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Correction

Correction to: A General Approach to Equivariant Biharmonic Maps

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In this erratum first we amend the stability study of some proper biharmonic maps $\varphi_\alpha : T^2 \to S^2$ (Theorem 3.2 of [1]). We also correct the proof of a claim in Example 3.5 of [1], showing that biharmonic maps do not satisfy the classical Sampson's maximum principle for harmonic maps.

1. Equivariant biharmonic maps and applications

We use the notation of Example 3.1 of [1]. Theorem 3.2 of [1] is not correct and must be replaced by

Theorem 1.1. *Let* $\varphi_\alpha : T^2 \to S^2$ *be a proper biharmonic map as in equation* (3.8)(ii) *of* [1]*. Then* φ_α *is an unstable critical point.*

Proof. It suffices to prove that φ_α is unstable with respect to equivariant variations. To this purpose, we compute the second variation of the reduced bienergy functional (we denote by α^* the constant function $\alpha \equiv \pi/4$):

$$\begin{aligned}
\nabla^2 E_2^{\varphi}(\alpha^*)\,(V,V) &= \frac{\mathrm{d}^2}{\mathrm{d}h^2}\,\left[E_2^{\varphi}(\alpha^* + h\,V)\right]|_{h=0}\\
&= \int_0^{2\pi} \frac{\mathrm{d}^2}{\mathrm{d}h^2}\left[(h\,\ddot{V})^2 + \frac{k^4}{4}\sin^2\left(\frac{\pi}{2} + 2h\,V\right) - h\,\ddot{V}\,k^2\,\sin\left(\frac{\pi}{2} + 2h\,V\right)\right]\Big|_{h=0}\,\mathrm{d}\theta\\
&= \int_0^{2\pi} \frac{\mathrm{d}^2}{\mathrm{d}h^2}\left[h^2\,\ddot{V}^2 + \frac{k^4}{4}\cos^2(2h\,V) - h\,\ddot{V}\,k^2\,\cos(2h\,V)\right]\Big|_{h=0}\,\mathrm{d}\theta\\
&= \int_0^{2\pi}\left[2\,\ddot{V}^2 - 2\,V^2\,k^4\right]\mathrm{d}\theta.
\end{aligned}$$

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By taking $V \equiv 1$, we conclude from the last equality that φ_{α^*} is unstable, as required to end the proof. The case $\alpha \equiv 3\,\pi/4$ is analogous. $\square$

As a consequence of Theorem 1.1, Remark 3.3 of [1] should be deleted.

Next, we use the notation of Example 3.5 of [1]. The claim of Example 3.5 of [1], stating that biharmonic maps do not verify Sampson's maximum principle for harmonic maps, is correct. However, in order to prove it, we use the function

$$\alpha(r) = r\,e^{-\sqrt{\lambda}\,r}\ , \qquad r \in \mathbb{R} \tag{1.1}$$

instead of the one which appeared in (3.21) of [1]. Indeed, the function in (1.1) admits a *strictly positive interior maximum point* at $r_0 = (1/\sqrt{\lambda}) > 0$. Thus, the image through φ_α of an open set $S^m \times (r_0 - \varepsilon, r_0 + \varepsilon)$ is contained in the concave side of $S = \partial B_{\alpha(r_0)}(O)$ provided that $\varepsilon > 0$ is small.

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