniform flow. This additional phase lag was not used internal to the theory but applied after the theoretical calculation was completed. The section of this report entitled "A New Transonic Effects Method" describes how this acoustic type of information can be used internally with the theory so that the pressure amplitudes are also effected. This approach has not yet been tried.

A possible explanation of the phase angle differences between theory and experiment might be due to the fact that signal fronts, which emanated from the control surface, do not exactly travel normal to the flow as assumed in the Present Method. Tijdeman and Bergh have shown that the wave fronts are actually inclined to the flow to a considerable degree, within the supersonic zone. This being the case the wave fronts impinge on the forward part of the airfoil (forward of the shock wave) with very little longitudinal time delay. This would explain the flattening of the phase angle curve in front of the shock.

Thus far detailed pressure distributions have been discussed. Attention is now focused on the forces and moments these pressures produce. Figures 9 through 12 present comparisons of the present method with the classic theory and experimental data. In figure 9 the Traci, Farr, Albano theory is also plotted. This figure shows that the Present Method $\left(M_{\infty}\right)$ is in better agreement with the data than is the classic method. As expected the Present Method and classic theory tend to coalesce at low Mach Numbers, out of the transonic region. The transonic peak lift, predicted by the Present Method, occurs earlier, as Mach Number is increased, than does the experimental data. Also the dip occurring after this peak is not nearly as deep as shown by the experimental data.

Both theories show values of lift coefficient that are higher than the
experimental values. This is due to reduced flap effectivity caused by the viscous boundary layers.

Figure 10 presents a comparison similar to the previous figure except that pitching and hinge moment coefficients are considered. Of particular note is the over prediction of hinge moment by both theories. Again this is due to viscous effects on the flap.

The last two figures have dealt with force and moment coefficients in steady flow for an airfoil with a deflected flap. Figures 11 and 12 present the same data for the oscillatory case. (The reduced frequency varies from 0.098 at $M_{\infty}=0.5$ to 0.057 at $M_{\infty}=0.901$ in these figures.) Generally speaking the same trends and conclusions hold for these figures as for the previous two figures.

## Subsonic Cases

Oscillating Wing with Control Surface. - Extensive low speed wind tunnel measurements of static and oscillatory pressure data have been made by Hertrich (refs. 14 and 15) on straight and swept wings with a full span control surface. The wings had no taper and the control surface had a $30 \%$ chord; two aspect ratios, 2.5 and 3.1 , were tested by changing the exposed span in the tunnel. The swept wings had a sweep angle of $25^{\circ}$. A later oscillatory test of the swept wing was made by Förshing, Triebstein, and Wagener (ref. 16) in which the full span control surface was split approximately in half (the inboard flap had $46.59 \%$ of the span ( and the aspect ratio was set at 2.94 .

The pressure data from the first tests (refs. 14 and 15) were integrated by Hertrich to obtain lift and moment coefficients and the static values for the swept wing with aspect ratio 3.1 have been used here as constraints to determine correction factors. The static values corrected for wind-tunnel wall interference aré: lift curve slope $C_{L_{\alpha}}=L / q S=3.13$ per radian, pitching moment curve slope $C_{m_{\alpha}}=M / q S^{-} \alpha={ }_{0}^{\alpha} .148$ per radian, flap lift effectiveness $C_{L_{\delta}}=L / q S \delta=1.95$ per ${ }^{\alpha}$ radian, flap pitching moment effectiveness $C_{m_{\delta}}=M / q S \bar{c} \delta=-0.432$ per radian, and the flap hinge moment coefficient $C_{h_{\delta}}=H / q S^{-} \delta=-0.0350 \cos 25^{\circ}=-0.03172$ per radian where the sweep correction is added to give the moment about the hinge axis; the reference area is $S=0.564 \mathrm{~m}^{2}$, the reference chord is $\bar{c}=0.6 \mathrm{~m}$, and the pitch axis is located at $61.5 \%$ of the root chord.

Rolling moment coefficients were not derived from the pressure data and neither was the hinge moment due to angle of attack $C_{h_{\alpha}}$. The available data permitted a maximum of five constraints, and two sets of constraints were investigated; the first set used two constraints from the angle of attack data, $C_{L_{\alpha}}$ and $C_{m_{\alpha}}$, and the second set used all five constraints. The use of the flap rotation data alone without the angle of attack data was not considered.

The theoretical basis for the correction factors is the Doublet-Lattice Method (DLM) of reference 17. The idealization of the lifting surface consists of 110 boxes resulting from 11 strips and 10 equal chordwise divisions. The chordwise division on each strip result in 7 boxes on the primary surface and 3 boxes on the control surface. The strip widths $\Delta y_{i}$ are chosen so that the strip centerlines fall along the lines of pressure taps. The span of 0.940 m is divided into the following strip widths from root to tip: $\Delta y_{1}=0.110 \mathrm{~m}$, $\Delta y_{2}=0.080 \mathrm{~m}, \Delta y_{3}=0.075 \mathrm{~m}, \Delta \mathrm{y}_{4}=\Delta \mathrm{y}_{5}=\Delta \mathrm{y}_{6}=\Delta \mathrm{y}_{7}=\Delta \mathrm{y}_{9}=\Delta \mathrm{y}_{10}=0.090 \mathrm{~m}$, and $\Delta \mathrm{y}_{11}=0.045 \mathrm{~m}$. The pressure stations correspond to the strips as follows: pressure station VII is on Strip 1, VI on Strip 4, V on 6, IV on 8, III on 9, and finally station II is on Strip 10. Pressure station I is too close to the tip to permit a meaningful calculation.

The theoretical pressure distributions are compared to the experimental measurements in figures 13 through 24. The theoretical estimates of the five constraint parameters are: $C_{L_{\alpha}}=3.207462 . \quad C_{m_{\alpha}}=0.179494, C_{L_{\delta}}=2.131577$, $C_{m_{\delta}}=-0.463554$, and $C_{h_{\delta}}=-0.057784$. Three additional parameters are also of ${ }^{\delta}$ interest. These are ${ }^{\delta}$ the locations of the spanwise aerodynamic centers for angle of attack, $\bar{y}_{\alpha} / s$, and for flap deflection, $\bar{y}_{\delta} / s$, and the hinge moment coefficient for angle of attack, $C_{h_{\alpha}}$. Their theoretical estimates are $\bar{y}_{\alpha} / \mathrm{s}=0.452071, \bar{y}_{\delta} / \mathrm{s}=0.464614$ and $C_{h_{\alpha}}=-0.021034$.

A typical set of correction factors is shown in table I; it is for a premultiplying matrix and is based on five constraints. The factors are listed in order from leading edge to trailing edge on each strip beginning at the root; factors 1 to 10 are on Strip 1, and factors 101 to 110 are on Strip 11. The first seven factors on each strip apply to the primary surface and the last three apply to the control surface. The general trends seen in table I are a spanwise increase in factors from root to tip and a chordwise increase toward the hinge line. The minimum correction factor in table I is a 0.255873 for Box No. 10 and the maximum factor is 2.00893 for Box No. 108.

TABLE I
CORREATICN FACTORS ** PREMLLTIPLIER CASE

| 00000000 00000000 |  |
| :---: | :---: |
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TABLE II
Type of
Correction
Matrix
Number
of
Constraints

|  | 0 | 0.452071 | 0.464614 | -0.021034 | -0.057784 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| None | 2 | 0.456751 | 0.469814 | -0.021746 | -0.059959 |
| Pre- | 5 | 0.484939 | 0.522318 | -0.010117 | -0.031721 |
| Pre- | 2 | 0.453096 | 0.465632 | -0.022409 | -0.057711 |
| Post- | 5 | 0.469176 | 0.491720 | +0.013014 | -0.031721 |
| Post- |  |  |  |  |  |

The modified static pressure distributions for angle of attack are shown in figures 13 and 14, and for flap deflection are in figures 15 and 16. Perusal of these figures indicates the following results. For the angle of attack loading, both the premultiplying and postmultiplying corrections move the theoretical results slightly away from the experimental data, the postmultiplier causing a little greater change. The effect of five constraints is greater than that of two. For the flap loading, both the pre- and postmultipliers based on two constraints have small effect. The corrections based on five constraints improve the correlations on the control surface but increase the discrepancies on the wing. The postmultiplier causes a much larger change and, although the data show a pressure reversal near the trailing edge, the postmultiplier exaggerates this reversal to the extent that the sign of the hinge moment is reversed. Table II shows the effects of the four correction matrices on the aerodynamic centers and hinge moments. of the correction matrices resulted in an outboard shift of the aerodynamic centers, the largest shift coming from the 5 -constraint premultiplier.

Two constraints did not improve the hinge moment prediction; and the 5constraint postmultiplier lead to an unreasonable prediction of $\mathrm{C}_{\mathrm{n} \alpha}$. The effect of additional constraints based on estimates is a topic deserving further investigation.

We can anticipate similar discrepancies when the correction factors derived from static data are applied to the oscillatory cases, and, indeed, they are shown in figures 17 through 20 for the angle of attack oscillating at $k_{r}=0.622$, and in figures 21 through 24 for the flap oscillating at $k_{r}=0.752$. The theoretical loading for the oscillating angle of attack is not changed significantly by either the premultiplier or the postmultiplier based on two constraints and both the real and imaginary parts are affected about the same. In some regions the theory is shifted toward the data and in others the theory is moved away from the data. The effects of five constraints are more extreme. The 5 -constraint premultiplier improves the correlation for the real part but only improves the agreement for the imaginary part on the control surface while diverging on the wing. The 5 -constraint postmultiplier
is substantially worse in correcting the real part but is no worse than the premultiplier in modifying the imaginary part. Again, the theoretical load distribution from the oscillating control surface is not changed significantly by either the 2-constraint pre- or postmultipliers. However, some improvement is noted with the 5 -constraint corrections although it is only slight. As in the static case, an outboard shift in loading occurs with all correction matrices and for both modes of motion.

The above applications of correction matrices have achieved very limited, if any, success. The lack of improvement in the most elementary case, however, is rather puzzling. This was the case of the static loading at angle of attack for which the correction factors were derived using the two constraints of lift and pitching moment. The pitching moment constraint was expected to shift the theoretical chordwise center of pressure in such a manner that the predicted pressure distribution would be closer to the experimental data. Two explanations for the lack of improvement appear possible. The first is that the theoretical loading.in the leading edge region differs so much from the data that it dominates the correction factor calculation and results in a distorted loading. The second possibility is that the limited number of pressure taps near the leading edge prevented an accurate evaluation of the leading edge contribution to the pitching moment. A strain gage measurement of pitching moment would have shed some light on this possible difference.

A number of options were not pursued with these data which may have shown better correlation. First, only one configuration was studied here, the swept wing with aspect ratio 3.1 ; as noted above, straight wings with two aspect ratios and a swept wing with another aspect ratio were also tested. Next, only the reported integrated loads were used as constraints: the two angle of attack coefficients and the three additional control surface coefficients. The three control surface coefficients were not used as constraints by themselves, nor were additional constraints used based on estimates of rolling or bending moments. The new postmultiplying matrix was also not investigated. Finally, it would have been interesting to apply complex correction factors derived from the oscillatory angle of attack data at
$k_{r}=0.622$ to the oscillatory control surface data at $k_{r}=0.752$; however, this would have required integration of the published oscillatory pressure data to obtain the complex constraint coefficients.

Wing With and Without Leading Edge Droop. - Trailing edge control surfaces are studied in several other sections of this report. In this section, an attempt is made to study leading edge control surfaces. Usable data for such devices is very scarce. Several references have been investigated; however, only reference 18 proved in any way useful. The leading edge device described in this reference is a wing droop of $6^{\circ}$. The droop was applied to the first $19 \%$ of the wing chord along its entire span (see fig. 25). The idealization shown in this figure is for the Doublet Lattice Method (DLM). The fuselage was simplified as simply a wing extension to the centerline.

A steady case at $M=0.80$ is considered and the uncorrected calculated results using the DLM for $\alpha=4^{\circ}$, (no droop) agree very well with the experimental data (see fig. 25). Only a lateral shift in the center of pressure seems evident. Correction factors were developed for this case to correct this slight deviation in the theory. The constraints used are lift, pitching moment and bending moment coefficients. These coefficients were summed on strips outboard of the station $y /(b / 2)=0.11$ and are defined as:

$$
\begin{gathered}
C_{L}=\frac{L}{q A}, \frac{A}{c_{\text {root }}^{2}}=1.28 \\
C_{M}=\frac{M}{q A_{c}^{2}}, c / C_{\text {root }}=.815 \text { (moment taken about } \times / c_{\text {root }}=1.0 \text { ) } \\
C_{B}=\frac{B}{q A b / 2}, b / 2 / c_{\text {root }}=1.6 \text { (moment taken about } \times \text {-axis) }
\end{gathered}
$$

For the various modes the coefficients are:

|  | Pitch $\alpha=4^{\circ}$ | $\alpha=10^{\circ}$ | $6^{\circ}$ Droop |
| :--- | ---: | :--- | :--- |
| $C_{L}$ | .2282 | .5356 | -.0215 |
| $C_{M}$ | -.0233 | -.065 | -.008 |
| $C_{B}$ | .0563 | 0.128 | -.00536 |

A premultiplier and a postmultiplier (new type) were tried with equally good results on the span loading. Figure 26 illustrates the effect of the correction factors on the spanwise distribution of aerodynamic center. The correction factors increased the accuracy of the aerodynamic center inboard, but decreased it outboard.

The single mode application of both pre- and postmultipliers, also produces good results for leading edge droop span loadings (fig. 27). Notice that the unmodified results are approximately half of the experimental values. The experimentai data were difficult to read on the plots (open squares). Thus, the pressure distributions were integrated to produce the darkened squares. The pressure distributions themselves were difficult to integrate accurately since there were down loads at the nose and uploads near the bend in the chord, such that the total loads were small. If the correction factors possessed only a slight variation in the chordwise direction, the balance of integrated load could shift drastically as a percent of the total.

The flow field near the wing changes at approximately $\alpha=8^{\circ}$. Here the flow is to a large extent separated from the upper outboard surface. A comparison of uncorrected theory and experimental data, for the case of $\alpha=10^{\circ}$, no wing droop, in figure 28, shows a loss of lift outboard of the 40\% semi-span. Application of both pre- and postmultipliers (New), using $C_{L}, C_{M}$ and $C_{B}$ (bending moment at the centerline), show a much improved prediction of span loading. Also, shown in this figure is the application of the premultiplier correction factors, obtained at $\alpha=4^{\circ}$ to the $\alpha=10^{\circ}$ case (diamond symbols). The span loads are improved which shows that data obtained at one angle of attack can be profttably applied to other angles of attack. The corrections generated at $\alpha=4^{\circ}$ are not as large as those generated at
$\alpha=10^{\circ}$ because flow separation exist in the latter case. However, both. corrections are in the same direction. Therefore, application of correction factors for $\alpha=4^{\circ}$ improves the results for the $10^{\circ}$ case. In general, the reverse may not hold; i.e., the correction factors obtained at $\alpha=10^{\circ}$ (or larger angles) may be too large and an excessive correction may result leaving the corrected data further from the experimental data than there were originally. It does seem safe, however, to apply correction factors obtained at one angle of attack to other nearby angles if the flow is qualitatively similar (e.g., no great changes in flow pattern).

In this section applications of the correction factor technique are made to the same cases considered in the "Local Mach Number Studies" section. Specifically a two-dimensional symmetric airfoil (NACA 65A006) with an oscillating $25 \%$ chord flap is used.

Figure 29 illustrates the difference in results obtained when the classic theory (subsonic compressible) and the new transonic theory (Present Method $\left(M_{\infty}\right)$ ) are corrected. A premultiplying set of correction factors were obtained using three constraints; lift, moment ( $1 / 4$ chord) and control surface hingemoment ( $3 / 4$ chord). Each theory was corrected to the proper experimental constraints, i.e.,

$$
\begin{gathered}
c_{\ell}=4.93 \\
c_{m_{1 / 4}}=-1.57 \\
\cdots \\
c_{h_{3 / 4}}=-0.053
\end{gathered}
$$

where the characteristic length is the chord and the downwash over the control surface is unity. The classic theory does not have the bulge in pressure, near the compression (or shock) region for the steady flow as does the experimental data and applying correction factors will not make it appear. Thus correction factors can not make a qualitative feature appear where none existed before. The corrected classic theory does not compare well with the experimental data and the correction factors themselves, ( $1+\varepsilon$ ), show fairly large deviations from unity especially near the leading and trailing edges.

The Present Method $\left(M_{\infty}\right)$ however possesses a qualitative similarity with the experimental data and thus it is a better candidate for correction. Figure 29 shows such a correction. The bulge in pressure as calculated by the present method is amplified as it should be. The loading on the flap however is reduced, again as it should be, however the shape of the flap load is distorted. The correction factors themselves are better behaved for the

Present Method, $\left(M_{\infty}\right)$, showing large deviations from unity only on the flap surface.

Figure 30 presents the results of applying the premultiplying correction factors, obtained for the steady case, to an unsteady case. That is, the correction factors shown at the bottom of figure 29 are applied to the oscillatory results of the Present Method $\left(M_{\infty}\right)$ and the classic theory ( $k_{r}=\omega \bar{C} / 2 U_{\infty}=0.059$ ). Since the correction factors are real they do not effect the phase angles of the pressures but only the amplitudes, $\left|\Delta C_{p}\right|$. Also shown in the figure is a pressure distribution corrected using factors based on the complex lift moment ( $1 / 4$ chord) and hinge moment obtained for the unsteady case. Specifically

$$
\begin{array}{ll}
c_{\ell}=3.5-i 1.18 & M_{\infty}=0.875 \\
c_{m_{1 / 4}}=-1.66+i 0.07 & k_{r}=0.059 \\
c_{h_{3 / 4}}=-0.057-i 0.016 &
\end{array}
$$

The correction factors obtained in this manner produce pressures that are close to those obtained using the steady correction factors (except near the flap) even though the constraints in lift are considerably different in the two cases. There is one slight anomaly in the phase angle for the complex constraint case ( $k_{r}=0.059$ ) and it exists on the last two pressure points on the flap. The phase angle there is quite large however these angles do not have a large effect since the amplitude of pressures is very small there.

The question arises; to what extent can static correction factors be applied with accuracy to the oscillatory case? Figures 31 and 32 illustrate the effect of static correction factors on lift, moment and hinge moment coefficient versus reduced frequency for a Mach Number of 0.85 . Considering first the lift coefficient it is noticed that the accuracy of the imaginary part is increased up to $k_{r}=0.2$. Beyond this point application of correction factors decreases the accuracy of the theory. For the real part of the lift the corrected theory is more accurate only below a reduced frequency of 0.06 . For the pitching moment and hinge moment the cross over point is roughly
$k_{r}=0.1$. On the average then the static correction factors are useful up. to about a reduced frequency of 0.1 . Beyond this point it is better to use the original theory.

It is probably true that the accuracy of extrapolating correction factors versus reduced frequency depends on Mach Number. Figure 33 gives an indication that as Mach Number is reduced the accuracy increases. Specifically, static correction factors have been applied to the oscillatory case ( $k_{r}=0.098$ ) with very good results. Both amplitude and phase angle are improved.

The theory used in figures 31,32 and 33 is a variation of the new transonic method presented previously. Specifically the variation utilizes an average local Mach Number ( $\bar{M}_{\infty}$ ) in place of the free stream Mach Number, $M_{\infty}$. Figures 31 and 32 show the application of two separate types of correction factors; a premultiplier type, the type used in figures 29 and 30 , and a postmultiplier type. The postmultiplier is actually an additive viscous type of correction. It can be seen that this correction does not extrapolate to higher frequencies as well as the premultiplier type (as far as the lift coefficient is concerned). As the frequency is increased the experimental data approach the unmodified theory. One interpretation of this fact is that as the frequency is increased viscous effects are reduced.

The postmultiplying correction factor (designated as "New Post") used in figures 31 and 32 is the new postmultiplying correction factor discussed in the Theoretical Development section. The reason a new type of postmultiplier was needed is because the original one seemed to fail when control surfaces are considered. Postmultipliers correct the downwash matrix. When all downwash values are non zero, e.g., wing pitch, the method seems to work. However, when this is not the case, e.g., control surface deflections, the method fails entirely. The corrected downwash values are either large and erratic themselves or they cause large and erratic pressures due to the modified downwash.

It was hoped that the introduction of correction factor mode shapes would smooth out the corrected downwash and produce accurate results. This did not
happen. Even though smooth, well behaved functions were used the results were unrealistic. Although not tried, it seems that limiting the maximum and minimum values of the correction factors probably would not help very much either.

This failure of the postmultiplying correction factors led to an interesting investigation and subsequent development of the "New Postmultiplier". The investigation consisted in finding out what downwash in the theory would produce the experimental pressure distribution. Specifically the theoretical influence coefficient matrix was multiplied by the vector of experimental pressures to produce a vector of downwash values.

Figure 34 shows the results of this type of analysis (designated as Experimental) for a steady subsonic case ( $M_{\infty}=0.5$ ). Also shown is the theoretical downwash, i.e., unity over the flap. One thing is noticed immediately, there is a change in downwash ahead of the flap even though it is theoretically zero there. This downwash change is like a negative pitch of the entire airfoil. Figure 35 shows the camber (designated $M_{\infty}=0.5$ ) associated with the downwash given in figure 34 and indeed it is like a negative or nose down pitch. This fact suggests that an additive type of correction factor, whereby all downwash values are changed, is necessary. This resulted in the development of the "New Postmultiplier" as described in the Theoretical Development section. This name is somewhat of a misnomer since the correction factor is additive and not multiplicative although the correction factors are proportional to the theoretical pressures.

The results of applying the new postmultiplying correction factors are also shown in figure 34. Again lift, moment (c/4) and hinge moment (c3/4) coefficient were used as the constraints
$c_{\ell}=3.2$
$c_{m_{1 / 4}}=-.70$
$c_{h_{3 / 4}}=-.0528$

The corrected values of downwash (circular symbols) agree well with the experimentally deduced downwash. The disagreements at the leading edge of the air-
foil and ahead of the flap are due to the fact that downwash is a sensitive function of pressure and slight variations cause large variations in downwash. With this in mind the agreement is very good especially over the flap itself.

Applying this corrected downwash to the theory produces the results given in figure 36 for the pressure distribution. The results of the New Postmultiplier agree very well with the experimental pressures. For reference, corrections by a premultiplier are also shown and these are also very good. The uncorrected theory is also presented for reference.

At the low Mach Numbers used in the last few figures ( $M_{\infty}=0.5$ ) transonic effects are not present and any differences between theory and experiment are, in all probability, due to viscous effects. Figure 35 has shown that viscous effects modify not only the downwash over the flap but also over the forward part of the airfoil as well. This comes about due to the fact that the deflected flap causes an induced upwash over the forward portion of the airfoil which in turn generates a difference in boundary layer displacement thickness on the upper and lower surfaces. This difference in displacement thicknesses causes an effective nose down pitch.

It stands to reason that the correction factors generated at $M_{\infty}=0.5$ could be used to increase the accuracy of the theory at all Mach Numbers since viscous effects are present at all Mach Numbers. Figure 35 shows the effective cambers at $M_{\infty}=0.5$ and $M_{\infty}=0.875$ using the $\left(\bar{M}_{\infty}\right)$ variation of the new transonic theory. Notice that the transonic camber can be thought of as composed of two pieces; one viscous piece very similar to that found at $M_{\infty}=0.5$ and one transonic piece with the shape of a bump. This indicates that the accuracy of the corrected camber (or downwash) at transonic Mach Numbers can be increased if the subsonic ( $M_{\infty}=0.5$ ) results are known and used since it represents one part of the correction.

Figures 37, 38, 39 and 40 give examples of applying correction factors obtained at $M_{\infty}=0.5$ to other Mach Numbers for both pressures and aerodynamic coefficients. Specifically figures 37 and 38 present the results for Mach Numbers of 0.85 and 0.875 respectively. Up to three separate corrected
pressure distributions are shown in each figure. One is the result of applying a premultiplying correction factor matrix to the $\left(M_{\infty}\right)$ variation of the new transonic method. A second is the result of applying a new postmultiplier to the same theory; and third is the result of applying a new postmultiplier to the $\left(\bar{M}_{\infty}\right)$ variation of the new transonic method. The last pressure distribution is seen to be the most accurate and a definite improvement over the unmodified $\left(\bar{M}_{\infty}\right)$ theory (see fig. 4).

Figures 39 and 40 give a clear picture of the effect of applying corrections obtained at $M_{\infty}=0.5$ to other Mach Numbers. Figure 39 presents the lift coefficients associated with corrected and uncorrected pressure distributions. Two types of corrections are used; both pre- and postmultiplier (New). The theory used is the $\left(\bar{M}_{\infty}\right)$ variation of the new transonic method. Figure 40 presents similar results for the pitching moment and hinge moment coefficients. The corrections developed at $M_{\infty}=0.5$ greatly improve the theory as far as the lift coefficent is concerned. The pitching moment is not changed much because it was very close to the data to begin with. The hinge moment also is not changed much.

Figures 39 and 40 show that corrections obtained at low Mach Numbers can be applied to the theories to improve accuracy at higher Mach Numbers.

Figure 41 presents the results of correcting the theory with both a postmultiplier (New) and a premultiplier. First the theory is corrected using a new postmultiplier obtained at $M_{\infty}=0.5$. This represents a viscous type of correction. A premultiplier is then applied to the previously corrected results to account for transonic effects. This process produces a pressure distribution that approaches the data more closely than any of the others when it is combined with the $\left(\bar{M}_{\infty}\right)$ variation of the new transonic method.

Figure 42 presents typical correction factors for the steady two-dimensional cases considered in this section. The theory used is the Present Method $\left(M_{\infty}\right)$. There is a greater change in premultiplying correction factors between $M_{\infty}=0.85$ and 0.875 than there is between $M_{\infty}=0.5$ and 0.85 .

## Supersonic Case

The arrow wing, shown in figure 43, has been chosen to illustrate the application of the correction factor technique to the supersonic case. The Douglas Supersonic Doublet Method (SDM) (ref. 20) has been used to determine the theoretical loads. The box idealization used is also shown in the figure. Notice that the tip of the wing has been clipped to reduce the number of boxes. Two modes are considered; (1) pitch ( $\alpha=4^{\circ}$ ) and (2) camber. The wing is operating at a Mach Number of 2.05 and a reduced frequency of zero.

Figure 43 presents a comparison of uncorrected theory (dotted line), corrected theory and experimental data. The experimental values lie below the theoretical (uncorrected) values over the entire span.

Four different methods of correcting the theory were tried. A pre- and postmultiplier (New) were applied using the pitch mode only and the results are very encouraging. The only real difference between the theory, corrected in this manner, and the experimental data appears at the wing tip. A multiple mode case was tried using the pitch and camber modes and the results are good but not as good as the previous two corrections. The fourth method is the application, to the pitch case, of a premultiplier correction factor matrix that was derived for the camber case. Thus a correction factor derived for one mode (camber) is applied to another mode (pitch). The results are not very accurate on the inboard part of the wing but agree as well as the other methods on the outboard part. The constraints used are summarized as follows:

\[

\]

$$
C_{B}=\frac{B}{q A b / 2}
$$

$$
(b / 2) / c_{\text {root }}=0.564
$$

$$
\mathrm{B} \equiv \text { Moment about } \mathrm{x} \text {-axis }
$$

A similar set of corrections were applied to the camber case and the results are shown in figure 44. Specifically pre- and postmultiplying correction factors were obtained using the camber mode. In addition a multiple mode (pitch, camber) premultiplying correction factor matrix was derived using six constraints; i.e., $C_{L}, C_{M}, C_{B}$ for both modes. All three of these corrections give approximately the same good results except right at the wing tip.

Two other types of correction factors are applied to the theory and these refer to applying the correction factors derived for pitch to the camber mode. On the inboard portion of the wing these correction factors over-correct the theory, but are accurate on the outboard portion of the wing. On the inboard portions of the wing the correction factors move the corrected theory further from the data than it was originally in its uncorrected state.

Figure 45 illustrates how correction factors modify the pressure distribution over the wing (in pitch) using a premultiplier. It seems that the reduction in lift due to the correction factors is taken out at the trailing edge rather than the leading edge as the experimental pressures would indicate. This fact might be explained if the experimental pitching moment were inaccurate.

The postmultiplier does not directly modify the pressures but modifies the downwash. Modified downwash can be expressed in terms of modified camber. It is of interest to know how the postmultiplier (New) modifies the wing camber and figure 46 presents such a modification. The camber is reduced, by the correction factors, over most of the wing just as expected, since the boundary layer and separation regions act to reduce the effective wing camber. The postmultiplier, then, acts in a way that is consistent with physical processes.

## RECOMMENDATIONS FOR DATA ACQUISITION

Most of the data utilized in this study were pressure data, and the correction factors were derived using constraints that were obtained in some instances from integrations of the pressures. Certain errors are associated with integrations of pressures to obtain generalized forces, arising primarily from the limited number of pressure pickup points on a practical model. The forces and moments should be measured directly in addition to the pressures. Control surface hinge moments should be measured and so should rolling moments, i.e., root bending moments, because of the importance of the spanwise aerodynamic center location. For swept surfaces it would be desirable to measure not only pitching and rolling moments in the streamwise coordinate system but also root bending moment and torque about some swept coordinate system, e.g., the $25 \%$ or $50 \%$ chord lines.

Two significant deficiencies were observed in available experimental data besides the absence of combined pressure and force data. One was a lack of any systematic variation in reduced frequency in covering the range from steady flow to high frequency, i.e., $k_{r}$ of order unity. The correction factors are frequency dependent, and it is not reliable to use factors derived from low frequency data to predict pressure distributions at high frequencies. The second deficient area is the effect of Reynolds number. An important source of discrepancy between theory and test is the neglect of viscosity in the theory. When extensions of oscillatory lifting surface theory are made to account for viscous effects, data will be needed to verify the accuracy of the improved theory. However, these data are also needed to determine the accuracy with which correction factors derived from data at one Reynolds number can be used to predict pressures at another Reynolds number. This is particularly important for trailing edge control surfaces.

A number of suggestions can be made for future wind tunnel tests in addition to those indicated above. Leading edge control surfaces should be tested; spoilers might also be considered. Very little data are available for these configurations. Models should be designed so that components can be
tested in their principal modes of motion. Complete models usually have moveable control surfaces but a moveable fin, horizontal tail, and engine pylon should also be considered to distinguish between component loads and interference loads. More oscillatory transonic data are needed. In twodimensions, pitch data should be measured in addition to control surface data; in three-dimensions, data on both straight and swept wings are required.

## CCNCLUDING REMARKS

## CONCLUSIONS

The basic conclusions arrived at as a result of the calculations and correlations presented are outlined as follows:

## (1) One Set of Correction Factors Is Not Good For All Modes

Application of correction factors, determined from one mode, to other modes has not met with much success. Specifically, correction factors obtained using a pitch mode can not be applied to pressures due to control surface deflections. The converse is also true. In addition, application of correction factors, obtained using a pitch mode, to pressures due to a camber mode (and vice versa) have not proved to be very accurate either. Bergh and Zwaan, reference 6 , on the other hand have concluded that correction factors calculated using a pitch mode can be applied to a roll mode. These two modes, however, are similar. One of them has a constant angle-of-attack along the span while the other has a linearly varying angle-of-attack along the span.

Further study is required to find out what types of correction factors are required for the various modes encountered in flutter and other dynamic aeroelastic analyses. The practical method of implementing such a correction procedure also requires further study.

The fact that one set of correction factors can not be applied to all modes has certain implications for testing procedures. It may be that more rigid body types of modes will be required (e.g., pylon yaw, wing alone pitch, tail alone pitch, outer wing pitch, inner wing pitch, fuselage alone pitch, etc.) than are now considered.

A second approach to the problem of obtaining one set of correction factors for both a pitch and a control surface mode was attempted using the "multiple mode" capability of the program. This capability allows the theory to be constrained to produce the correct lift and moment coefficient, etc., for each of several modes. The resulting span loading and/or pressures were not improved for either the pitch or control surface modes.

Even within a single mode problems can occur for different amplitudes. For instance high angle-of-attack flow fields can be basically different (separated) from those at low angles-of-attack. However correction factors obtained at low angles-of-attack can result in improved predictions for all angles-of-attack. The basic reason being that the viscous corrections, although smaller for the unseparated case, are still in the same direction as that for the separated case.

## (2) A Bending Moment Constraint Is Needed For Swept Wings

Lift and pitching moment constraints are not enough for the swept wing case. A bending moment constraint is also required so that the loading is not shifted outboard to accommodate an aft shift in aerodynamic center of pressure. Without the constraint on the bending moment the correction factors will cause the wing loading to be moved toward the wing tip instead of moving the load aft along the chordline.
(3) Correction Factors Can Be Extrapolated More Accurately To Other Mach Numbers Than To Other Frequencies

When correction factors are determined at low Mach Numbers they are caused primarily by viscous effects. Since viscous effects exist at all Mach Numbers an increase in accuracy will result if the low Mach Number correction factors are applied to the high Mach Number cases.

Extrapolation in frequency has not been as successful as extrapolation in Mach Number. For the two dimensional case studied it seems that extrapolation further than $\Delta k_{r}=0.1$ (based on the half chord and a Mach Number of 0.85 ) will lead to a decrease in accuracy as $k_{r}$ is increased. It is believed that extrapolation in reduced frequency is more accurate at lower Mach Numbers.

It appears that as the frequency is increased the viscous effects are reduced. This is an important fact and if steady wind tunnel results are to be used for correcting data then a good estimate of the reduction in the viscous effect must be known. One way to accomplish this without testing every configuration is to test a representative sample of configurations over
a range of frequencies and construct general trends to be used in conjunction with steady data to estimate the frequency effect on correction factors.
(4) Qualitative Features Missing From The Theory Can Not Be Generated By Correction Factors

Correction factors tend to produce quantitative changes to the theory and not qualitative ones. For instance, in transonic flow, the bulge in pressure at the shock location cannot be induced with correction factors if one did not exist in the basic theory.

## (5) Downwash Correction Factors Must Be Additive, Not Multiplicative

It was found that postmultiplying correction factors did not work for control surface modes (they did work for pitch modes however). That is, scaling the downwash to reproduce the imposed experimental constraints ( $C_{L}, C_{M}$, etc.) led to unusable results. Smoothing of the results was obtained by introducing correction factor modes; however, the levels of correction were still unrealistic. An analysis was performed to see what downwash was required to produce the experimental pressures and it became evident that the downwash had to be corrected everywhere and not just on the control surface. This suggested an additive downwash correction. A new method was developed and executed successfully.

Downwash correction factors essentially reflect the physical fact that viscous effects tend to change the effective airfoil camber (and thus the downwash). The camber is changed over the entire airfoil.
(6) Premultiplying And (New) Postmultiplying Correction Factors Are Equally Accurate

In the cases studied the accuracy of the corrected theory is improved equally well (approximately) by either type of correction factor matrix. The downwash correcting factors (New Postmultipliers) are physically more meaningful if interpreted as viscous corrections while premultipliers are more meaningfully intepreted as compressbility corrections.

Since transonic experimental data reflect both viscous and compressibility effects a very accurate way to obtain correction factors, if data permit, is to combine pre- and postmultipliers together. First a postmultiplier (New) is developed at low Mach Number where viscous effects dominate. This correction is then applied to the transonic case. The modified theory is then corrected further for transonic effects using a premultiplier. Correction factors produced in this way are more accurate than most.

## (7) New Transonic Method Useful But Requires Further Investigation

Various methods of applying local Mach Number were tried. Simple procedures based on the substitution of the local steady Mach Number (or some average between surface and free stream) for the freestream value in the boundary conditions, kernel, and pressure equations have been tried. The results have only shown minor changes and have not even given qualitatively good results.

A new method was developed at Douglas (under the McDonnell Douglas IRAD program) and is based on a transformation of the distance between sending and receiving points based on acoustic travel time between the two points. This method was implemented for the two-dimensional case and correlated in the present study. The results are encouraging since the predicted pressures are qualitatively similar to the experimental data. That is, the new method predicts a bump in the pressure which is centered at the shock wave location and predicts a lowering of the pressure forward of the shock wave. This bump, however, is forward of the experimentally observed bump, and is usually smaller in amplitude.

The theoretically determined phase angles of the pressures are not in good agreement with the experimental data forward of the 40 percent point (for the case of a control surface rotation). One possible reason for this is the fact that the wave fronts emanating from points on the flap tend to move up and over the shock wave and arrive at the forward portions of the airfoil in nearly a horizontal configuration. On the other hand, in the theory, the paths of the wave fronts are assumed to be normal to the free stream flow with the wave fronts vertical, and this difference causes the phase angles to be greater for the theory than for the data. Further investigation of this
discrepancy and its solution is required. It may be possible to use a more exact phase lag time computation in the theory, such as the one used by Tijdeman and Bergh in reference 11.

When trying to decide which theory is best it is important to account for viscous effects. Without such a correction the new transonic method, designated as $\left(M_{\infty}\right)$, seems best. However, when viscous effects are accounted for, the variation designated $\left(M_{\infty}\right)$ is best.

With the current method of computation the new transonic method is probably not reliable past $M_{\infty}=0.90$. This may not be too restrictive since most wings are swept which reduces the normal Mach Numbers to values lower than 0.90 .

## (8) Overview of Conclusions

The concept of correcting theoretical pressure or load distributions so that they reflect associated experimental data works well with the correction factor technique, especially if the proper experimental data are available (e.g. bending moments.) It was hoped however that a set of correction factors, once developed, would be applicable to a wide variety of other cases. The range of applicability however has not been as wide as hoped for. Success in extrapolating correction factors was obtained for Mach Number and to a limited extent for frequency. Attempts to apply correction factors to dissimilar mode shapes however has not met with much success. Therefore more than one set of correction factors is required. The use of several sets of correction factors to correct oscillatory aerodynamic generalized forces for use in dynamic aeroelastic analyses requires further investigation.

Also concluded from the present analysis is that correction factors can not change the character of the load distribution. If a fundamental feature is missing from the theoretical loading then the correction factors will not make it appear. Thus theoretical methods must possess at least qualitative accuracy.

## Recommendations for Further Studies

Further studies may be profitably pursued in several areas. The successes and failures of the correction factor technique, presented here, furnish a guide to such studies.

First, it seems advisable to exercise more of the various options in the present method and include more types of force data (integrated from pressure data) for some of the cases treated in this report. For instance such a case would be the Hertrich wing (refs. 14, 15). It would also be desirable to obtain new data similar to that obtained by Hertrich, in which both force and pressure data are available. It would be interesting to compare force data with integrated pressure data.

Second, it is now clear that one set of correction factors is not sufficient for all deflection modes. Thus a method for including multiple sets of correction factors into the determination of generalized oscillatory aerodynamic forces for various modes is required. This method may require special testing procedures whereby each major component or subcomponent is systematically given a rigid body rotation.

Third, the studies on viscous effects initiated in this report should be continued. Specifically the technique of determining theoretical camber lines that reproduce experimental pressure distributions seems valuable and could lead to a semiempirical method for viscous effects when combined with boundary layer theory.

Fourth, the new transonic method illustrated in this report should be refined and extended. Initially the two-dimensional capability should be refined in the areas of; 1) pressure phase angle and, 2) unsteady shock wave motion. Subsequent to this a three-dimensional method should be developed.

In addition, an investigation should be undertaken to explore the possibility of developing a semiempirical transonic method. The local steady Mach Number can be used as an adjustable parameter so as to produce the pressure distribution changes necessary to satisfy experimental constraints (lift moment etc.).

## Introduction

As described in the Theoretical Development Section this method generates a set of correction factors that can be applied to a set of data (e.g., theoretical pressure) such that the data satisfies certain imposed (e.g., experimental) constraints.

For convenience this data will be referred to as pressure data since this is the most common application. However the correction factor procedure is not restricted to pressures and can be applied to other data sets (e.g., span loads, etc.).

For this procedure it is assumed that one or more theoretical pressure distributions, $\Delta C_{p_{j}}$, $(j=1$, number of pressure modes) are input. Associated with these pressures are: an area distribution, $\Delta A$, a set of coordinates, ( $x, y, z$ ), and a dihedral angle distribution $\bar{\gamma}$ which are input via cards, tape or both. As an option the aerodynamic influence coefficient matrix, $[A]=[D]^{-1}$, along with one or more normalwash distributions, $w_{j}$, can be input in place of $\Delta C_{p_{j}}$; and as a matter of fact these must be input for postmultiplier correctioh matrices (i.e., correction factor matrices for the normalwash).

Constraint data (experimental data) are input as force or moment coefficients. If a force coefficient, $C_{e}$, is considered it is defined as

$$
\begin{equation*}
C_{e}=\frac{1}{\tilde{c}} \sum_{a}^{b} \Delta A \Delta C_{p} \vec{n} \cdot \vec{i}_{a} \quad \text { (force coef.) } \tag{87}
\end{equation*}
$$

where $\tilde{c}$ is a constant used to convert the dimensional sum into a coefficient form. For example if $C_{e}=C_{L}$ then $\tilde{c}$ is equal to the reference area. The limits of the sum are also input to the program. The unit vector $\vec{i}_{a}$ is in the direction of an input axis. A set of axes are input for use in the constraining and monitoring features of the program. Each axis can be input in one of two
ways; (1) a point and a direction or (2) by two points. The unit vector $\overline{\mathrm{i}}_{\mathrm{a}}$ is calculated as follows:

$$
\begin{equation*}
\vec{i}_{a}=\vec{i} \cos \alpha+\vec{j} \cos \beta+\vec{k} \cos \gamma \tag{88}
\end{equation*}
$$

The unit vector $\vec{n}$ is in the direction of the lifting pressure which is given in terms of the dihedral angle of the lifting surface.

$$
\begin{equation*}
\vec{n}=\vec{j}(-\sin \bar{\gamma})+\vec{k}(\cos \bar{\gamma}) \tag{89}
\end{equation*}
$$

where $\vec{j}$ and $\vec{k}$ are unit vectors in the $y$ and $z$ directions respectively and where a right handed system is employed where $z$ is up, $y$ is out the starboard wing and $x$ is aft.

If a moment coefficient, $C_{e}$, is considered then it is defined as follows:

$$
\begin{equation*}
c_{e}=\frac{1}{\tilde{c}} \sum_{a}^{b} \Delta A C_{p}(\vec{r} \times \vec{n}) \cdot \vec{i}_{a} \text { (mement coef.) } \tag{90}
\end{equation*}
$$

where

$$
\begin{equation*}
\vec{r}=\left(x-\xi^{(1)}\right) \vec{i}+\left(y-n^{(1)}\right) \vec{j}+\left(z-5^{(1)}\right) \vec{k} \tag{91}
\end{equation*}
$$

and where $\xi^{(1)}, \eta^{(1)}, \zeta^{(1)}$ are the coordinates of the first end point of the axis considered and where $\vec{i}_{a}$ is its direction. The constant $\tilde{c}$ for the case of $C_{M}$ has the dimensions of volume.

The program has various other capabilities and one of these is its ability to monitor the corrected or uncorrected pressures. The integrations performed in equations (87) and (90) can be performed using data without reference to constraints. Thus if span loads are desired for data that has been corrected (or uncorrected) then the proper summations are activated in the program in a manner similar to that for constraining the data.

The program also has the capability to use correction factor modes. That
is, the actual correction factors $\{\varepsilon\}$ are related to a set of modal coordinates $\{\bar{\varepsilon}\}$ as follows:

$$
\begin{equation*}
\{\varepsilon\}=[\phi]\{\bar{\varepsilon}\} \tag{92}
\end{equation*}
$$

The modal matrix, $\phi$, is either input directly by cards or certain built in modes can be activated.

The program has the capability to limit the excursion of any or all correction factors. The upper and lower bounds are simply input for the correction factors that are to be limited. If correction factor modes are used then the limits are placed on the modal coordinates, $\bar{\varepsilon}$, and not on the correction factors themselves.

In addition to limits, a factor $\tilde{a}$ is input for each constraint to indicate its "constraining power". The term $\tilde{a}$ ranges from 0 to 1.0. If $\tilde{a}$ is 1.0 the constraint. has full power and is 100 percent effective as a constraint. If $\tilde{a}$ is 0 then the constraining power is zero and the "constraint" has no effect. For values of $\tilde{a}$ anywhere in between the constraint is said to be an estimate.

Finally, the program can be used to apply previously obtained correction factors to input pressure distributions. The program can also be used simply to monitor existing data without any constraints.

One nomenclature problem which might cause confusion is the fact that the normalwash $w$ is called $W$ in the program, while the correction factors $W$ are called CF.

The following table provides an overview of the card input data grouped according to their functions.in the program. The layout of the input sheets and a detailed description of each input item are also given following the table.

Overview of Input Data

| ITEMS | $\begin{aligned} & \text { CARD } \\ & \text { NO. } \end{aligned}$ | $\begin{aligned} & \text { WHEN } \\ & \text { NEEDED } \end{aligned}$ | COMMENTS |
| :---: | :---: | :---: | :---: |
| CONTROL | 1,2, | Always | Header card, control dimensions |
| DATA | 3, 4 |  | and control flags. |
| GEOMETRY | 5, 6 |  | Geometry data is input on one card |
| AND PRESSURES |  | FLAGP $=3$ | per $i, i=1$, NP (NP = number of pressures). Pressures are input either 6 real |
|  |  |  | numbers per card (when FLAGI=1), or 3 complex numbers per card (when FLAGI=0). Repeat pressure input per pressure mode, symmetric modes first, antisymmetric modes (if any) last. |
| AXIS DATA | 7,8 | Always | Axis data, 2 cards per $i$, $i=1$, NAXIS (NAXIS=number of input axes) |
| CONSTRAINT | 9, 10 | If | Constraint data is input in a minimum |
| DATA | 11, 12 | NC $\neq 0$ | of four cards per $\mathbf{i}, i=1$, NC (NC= number of constraints) |
| MONITOR | 13, 14 | If | Monitor data is input in a minimum |
| DATA | 15, 16 | NMON $\neq 0$ | of four cards per $i$, <br> $i=1$, $N M \Phi N$ ( $N M \emptyset N=$ the number of monitored aerodynamic parameters) |
| LIMITS DATA | 17 | If <br> NELIMS $\neq 0$ | Minimum and maximum limit values on $\varepsilon$; repeat per $\mathbf{i}, \mathbf{i}=1$, NELIMS <br> (NELIMS=number of min. and max. limiting value pairs) |


| ITEMS | $\begin{aligned} & \text { CARD } \\ & \text { NO. } \end{aligned}$ | $\begin{aligned} & \text { WHEN } \\ & \text { NEEDED } \end{aligned}$ | COMMENTS |
| :---: | :---: | :---: | :---: |
| CORRECTION <br> FACTOR <br> MODES <br> DOWNWASH <br> DATA | $\begin{aligned} & 18,19 \\ & 20,21 \end{aligned}$ $22,23$ | If <br> NEM $\neq 0$ <br> If <br> FLAGW=1 | Correction factor modes may be input according to two options depending on the flag TYPE (see detail description of data) either in cards 18, 19 and 21, or in cards 18, 20 and 21. Repeat per $\mathbf{i}, \mathbf{i}=1$, NEM (NEM=number of correction factor modes) <br> Downwash data is input in a minimum of two cards per mode (see detail description of data). Input symmetric modes first, antisymmetric modes (if any) last. |

Computer program requires less than 200K OCTAL storage.
$\infty$
$\infty$
$\therefore$
$\therefore$
$\curvearrowleft$
$\vdots$
$\vdots$

PUNCH IN ALL CARDS CASE |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |


DIRECTIONS FOR KEYPUNCH DO NOT PUNCH BLANK COLUMNS_ NO UNDERPUNCHES IN SIGN FIELDS.
MODES WASH)DATA



$$
\begin{array}{l|l|}
\therefore & 0 \\
& \\
\therefore & 0 \\
\therefore & - \\
\therefore & -1 \\
&
\end{array}
$$




## Description of Input Data

## Control Data

These data items are required for all cases. They consist of a header, control numbers, flags, and tape (or scratch unit) numbers.

| CARD | ITEM | MNEMONIC | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1 | Header | HEADER | Alpha-numeric description of case in card columns 1 through 60 |
| 2 | NP | $N P$ | Number of $\Delta C_{p}$ elements, where $\Delta C_{p}$ may represent any type of quantity ( $N P \leq 350$ ) |
| 2 | NC | NC | Number of constraints to be applied to the $\Delta C_{p}$ values $(N C \leq 35)$ |
| 2 | NEM@DES | NEM | Number of correction factor modes if any (NEM $\leq 100$ ) |
| 2 | NELIMS | NELIMS. | Number of input cards giving the minimum and maximum values of $\varepsilon$ (NELIMS $\leq 100$ ). |
| 2 | NMONITOR | NMIN | Number of sets of monitoring data used to integrate $\Delta C_{p}$ into aerodynamic parameters ( $N M O N \leq 35$ ) |
| 2 | NAXIS | NAXIS | Number of axes input for use in integrating the $\Delta C_{p}$ data into forces and moments for constraint and monitoring purposes (NAXIS $\leq 25$ ) |
| 3 | FLAGB | $\begin{gathered} \text { FLAGB }, \\ \text { IFB } \end{gathered}$ | FLAGB $=0$, correction matrix calculation <br> FLAGB $=1$, monitor data only <br> FLAGB=2, apply input correction factor matrices <br> to input pressure distribution |
| 3 | FLAGP | FLAGP, <br> IFP | FLAGP $=0$, geometry data and $\Delta C_{p}$ are input from tapes; calculate premultiplying correction factors. (See Tape Description section for format) FLAGP $=1$, geometry data and $D^{-1}$ (inverse aero matrix) are input from tapes, $W$ (normalwash) input either from tape or on cards (see FLAGW below), calculate post-multiplying correction factors |


| CARD | ITEM | MNEMONIC | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 3 | FLAGT | FLAGT | FLAGP=2, input as for $\operatorname{FLAGP}=1$; but calculate pre-multiplying correction factors <br> FLAGP=3, geometry data and $\Delta C_{p}$ input on cards; calculate pre-multiplying correction factors FLAGP $=4$, geometry data and $D^{-1}$ input from tapes, W input either from tape or on cards; calculate modified post-multiplying correction factors <br> FLAGT=0, weights for minimization process are absolute values of forces for unit deflections FLAGT=1, weights are unity |
| 3 | FLAGW | FLAGW | FLAGW=0, normalwash matrix, $W^{*}$, is input from tape, if needed <br> FLAGW $=1$, normalwash matrix, $W$, is input on cards |
| 3 | FLAGI | FLAGI | FLAGI $=0, \Delta C_{p}$ values are input as complex numbers (either from tape or on cards) <br> FLAGI $=1, \Delta C_{p}$ values are input as real numbers (i.e. not complex) |
| 3 | IPRINT | IPRINT | Detail print flag; <br> IPRINT $=1$, print rows of the <br> SAI matrix, and rows of the <br> SAN matrix (if any) <br> IPRINT $=0$, bypass printing <br> of same |

* Cap. $W$ is normalwash in the computer program where correction factors are called CF.

| CARD | ITEM | MNEMONICS | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 4 4 | TS <br> TA | NMSYM <br> NMASYM | Number of symmetric pressure modes $(\text { NMSYM }, \text { NMASYM } \leq 10)$ <br> Number of antisymmetric pressure modes |
|  |  |  | Note that all data items in cards 2 through 4 are input as integers, right-justified* in their respective fields of ten card columns each (format I10) as shown on the input sheets. |

* Right justified means input ending in the last (or right-most) card column of the field.


## Geometry and Pressure Data

Data items defining the geometry of a case are usually available on tape; similarly, pressure data (if needed; see item FLAGP under Control Data) are usually input from tape. However, if this is not the case, these data items may be input from cards by specifying FLAGP=3, as shown below.

| CARD | ITEM | MNEMONICS | DESCRIPTION |
| :---: | :---: | :---: | :---: |
|  |  |  | The following two cards are input only if FLAGP=3. |
| 5 | ${ }^{\boldsymbol{i}}$ | $x$ |  |
| 5 | $y_{i}$ | $Y$ | $x, y, z$ coordinates of pressure point $i$ |
| 5 | $z_{i}$ | Z |  |
| 5 | $\bar{\gamma}_{i}$ | GMA | Dihedral angle of pressure point i |
| 5 | $\Delta A_{i}$ | DELA | Area of box over which the pressure acts. Repeat card 5 for all points, $\mathbf{i = 1}$, NP |
| 6 | $\Delta C_{p}$ | DCP | Array of the $\Delta C_{p}$ values (lifting pressures) either 3 complex numbers per card (when FLAGI $=0$ ), or 6 real numbers per card (when FLAGI=1; see Control Data) |
|  |  |  | The format used for cards 5 and 6 is 6F10.0. |

## Axis Data

The following data items are required for all cases. These input data are used to describe an axis in space. Axes can be described by either two endpoints or by one endpoint and a set of direction cosines. These axes are used in the integration of the pressures into force or moment coefficients. Forces are resolved in the direction of the axis, while moments are taken about the axis.

| CARD | ITEM | MNEMONIC | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 7 | Axis number | IAX | Axis number |
| 7 | Axis type | IFA | IFA $=0$, axis endpoints are input IFA=1, a point and direction cosines are input |
| 8 | $\xi 1$ | XI] |  |
|  | 71 | ETAI | Axis endpoint coordinates |
|  | $\zeta 1$ | ZETAI |  |
| 8 | $\xi 2 / \cos \alpha$ | XI2 | Second axis endpoint coordinates if IFA=0; direction cosines if IFA=1 |
|  | $n 2 / \cos \beta$ | ETA2 |  |
|  | $52 / \cos \gamma$ | ZETA2 |  |
|  |  |  | Card 7 format is 6110; card 8 format is 6F10.0 |

## Constraint Data

The correction factors modify the theoretical values of $\Delta C_{p}$ by a minimum amount so that specified forces and moments are reproduced. For example, if the total lift is known experimentally, then several data items must be input specifying the actual value of the lift coefficient and describing the way the $\Delta C_{p}$ values are to be integrated to obtain this coefficient. The lift coefficient is then called a constraint on the theoretical data.

| CARD | ITEM | MNEMONIC | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 9 | Axis number F-M Flag | JAX IFF | Number of the axis to be used for calculating the constraint force or moment <br> IFF $=0$, the constraint, $C_{e}$, is a force in the direction of the axis; IFF=1, the constraint, $C_{e}$, is a moment about the axis (right-hand-rule). Card format is 6I10 |
| 10 | $\delta$ | NDI | $\delta=1$, symmetric pressure mode to be used; $\delta=-1$, antisymmetric pressure mode to be used |
| 10 | Press. <br> mode <br> ã <br> $\tilde{c}$ | MI AIT | Pressure mode number to be used with constraint $\mathrm{C}_{\mathrm{e}}$ <br> Constraining effectiveness of $C_{e}$; $0 \leq \tilde{a} \leq 1$. <br> If $\tilde{a}=1, C_{\theta}$ is a constraint; <br> if $\tilde{a}<1, C_{e}^{e}$ is only an estimate, and the <br> resulting weighted (corrected) theory will <br> only approximately reproduce $C_{e}$. If $\tilde{a}=0$, <br> then $C_{e}$ will not affect data. <br> Constant used to nondimensionalize integrated data. $\begin{aligned} & \text { If } c_{e} \text { is a force, } \tilde{c}=\text { Area; } \\ & \text { if } c_{e} \text { is a moment, } \tilde{c}=\text { Area } \times \text { length. } \end{aligned}$ |


| CARD | ITEM | MNEMONIC | DESCRIPTION |
| :---: | :--- | :--- | :--- |
| 10 | $C_{e}$ | CIE | Experimental (or any other) constraint on <br> the data. Card format is 2I10, 4F10.0. |
| 11 | LIMI1, <br> LIMI2 | LIMI(1) <br> LIMI(2) | Identification of a range of $\Delta C_{p}$ values <br> (or boxes) from LIMI1 to LIMI2 defining <br> the limits of integration for the pressures. <br> There may be as many sets of ranges input <br> as needed. Card format is 6I10. |
| 12 | -1 | -1 | The number -1; end indicator for the <br> sets of data LIMI1, LIMI2 |

## Monitor Data

The following data are input only if the control data item NM@IITØR has a value different from zero. These data are used for the integration of the $\Delta C_{p}$ values into some meaningful parameters as a check on the effect of the correction factors on theory, whenever $\operatorname{FLAGB}=0$. Since often it is desirable to monitor the unmodified data as well, the setting FLAGB = 1 is designed to integrate the $\Delta C_{p}$ values into parameters without calculating the correction factors. Another monitoring option can be activated by the setting FLAGB $=2$; in this case the correction factors are input from tape (FT16) saved in a previous run and the weighted $\Delta C_{p}$ values are integrated into parameters as specified by the monitor data.

| CARD | ITEM | MNEMONIC | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 13 13 | Axis <br> number <br> F-M flag | NAX IFN | Axis number used in the integration of the $\Delta C_{p}$ values into forces and moments <br> IFN=0, parameter to be determined is a force <br> IFN $=1$, parameter to be determined is a moment. <br> Card format is 6I10. |
| 14 | $\delta$ | NDN | $\delta=1$, symmetric pressure mode used $\delta=-1$, antisymmetric pressure mode used |
| 14 | Press. <br> mode | MN | Pressure mode number |
| 14 | ã | ANT | Not used |
| 14 | $\tilde{\mathrm{c}}$ | CNT | Constant used to nondimensionalize integrated data |
| 14 | LABEL | LABEL | Alphameric identifier of the integrated parameter (ten characters long) Card format is $2110,2 \mathrm{~F} 10.0$, 10 Al |


| CARD | ITEM | MNEMONIC | DESCRIPTION |
| :---: | :--- | :--- | :--- |
| 15 | LIMN1, | LIMN(1) <br> LIMN2 | Identification of a range of $\triangle C_{p}$ values <br> defining the limits of integration for <br> the pressures. There may be as many sets <br> of ranges input as needed. <br> Card format is 6I10. |
| 16 | -1 | -1 | The number -1; end indicator for the <br> sets of data LIMN1, LIMN2 |

## Limits Data

It is sometimes desirable to place a restriction on the range of values of $\varepsilon$ by specifying a minimum and a maximum bound on $\varepsilon$. In this case the control data item NELIMS is input as the number of $\varepsilon$ limit pairs to be supplied, which are input as shown below. Note that this input (card 17) is omitted when NELIMS $=0$.

| CARD | ITEM | MNEMONIC | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 17 17 | LIM 1 <br> LIM 2 $\begin{aligned} & \bar{\varepsilon}_{\text {min }} \\ & \bar{\varepsilon}_{\text {max }} \end{aligned}$ | LIMK (1) <br> LIMK (2) <br> EBMIN <br> EBMAX | A range of boxes, or $\Delta C_{p}$ elements, over which a limit is placed on $\varepsilon$ <br> Card format is 2110, 4F10.0 |
| $\cdot$. |  |  | Card 17 is repeated for all sets of ranges, i.e., NELIMS times. |

## Correction Factor Modes Data

In many instances it is desirable to restrict the incremental correction factors $\{\varepsilon\}$ to a linear combination of a set of modes, $\{\varepsilon\}=[\phi]\left\{\varepsilon_{g}\right\}$. The mode shapes $[\phi]$ can be input directly per box, and per mode, or the mode shapes may be selected from a set of functions. This set of data is input only if the control data item NEM is different from zero.


| CARD | ITEM | MNEMONIC | DESCRIPTION |
| :--- | :--- | :--- | :--- |
| 20 | LIMLI | LIML(1) <br> LIML2 | Omit card 20 if TYPE $=0$. <br> Range of boxes or $\varepsilon$ 's over which <br> the current $\varepsilon$-mode applies. There <br> may be as many sets of ranges input <br> as needed. <br> Card format is 6I10. |
| 21 | -1 | -1 | The number -1; end indicator of <br> data set for the current $\varepsilon$-mode <br> (MøDENØ) |

## Normalwash Data

If the normalwash matrix [W] is needed (see control data FLAGP), and it is not available on tape, the control flag FLAGW must be input as 1 , and then the normalwash values are card input as shown below.

| CARD | ITEM | MNEMONIC | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 22 | MDDE | M9DE (J) | Mode number for the current set of $W$ values |
| 22 | $\delta$ | IDELW | Symmetry flag to aid in identifying the mode; note that $\delta=1$ type values are expected to precede the $\delta=-1$ type values |
| 22 | LIMW1 <br> LIMW2 | $\operatorname{LIMW}(1)$ <br> LIMW(2) | A range of boxes over which the $W$ value applies |
| 22 | W | WIN | Normalwash, W, for the above range of boxes. <br> Card format is 4I10, 2F10.0. <br> Repeat card 22 as needed. <br> Note that only the non zero $W$ values need be input. |
| 23 | -1 | -1 | The number -1 ; end indicator for the normalwash input data |

## Tape Description

Program EIGC uses a minimum of four, and a maximum of twelve tapes and/or utility (scratch) units depending on the type of the case considered. In addition NPIT $=5$ and NPDT $=6$ are used throughout the program as the system input/output units respectively. These, as well as all tapes and utility units are defined in subroutine WEYT by means of a DATA statement specification under their respective names. The following table gives a summary of tape names and their use; the formats of those tapes that may be specified as input/output units are described in subsequent tables.

Summary of Tape Units

| NAME | UNIT | $\begin{aligned} & \text { WHEN } \\ & \text { NEEDED } \end{aligned}$ | $\begin{aligned} & \text { USER } \\ & \text { SUBROUTINES } \end{aligned}$ | DESCRIPTION OF CONTENTS |
| :---: | :---: | :---: | :---: | :---: |
| NUTLI | 1 | Always | WEYT, WSWA, SDBL, EPSJ | Miscellaneous intermediate |
| NUTL2 | 2 | Always | WEYT, SDBL, DCPT, CEMN | solutions |
| NTSAIJ | 3 | If $N C \neq 0$ | WEYT, SAIJ, DELC, SDBL | SAI matrix rows |
| NTSANJ | 4 | If $\mathrm{NM} \supseteq \mathrm{N} \neq 0$ | WEYT, SAIJ, CEMN | SAN matrix rows |
| NTPHIJ | 8 | If NEM $\ddagger 0$ | WEYT, PHIJ, <br> SDBL, EPSJ | ¢ matrix columns |
| MASTSB | 9 | Always | $\begin{aligned} & \text { WEYT, SDBL, } \\ & \text { GINV } \end{aligned}$ | S matrix rows |
| NEWTSB | 10 | If NELIMS | WEYT, M@DF, GINV | The modified S matrix rows |
| NTGEQM | 11 | If FLAGP $\neq 3$ | WEYT | Geometry arrays; input tape |
| NTDCP | 12 | Always | WEYT, DCPB | ```\DeltaC p matrix columns; either input tape or scratch unit depending on FLAGP``` |
| NTAPW | 13 | If FLAGP | WEYT, WSWA | ```W (normalwash) columns; input tape if NTAPW = 0, scratch unit otherwise``` |
| NTAPDI | 14 | $\neq 0,3$ | WEYT, DCPB, SDBL, DCPT | Inverse downwash factor matrix $[D]^{-1}$ |
| NEWDCP | 15 | $\begin{aligned} & \text { If FLAGI=1 } \\ & \text { and FLAGP } \\ & =0 \end{aligned}$ | WEYT, DCPB | Complex $\Delta C_{p}$ columns, when $\Delta C_{p}$ is input as a real matrix |
| NTAPCF | 16 | If FLAGB $\ddagger 1$ | WEYT, DCPT | CF, the correction factor matrix; NTAPCF is output tape (or scratch unit) for FLAGB $=0$ cases; NTAPCF is an input tape for $\operatorname{FLAGB}=2$ cases |


| RECORD | WORD | ITEM | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1 | 1 | LENGTH | Length of arrays in records <br> 2 through $6 \quad(L E N G T H ~=~ N P) ~$ |
| 2 | $1-N P$ | $X$ | x-coordinate array |
| 3 | $1-N P$ | $Y$ | $y$-coordinate array |
| 4 | $1-N P$ | $Z$ | z-coordinate array |
| 5 | $1-N P$ | GMA | Dinedral angle $(\bar{r})$ array |
| 6 | $1-N P$ | DELA | Array of box areas |

Input Tape (or Scratch Unit) NTDCP

| RECORD | WORD | ITEM | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1 | 1 | $\cdots N P$ | Row dimension of the $\Delta C_{p}$ matrix <br> (column length) |
| 2 | 3 | NASYM | Number of $\Delta C_{p}$ columns for <br> symmetric modes <br> Number of $\Delta C_{p}$ columns for <br> antisymmetric modes |
| 2 | $1-N P$ | DCP | $\Delta C_{p}$ column for first symmetric mode |
| $\vdots$ | $1-N P$ | DCP | $\Delta C_{p}$ column for last antisymmetric <br> mode* |
| $1+N S Y M$ <br> +NASYM | $\vdots$ |  |  |

* Note that if NASYM $=0$, the last $\Delta C_{p}$ column refers to last symmetric mode

Input Tape (or Scratch Unit) NTAPW

| RECORD | WORD | ITEM | DESCRIPTION |
| :---: | :---: | :---: | :--- |
| 1 | 1 | NP | Row dimension of the W <br> (nommalwash) matrix |
| 2 | NSYM | Numbers of W columns for <br> symmetric modes <br> Number of W columns for <br> antisymmetric modes |  |
| 2 | $1-$ NP | W | W column for first symmetric <br> mode |
| $\vdots$ | $\vdots$ | $\vdots$ | W column for last anti- <br> symmetric mode (if any) |
| $1+$ NSYM | $1-$ NP | W |  |
| +NASYM |  |  |  |

Input Tape NTAPDI

| RECORD | WORD | ITEM | DESCRIPTION |
| :---: | :---: | :---: | :---: |
| 1 | $\begin{aligned} & 1 \\ & 2 \\ & 3 \end{aligned}$ | $N P$ <br> NP NCQL2 | Row dimension of matrix DI <br> Colunn dimension of matrix DI <br> NCDL2 $=$ NP if both symmetric <br> and antisymmetric DI matrices are <br> on tape; NCøL2 $=0$ otherwise |
| 2 | 1-NP | DI | First row of DI, the inverse downwash factor matrix, $\left[\mathrm{D}^{-1}\right]$ for symmetry |
| : | : | : | : |
| $1+N P$ | $1-N P$ | DI | Last DI-row for symmetry |
| $2+N P$ | 1-NP | DI | The following records may be omitted when antisymmetric modes are not desired. <br> First DI-row for antisymmetry |
| $1+2 N P$ |  | DI | Last DI-row for antisymmetry |

Input/Output Tape NTAPCF

| RECORD | WORD | ITEM | DESCRIPTION |
| :---: | :---: | :---: | :--- |
| 1 | 1 | CØDE | Alphameric identifier of tape, <br> 4 characters in length, left justified; <br> CøDE = PRE, for pre-multiplier <br> cases, CDDE = PøST for post- <br> multiplier cases |
| 2 | 1 | LENGTH | Length of array CF. LENGTH <br> should be equal to NP. <br> Number of symmetric modes for case |
| 3 | NMSYM | Number of antisymmetric modes for case <br> Note that the last two items are not <br> used when tape NTAPCF is an input tape |  |
| 3 | $1-N P$ | CF | Array of the complex correction factors |

Test Cases. - The use of the program will be illustrated by two test cases. The first will be a premultiplier and will exercise most features of the program so that their use can be illustrated. The second test case will illustrate the use of the new postmultiplier, tape input, and downwash input on cards.

The theoretical pressures are taken from a two-dimensional analysis of an airfoil with a $25 \%$ chord flap. The new transonic procedure discussed previously will be used for the airfoil operating at a Mach Number of 0.875 and a reduced frequency of 0.0 . Figure 47 illustrates the geometry, pressures and axes data for the airfoil for control surface rotation (Mode 1) and pitch (Mode 2). Also shown on the figure are the theoretical and experimental values of $c_{\ell}, c_{m l / 4}$ and $c_{h 1 / 4}$ for mode 1 and $c_{\ell}$ for mode 2 . The experimental values are used as the constraints. An experimental value for $c_{\ell}$ for mode 2 is not available thus an estimate is given in its place in the figure.

Test Case 1. - Table III presents the input cards for the first test case. The number of pressures, NP, is 19; the number of constraints, NC, is 4. For this case 19 correction factor modes, $\phi$, will be used, thus NEM $=19$. In addition limits will be placed on the values of $\bar{\varepsilon}$. These limits will be described by one card thus NELIMS $=1$. The number of axes, NAXIS, is 3 . The program is able to monitor the corrected data, and in this test case the number of coefficients to be monitored, NMON, is 4 and they are $c_{\ell}, c_{m l} / 4$ and $c_{h 3 / 4}$ for mode 1 and $c_{\ell}$ for mode 2. Thus the monitored coefficients should reproduce the input constraints. This, in fact, is the case as the output shows in Table IV.

Since correction factors are to be calculated rather than data monitored only, FLAGB $=0$. Also since the geometrical data and pressure data are to be card input and a premultiplier is to be calculated FLAGP $=3$. The usual weight factor $T$, (the absolute value of the force on an element) is not used, thus FLAGT $=1$. Normalwash values are not input thus FLAGW $=0$. Only real values of pressure are used thus FLAGI $=1$; the detail print flag is input as IPRINT $=1$. In this example there are two modes (call them symmetric) thus

NMSYM $=2$ and NMASYM $=0$. This marks the end of the control data.

The geometry data are taken from figure 47 and are given on cards designated as type 5. The $1 / 4$-chord point of each box is input along with its area, $\Delta A=\Delta X$. The pressures at each $1 / 4$-chord point of each box and for each mode are taken from figure 47 and are given on cards designated as type 6 .

The axis data are encountered next. IAX identifies the axis number and IFA identifies how it is input (whether by two points or a point and a direction). In this case a point $\left(\xi^{(1)}, \eta^{(1)}, \zeta^{(1)}\right)$ and direction $(\cos \alpha, \cos \beta$, $\cos \gamma$ ) are input thus IFA $=1$. These points and directions are taken from figure 47 and are input on card designated as type 8.

The constraint data is next. Input are four constraints $c_{\ell}, c_{m l / 4}$ and $c_{h 3 / 4}$ for mode 1 and $c_{2}$ for mode 2 taken from the experimental values of these parameters given on figure 47. Each constraint has a 9 and 10 type card. JAX identifies the axis to be used with the constraint (axis 1 for $c_{\ell}$, axis 2 for $c_{m l / 4}$ and axis 3 for $c_{h 3 / 4}$ ). The flag IFA identifies the coefficient type to be calculated whether the force type (IFA $=0$ ) or moment type (IFA $=1$ ). The terms MI and NDI denote the mode to be used. In this case modes 1 and 2 are symmetric. The constraining power AIT is taken as 1.0 for the constraints of mode 1 to ensure a full constraint. However AIT for $c_{\ell}$ of mode 2 is taken as .95 since this is an estimate. The nondimensionalizing constant CIT is the chord for the $c_{\ell}$ constraint and the chord squared for $c_{m 1 / 4}$ and $c_{h 3 / 4}$. The limits of integration LIMI1, LIMI2 span the entire surface for $c_{\ell}$ and $c_{\mathrm{ml} / 4}$ (from box. 1 to box 19) but only range over the control surface (box 13 to box 19) for $\mathrm{c}_{\mathrm{h} 3 / 4}$.

The monitor data found on card types $13,14,15,16$ are almost identical to that of the constraint data because in this case $c_{\ell}, c_{m 1 / 4}$ and $c_{h 3 / 4}$ are the parameters to be monitored. Of course they could be any quantity or for that matter no quantities if monitoring is not desired. The only real difference between monitor data and constraint data is that an alpha-numeric identifier is input in place of the constraints for the monitor data.

As an example of the use of limiting values on $\bar{\varepsilon}$, card type 17 is input for this test case. Specifically it is required that

$$
-0.7 \leq \varepsilon \leq 1.5
$$

hold for all values of $\bar{\varepsilon}, 1$ through 19 (LIMK1 $=1$, LIMK2 $=19)$.
As a simple example of the use of correction factor mode shapes, $\phi$, an identity matrix will be used;

$$
[\phi]=[I\rfloor \quad(I T Y P E=0)
$$

Card types 19 and 21 are used to input these modes.
The program output for this case is given in Table IV. The printed output, which fits on $81 / 2 \times 11$ sheets, contains most of the input. Integration matrices are then printed along with other intermediate steps in the process of solution. At the end of the printout a summary of the geometry data, incremental correction factors, $\varepsilon$, and modified pressures are printed. Next are the correction factors $\varepsilon+1$ and finally the aerodynamic parameters, calculated using the modified pressures, that have been monitored by the program.

Test Case 2. - Table $V$ presents the input sheets for the second test case. This test case is the same as the test case 1 with the following exceptions: (1) the geometry and $[\mathrm{D}]^{-1}$ are input from tape; (2) a new postmultiplier is developed; (3) one mode is used with three constraints; (4) correction factor modes are not used and (5) limits on incremental correction factors, $\varepsilon$, are not imposed.

For this case changes from test case 1 occur in the control data (cards 1 through 4). First, no correction factor modes (NEM $=0$ ) are to be used. Second, the card giving limits on $\varepsilon$ is omitted, thus NELIMS $=0$. Third, $[0]^{-1}$ and the geometry are to be read in on tapes and the new postmultiplying
correction factor is desired, thus FLAGP $=4$. In this case normalwash values are to be card read and so FLAGW $=1$. Also only one mode is to be used (control surface rotation), thus $N M S Y M=1$. The geometry data remains the same as in test case 1.

Finally the normalwash is input on card type 22. The mode is a control surface rotation, thus $W^{*}=1.0$ over boxes 13 through 19 , i.e., $\operatorname{LIMW1}=13$, LIMW2 $=19 . \quad$ The program output is given in Table VI.

[^0]TABLE III
INPUT DATA - TEST CASE I



$$
\begin{aligned}
& \begin{array}{llllll}
0 & 0 & 0 & 0 & 0 & 0 \\
\dot{0} & 0 & \dot{0} & 0 & 0 & \dot{0}
\end{array} \\
& \begin{array}{lllllll}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array} \\
& \begin{array}{lllllll}
\vdots & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
\end{aligned}
$$

$$
\begin{aligned}
& \overrightarrow{1} \overrightarrow{1} \overrightarrow{1} \overrightarrow{1} \overrightarrow{1} \overrightarrow{1}
\end{aligned}
$$

## QUTPUT LISTING - TEST CASE 1



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\end{aligned}
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$$
\begin{aligned}
& \begin{array}{lllll}
= & 0 & 0 & 0 & 0 \\
- & 0 & 0 & 0 & 0 \\
- & 0 & 0 & 0 & 0 \\
\vdots & 0 & 0 & 0 & 0 \\
1 & -1 & -1 & 0 & 0
\end{array}
\end{aligned}
$$

$$
\begin{aligned}
& \underset{\sim}{\sim}
\end{aligned}
$$

$$
\begin{aligned}
& \begin{array}{l}
A-W I G(N) \\
1.000000 \\
1.000000 \\
1.000000 \\
1.000000
\end{array} \\
& \begin{array}{ccccccc}
\text { THE } 4 \text { SETS OF INPUT DATA FOR MCNITORING } & \\
\text { AXISNO(N) } & \text { FLAGF(N) } & \text { DELTA } & \text { M } & \text { LIMI } & \text { LIM2 } \\
1 & 0 & 1 & 1 & & 19 \\
2 & 1 & 1 & 1 & 1 & 19 \\
3 & 1 & 1 & 1 & 1 & 19 \\
1 & 0 & 1 & 2 & 13 & 19 \\
& & & & & 1 & 19
\end{array}
\end{aligned}
$$



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\text { EPSILON-PAP-MAX } \\
\text { REAL } \\
1.500000
\end{array} \\
& \begin{array}{rr}
\text { FPSILCN-RAF-MIN } \\
\text { IMAG. } \\
-0.700000 & 0.0
\end{array} \\
& \begin{array}{c}
\text { LIM-2 } \\
19
\end{array}
\end{aligned}
$$

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| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0000 | 0000 | 0000 | 0000 | 0000 | 0000 | 0006 |


|  | 0 | 0 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
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| 0.000 | 0000 | 0000 | -000 | 0000 | 0000 | 000 |
| 0000 | 000 | 000 |  |  |  |  |



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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 0 0 8 0 |  |  |  | 0 0 8 8 |  |
| 0000 | 00000 | 00000 | 0000 | 00000 | 00000 | 00000 |
| -0000 | -1000 | $00^{\circ}$ | -000 | 0000 | 0.000 | cocco |


| $\begin{aligned} & \text { ocmo } \\ & \text { oc } \end{aligned}$ | $\begin{aligned} & 0000 \\ & 0000 \end{aligned}$ | $\begin{aligned} & 0000 \\ & 0000 \end{aligned}$ | 9000 0000 | -000 | $\begin{aligned} & 0000 \\ & 0000 \end{aligned}$ | 0000 $00^{\circ \circ}$ | $\begin{aligned} & 0000 \\ & 0.000 \end{aligned}$ | $\begin{aligned} & 900 \\ & 00 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 0 \\ 0 \\ 0 \\ 0 \\ 0.00 \\ 0-100 \end{gathered}$ | $\begin{aligned} & 0000 \\ & 0000 \end{aligned}$ | $\begin{aligned} & 0000 \\ & 0000 \end{aligned}$ | $\begin{aligned} & 0000 \\ & 0000^{\circ} \end{aligned}$ | $\begin{gathered} 0 \\ 0 \\ 0 \\ 0 \\ 0000 \\ 00-0 \end{gathered}$ | $\begin{aligned} & 0000 \\ & 0000 \end{aligned}$ | $\begin{aligned} & 0000 \\ & 0000 \end{aligned}$ | $\begin{aligned} & 0000 \\ & 0000 \end{aligned}$ |  |
| $\begin{aligned} & 00000 \\ & 00000 \end{aligned}$ | $00000$ | 00000 00000 | 00000 00000 | 00000 00000 | $\begin{aligned} & 00000 \\ & 00000 \end{aligned}$ | $\begin{aligned} & 00000 \\ & 0.0000 \end{aligned}$ | 00000 -••• 00000 | $\begin{aligned} & 000 \\ & 00- \end{aligned}$ |
| 00000 00000 | 00000 00000 | $\begin{aligned} & 00000 \\ & 0.000 \end{aligned}$ | $\begin{gathered} \circ \\ 8 \\ 0 \\ 0 \\ 0000 \\ 00-100 \end{gathered}$ | 20000 00000 | $\begin{aligned} & 00000 \\ & 0.0000 \end{aligned}$ | $00000$ | $\begin{array}{r} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 000 \\ 000 \\ 000 \end{array}$ | $\begin{aligned} & c=r \\ & 000 \end{aligned}$ |
|  |  |  |  | - |  |  |  |  |
| $\begin{aligned} & 00000 \\ & 0.000 \end{aligned}$ | 00000 $00^{\circ \circ}$ | $\begin{aligned} & 00000 \\ & 0.000^{\circ} \end{aligned}$ | $\begin{aligned} & 00000 \\ & 0.000 \end{aligned}$ | $\begin{aligned} & 00000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 00000 \\ & 0.0000 \end{aligned}$ | 00000 $0^{\circ} 0^{\circ} 0^{\circ}$ | 00000 $0^{\circ 000} 0^{\circ}$ | $\begin{aligned} & 000 \\ & 00 \\ & \hline 0 . \end{aligned}$ |
| 00000 -0000 | $\begin{aligned} & 00000 \\ & 0.0000^{\circ} \end{aligned}$ |  | $\begin{aligned} & 00000 \\ & .0000 \end{aligned}$ | $\begin{aligned} & 0000 \\ & 0.000 \end{aligned}$ | $\begin{aligned} & 00009 \\ & 00000 \end{aligned}$ |  | $00000$ | $\begin{aligned} & 000 \\ & \dot{\circ} \mathrm{c} \end{aligned}$ |





| The | 2 COLIJMNS CF | THE | DELTA-CP-RAR | MATRIX |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{\rightharpoonup}{\mathrm{O}} \operatorname{colu}$ | 1 |  |  |  |  |
|  | $0.701000 F+01$ | 0.0 | 2 | $0.245000 \mathrm{E}+01$ | 0.0 |
| 4 | $0.128000 \mathrm{~F}+01$ | 0.0 | 5 | $0.114000 \mathrm{~F}^{+01}$ | 0.0 |
| 7 | $0.248400 \mathrm{E}+01$ | 0.0 | $1{ }^{8}$ | $0.969000 F+01$ $0.126500+02$ | 0.0 |
| 13 | -178800E+02 | 0.0 | 14 | $0.125000 \mathrm{E}+02$ | 0.0 |
| 16 | $0.560000 \mathrm{E}+01$ | 0.0 | 17 | $0.341000 E+01$ | 0.0 |
| 19 | $0.561400 E+00$ | 0.0 |  |  |  |

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| 2 | $0.918750 \mathrm{E}-01$ | 0.0 |
| :--- | :--- | :--- |
| 5 | $0.103170 \mathrm{E}+00$ | 0.0 |
| 8 | 0.0 | 0.0 |
| 11 | $0.474375 \mathrm{E}+00$ | 0.0 |
| 14 | 0.0 |  |
| 17 | $0.170500 \mathrm{E}+00$ | 0.0 |
| 20 | 0.0 | 0 |
| 23 | 0.0 | 0.0 |
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|  | SYMMFTRIC DELTA-CP-TILD |  |  | MIOES 1 |  | RFAI EPS(J) IMAG. |  | DCP-TILDA(J) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $J$ | X (J) | $Y(J)$ | Z(J) | GAMMA (J) | DELTA-A (J) |  |  |  |  |
|  | -0.9936 | 0.0 | 0.0 | 0.0 | $0.0256$ | $-0.401236$ | 0.0 |  | $0.0$ |
| 2 | -0.9936 -0.9429 | 0.0 | 0.0 | 0.0 | $0.0750$ | $-0.422297$ | $0.0$ | $1.415373$ | $0.0$ |
| 3 | -0.8450 | 0.0 | 0.0 | 0.0 | 0.1190 | -0.406577 | 0.0 | 0.858811 | 0.0 |
| 4 | -0.7066 | $0 \cdot 0$ | 0.0 | 0.0 | 0.1550 | $-0.487404$ | . 0 | $0.656123$ | 0.0 0.0 |
| 5 | -0.5370 | 0.0 | $0 \cdot 0$ | 0.0 | 0.1810 | $-0.471456$ | 0.0 | $0.602541$ | 0.0 |
| 6 | -0.3479 | 0.0 | 0.0 | 0.0 | 0.1940 | $-0.391275$ | 0.0 | $0.771254$ | 0.0 |
| 7 | $-0.1521$ | 0.0 | $0 \cdot 0$ | 0.0 | 0.1940 | $0.030033$ | 0.0 | $\begin{array}{r} 2.558602 \\ 2429001 \end{array}$ | O. 0 |
| 8 | $0.0370$ | $0 \cdot 0$ | 0.0 | $0 \cdot 0$ | 0.1810 | $1.500000$ | 0.0 | $24 \cdot 224991$ | 0.0 |
| 9 | $0.2066$ | 0.0 | 0.0 | 0.0 | 0.1550 | $0.216393$ | 0.0 | $10.874557$ | 0.0 |
| 10 | 0.3450 | 0.0 | 0.0 | $0 \cdot 0$ | 0.1190 | $-0.426141$ | 0.0 | $5.876318$ | 0.0 |
| 11 | 0.4429 | 0.0 | $0 \cdot 0$ | 0.0 | $0.0750$ | $-0.637582$ | 0.0 | $\begin{array}{r} 4.584599 \\ 11.557776 \end{array}$ | 0.0 |
| 12 | 0.4936 | 0.0 | $0 \cdot 0$ | 0.0 | $0.0256$ | $-0.320454$ | 0.0 | $11.552276$ | 0.0 |
| 13 | $0.5063$ | $0 \cdot 0$ | 0.0 | 0.0 | $0.0248$ | $-0.342438$ | 0.0 | 11.757212 | 0.0 0.0 |
| 14 | $0.5545$ | 0.0 | 0.0 | 0.0 | $0.0694$ | $-0.700000$ | 0.0 | 3.750000 2.526000 | 0.0 0.0 |
| 15 | $0.6415$ | 0.0 | 0.0 | $0 \cdot 0$ | $0.1000$ | $-0.700000$ | 0.0 0.0 | $\begin{aligned} & 2.526000 \\ & 1.470000 \end{aligned}$ | 0.0 0.0 |
| 16 | $0.7500$ | 0.0 | 0.0 | 0.0 | $0.1110$ | $-0.700000$ | 0.0 | $\begin{array}{r} 1.579999 \\ 1.735744 \end{array}$ | 0.0 0.0 |
| 17 | $\begin{aligned} & 0.8585 \\ & 0.9455 \end{aligned}$ | 0.0 0.0 | 0.0 0.0 | $\begin{aligned} & 0.0 \\ & 0.0 \end{aligned}$ | $\begin{aligned} & 0.1000 \\ & 0.0694 \end{aligned}$ | $\begin{aligned} & -0.490984 \\ & -0.192810 \end{aligned}$ | 0.0 0.0 | 1.735744 1.444869 | 0.0 0.0 |
| 18 19 | 0.9455 0.9937 | 0.0 0.0 | 0.0 0.0 | 0.0 0.0 | $\begin{aligned} & 0.0694 \\ & 0.0248 \end{aligned}$ | $\begin{aligned} & -0.192810 \\ & -0.022389 \end{aligned}$ | 0.0 0.0 | 1.444869 0.548831 | 0.0 0.0 |

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table V
INPUT DATA - TEST CASE 2


## TABLE VI

gUTPUT LISTING - TEST CASE 2


```
M= O
    M
    N=\frac{~}{n}
    a
    M
L N
```

$$
\begin{aligned}
& \begin{array}{llll}
\dot{L} & & \\
\Sigma & 0 & 0 \\
0 & 0 & 0
\end{array}
\end{aligned}
$$

$$
\begin{aligned}
& \begin{array}{rccccccc}
\text { THE } 3 & \text { SETS OF INPIT DATA FOR ALL CONSTRAINTS } \\
\text { AXISNOIII } & \text { FLAGFII) } & \text { NELTA } & \text { M LIMI } & \text { LIMZ } \\
1 & 0 & 1 & 1 & & \\
2 & 1 & 1 & 1 & 1 & 19 \\
3 & 1 & 1 & 1 & 1 & 19 \\
& & & & & 13 & 19
\end{array}
\end{aligned}
$$

$$
\begin{aligned}
& \begin{array}{r}
C-W I G(N) \\
0.200000 F+01 \\
0.400000 E+01 \\
0.400000 E+01
\end{array} \\
& \begin{array}{l}
\triangle-W I G(N) \\
1.000000 \\
1.000000 \\
1.000000
\end{array}
\end{aligned}
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| DELTA-C |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\vec{\omega}-0.630051$ | ). 122550 | 0.5198 .34 | -0.018031 | 0.087867 | -0.001059 |  |  |
| THEDRFTICAL | c-values |  |  |  |  |  |  |
| 5.560051 | -). 127550 | $-2.0878 .34$ | ). 218931 | $-.7 .143867$ | 3.001059 |  |  |
| ONF S-BAR | PחW |  |  |  |  |  |  |
| -0.035068 | 0.00061 ? | -0.147536 | 0.002546 | -0.307465 | 0.005638 | -0.488549 | 0.009541 |
| -0.673939 | 0.013722 | -0.746481) | 0. 015813 | -).612399 | 0.312899 | -9.414697 | 0.008227 |
| -0.437368 | 0.008864 | -0.357380 | 0.007347 | -0.204170 | 0.004221 | -0.031777 | 0.000653 |
| -0.195771 | 0.004057 0.025356 | -0.404510 -1.426368 | 0.008493 0.032111 | -0.636052 -0.871721 | 0.013583 0.019694 | -0.893069 0.0 | 0.019254 |
| $\begin{array}{r} -1.143367 \\ 0.0 \end{array}$ | 0.025356 | -1.426368 | $2.03$ | -0.871721 |  |  |  |
| ONE S-BAR | ROW |  |  |  |  |  |  |
|  | -0.000047 | -3.0881.34 | -0. 0.10163 | -.). 170660 | -0.000274 | -0.239268 | -0.000325 |
| -0.266833 | -0.000338 | -0.212600 | -0.000411 | -0.085731 | -0.000688 | 0.005131 | -0.000793 |
| 0.039694 | -0.000944 | 0.054618 | -0.000838 | 0.739468 | -0.000504 | 0.006702 | -0.000080 |
|  |  |  |  |  |  |  |  |
| $\begin{aligned} & 0.043370 \\ & 0.453494 \\ & 0.0 \end{aligned}$ | $\begin{aligned} & -0.000781 \\ & -0.103788 \\ & 0.0 \end{aligned}$ | $\begin{aligned} & 0.1078 y^{4} \\ & 0.0 \\ & 0.0 \end{aligned}$ | $\begin{gathered} -0.004997 \\ 0.0 \end{gathered}$ | 0.379680 | -0.003102 | 0.0 | $0$ |
| ONE S-RAR | RTW |  |  |  |  |  |  |
| -0.000135 | -0.000005 | -0.000569 | -0.000020 | -0.001298 | -0.000043 | -0.002319 | -0.000069 |
| -0.003637 | -0.000n94 | -0.004861 | -0.000103 | -0.005300 | -0.000088 | -0.005218 | -0.000065 |
| -0.007625 | -0.000n69 | - 0.108914 | - 0.00055 |  |  |  |  |
| -0.010606 0.042317 | -0.000030 -0.000217 | -0.070465 0.085811 | -0.000059 -0.000315 | -0.018662 0.059176 | -0.000093 -0.000202 | O.0.003496 | -0.000144 0.0 |
| 0.04 | -0.0 | 0.0 | 0.0 |  |  |  |  |


| -S- MATPIX |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{r} 0.776227 F+01 \\ -0.163754 F+01 \\ -0.181983 F+00 \end{array}$ | 0.0$0.168500 F-01$ |  | $\begin{array}{r} -7.163754 \mathrm{~F}+01 \\ 0.108984 \mathrm{C}+01 \\ 0.909225 \mathrm{~F}-01 \end{array}$ |  | $\begin{array}{r} -0.168500 E-01 \\ 0.0 \\ -0.326475 E-93 \end{array}$ |  | $\begin{array}{r} -0.181983 F+00 \\ 0.909225 \mathrm{~F}-01 \\ 3.138484 \mathrm{~F}-01 \end{array}$ | $\begin{aligned} & -0.27393 n c-02 \\ & 0: 326475 \mathrm{~F}-03 \\ & 0.7 \end{aligned}$ |  |
|  |  |  |  |  |  |  |  |  |  |
| -DC- COLIJMN |  |  |  |  |  |  |  |  |  |
| -0.630051F+00 | $0.122550 E+00$ |  | $0.5198045+00$ |  | -0.180207E-01 |  | $0.878674 E-01$ | -0.105876F-02 |  |
| SOLUTION DF MATRIX FQ. |  |  |  |  |  |  |  |  |  |
| 0.951395 F-01 | 0.167979E-01 |  | -1.3.j4458E-01 |  | -0.258064F-02 |  | $0.779167 F+01$ | $0.180770 F+00$ |  |
| OUTPUUT DF T.FN. INVERSE (EPS-TILNA) |  |  |  |  |  |  |  |  |  |
|  | -0.000575 |  | 15273 | -0.00 |  | -0.033606 | -0.005084 | -0.057116 | -0.008387 |
| -0.084121 | -0.011875 |  | 02196 | - -0.01 |  | -0.996750 | - 2.011589 | -0.088 0.014564 | -0.008222 |
| -0.102091 | -0.009162 |  | 207497 | -0.01 |  | -0.211533 | -0.015178 | -0.065944 | -0.015800 |
| -0.102517 0.207560 | -0.005489 |  | 14896 | -0.01 |  | -0.366935 | - 0.005312 | 0.0 | 0.0 |
| 0.0 | 0.0 |  |  | 0.0 |  |  |  |  |  |



```
mNmosuNmogonmonomag
```





The computer program for generating correction factors (EIGC) consists of twenty subroutines. The MAIN of this program reads and writes the header card and reads the control dimensions for a case; the latter are used for dimensioning most of the complex arrays that are passed into Subroutine WEYT via the argument list. Subroutine WEYT is the actual working main of the program, which calls all the major subroutines, supplying these with the necessary information via their argument lists. The following is a detailed description of all subroutines of program EIGC including their flow charts, where applicable, given in alphabetical order. The computer program is written in the FORTRAN IV programming language.

SUBRØUTINE CEMN(NPQT, IGQ, MQDE, NTAPSA, NP, NMON, LABEL, NUTL,
SAI, DCPTIL, CE)

## Functional Description

This subroutine integrates the corrected pressures, $\Delta C_{p_{e}}$, into coefficients, $C_{e}$, which are used to monitor the results (See Eq. (3)). The integration procedure is identical to that required for obtaining the imposed constraints.

$$
\left\{C_{e}\right\}=[S]\left\{\Delta C_{p_{e}}\right\}
$$

The coefficients $C_{e}$ are part of the printed output.

Description of Argument List

| NPøT | Data set number of the system output data set |
| :---: | :---: |
| IG $\emptyset$ | 1 for symmetric modes |
|  | 2 for antisymmetric modes |
| MIDE | Mode number |
| NTAFSA | Data set (tape) number of tape containing the integration matrix [S.] |
| NP | Number of rows in the $\Delta C_{p}$ matrix |
| AMgN | Number of integration rows used for monitoring |
| LABEL | Alphanumeric label describing the aerodynamic |
|  | parameters |
| NUTL | Data set (tape) number of tape containing columns |
|  | of the weighted pressures, $\Delta C$ |
| SAI | A row of the integration matril [S] |
| DCPTIL | A column of the weighted pressures $\left\{\Delta C_{p}\right\}$ |
| CE | A column of the aerodynamic parameters $\left\{C_{e}\right\}$ |

[^4]Flow Chart


SUBRQUTINE DCPB(NTDCP, NTAPW, NTAPDI, IG , IFP, IFW, NRØW, NCQL, NMAX, DCP, CQL, WGRK)

## Functional Description

This subroutine computes the theoretical pressure distribution if it is not input. Specifically

$$
\left\{\Delta C_{p_{t}}\right\}=[D]^{-1}\{W\}
$$

where $W$ is the normalwash and $[D]^{-1}$ is the inverse of the aerodynamic influence coefficient matrix. This corresponds to Equation (1) where [ $D]^{-1}$ $=[A] . \quad \Delta C_{P_{t}}$ is called DELCPB in this subroutine.

## Description of Argument List

NTDCP Tape number containing the matrix of pressure coefficients, $\left[\Delta C_{p_{t}}\right]$, in column order

NTAPW

NTAPDI

IG0

IFP

I FW

NRØW
NCDL
NMAX
DCP
COL
WØRK Tape number containing the normalwash matrix, [W], in column order

Tape number containing the inverse-D matrix, ([A])
[DI], in row order
1 for symmetric modes,
2 for antisymmetric modes
Control flag (see input flag FLAGP).
IFP $=0,2,3$ means premultiplying correction
factors, IFP $=1,4$ means post-multiplying correction factors

Normalwash flag. IFW $=0$ means normalwash
is tape input (if any),
IFW = 1 means normalwash is card input
Number of rows in the $\Delta C_{p_{t}}$ matrix
Number of columns in the $\Delta C_{P_{t}}$ matrix
Maximum number of columns in the $\Delta C_{p_{t}}$ matrix
One column of the $\Delta C_{p_{t}}$ matrix (complex)
Temporary work array (complex)
The NROW $x$ NMAX complex array containing the $\Delta C_{p_{t}}$ matrix

Calling Subroutine
WEYT


SUBROUTINE DCPT (NPDT, FLAGB, IGn, MODE, NP, NSCRCH, NUTL, NTAPDI,

DI, EPS, DCPBAR, DCPTIL, WoRK, EB)

## Functional Description

This subroutine modifies the theory with the calculated correction factors. If a premultiplier is used the theoretical pressure, $\Delta C_{p_{t}}$, is modified to produce the modified pressures $\Delta C_{P_{e}}$ (see eqs.(2) and $t^{(5)}$ ).

$$
\left\{\Delta C_{p_{e}}\right\}=[1+\varepsilon]\left\{\Delta C_{p_{t}}\right\}
$$

If a postmultiplier is used then the downwash, $W$, is modified to produce the corrected pressures $\Delta C_{p}$ (see eqs. (5) and (28)).

$$
\left\{\Delta C_{p_{e}}\right\}=[D]^{-1}\lceil 1+\varepsilon\rfloor\{W\}
$$

If the new postmultiplier is used then

$$
\begin{aligned}
& \left\{\Delta C_{p_{e}}\right\}=[D]^{-1}\{W+[\bar{\phi}]\{\bar{\varepsilon}\}\} \\
& {[\bar{\phi}]=[\phi][\ell]} \\
& \{\ell\}=[\phi]^{\top}\left\{\Delta C_{p_{t}}\right\}
\end{aligned}
$$

(See eq. 65)
Also the correction factors, CF, are written on tape where

$$
C F=1+\varepsilon
$$

## Description of Arguments

| NPØT | Data set number of the system output data set |
| :--- | :--- |
| FLAGB | Option flag for correction matrix calculation and/or |
| monitoring of data |  |
| IG | 1 for symmetric modes <br> 2 |

MODE
NP
NSCRCH

NUTL

NTAPDI

NTAPW

NTAPCF

X
$Y$
Z

NMAX

NEM

Mode number
Number of row elements in the $\Delta C_{p}$ matrix Data set (tape) number containing the $\bar{\phi}$ matrix in row order (for FLAGP=4 cases only) Data set (tape) number on which the $\Delta C_{p_{e}}$ columns are saved

Data set (tape) number containing the $D^{-1}$ matrix rows (if needed) $\quad\left(D^{-1}=A\right)$

Data set (tape) number containing the $W$ matrix columns (if needed)

Data set (tape) number on which the matrix of correction factors, CF, is saved in column order
x coordiantes
$y$ coordinates ( of the pressure points or the $\Delta C_{P_{t}}$ $z$ coordinates

Dihedral angle array of the boxes over which the pressures act

Array of box areas
Column dimension of the two-dimensional complex array WØRK

W
DI
EPS
DCPBAR
DCPTIL
WgRK
EB $\bar{\varepsilon}$ array $(\varepsilon=\phi \bar{\varepsilon})$

Calling Subroutine WEYT


Flow Chart



## Elow Chart



## Functional Description

This subroutine forms the difference between the theoretical, $C_{t}$, and the experimental (constrained) $C_{e}$ coefficients (see Equations (9) and (10)).

$$
\left\{\Delta C_{e}\right\}=\left\{C_{e}\right\}-[S]\left\{\Delta C_{p_{t}}\right\}
$$

It also prints out $C_{t}\left(=[S]\left\{\Delta C_{p_{t}}\right\}\right)$ and $\Delta C$.

## Description of Argument List

| NTAPE | Tape containing rows of the integration matrix, [S] |
| :---: | :---: |
| NP®T | Data set number of the system output data set |
| NC | Number of constraints applied to $\Delta C_{p}$ values |
| NP | Number of row elements in the $\Delta C_{p}$ matrix |
| NMgDE | Number of modes |
| NMAX | Maximum number of columns in the two-dimensional |
|  | WØRK array |
| CIE | Array containing the input values |
|  | ${ }_{\text {Ce }}$ (experimental constraints) |
| DCI | The $\left[\Delta C_{e}\right.$ ] matrix |
| SAI | A row of the integration matrix [S.] |
| WORK | The NP by NMAX complex array containing the $\Delta C_{p_{t}}$ matrix |

Calling Subroutine WEYT


# SUBRØUTINE EDBL(NPØT, NELIMS, NP, NS, LIMK, JARR, NSM9D, EBMIN, EBMAX, EB, ELIM) 

## Functional Description

This subroutine compares the correction factors, $\bar{\varepsilon}$, with the input limits $\bar{\varepsilon}_{\min }, \bar{\varepsilon}_{\text {max }}$. If any $\bar{\varepsilon}$ falls outside of the limits it replaces $\bar{\varepsilon}$ with the closest limit. (The values of $\bar{\varepsilon}$ are correction factors if correction factor modes do not exist). This subroutine forms the final correction factor array $\bar{\varepsilon}$ (see paragraph below Eq. (60)).

## Description of Argument List

NP@T Data set number of the system output data set
NELIMS
NP
NS $\quad N S=N P+N C$ when $N E M \leq N P$
NS $=$ NEM + NC when NEM > NP
LIMK A two-dimensional array containing the first- and last box numbers that define a range of boxes (or $\Delta C_{p}$ ) over which
a limit is to be placed on $\varepsilon$
Array of the box numbers for which the $\varepsilon$ values are modified

NSMøD The number of $\varepsilon$ values which are modified due to the limits placed on these

EBMIN $\quad$ The minimum- and maximum value allowed for the values
EBMAX $\quad$ of $\varepsilon$ for boxes (or $\Delta C_{p}$ ) in the range defined by LIMK
EB
ELIM Array of the calculated $\bar{\varepsilon}$ values

Array of the $\varepsilon$ values that were modified due to the $\bar{\varepsilon}_{\text {min }}, \bar{\varepsilon}_{\text {max }}$ restrictions

Calling Subroutine WEYT

Flow Chart


Elow Chart


SUBRQUTINE EPSJ(NTPHIJ, NP, NEM, NS, EB, EPS, PHI)

## Functional Description

This subroutine relates $\varepsilon$ to $\bar{\varepsilon}$, as in Equation (53).
$\{\varepsilon\}=[\phi]\{\bar{\varepsilon}\}$
where $[\phi]$ are correction factor modes and where
$\varepsilon=\left\{\begin{array}{l}\varepsilon_{\mathrm{p}} \text { for premultiplying correction factors } \\ \varepsilon_{\mathrm{W}} \text { for postmuliplying correction factors }\end{array}\right.$

Description of Argument List

| NTPHIJ | Tape number containing the $\phi$ matrix |
| :--- | :--- |
| NP | Number of row elements in the $\phi$ matrix |
| NEM | Number of correction factor modes |
| NS | NS $=$ the greater of (NP+NC) and (NEM+NC) |
| EB | Array of the $\bar{\varepsilon}$ values |
| EPS | The final $\varepsilon$ array |
| PHI | A column of the matrix of weight factor mode |
|  | shapes, $\phi$ |

Calling Subroutine WEYT

## Elow Chart



SUBROUTINE GINV (NPQT, NTAPSB, NC, NS, AX, DC, ER, B, S, PB)

## Functional Description

This subroutine provides a general inverse of the following set of equations:

$$
N C\left\{\left\{\Delta C_{e}\right\}=[\underline{\tilde{S}}]\{\tilde{\varepsilon}\}\{N S\right.
$$

When NC $=$ NS (Direct Solution)

$$
\{\tilde{\varepsilon}\}=[\tilde{\bar{S}}]^{-1}\left\{\Delta C_{e}\right\}
$$

When $N C$ < NS (Minimization Solution, $\tilde{\varepsilon}^{\sim}{ }^{2}=\min$ (see Eq. (20))

$$
\begin{aligned}
& \{\tilde{\varepsilon}\}=[\tilde{\tilde{S}}]^{H}{ }_{\{\lambda\}} \\
& \left.\{\lambda\}=[\underline{\tilde{S}}][\tilde{\tilde{S}}]^{H}\right]^{-1}\left\{\Delta C_{e}\right\}
\end{aligned}
$$

When $N C>N S$ (Least Squares Solution $\left.\sum \Delta C_{e}{ }^{2}=\min .\right)$

$$
\begin{aligned}
& \{\tilde{\sim}\}+\left[[\tilde{\mathcal{S}}]^{H}[\underline{\bar{S}}]\right]^{-1}\left\{\Delta C_{e}\right\} \\
& \{\lambda\}=[\tilde{\mathcal{S}}]^{H}\left\{\Delta C_{e}\right\}
\end{aligned}
$$

In the above;

$$
[\underline{\bar{S}}]=[\overline{\bar{S}}]\left[\sqrt{T_{p}}\right]^{-1}
$$

where [ $\overline{\bar{S}}]$ given in Eq. (51). The term $\tilde{\varepsilon}$ is given in Eq. (22). The superscript $H$ indicates the Hermitian transpose.

## Description of Argument List

| NP®T | Data set number of the system output data set |
| :---: | :---: |
| NTAPSB | Tape number containing the $\frac{\tilde{S}}{\sim}$ matrix |
| NC | Number of constraints; number of rows in the $\stackrel{\sim}{S}$ matrix (SBB) |
| NS | $N S=$ the greater of ( $N P+N C$ ) and (NEM+NC); number of columns in the $\frac{\tilde{n}}{\bar{S}}$ matrix |
| NX | $N X=N S$ if $N E M=0, N X=N E M+N C$ otherwise |
| DC | The complex $\Delta C_{e}$ array |
| EB | The complex array $\{\varepsilon\}$, output of subroutine GINV |
| B | An array of intermediate solutions (complex) |
| S | A complex two-dimensional work array of dimension |
|  | NC by NC |
| SBE | The complex NC by NS matrix, $\frac{\sim}{S}$ |

Calling Subroutine
Called Subroutine

WEYT
MIS2, the standard IBM system subroutine for solving complex matrix equations


> SUBRQUTINE MATM(NT, IG@, NR, NC, NMAX, A, C, B)

## Functional Description

Subroutine MATM is essentially a matrix multiplication routine. It obtains the DI matrix ([A]) rows from tape NT and the $W$ matrix from the two-dimensional array $B$. The results of the matrix multiplication, $\Delta C_{p_{t}}$, are saved in array $B$ which is returned to the calling routine, DCPB , via the argument list.

## Description of Argument List

NT Tape number containing the inverse-D matrix

IG

NR
NC
NMAX
A
C

B
Two-dimensional complex array in which the $\Delta C_{P_{t}}$ matrix is stored

Calling Subroutine DCPB

SUBRQUTINE M $\operatorname{LDF}$ (NC, NS, MASTSB, NEWTSB, JARR, SQRTT, ELIM, SBB, DCI, DCM9D)

## Functional Description

When some of the values of $\bar{\varepsilon}$ have exceeded their limits and have been replaced by the limit values, these new values of $\bar{\varepsilon}$ (called $\varepsilon_{d}$ in Equation (56) are then considered fixed and known. However the constraints are now not satisfied and a change in the constraint $\Delta C_{e}$, i.e., $\Delta C_{\text {mod }}$, is calculated (see Equation (59)).

$$
\Delta C_{\text {mod }}=\Delta C_{e}-\left[\overline{\bar{S}}_{d}\right]\left\{\varepsilon_{d}\right\}
$$

Since the new values of $\bar{\varepsilon}$, i.e., $\varepsilon_{d}$, can not influence solution further the $\overline{\bar{S}}$ matrix must be changed to delete the influence of $\varepsilon_{d}$. Thus the elements of $\overline{\bar{S}}$ that give the influence of $\varepsilon_{d}$, i.e., $\overline{\bar{S}}_{d}$, must be eliminated resulting in $\overline{\bar{S}}_{u}$. This subroutine forms $\overline{\overline{\mathrm{S}}}_{u}$, or in the notation of the computer program $\left[\overline{\bar{S}}_{\text {mod }}\right]$.

Description of Argument List

NC Number of constraints - dimension of the complex
arrays DCI and DCMOD
NS
Dimension of the complex array SBB
MASTSB
Tape number containing SBB arrays (i.e., rows of the $\bar{S}$ matrix)

NEWTSB

JARR

SQRTT
ELIM
SBB
Tape number containing the modified SBB arrays (i.e. rows of the $\overline{\bar{S}}_{\text {mod }}$ matrix) $\quad \overline{\bar{S}}_{\text {mod }}=\overline{\bar{S}}_{u}$
Array of the element numbers for which the $\overline{\bar{S}}$-values
are replaced by zeroes
$\sqrt{T_{j}} \quad-$ see Equations (23) and (34)
${ }^{\varepsilon} \mathrm{limj}$, array of the modified $\varepsilon$ values
Complex array containing rows of the $\overline{\bar{S}}$ matrix
DCI
Complex array containing $\Delta C_{e}$
DCM@D
Complex array containing $\Delta C_{\text {mod }}$

Calling Subroutine WEYT

Elow Chart


SUBRQUTINE PHIJ(NPIT, NPQT, NTPHIJ, NEM, NP, KЯDE, MロDES, $X, Y, z$, PHI)

## Functional Description

This subroutine forms the correction factor modes. If $\phi$ is input element by element (TYPE $=0$ ) then this subroutine simply arranges the data into arrays. If TYPE $=0$ then modes are calculated as follows:

$$
\begin{array}{lr}
\left(x_{j}-a_{\ell}\right)^{n_{\ell}} & \text { TYPE }=1 \\
\left(y_{j}-a_{\ell}\right)^{n_{\ell}} & 2 \\
\left(z_{j}-a_{\ell}\right)^{n_{\ell}} & 3 \\
\exp \left[b_{\ell}\left(x_{j}-a_{\ell}\right)^{n_{\ell}}\right] & 4 \\
\exp \left[b_{\ell}\left(y_{j}-a_{\ell}\right)^{n_{\ell}}\right] & 5 \\
\exp \left[b_{\ell}\left(z_{j}-a_{\ell}\right)^{n_{\ell}}\right] & 6
\end{array}
$$

where a b $n$ are input per mode and where $\phi_{j \ell}=0$ over boxes are not considered.

| NPIT | Data set number of the system input data set |
| :---: | :---: |
| NP®T | Data set number of the system output data set |
| NTPHIJ | Tape number containing columns of the NP by NEM |
|  | ¢ matrix |
| NEM | Row dimension of the $\phi$ matrix |
| NP | Column dimension of the $\phi$ matrix |
| KDDE | -1 (end indicator of card input sets) |
| MODES | not used |
| X | $x$ ) |
| $Y$ | $y$ ( coordinates of the pressure points or |
| 2 | $z$ ) of the $\Delta C_{p}{ }^{\prime} s$ |
| PHI | Complex array containing one column of the $\phi$ mat |

Calling Subroutine ..... WEYT


## Functional Description

This subroutine positions tapes of a certain uniform format for reading; see Tape Description for NTDCP, NTAPW and NTAPDI. Description of Argument List
$\begin{array}{ll}\text { NT } & \text { Tape number to be positioned for reading } \\ \text { IGØ } & 1 \text { for symmetric modes, } 2 \text { for antisymmetric modes }\end{array}$

Calling Subroutines $\quad$ DCPB, DCPT, MATM, SBAR

## Functional Description

This subroutine reads arrays of real numbers $A$ of length $N$ from tape NTAPE one record at a time. It is used for the reading of the geometry arrays when these are input from tape NTGEDM.

## Description of Argument List

NTAPE Tape number

A
Array to be read from tape
$N$
Length of array $A$

## Calling Subroutine WEYT

SUBRQUTINE SAIJ(NPIT, NPQT, NTSAIJ, NTSANJ, NC, NP, NMQN, NAXIS,
AIT, CIE, $X, Y, Z, C G, S G$, DELA, FLAGA, FLAGF,
KgDE, IPRINT, LABEL, SAI)

Functional Description
This subroutine sets up proper argument lists for SROW so that integration matrices, [S], can be calculated for both constraining and monitoring purposes.

NPIT Data set number of the system input data set

NPØT
NTSAIJ

NTSANJ

NC
NP
NMGN
NAXIS
AIT
CIE
$X, Y, Z$
CG, SG
DELA
FLAGA

FLAGF Data set number of the system output data set Tape number containing the integration matrix rows, $S A_{i j}$, for constraints Tape number containing the inegration matrix rows, $S A_{h_{j}}$, for monitoring Number of constraints

Number of $\Delta C_{p}$ elements
Number of sets of the monitoring data
Number of axes
Constraining effectiveness of the experimental data, $\tilde{\mathbf{a}}_{\mathbf{i}}$
Experimental (or any other) constraint on the data, $C_{e}$
Coordinates of the pressure points (boxes)
Cosine-, sine of box dihedral angles
Box areas
Axis flag
$=0$, axis endpoints are input
$=1$, direction cosines are input
Force/moment flag
$=0, C_{e}$ is a force in the direction of axis
$=1, C_{e}$ is a moment about axis

KDDE
IPRINT


SAI
$=-1$
Detail print flag
IPRINT $=1$, print $S A_{i j}$
and $S A_{n j}$ rows
IPRINT $=0$, bypass print
Alphameric identifier of the integrated parameters Complex array containing a row of either one of the integration matrices $S A_{i j}$ or $S A_{n j}$

Calling Subroutine WEYT
Called Subroutine SROW

## Flow Chart



SUBRDUTINE SBAR(NTSBIJ, NTAPDI, NC, NP, NS, FLAGP, FLAGT, FLAGW,

$$
I, I G \emptyset, S Q R T T, A I W, S A I, D I, W, D E L C P B, S B I)
$$

## Functional Description

This subroutine solves for the matrix SBI where

$$
\left.S B I=[\bar{S}] \Gamma \sqrt{T^{\star}}\right]^{-1}
$$

The matrix $[\bar{S}]$ contains all the capability of the program except modes and limits. This capability is outlined in Eqs. (45), (26) and (9) for premultipliers and (45), (39) and (30) for postmultipliers. The weights $\mathrm{T}^{*}$ are defined below Equation (47).

$$
S B I= \begin{cases}\overline{S A}_{i j} & \text { when } j=1,2 \cdots N P, i=1,2 \cdots N C \\ W T_{i} & \text { when } j=N P+i \\ 0 & \text { otherwise }\end{cases}
$$

where $N P=$ number of pressure values and $N C=$ number of constraints.

$$
\begin{aligned}
& \overline{S A}_{\mathbf{i j}}=\left\{\begin{array}{l}
S A_{i j} \Delta C_{p_{j}} / \sqrt{T_{j}} \text { when } \Delta C_{p} \text { available } \\
\sum S A_{i_{\ell}} D I_{\ell j} W_{j} / \sqrt{T_{j}} \text { when } \Delta C_{p_{j}} \text { not available } \\
W T_{i}
\end{array}= \begin{cases}\left(1-\tilde{a}_{\mathbf{i}}\right) / \tilde{a}_{\mathbf{i}} & \tilde{a}_{\mathbf{i}}>0.0001 \\
10^{4} & \tilde{a}_{\mathbf{i}} \leq .0001\end{cases} \right. \\
& T_{j}=\left\{\begin{array}{cc}
\left|\Delta C_{p_{i}}\right| \Delta A & \text { FLAGP } \neq 1 \\
o r & \text { FLAGP }=1 \\
|W| & \text { FLAGT }=0 \\
1.0 & \text { FLAGT }=1
\end{array}\right.
\end{aligned}
$$

## Description of Argument List

| NTSBIJ | Tape number containing rows of the integration matrix without weight factor modes, $\overline{\mathrm{S}}$ |
| :---: | :---: |
| NTAPDI | Tape number containing rows of the inverse-D matrix |
| NC | Number of constraints |
| NP | Number of $\Delta C_{p}$ elements |
| NS | Length of the SBI matrix rows |
| FLAGP | Option flag for $\Delta C_{p}$ and/or pre- or post-multiplying correction factors |
| FLAGT | Option flag for weights |
| FLAGW | Option flag for normalwash input |
| I | An intermediate index |
| IGD | 1 for symmetric modes |
|  | 2 for antisymmetric modes |
| SQRTT | $=\sqrt{T}$; see equations |
| AIW | Constraining effectiveness $\tilde{a}_{\mathbf{i}}$ |
| SAI | A row of the integration matrix $S_{i j}$ |
| DI | A row of the inverse-D matrix |
| W | A column of the normalwash matrix |
| DELCPB | A column of the $\Delta C_{p_{e}}$ matrix (lifting pressure coefficients, either input, $\Delta C_{p}$, or computed as $[D]^{-1}\{W\}$ ) |
| SBI | A row of the integration matrix without weight factor modes, $\left.[\bar{S}] \Gamma \sqrt{T_{*}}\right]^{-1}, T^{*}= \begin{cases}T & \text { for constraints } \\ \frac{1-\tilde{a}}{\tilde{a}} & \text { for estimates }\end{cases}$ |



SUBROUTINE SDBL(NSCRCH, NUTL, MASTSB, NTPHIJ, NTAPW, NTSAIJ,
NTAPDI, IGף, FLAGW, FLAGP, FLAGT, NC, NP, NS,
NEM, SQRTT, AIT, DELA, SBB, SBI, SAI, DI, W, PHI, DELCP)

## Functional Description

This subroutine calculates $[\stackrel{B}{S}]$ described in Equation (54). The quantity calculated in this routine includes estimates and thus

$$
[\overline{\bar{S}}]=[S B B]=[\bar{S}]\left[\begin{array}{ccc}
\phi & 10 \\
-1 & - & - \\
0 & 1
\end{array}\right]
$$



| Calling Subroutine | WEYT |
| :--- | :--- | :--- |
| Called Subroutine | SBAR |

Flow Chart


## Flow Chart


$X, Y, Z, D E L A, L I M I, ~ I I M A X, I, N P, I R, X I 2$,
ETA2, ZETA2, SAI)

## Functional Description

This subroutine constructs the integration matrix [S] described in Equation (3) a row at a time.

$$
\begin{aligned}
& S_{i j}=\text { SAIJ }= \begin{cases}A_{i j} \Delta A_{j} & \text { for force calc. } \\
B_{i j} \Delta A_{j} & \text { for moment calc. }\end{cases} \\
& A_{i j}=\left[-\cos \beta_{i} \sin \bar{\gamma}_{j}+\cos \gamma_{i} \cos \bar{\gamma}_{j}\right] / \tilde{c}_{i} \\
& B_{i j}=\left(\cos \alpha_{i}\left[\left(y_{i}-\eta_{i}{ }^{(1)}\right)\left(\cos \bar{\gamma}_{j}\right)+\left(z_{j}-\zeta_{i}{ }^{(1)}\right) \sin \bar{\gamma}_{j}\right]\right. \\
& -\cos \beta_{i}\left(x_{j}-\xi_{i}{ }^{(l)}\right) \cos \bar{\gamma}_{j} \\
& \left.-\cos \gamma_{j}\left(x_{j}-\xi_{i}{ }^{(1)}\right) \sin \bar{\gamma}_{j}\right) / / \tilde{c}_{\mathbf{i}}
\end{aligned}
$$

where $\cos \alpha_{i}, \cos \beta_{i}, \cos \gamma_{i}$ are the direction cosines of the input axis and where $x_{j}, y_{j}, z_{j}, \bar{\gamma}_{j}$ are the coordinates and dihedral of the aerodynamic box and $\xi_{i}{ }^{(1)} \eta_{i}{ }^{(1)}{ }_{\zeta_{i}}$ (1) are the coordinates of the one edge of the input axis. SAIJ is of course zero on boxes that are not to be integrated.

## Description of Argument List

| FLAGA | Axis input option flag |
| :---: | :---: |
|  | $=0$, axis endpoints are input |
|  | $=1$, direction cosines are input |
| FLAGF | Force/moment flag |
|  | $=0, c_{i}{ }^{(e)}$ is a force in direction of axis $=1, C_{i}{ }^{(e)}$ is a moment about axis |
| XII |  |
| ETA1 | Axis endpoint coordinates, $\xi^{(1)}, \eta^{(1)}, \zeta^{(1)}$ |
| ZETA1 |  |
| CG, SG | Cosine-, sine of box dihedral angles |
| CTIL | Constant used to nondimensionalize integrated data ( $\tilde{c}$ ) |
| $X, Y, Z$ | Coordinates of the pressure points (boxes) |
| DELA | Box areas |
| LIMI | First-, last box numbers for the integration of the |
|  | $\Delta C_{p}$ values |
| IIMAX | Number of LIMI sets input for one constraint |
| I | Intermediate index |
| NP | Number of $\Delta C_{p}$ values |
| IR | Row index of the $S_{i j}$ matrix |
| XI2 7 l |  |
| ETA2 | direction cosines when FLAGF $=1$ |
| ZETA2 |  |
| SAI | A row of the integration matrix $S$ |

Flow Chart


> SUBRQUTINE WEYT(NP, NC, NEM, NELIMS, NMDN, NAXIS, NMIN, NMAX, NS 2
> NPIT, NPGT, W, DI, DCP, EPS, PHI, SAI, DCPTIL,
> DELCPB, CQL, CIE, DCM@D, EBMIN, EBMAX, EB, ELIM,
> SBB, SBI, S, SBMAT, DCI, W@RK)

## Functional Description

This subroutine is the core of the correction factor method. All logic for the method is established here. This subroutine uses input to decide what is to be done and sets up the argument lists for and executes the calls to all required subroutines. The following flow charts document the logic flow of this subroutine. This subroutine sets up the logic for various forms of input data and various types of calculations. The input data ranges over geometry, pressures, downwashes, aerodynamic influence matrices, previously generated correction factors, integration matrix data, etc. This data can enter the program by cards, tapes or both.

There are three basic computational branches; (1) correction factor calculation, (2) monitoring of data (integration of pressures into aerodynamic parameters) and (3) application of previously generated correction factors to pressure distributions. Within branch (1) there exists a choice of what type of correction factors to generate, premultiplier, postmuliplier and new postmultiplier. Also a choice as to the type of weighting to be used (i.e. the $T$ ) is available. The program also tests to see if limits are placed on the correction factors and if modes are used. The constraining power $\tilde{a}$ is always input since a constraint is simply $\tilde{a}=1.0$.

## Description of Argument List

$N P \quad$ Number of $\Delta C_{p}$ elements
NC Number of constraints
NEM Number of correction factor modes
NELIMS Number of input cards for the
EBMIN, EBMAX pairs
Number of sets of monitoring data
Number of axes for use in the inegration of the $\Delta C_{p_{t}}$ values
NMIN
NMAX $\quad=\max (N C$, NMIN, 10)
NS $\quad=\max (N P+N C, N E M+N C)$
NPIT Data set number of the system input data set
NPØT Data set number of the system output data set
W
DI
DCP
EPS
PHI
SAI
DCPTIL
DELCPB
A column of the normalwash matrix
A row of the inverse-D matrix, (A matrix)
A column of the theoretical $\Delta C_{p_{t}}$ matrix
Incremental weight factors array, $\varepsilon$
A column of the weight factor mode shape matrix, $\phi$
Integration matrix row array, [S]. SAN = [S] for monitoring A column of pressures modified by weight matrix, $\ddot{i}^{\Delta} C_{P_{e}}$ A column of the unmodified lifting pressure coefficients, $\Delta C_{P_{t}}$
(either input, $\Delta C_{P_{t}}$, or computed as $[D]^{-1}\{W\}$ )

CDL Complex array for intermediate use
CIE Array of the experimental constraints, $\mathrm{C}_{\mathrm{e}_{\mathrm{i}}}$

DCMØD

EB

SBB
SBI

S

SBMAT
DCI
WØRK

EBMIN, EBMAX Minimum-, maximum values allowed for the $\varepsilon$ array to take

ELIM Array of the modified $\varepsilon$ values
Array of the modified $\Delta C_{e}$ values
$\bar{\varepsilon}$ array (incremental weight factors) ( $\varepsilon=\phi \bar{\varepsilon}$ )

A row of the $\overline{\bar{S}}$ matrix
A row of the $[\bar{S}]\left\lceil\sqrt{T^{*}}\right]^{-1}$ matrix
A two-dimensional complex work array of dimension NC by NC
$\overline{\bar{S}}$ matrix of maximum dimension NC by NS
Array of the $\Delta C_{e}$ values
A two-dimensional complex array of dimension NP by NMAX in which the $\Delta C_{P_{t}}$ matrix is stored

Calling Subroutine
Called Subroutines

MAIN
CEMN, DCPB, DCPT, DELC, EDBL, EPSJ,
GINV, M $\emptyset D F$, PHIJ, RECD, SAIJ, SDBL, WSWA

Flow Charts (Read in geometry and pressures)


Flow Charts (Calculation of $S, \phi, W$ )


Flow Charts (Basic method)


Flow Charts (Accounting for limits)


SUBROUTINE WSWA(NPIT, NPЯT, NUTL1, NTAPW, KøDE, NP, NCØL,

> NMAX, NMSYM, NMASYM, WL

## Functional Description

Subroutine WSWA is called from WEYT only if the input flag FLAGW $=1$. It reads and prints the mode number and symmetry flag identifying the mode, the range of boxes over which the input $W$ value applies, and the normalwash, $W$ for this range of boxes. This card input is repeated for all ranges as needed, but only the non-zero $W$ values are required as input. The complete W matrix is assembled from the input and is saved on tape NTAPW in column order.

## Description of Argument List

| NPIT | Data set number of the system input data set |
| :--- | :--- |
| NPØT | Data set number of the system output data set |
| NUTL1 | Utility (scratch) tape number |
| NTAPW | Tape number containing columns of the W matrix |
| KØDE | $=-1$ |

NP
NCDL
NMAX
NMSYM
NMASYM
W
Number of row elements in the $W$ matrix
Number of columns in the $W$ matrix
Maximum number of columns in the $W$ matrix
Number of symmetric modes
Number of antisymmetric modes
Two-dimensional complex array containing the $W$ matrix

Calling Subroutine
WEYT

Flow Chart


## Functional Description

This subroutine initializes a complex array WQRK of length LENGTH to zeroes. In addition to this, when the argument ITAPE $\neq 0$, the complex zeroes stored in WøRK are written on tape ITAPE as many times as specified by the argument L $\emptyset \emptyset P$.

Description of Argument List

WØRK Complex array to be initialized to zeroes
LENGTH Length of the complex array WØRK
LODP Number of times the array W@RK is to be written on
tape ITAPE (only if ITAPE $\neq 0$ )
ITAPE
Tape number on which the array WGRK is saved LøøP-times (if any)

Calling Subroutines EPSJ, MAIN, MATM, SDBL

CORRECTION FACTORS (EIGCI 02/03/76
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REAU GEOMETRY ARRAYS AND DELTA-CP MATRIX FROM CARDS


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| :--- | :--- | :--- | :--- | :--- |
| $n$ | 0 | 0 |  |  |
| $n$ | $n$ | $n$ | $m$ | $m$ |
| $m$ | $m$ |  |  |  |

CORRECTION FACTORS (EIGCI 02/U3/76
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Figure 3. - Effect of applying local Mach Number to the boundary $\therefore$ conditions and the kernel of the classic theory.


Figure 4. - Comparison of classic theory and two variations of the present method with experimental data for the steady case.


Figure 5. - Comparison of classic theory and the present method $\left(M_{\infty}\right)$ (with and without second order Bernoulli correction) with experimental data.


Figure 6. - Comparison of various theories with experimental data for the 244 oscillatory control surface case.


Figure 7. - Comparison of classic theory and the present method $\left(M_{\infty}\right)$ with experimental data for the oscillatory case.
Phase Angle (Deg)


Figure 8. - Comparison of two variations of the Present transonic


Figure 9. :- Comparison of experimental and theoretical lift coefficient for an airfoil with a deflected 25\% chord flap.


Figure 10 - Comparison of experimental and theoretical pitching (about $\mathrm{c} / 4$ ) and hinge moment (about $3 \mathrm{c} / 4$ ) coefficient for an airfoll with a deflected $25 \%$ chord flap.


Figure 11. - Comparison of experimental and theoretical lift coefficient for ah argot with deflected $25 \%$ chord pap.


Figure 12. - Comparison of experimental and theoretical pitching moment (about $c / 4$ ) and hinge moment (about 3c/4) coefficient for an airfoil with an oscillating control surface.


Figure 13. - Experimental, Theoretical and Corrected Pressure Loadings for Angle of Attack Using


Figure 16. - Experimental, Theoretical and Corrected Pressure Loadings for Flap Rotation Using
Postmultiplying Correction Factors











Experiment [18]


Figure 28. - Comparison of experimental data with corrected and uncorrected theory for a swept wing operating at subsonic speeds at $\alpha=10^{\circ}$.
$\because 10$
(


Figure 30. - Comparison of experimental data with corrected theory; the cerrectfons: being based on static and oscillatory data.


Figure 31. - Effect of applying static correction factors to the Present Method ( $\bar{M}_{\infty}$ ) for the oscillatory case.


Figure 32. - Effect of applying static correction factors to the Present Method ( $\bar{M}_{\infty}$ ) for the oscillatory case.


Figure 33. - Application of static correction factors to the oscillatory case.

(Present Method (M)
Figure 35. - Comparison of the apparent camber distribution induced at sub- and transonic two-dimensional section.


Figure 36. - Comparison of data with premultiplier and postmultiplier (New) corrected theory.


Figure 37. - Application of correction factors obtained at subsonic Mach Number ( $M_{\infty}=0.5$ ) to the transonic case ( $M_{\infty}=0.85$ ).


Figure 38. - Application of correction factors obtained at a subsonic Mach Number ( $M_{\infty}=0.5$ ) to theory for the transonic case ( $M_{\infty}=0.875$ ).


Figure 39. - Effect of applying correction factors obtained at $M_{\infty}=0.5$ to other Mach Numbers for the Present Method $\left(\bar{M}_{\infty}\right)$.


Figure 40. - Effect of applying correction factors obtained at $M_{\infty}=0.5$ to other Mach Numbers for the Present Method $\left(\bar{M}_{\infty}\right)$


Figure 41. - Results of applying postmultiplying correction factors (New) obtained at $M=0.5$ and premultiplying factors obtained at $M=0.875$ to theory.


Figure 42. - Comparison of incremental correction factors obtained at various Mach Numbers using the Present Method ( $M_{\infty}$ ).





$$
M_{\infty}=2.05
$$

$$
k_{r}=0.0
$$






Figure 45. - Comparison of experimental data with corrected and uncorrected tneory for an Arrow wing in pitch.

$$
M_{\infty}=2.05
$$

$$
k_{r}=0.0
$$




Figure 46. - Camber changes inferred by New Postmultiplying correction factors as applied to an Arrow wing operating at supersonic speeds and $a=10^{\circ}$.


Axes Data

| Axis | Origin |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
|  | $\xi^{(1)}$ | $\eta^{(1)}$ | $5^{(1)}$ | $\cos \alpha$ | $\cos \beta$ | $\cos \gamma$ |
|  | 0 | 0 | 0 | 0 | 0 | 1 |
| -.5 | 0 | 0 | 0 | 1 | 0 |
| 3 | +.5 | 0 | 0 | 0 | 1 | 0 |


| Lift and Moment Data |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Flap |  |  | Pitch |
| $\mathrm{C}_{\ell}$ | Exp. | Theo. | Estimate | Theo. |
| $\mathrm{c}_{m 1 / 4}$ | -1.57 | 5.56 | -2.09 |  |
| $c_{h 3 / 4}$ | -0.053 | -.141 |  |  |

Figure 47. - Graphical and Tabulated Data for Program Test Cases

*For sale by the National Technical Information Service, Springfield, Virginia 22151


[^0]:    * Remember $W$ is downwash in this computer program

[^1]:    LIST OF INPUT/OUTPUT TAPFS - $\qquad$
    

[^2]:     THM-HmmmamminNN

[^3]:    

[^4]:    Calling Subroutine
    WEYT

