Correction, Generalisation and Validation of the "Max-Min d-Cluster Formation Heuristic"*

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Introduction and Abstract

The justification for using mutihop clusters may be found in [1]. In the well known heuristic proposed in [2], the *d-dominating* set of clusterheads is first selected by using nodes identifiers and then clusters are formed. In this paper we generalise this algorithm in order to select nodes depending of a given criterion (as the the degree, density or energy of nodes). The first section of this paper simplifies and proves the correctness of our generalised algorithm to select clusterheads. The cluster formation process proposed in [2] is extensively studied in the second section and is proved to be false.

1 Formation of d-Dominating Sets Based on a Given Criterion

Due to a lack of room, proofs of this section were published in [3].

Let $\mathcal{G} = \{V, E\}$ be a graph with sets of vertices V and edges E . Clusterheads form a subset, S of V which is a *d-dominating* set over $\mathcal{G}$. Let us consider $x \in V$, $\mathcal{N}_i(x)$ is the set of neighbours which are less than i hops from x. Let Y be a set on which a total order relation is defined. Let v be an injective function of V in Y and $X = v(Y)$. Our generalised algorithm iterates $2d$ runs. Each node updates two lists : *Winner* which is a list of elements of X and *Sender* which is a list of elements of V. Let us note $W_k(x)$ and $S_k(x)$ the images in x of the functions W_k and S_k, defined by induction.

Initial Phase ($k = 0$). $\forall\, x \in V \quad W_0(x) = v(x) \quad S_0(x) = x$.
Max Phase ($k \in [\![1, d]\!]$). For $x \in V$, let $y_k(x)$ be the only node of $\mathcal{N}_1(x)$ which is such that $\forall\, y \in \mathcal{N}_1(x) \setminus \{y_k(x)\} \quad W_{k-1}(y_k(x)) > W_{k-1}(y)$. W_k and S_k are derived from : $\forall\, x \in V \quad W_k(x) = W_{k-1}(y_k(x)) \quad S_k(x) = y_k(x)$.
Min phase ($k \in [\![d + 1, 2d]\!]$). For $x \in V$, let $y_k(x)$ be the only node of $\mathcal{N}_1(x)$ which is such that $\forall\, y \in \mathcal{N}_1(x) \setminus \{y_k(x)\} \quad W_{k-1}(y_k(x)) < W_{k-1}(y)$ W_k and S_k are derived from: $\forall\, x \in V \quad W_k(x) = W_{k-1}(y_k(x)) \quad S_k(x) = y_k(x)$.

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Definition 1. *Let S be the set defined by $S = \{x \in V,\ W_{2d}(x) = v(x)\}$* [1].

Theorem 1. *Each node $x \in V \setminus S$ may determine one node of S at least which is in $\mathcal{N}_d(x)$. It needs only to derive it from its* Winner *list. If $W_{2d}(x) = v(x)$ then x defines itself as a dominating node* (Rule 1)*. If node x finds a $v(y)$ value which appears once in each of the two phases at least, then $y \in S \cap \mathcal{N}_d(x)$. If node x find several pairs, the node y with the smallest value $v(y)$ is chosen* (Rule 2)*. If not, let y be the node such that $v(y) = W_d(x)$. Then $y \in S \cap \mathcal{N}_d(x)$* (Rule 3)*.*

Corollary 1. *S is a d-dominating set for the graph $\mathcal{G}$.*

This definition of S (see Def. 1) is different from the definition given in [2] where S is defined as: $S' = \{x \in V,\ \exists k \in [\![d+1, 2d]\!]\ W_k(x) = v(x)\}$.

Theorem 2. *$S = S'$.*

2 Cluster Formation

To join a clusterhead $c(x)$, nodes must establish a path to reach it provided all nodes in the path belong to the same cluster. Therefore, it is necessary to find an algorithm to partition the topology in connected components, called clusters. In this section we shall study the formation of these clusters.

2.1 The Solution Proposed in 'Max-Min d-Cluster' Formation is False

The authors of [2] proposed a formation of above path. We now prove that there exist some cases for which the formation of the path is not valid.

Max-Min d cluster formation proposal. Let x be a node and let y be the corresponding dominated node as defined in Theorem 1 ($y = c(x)$) and let $k \in [\![1, d]\!]$ be such as $W_k(x) = v(y)$. x chooses then $S_k(x)$ as father[2]. It may be that : $c(p(x)) \neq c(x)$. Therefore, in some cases it is necessary to use an additional rule to make sure that node $c(p(x)) = c(x)$. This rule is named *convergecast* in paper [2] and introduces a new necessary condition which is : $\forall x \in E\ p(p(x)) \neq x$. If not , the rule would lead to an infinite loop. However, this condition cannot be observed, as shown in the following example.

On an example where the algorithm leads to a bug. The network is shown in Fig. 1 and the results of father and clusterhead selection algorithm (with $d = 5$) are given by Table 1.

The cluster formation proposed in [2] leads to an infinite loop as $c(p(3)) = c(5) = 11$, $c(3) = 10$, and $p(p(3)) = 3$. Hence, the use of the *convergecast* rule is not possible. The next paragraph proves that this phenomenon is due to the use of the *Rule 2.*

[1] This definition is not the same as the one provided in [2] but both definitions are equivalent (see Theorem 2).

[2] By definition, $S_k(x) \in \mathcal{N}_1(x)$.

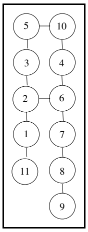

Fig. 1. Topology

Table 1. 5-Max-Min results

Node	1	2	3	4	5	6	7	8	9	10	11
Max1	11	6	5	10	10	7	8	9	9	10	11
Max2	11	11	10	10	10	10	9	9	9	10	11
Max3	11	11	11	10	10	11	10	9	9	10	11
Max4	11	11	11	11	11	11	11	10	9	10	11
Max5	11	11	11	11	11	11	11	11	10	11	11
Min1	11	11	11	11	11	11	11	10	10	11	11
Min2	11	11	11	11	11	11	10	10	10	11	11
Min3	11	11	11	11	11	10	10	10	10	11	11
Min4	11	10	11	10	11	10	10	10	10	11	11
Min5	10	10	10	10	11	10	10	10	10	10	11
Clusterhead	11	11	10	10	11	10	10	10	10	10	11
Father	11	1	5	10	3	4	6	7	8	10	11

Notice that if a node i is such that $v(c(i)) < W_d(i)$ then the *Rule 2* was used.

Necessary condition: *Rule 2* was used. For two nodes i and j, let us note $d(i,j)$ the distance in hops. Now, let x, y and z be the three nodes. Then, for any node such that $c(i) \neq p(i)$, $d(i,c(i)) = d(p(i),c(i)) + 1$ since $p(i)$ is the node allowing i to know $c(i)$. Let i and j the be such as $p(i) = j$ and $p(j) = i$; i and j are thus not clusterhead since each one have a different father. The preceding equality applied to i and j implies that $d(i,c(i)) = d(j,c(i)) + 1$ and $d(j,c(j)) = d(i,c(j)) + 1$. Assume that $c(i) = c(j) = l$, then $d(i,l)$=$d(j,l)$+1 and $d(j,l) = d(i,l) + 1$ which is absurd, so $c(i) \neq c(j)$. Suppose, without any generality restriction, that $v(c(i)) > v(c(j))$. Node i belongs obviously in the d hops neighbourhood of $c(i)$. Therefore, $p(i)$ also is in the d hops neighbourhood of $c(i)$ and therefore $j \in \mathcal{V}_d(c(i))$. Thus $c(i) \in \mathcal{V}_d(j)$. Then, $W_d(j) \geq v(c(i))$ and then $W_d(j) > v(c(j))$. Hence, *Rule 2* was used according to what precedes.

Sufficient condition: *Rule 2* was used. If a node i is not a clusterhead, then $v(c(i)) = W_d(i)$ (*Rule 3*). Let i be a node which belongs to a loop. Without any generality restriction, let us show that a loop with a length 5 cannot occur. Let j, k, l, m and i be the father of i, j, k, l and m respectively. Since, j is father of i, j belongs to the d hop neighbourhood of $c(i)$. So, $W_d(j) \geq v(c(i))$. But $v(c(i)) = W_d(i)$ thus $W_d(j) \geq W_d(i)$. It may be deduced that $W_d(i) = W_d(j) = W_d(k) = W_d(l) = W_d(m)$ then $c(i) = c(j) = c(k) = c(l) = c(m) = c$. Therefore, by applying to each node the general equality $d(i,c(i)) = d(p(i),c(i)) +$ 1 since no node among i, j, k, l is clusterhead : d(i,c) = d(j,c)+1, d(j,c) = d(k,c)+1, d(k,c) = d(l,c)+1, d(l,c) = d(m,c)+1, d(m,c) = d(i,c)+1, which is absurd. The same kind of demonstration can be applied for any other loop of any given length. Hence, if the *Rule 2* is removed there is no loop.

The following example shows that the suppression of the *Rule 2* leads to new problems. The network is shown in Fig. 2 and the results of father and clusterhead selection algorithm (with $d = 2$) are given by Table tab:2.

It can be noticed that node node 2 is not a clusterhead and $c(p(1)) = c(2) = 5$ whereas $c(1) = 4$. Therefore, there is another problem which is not solved by *convergecast* rule as it is not possible to go from sons to fathers and to be sure to go through son's clusterhead before the father be attached to another clusterhead.

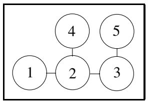

Fig. 2. Topology

Table 2. 2-Max-Min results

Node	1	2	3	4	5
Max1	2	4	5	4	5
Max2	4	5	5	4	5
Min1	4	4	5	4	5
Min2	4	4	4	4	5
Clusterhead	4	5	5	4	5
Father	2	3	5	4	5

2.2 Another Proposal for the Formation of the Cluster

If a node i is a clusterhead after application of the *Rule 1*, then node i informs its neighbours that it is a clusterhead. The unclustered neighbours choose i as clusterhead and transmit a message to their neighbours to inform them that they are at one hop from the clusterhead i. The unclustered neighbours of these nodes choose i as clusterhead by attaching themselves to one of node i neighbours, and inform their neighbours that they are at 2 hops from i. This process is repeated d times so as not to exceed d hops. It guarantees that there is no loop and that all the connected components are tree clusters with a clusterhead root.

3 Conclusion

In this paper, we simplified (cf. Theorem 2) the heuristic presented in the paper [2]. We generalized this heuristic to any given criterion and not only to the identifier of the nodes. This allows to take into consideration other factors influencing the performance of the network. For example, the energy of a wireless sensor network benefits from a hierarchical routing introduced by the determination of clusters with a maximum depth d (cf. paper [1]). In the second part, we gave an example which shows that the cluster formation process proposed in [2] is not always valid. This is an important result since Amis et al. algorithm is well known. We then suggested a correct cluster formation process.

References

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