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CORRECTION

A. Arbind · J. N. Reddy

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Correction in Appendix B

The correction in Eq. (B.1)

$$\mathbf{e}_s = \frac{\partial \mathbf{R}}{\partial s} = \xi\, \mathbf{T} + \tau \mathbf{e}_\theta, \quad \text{with} \quad \xi = 1 - r\kappa\cos\theta \tag{1}$$

Because of this change, the follow-up corrections in the equations are as follows:
In Eq. (B.2)

$$g_{ss} = \xi^2 + r^2\tau^2, \quad g_{s\theta} = g_{\theta s} = r^2\tau, \quad g_{rr} = 1, \quad g_{\theta\theta} = r^2 \tag{2}$$

In Eq. (B.3)

$$\mathbf{e}^s = g^{sk}\mathbf{e}_k = \frac{1}{\xi}\mathbf{T}, \quad \mathbf{e}^r = \mathbf{e}_r, \quad \mathbf{e}^\theta = -\frac{\tau}{\xi}\mathbf{T} + \frac{1}{r^2}\mathbf{e}_\theta \tag{3}$$

In Eq. (B.4)

$$\mathbf{e}_s = \xi\, \mathbf{T} + \tau\, \mathbf{e}_\theta \tag{4}$$

In Eq. (B.9)

$$\begin{aligned}
\frac{\partial \mathbf{e}_\theta}{\partial s} &= \frac{r}{\xi}\hat{\kappa}_2\mathbf{e}_s - \frac{r}{\xi}\hat{\kappa}_2\tau\mathbf{e}_\theta - r\tau\,\mathbf{e}_r \\
\frac{\partial \mathbf{e}_r}{\partial s} &= -\frac{1}{\xi}\hat{\kappa}_1\mathbf{e}_s + \left(\frac{1}{\xi}\hat{\kappa}_1\tau + \frac{\tau}{r}\right)\mathbf{e}_\theta \\
\frac{\partial \mathbf{e}_s}{\partial s} &= \frac{1}{\xi}\left(\frac{\partial \xi}{\partial s} + r\hat{\kappa}_2\tau\right)\mathbf{e}_s + \left(\xi\hat{\kappa}_1 - r\tau^2\right)\mathbf{e}_r + \left(\frac{\partial \tau}{\partial s} - \frac{1}{\xi}\frac{\partial \xi}{\partial s}\tau - \frac{\xi}{r}\hat{\kappa}_2 - \frac{r}{\xi}\hat{\kappa}_2\tau^2\right)\mathbf{e}_\theta
\end{aligned} \tag{5}$$

where

$$\hat{\kappa}_1 = \kappa\cos\theta, \quad \hat{\kappa}_2 = \kappa\sin\theta$$

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A. Arbind · J. N. Reddy (✉)
Advanced Computational Mechanics Laboratory, Department of Mechanical Engineering, Texas A&M University, College Station, TX, USA
E-mail: jnreddy@tamu.edu

In Eq. (B.10),

$$\Gamma^s_{ss} = \frac{1}{\xi}\left(\frac{\partial \xi}{\partial s} + r\hat{\kappa}_2\tau\right), \quad \Gamma^r_{ss} = \left(\xi\hat{\kappa}_1 - r\tau^2\right), \quad \Gamma^\theta_{ss} = \left(\frac{\partial \tau}{\partial s} - \frac{1}{\xi}\frac{\partial \xi}{\partial s}\tau - \frac{\xi}{r}\hat{\kappa}_2 - \frac{r}{\xi}\hat{\kappa}_2\tau^2\right)$$
$$\Gamma^s_{rs} = \Gamma^s_{sr} = -\frac{1}{\xi}\hat{\kappa}_1, \quad \Gamma^\theta_{rs} = \Gamma^\theta_{sr} = \left(\frac{1}{\xi}\hat{\kappa}_1\tau + \frac{\tau}{r}\right), \quad \Gamma^s_{\theta s} = \Gamma^s_{s\theta} = \frac{r}{\xi}\hat{\kappa}_2, \quad \Gamma^\theta_{\theta s} = \Gamma^\theta_{s\theta} = -\frac{r}{\xi}\hat{\kappa}_2\tau \tag{6}$$

Equation (B.11) becomes

$$\begin{aligned}
\nabla \mathbf{u} &= \left(\frac{\partial u^j}{\partial z^i} + \Gamma^j_{ik}u^k\right)\mathbf{e}^i\mathbf{e}_j \\
&= \left(\frac{\partial u^s}{\partial s} + \frac{1}{\xi}\left(\frac{\partial \xi}{\partial s} + r\hat{\kappa}_2\tau\right)u^s - \frac{1}{\xi}\hat{\kappa}_1\, u^r + \frac{r}{\xi}\,\hat{\kappa}_2\, u^\theta\right)\mathbf{e}^s\mathbf{e}_s \\
&\quad + \left(\frac{\partial u^r}{\partial s} + \left(\xi\hat{\kappa}_1 - r\tau^2\right)\, u^s - \tau\, r\, u^\theta\right)\mathbf{e}^s\mathbf{e}_r \\
&\quad + \left(\frac{\partial u^\theta}{\partial s} + \left(\frac{\partial \tau}{\partial s} - \frac{1}{\xi}\frac{\partial \xi}{\partial s}\tau - \frac{\xi}{r}\hat{\kappa}_2 - \frac{r}{\xi}\hat{\kappa}_2\tau^2\right)\, u^s + \left(\frac{1}{\xi}\hat{\kappa}_1\tau + \frac{\tau}{r}\right)u^r - \frac{r}{\xi}\hat{\kappa}_2\tau\, u^\theta\right)\mathbf{e}^s\mathbf{e}_\theta \\
&\quad + \left(\frac{\partial u^s}{\partial r} - \frac{1}{\xi}\hat{\kappa}_1\, u^s\right)\mathbf{e}^r\mathbf{e}_s + \frac{\partial u^r}{\partial r}\mathbf{e}^r\mathbf{e}_r + \left(\frac{\partial u^\theta}{\partial r} + \left(\frac{1}{\xi}\hat{\kappa}_1\tau + \frac{\tau}{r}\right)\, u^s + \frac{u^\theta}{r}\right)\mathbf{e}^r\mathbf{e}_\theta \\
&\quad + \left(\frac{\partial u^s}{\partial \theta} + \frac{r}{\xi}\,\hat{\kappa}_2\, u^s\right)\mathbf{e}^\theta\mathbf{e}_s + \left(\frac{\partial u^r}{\partial \theta} - \tau\, r\, u^s - r\, u^\theta\right)\mathbf{e}^\theta\mathbf{e}_r \\
&\quad + \left(\frac{\partial u^\theta}{\partial \theta} - \frac{r}{\xi}\hat{\kappa}_2\tau\, u^s + \frac{u^r}{r}\right)\mathbf{e}^\theta\mathbf{e}_\theta
\end{aligned} \tag{7}$$

In Eq. (B.12)

$$\hat{\mathbf{e}}_s = \mathbf{T}. \tag{8}$$

In Eq. (B.13)

$$u^s = \frac{\hat{u}^s}{\xi}, \quad u^\theta = \frac{\hat{u}^\theta}{r} - \frac{\tau}{\xi}\hat{u}^s \tag{9}$$

In Eq. (B.14)

$$\begin{aligned}
\nabla \mathbf{u} &= \left(\frac{1}{\xi}\frac{\partial \hat{u}^s}{\partial s} - \frac{\tau}{\xi}\frac{\partial \hat{u}^s}{\partial \theta} - \frac{1}{\xi}\hat{\kappa}_1\, \hat{u}^r + \frac{1}{\xi}\,\hat{\kappa}_2\hat{u}^\theta\right)\hat{\mathbf{e}}_s\hat{\mathbf{e}}_s + \frac{1}{\xi}\left(\frac{\partial \hat{u}^r}{\partial s} - \tau\frac{\partial \hat{u}^r}{\partial \theta} + \hat{\kappa}_1\hat{u}^s\right)\hat{\mathbf{e}}_s\hat{\mathbf{e}}_r \\
&\quad + \left(\frac{1}{\xi}\frac{\partial \hat{u}^\theta}{\partial s} - \frac{\tau}{\xi}\frac{\partial \hat{u}^\theta}{\partial \theta} - \frac{1}{\xi}\hat{\kappa}_2\hat{u}^s\right)\hat{\mathbf{e}}_s\hat{\mathbf{e}}_\theta + \frac{\partial \hat{u}^s}{\partial r}\hat{\mathbf{e}}_r\hat{\mathbf{e}}_s + \frac{\partial \hat{u}^r}{\partial r}\hat{\mathbf{e}}_r\hat{\mathbf{e}}_r + \frac{\partial \hat{u}^\theta}{\partial r}\hat{\mathbf{e}}_r\hat{\mathbf{e}}_\theta \\
&\quad + \frac{1}{r}\frac{\partial \hat{u}^s}{\partial \theta}\hat{\mathbf{e}}_\theta\hat{\mathbf{e}}_s + \left(\frac{1}{r}\frac{\partial \hat{u}^r}{\partial \theta} - \frac{\hat{u}^\theta}{r}\right)\hat{\mathbf{e}}_\theta\hat{\mathbf{e}}_r + \left(\frac{1}{r}\frac{\partial \hat{u}^\theta}{\partial \theta} + \frac{\hat{u}^r}{r}\right)\hat{\mathbf{e}}_\theta\hat{\mathbf{e}}_\theta
\end{aligned} \tag{10}$$

In Eq. (B.15)

$$\begin{aligned}
(\nabla \mathbf{u})^{\mathrm{T}} &= \left(\frac{1}{\xi}\frac{\partial \hat{u}^s}{\partial s} - \frac{\tau}{\xi}\frac{\partial \hat{u}^s}{\partial \theta} - \frac{1}{\xi}\hat{\kappa}_1\, \hat{u}^r + \frac{1}{\xi}\,\hat{\kappa}_2\hat{u}^\theta\right)\hat{\mathbf{e}}_s\hat{\mathbf{e}}_s + \frac{\partial \hat{u}^s}{\partial r}\hat{\mathbf{e}}_s\hat{\mathbf{e}}_r \\
&\quad + \frac{1}{r}\frac{\partial \hat{u}^s}{\partial \theta}\hat{\mathbf{e}}_s\hat{\mathbf{e}}_\theta + \frac{1}{\xi}\left(\frac{\partial \hat{u}^r}{\partial s} - \tau\frac{\partial \hat{u}^r}{\partial \theta} + \hat{\kappa}_1\hat{u}^s\right)\hat{\mathbf{e}}_r\hat{\mathbf{e}}_s + \frac{\partial \hat{u}^r}{\partial r}\hat{\mathbf{e}}_r\hat{\mathbf{e}}_r + \left(\frac{1}{r}\frac{\partial \hat{u}^r}{\partial \theta} - \frac{\hat{u}^\theta}{r}\right)\hat{\mathbf{e}}_r\hat{\mathbf{e}}_\theta \\
&\quad + \left(\frac{1}{\xi}\frac{\partial \hat{u}^\theta}{\partial s} - \frac{\tau}{\xi}\frac{\partial \hat{u}^\theta}{\partial \theta} - \frac{1}{\xi}\hat{\kappa}_2\hat{u}^s\right)\hat{\mathbf{e}}_\theta\hat{\mathbf{e}}_s + \frac{\partial \hat{u}^\theta}{\partial r}\hat{\mathbf{e}}_\theta\hat{\mathbf{e}}_r + \left(\frac{1}{r}\frac{\partial \hat{u}^\theta}{\partial \theta} + \frac{\hat{u}^r}{r}\right)\hat{\mathbf{e}}_\theta\hat{\mathbf{e}}_\theta
\end{aligned} \tag{11}$$

In Eq. (B.17), the strain components in terms of the components of displacement field are:

$$
\begin{aligned}
E_{ss} &= \left(\frac{1}{\xi}\frac{\partial \hat{u}^s}{\partial s} - \frac{\tau}{\xi}\frac{\partial \hat{u}^s}{\partial \theta} - \frac{1}{\xi}\hat{\kappa}_1\,\hat{u}^r + \frac{1}{\xi}\,\hat{\kappa}_2\hat{u}^\theta\right) + \frac{1}{2}\Bigg[\left(\frac{1}{\xi}\frac{\partial \hat{u}^s}{\partial s} - \frac{\tau}{\xi}\frac{\partial \hat{u}^s}{\partial \theta} - \frac{1}{\xi}\hat{\kappa}_1\,\hat{u}^r + \frac{1}{\xi}\,\hat{\kappa}_2\hat{u}^\theta\right)^2 \\
&\quad + \frac{1}{\xi^2}\left(\frac{\partial \hat{u}^r}{\partial s} - \tau\frac{\partial \hat{u}^r}{\partial \theta} + \hat{\kappa}_1\hat{u}^s\right)^2 + \left(\frac{1}{\xi}\frac{\partial \hat{u}^\theta}{\partial s} - \frac{\tau}{\xi}\frac{\partial \hat{u}^\theta}{\partial \theta} - \frac{1}{\xi}\hat{\kappa}_2\hat{u}^s\right)^2\Bigg] \\
E_{rr} &= \frac{\partial \hat{u}^r}{\partial r} + \frac{1}{2}\left[\left(\frac{\partial \hat{u}^s}{\partial r}\right)^2 + \left(\frac{\partial \hat{u}^r}{\partial r}\right)^2 + \left(\frac{\partial \hat{u}^\theta}{\partial r}\right)^2\right] \\
E_{\theta\theta} &= \left(\frac{1}{r}\frac{\partial \hat{u}^\theta}{\partial \theta} + \frac{\hat{u}^r}{r}\right) + \frac{1}{2}\left[\left(\frac{1}{r}\frac{\partial \hat{u}^s}{\partial \theta}\right)^2 + \left(\frac{1}{r}\frac{\partial \hat{u}^r}{\partial \theta} - \frac{\hat{u}^\theta}{r}\right)^2 + \left(\frac{1}{r}\frac{\partial \hat{u}^\theta}{\partial \theta} + \frac{\hat{u}^r}{r}\right)^2\right] \\
2E_{r\theta} &= \frac{1}{r}\frac{\partial \hat{u}^r}{\partial \theta} - \frac{\hat{u}^\theta}{r} + \frac{\partial \hat{u}^\theta}{\partial r} + \frac{1}{r}\frac{\partial \hat{u}^s}{\partial r}\frac{\partial \hat{u}^s}{\partial \theta} + \frac{\partial \hat{u}^r}{\partial r}\left(\frac{1}{r}\frac{\partial \hat{u}^r}{\partial \theta} - \frac{\hat{u}^\theta}{r}\right) + \frac{\partial \hat{u}^\theta}{\partial r}\left(\frac{1}{r}\frac{\partial \hat{u}^\theta}{\partial \theta} + \frac{\hat{u}^r}{r}\right) \\
2E_{\theta s} &= \frac{1}{\xi}\frac{\partial \hat{u}^\theta}{\partial s} - \frac{\tau}{\xi}\frac{\partial \hat{u}^\theta}{\partial \theta} - \frac{1}{\xi}\hat{\kappa}_2\hat{u}^s + \frac{1}{r}\frac{\partial \hat{u}^s}{\partial \theta} + \frac{1}{r}\frac{\partial \hat{u}^s}{\partial \theta}\left(\frac{1}{\xi}\frac{\partial \hat{u}^s}{\partial s} - \frac{\tau}{\xi}\frac{\partial \hat{u}^s}{\partial \theta} - \frac{1}{\xi}\hat{\kappa}_1\,\hat{u}^r + \frac{1}{\xi}\,\hat{\kappa}_2\hat{u}^\theta\right) \\
&\quad + \frac{1}{\xi}\left(\frac{1}{r}\frac{\partial \hat{u}^r}{\partial \theta} - \frac{\hat{u}^\theta}{r}\right)\left(\frac{\partial \hat{u}^r}{\partial s} - \tau\frac{\partial \hat{u}^r}{\partial \theta} + \hat{\kappa}_1\hat{u}^s\right) + \left(\frac{1}{r}\frac{\partial \hat{u}^\theta}{\partial \theta} + \frac{\hat{u}^r}{r}\right)\left(\frac{1}{\xi}\frac{\partial \hat{u}^\theta}{\partial s} - \frac{\tau}{\xi}\frac{\partial \hat{u}^\theta}{\partial \theta} - \frac{1}{\xi}\hat{\kappa}_2\hat{u}^s\right) \\
2E_{sr} &= \frac{1}{\xi}\left(\frac{\partial \hat{u}^r}{\partial s} - \tau\frac{\partial \hat{u}^r}{\partial \theta} + \hat{\kappa}_1\hat{u}^s\right) + \frac{\partial \hat{u}^s}{\partial r} + \frac{\partial \hat{u}^s}{\partial r}\left(\frac{1}{\xi}\frac{\partial \hat{u}^s}{\partial s} - \frac{\tau}{\xi}\frac{\partial \hat{u}^s}{\partial \theta} - \frac{1}{\xi}\hat{\kappa}_1\,\hat{u}^r + \frac{1}{\xi}\,\hat{\kappa}_2\hat{u}^\theta\right) \\
&\quad + \frac{1}{\xi}\frac{\partial \hat{u}^r}{\partial r}\left(\frac{\partial \hat{u}^r}{\partial s} - \tau\frac{\partial \hat{u}^r}{\partial \theta} + \hat{\kappa}_1\hat{u}^s\right) + \frac{\partial \hat{u}^\theta}{\partial r}\left(\frac{1}{\xi}\frac{\partial \hat{u}^\theta}{\partial s} - \frac{\tau}{\xi}\frac{\partial \hat{u}^\theta}{\partial \theta} - \frac{1}{\xi}\hat{\kappa}_2\hat{u}^s\right)
\end{aligned} \tag{12}
$$

Corrections in Sects. 2 and 4

In Eq. (8)

$$
\begin{aligned}
E_{ss} &= \left(\frac{1}{\xi}\mathbf{A}_s\boldsymbol{\Phi}_{s,s} - \frac{\tau}{\xi}\mathbf{A}_{s,\theta}\boldsymbol{\Phi}_s - \frac{1}{\xi}\hat{\kappa}_1\,\mathbf{A}_r\boldsymbol{\Phi}_r + \frac{1}{\xi}\,\hat{\kappa}_2\mathbf{A}_\theta\boldsymbol{\Phi}_\theta\right) \\
&\quad + \frac{1}{2}\Bigg[\left(\frac{1}{\xi}\mathbf{A}_s\boldsymbol{\Phi}_{s,s} - \frac{\tau}{\xi}\mathbf{A}_{s,\theta}\boldsymbol{\Phi}_s - \frac{1}{\xi}\hat{\kappa}_1\,\mathbf{A}_r\boldsymbol{\Phi}_r + \frac{1}{\xi}\,\hat{\kappa}_2\mathbf{A}_\theta\boldsymbol{\Phi}_\theta\right)^2 \\
&\quad + \frac{1}{\xi^2}\left(\mathbf{A}_r\boldsymbol{\Phi}_{r,s} - \tau\mathbf{A}_{r,\theta}\boldsymbol{\Phi}_r + \hat{\kappa}_1\mathbf{A}_s\boldsymbol{\Phi}_s\right)^2 + \left(\frac{1}{\xi}\mathbf{A}_\theta\boldsymbol{\Phi}_{\theta,s} - \frac{\tau}{\xi}\mathbf{A}_{\theta,\theta}\boldsymbol{\Phi}_\theta - \frac{1}{\xi}\hat{\kappa}_2\mathbf{A}_s\boldsymbol{\Phi}_s\right)^2\Bigg] \\
E_{rr} &= \mathbf{A}_{r,r}\boldsymbol{\Phi}_r + \frac{1}{2}\left[\left(\mathbf{A}_{s,r}\boldsymbol{\Phi}_s\right)^2 + \left(\mathbf{A}_{r,r}\boldsymbol{\Phi}_r\right)^2 + \left(\mathbf{A}_{\theta,r}\boldsymbol{\Phi}_\theta\right)^2\right] \\
E_{\theta\theta} &= \left(\frac{1}{r}\mathbf{A}_{\theta,\theta}\boldsymbol{\Phi}_\theta + \frac{\mathbf{A}_r\boldsymbol{\Phi}_r}{r}\right) + \frac{1}{2}\Bigg[\left(\frac{1}{r}\mathbf{A}_{s,\theta}\boldsymbol{\Phi}_s\right)^2 + \left(\frac{1}{r}\mathbf{A}_{r,\theta}\boldsymbol{\Phi}_r - \frac{\mathbf{A}_\theta\boldsymbol{\Phi}_\theta}{r}\right)^2 \\
&\quad + \left(\frac{1}{r}\mathbf{A}_{\theta,\theta}\boldsymbol{\Phi}_\theta + \frac{\mathbf{A}_r\boldsymbol{\Phi}_r}{r}\right)^2\Bigg] \\
2E_{r\theta} &= \frac{1}{r}\mathbf{A}_{r,\theta}\boldsymbol{\Phi}_r - \frac{\hat{u}^\theta}{r} + \mathbf{A}_{\theta,r}\boldsymbol{\Phi}_\theta + \frac{1}{r}\mathbf{A}_{s,r}\boldsymbol{\Phi}_s\mathbf{A}_{s,\theta}\boldsymbol{\Phi}_s + \mathbf{A}_{r,r}\boldsymbol{\Phi}_r\left(\frac{1}{r}\mathbf{A}_{r,\theta}\boldsymbol{\Phi}_r - \frac{\hat{u}^\theta}{r}\right) \\
&\quad + \mathbf{A}_{\theta,r}\boldsymbol{\Phi}_\theta\left(\frac{1}{r}\mathbf{A}_{\theta,\theta}\boldsymbol{\Phi}_\theta + \frac{\hat{u}^r}{r}\right) \\
2E_{\theta s} &= \frac{1}{\xi}\mathbf{A}_\theta\boldsymbol{\Phi}_{\theta,s} - \frac{\tau}{\xi}\mathbf{A}_{\theta,\theta}\boldsymbol{\Phi}_\theta - \frac{1}{\xi}\hat{\kappa}_2\mathbf{A}_s\boldsymbol{\Phi}_s + \frac{1}{r}\mathbf{A}_{s,\theta}\boldsymbol{\Phi}_s \\
&\quad + \frac{1}{r}\mathbf{A}_{s,\theta}\boldsymbol{\Phi}_s\left(\frac{1}{\xi}\mathbf{A}_s\boldsymbol{\Phi}_{s,s} - \frac{\tau}{\xi}\mathbf{A}_{s,\theta}\boldsymbol{\Phi}_s - \frac{1}{\xi}\hat{\kappa}_1\,\mathbf{A}_r\boldsymbol{\Phi}_r + \frac{1}{\xi}\,\hat{\kappa}_2\mathbf{A}_\theta\boldsymbol{\Phi}_\theta\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{\xi}\left(\frac{1}{r}\mathbf{A}_{r,\theta}\mathbf{\Phi}_r-\frac{\mathbf{A}_\theta\mathbf{\Phi}_\theta}{r}\right)\left(\mathbf{A}_r\mathbf{\Phi}_{r,s}-\tau\mathbf{A}_{r,\theta}\mathbf{\Phi}_r+\hat{\kappa}_1\mathbf{A}_s\mathbf{\Phi}_s\right)\\
&+\left(\frac{1}{r}\mathbf{A}_{\theta,\theta}\mathbf{\Phi}_\theta+\frac{\mathbf{A}_r\mathbf{\Phi}_r}{r}\right)\left(\frac{1}{\xi}\mathbf{A}_\theta\mathbf{\Phi}_{\theta,s}-\frac{\tau}{\xi}\mathbf{A}_{\theta,\theta}\mathbf{\Phi}_\theta-\frac{1}{\xi}\hat{\kappa}_2\mathbf{A}_s\mathbf{\Phi}_s\right)\\
2E_{sr}=&\ \frac{1}{\xi}\left(\mathbf{A}_r\mathbf{\Phi}_{r,s}-\tau\mathbf{A}_{r,\theta}\mathbf{\Phi}_r+\hat{\kappa}_1\mathbf{A}_s\mathbf{\Phi}_s\right)\\
&+\mathbf{A}_{s,r}\mathbf{\Phi}_s+\mathbf{A}_{s,r}\mathbf{\Phi}_s\left(\frac{1}{\xi}\mathbf{A}_s\mathbf{\Phi}_{s,s}-\frac{\tau}{\xi}\mathbf{A}_{s,\theta}\mathbf{\Phi}_s-\frac{1}{\xi}\hat{\kappa}_1\,\mathbf{A}_r\mathbf{\Phi}_r+\frac{1}{\xi}\,\hat{\kappa}_2\mathbf{A}_\theta\mathbf{\Phi}_\theta\right)\\
&+\frac{1}{\xi}\mathbf{A}_{r,r}\mathbf{\Phi}_r\left(\mathbf{A}_r\mathbf{\Phi}_{r,s}-\tau\mathbf{A}_{r,\theta}\mathbf{\Phi}_r+\hat{\kappa}_1\mathbf{A}_s\mathbf{\Phi}_s\right)\\
&+\mathbf{A}_{\theta,r}\mathbf{\Phi}_\theta\left(\frac{1}{\xi}\mathbf{A}_\theta\mathbf{\Phi}_{\theta,s}-\frac{\tau}{\xi}\mathbf{A}_{\theta,\theta}\mathbf{\Phi}_\theta-\frac{1}{\xi}\hat{\kappa}_2\mathbf{A}_s\mathbf{\Phi}_s\right)
\end{aligned}
\tag{13}
$$

Equations (10) and (11) should be replaced by:

$$
\mathbf{A}_1=\begin{bmatrix}
-\frac{\tau}{\xi}\mathbf{A}_{s,\theta} & -\frac{1}{\xi}\hat{\kappa}_1\,\mathbf{A}_r & \frac{1}{\xi}\,\hat{\kappa}_2\mathbf{A}_\theta\\
\mathbf{0} & \mathbf{A}_{r,r} & \mathbf{0}\\
\mathbf{0} & \frac{1}{r}\mathbf{A}_r & \frac{1}{r}\mathbf{A}_{\theta,\theta}\\
\frac{1}{r}\mathbf{A}_{r,\theta} & \mathbf{0} & -\frac{1}{r}\mathbf{A}_\theta+\mathbf{A}_{\theta,r}\\
+\frac{1}{r}\mathbf{A}_{s,\theta}-\frac{1}{\xi}\hat{\kappa}_2\mathbf{A}_s & \mathbf{0} & -\frac{\tau}{\xi}\mathbf{A}_{\theta,\theta}\\
\frac{\hat{\kappa}_1}{\xi}\mathbf{A}_s+\mathbf{A}_{s,r} & -\frac{\tau}{\xi}\mathbf{A}_{r,\theta} & \mathbf{0}
\end{bmatrix},\quad
\mathbf{A}_2=\frac{1}{\xi}\begin{bmatrix}
\mathbf{A}_s & \mathbf{0} & \mathbf{0}\\
\mathbf{0} & \mathbf{0} & \mathbf{0}\\
\mathbf{0} & \mathbf{0} & \mathbf{0}\\
\mathbf{0} & \mathbf{0} & \mathbf{0}\\
\mathbf{0} & \mathbf{0} & \mathbf{A}_\theta\\
\mathbf{0} & \mathbf{A}_r & \mathbf{0}
\end{bmatrix}
$$

$$
\mathbf{A}_{nl_s}=\begin{bmatrix}
\begin{array}{c}\frac{1}{\xi^2}(u_{s,s}-\tau u_{s,\theta}\\-\hat{\kappa}_1u_r+\hat{\kappa}_2u_\theta)\mathbf{A}_s\end{array} & \frac{1}{\xi^2}(u_{r,s}-\tau u_{r,\theta}+\hat{\kappa}_1u_s)\mathbf{A}_r & \frac{1}{\xi^2}(u_{\theta,s}-\tau u_{\theta,\theta}-\hat{\kappa}_2u_s)\mathbf{A}_\theta\\
\mathbf{0} & \mathbf{0} & \mathbf{0}\\
\mathbf{0} & \mathbf{0} & \mathbf{0}\\
\mathbf{0} & \mathbf{0} & \mathbf{0}\\
\frac{1}{r\xi}u_{s,\theta}\mathbf{A}_s & \frac{1}{r\xi}(u_{r,\theta}-u_\theta)\mathbf{A}_r & \frac{1}{r\xi}(u_{\theta,\theta}+u_r)\mathbf{A}_\theta\\
\frac{1}{\xi}u_{s,r}\mathbf{A}_s & \frac{1}{\xi}u_{r,r}\mathbf{A}_r & \frac{1}{\xi}u_{\theta,r}\mathbf{A}_\theta
\end{bmatrix}
$$

$$
\mathbf{A}_{nl}=\begin{bmatrix}
\begin{array}{c}-\frac{\tau}{\xi}(\frac{1}{\xi}u_{s,s}-\frac{\tau}{\xi}u_{s,\theta}\\-\frac{1}{\xi}\hat{\kappa}_1\,u_r+\frac{1}{\xi}\,\hat{\kappa}_2u_\theta)\mathbf{A}_{s,\theta}\\+\frac{\hat{\kappa}_1}{\xi^2}(u_{r,s}-\tau u_{r,\theta}+\hat{\kappa}_1u_s)\mathbf{A}_s\\-\frac{\hat{\kappa}_2}{\xi^2}(u_{\theta,s}-\tau u_{\theta,\theta}-\hat{\kappa}_2u_s)\mathbf{A}_s\end{array} &
\begin{array}{c}-\frac{\hat{\kappa}_1}{\xi}(\frac{1}{\xi}u_{s,s}-\frac{\tau}{\xi}u_{s,\theta}\\-\frac{1}{\xi}\hat{\kappa}_1\,u_r+\frac{1}{\xi}\,\hat{\kappa}_2u_\theta)\mathbf{A}_r\\-\frac{\tau}{\xi^2}(u_{r,s}-\tau u_{r,\theta}+\hat{\kappa}_1u_s)\mathbf{A}_{r,\theta}\end{array} &
\begin{array}{c}\frac{\hat{\kappa}_2}{\xi}(\frac{1}{\xi}u_{s,s}-\frac{\tau}{\xi}u_{s,\theta}\\-\frac{1}{\xi}\hat{\kappa}_1\,u_r+\frac{1}{\xi}\,\hat{\kappa}_2u_\theta)\mathbf{A}_\theta\\-\frac{\tau}{\xi^2}(u_{\theta,s}-\tau u_{\theta,\theta}-\hat{\kappa}_2u_s)\mathbf{A}_{\theta,\theta}\end{array}\\
u_{s,r}\mathbf{A}_{s,r} & u_{r,r}\mathbf{A}_{r,r} & u_{\theta,r}\mathbf{A}_{\theta,r}\\
\frac{1}{r^2}u_{s,\theta}\mathbf{A}_{s,\theta} &
\begin{array}{c}\frac{1}{r^2}(u_{r,\theta}-u_\theta)\mathbf{A}_{r,\theta}\\+\frac{1}{r^2}(u_{\theta,\theta}+u_r)\mathbf{A}_r\end{array} &
\begin{array}{c}-\frac{1}{r^2}(u_{r,\theta}-u_\theta)\mathbf{A}_\theta\\+\frac{1}{r^2}(u_{\theta,\theta}+u_r)\mathbf{A}_{\theta,\theta}\end{array}\\
\frac{1}{r}(u_{s,r}\mathbf{A}_{s,\theta}+u_{s,\theta}\mathbf{A}_{s,r}) &
\begin{array}{c}\frac{1}{r}(u_{r,\theta}-u_\theta)\mathbf{A}_{r,r}\\+\frac{1}{r}u_{r,r}\mathbf{A}_{r,\theta}+\frac{1}{r}u_{\theta,r}\mathbf{A}_r\end{array} &
\begin{array}{c}-\frac{1}{r}u_{r,r}\mathbf{A}_\theta+\frac{1}{r}u_{\theta,r}\mathbf{A}_{\theta,\theta}\\+\frac{1}{r}(u_{\theta,\theta}+u_r)\mathbf{A}_{\theta,r}\end{array}\\
\begin{array}{c}\frac{1}{r\xi}(u_{s,s}-\tau u_{s,\theta}-\hat{\kappa}_1u_r\\+\hat{\kappa}_2u_\theta)\mathbf{A}_{s,\theta}-\frac{\tau}{r\xi}u_{s,\theta}\mathbf{A}_{s,\theta}\\+\frac{\hat{\kappa}_1}{r\xi}(u_{r,\theta}-u_\theta)\mathbf{A}_s\\-\frac{\hat{\kappa}_2}{r\xi}(u_{\theta,\theta}+u_r)\mathbf{A}_s\end{array} &
\begin{array}{c}-\frac{\hat{\kappa}_1}{r\xi}u_{s,\theta}\mathbf{A}_r-\frac{\tau}{r\xi}(u_{r,\theta}-u_\theta)\mathbf{A}_{r,\theta}\\+\frac{1}{r\xi}(u_{r,s}-\tau u_{r,\theta}+\hat{\kappa}_1u_s)\mathbf{A}_{r,\theta}\\+\frac{1}{r\xi}(u_{\theta,s}-\tau u_{\theta,\theta}-\hat{\kappa}_2u_s)\mathbf{A}_r\end{array} &
\begin{array}{c}-\frac{1}{r\xi}(u_{r,s}-\tau u_{r,\theta}+\hat{\kappa}_1u_s)\mathbf{A}_\theta\\+\frac{\hat{\kappa}_2}{r\xi}u_{s,\theta}\mathbf{A}_\theta\\+\frac{1}{r\xi}(u_{\theta,s}-\tau u_{\theta,\theta}-\hat{\kappa}_2u_s)\mathbf{A}_{\theta,\theta}\\-\frac{\tau}{r\xi}(u_{\theta,\theta}+u_r)\mathbf{A}_{\theta,\theta}\end{array}\\
\begin{array}{c}\frac{1}{\xi}(u_{s,s}-\tau u_{s,\theta}-\hat{\kappa}_1\,u_r\\+\hat{\kappa}_2u_\theta)\mathbf{A}_{s,r}-\frac{\tau}{\xi}u_{s,r}\mathbf{A}_{s,\theta}\\+\frac{\hat{\kappa}_1}{\xi}u_{r,r}\mathbf{A}_s-\frac{\hat{\kappa}_2}{\xi}u_{\theta,r}\mathbf{A}_s\end{array} &
\begin{array}{c}\frac{1}{\xi}(u_{r,s}-\tau u_{r,\theta}+\hat{\kappa}_1u_s)\mathbf{A}_{r,r}\\-\frac{\hat{\kappa}_1}{\xi}u_{s,r}\mathbf{A}_r-\frac{\tau}{\xi}u_{r,r}\mathbf{A}_{r,\theta}\end{array} &
\begin{array}{c}\frac{\hat{\kappa}_2}{\xi}u_{s,r}\mathbf{A}_\theta-\frac{\tau}{\xi}u_{\theta,r}\mathbf{A}_{\theta,\theta}\\+\frac{1}{\xi}(u_{\theta,s}-\tau u_{\theta,\theta}-\hat{\kappa}_2u_s)\mathbf{A}_{\theta,r}\end{array}
\end{bmatrix}
$$

In Eqs. (12), (14), (15), (19), (20), (27), (34), (37) and (38), dA should be replaced by ξdA.

Correction in Appendix D

The matrices $\tilde{\mathbf{P}}_i$ for $i = 1, 2, 3, 4$ in Eq. (38) can be given as follows:

$$
\tilde{\mathbf{P}}_1 = \frac{1}{\xi^2}\begin{bmatrix}
\begin{array}{c}\sigma_{ss}(\hat{\kappa}_1^2+\hat{\kappa}_2^2)\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_s \\ +\xi^2\sigma_{rr}\mathbf{A}_{s,r}^{\mathrm{T}}\mathbf{A}_{s,r} \\ +\left(\frac{\xi^2\sigma_{r\theta}}{r}-\xi\tau\sigma_{sr}\right)(\mathbf{A}_{s,r}^{\mathrm{T}}\mathbf{A}_{s,\theta} \\ +\mathbf{A}_{s,\theta}^{\mathrm{T}}\mathbf{A}_{s,r})+\left(\frac{\xi^2\sigma_{\theta\theta}}{r^2}\right. \\ \left.+\sigma_{ss}\tau^2-2\tau\sigma_{\theta s}\frac{\xi}{r}\right)\mathbf{A}_{s,\theta}^{\mathrm{T}}\mathbf{A}_{s,\theta}\end{array} &
\begin{array}{c}\left(\sigma_{ss}\hat{\kappa}_1\tau-\frac{\xi}{r}\sigma_{\theta s}\hat{\kappa}_1\right)(\mathbf{A}_{s,\theta}^{\mathrm{T}}\mathbf{A}_r \\ -\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_{r,\theta})-\frac{\xi}{r}\sigma_{\theta s}\hat{\kappa}_2\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_r \\ +\xi\sigma_{sr}\hat{\kappa}_1(\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_{r,r}-\mathbf{A}_{s,r}^{\mathrm{T}}\mathbf{A}_r)\end{array} &
\begin{array}{c}\left(\frac{\xi}{r}\sigma_{\theta s}\hat{\kappa}_2-\sigma_{ss}\hat{\kappa}_2\tau\right)(\mathbf{A}_{s,\theta}^{\mathrm{T}}\mathbf{A}_\theta \\ -\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_{\theta,\theta})-\frac{\xi}{r}\sigma_{\theta s}\hat{\kappa}_1\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_\theta \\ +\xi\sigma_{sr}\hat{\kappa}_2(\mathbf{A}_{s,r}^{\mathrm{T}}\mathbf{A}_\theta-\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_{\theta,r})\end{array} \\
\begin{array}{c}\left(\sigma_{ss}\hat{\kappa}_1\tau-\frac{\xi}{r}\sigma_{\theta s}\hat{\kappa}_1\right)(\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_{s,\theta} \\ -\mathbf{A}_{r,\theta}^{\mathrm{T}}\mathbf{A}_s)-\frac{\xi}{r}\sigma_{\theta s}\hat{\kappa}_2\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_s \\ +\xi\sigma_{sr}\hat{\kappa}_1(\mathbf{A}_{r,r}^{\mathrm{T}}\mathbf{A}_s-\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_{s,r})\end{array} &
\begin{array}{c}\left(\sigma_{ss}\hat{\kappa}_1^2+\frac{\xi^2}{r^2}\sigma_{\theta\theta}\right)\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_r \\ +\left(\sigma_{ss}\tau^2+\frac{\xi^2}{r^2}\sigma_{\theta\theta}\right. \\ \left.-\frac{2\xi}{r}\tau\sigma_{\theta s}\right)\mathbf{A}_{r,\theta}^{\mathrm{T}}\mathbf{A}_{r,\theta}+\left(\frac{\xi^2}{r}\sigma_{r\theta}\right. \\ \left.-\xi\sigma_{sr}\tau\right)(\mathbf{A}_{r,r}^{\mathrm{T}}\mathbf{A}_{r,\theta}+\mathbf{A}_{r,\theta}^{\mathrm{T}}\mathbf{A}_{r,r}) \\ +\xi^2\sigma_{rr}\mathbf{A}_{r,r}^{\mathrm{T}}\mathbf{A}_{r,r}\end{array} &
\begin{array}{c}-\sigma_{ss}\hat{\kappa}_1\hat{\kappa}_2\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_\theta+\left(\frac{\xi}{r}\tau\sigma_{\theta s}\right. \\ \left.-\frac{\xi^2}{r^2}\sigma_{\theta\theta}\right)(\mathbf{A}_{r,\theta}^{\mathrm{T}}\mathbf{A}_\theta-\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_{\theta,\theta}) \\ +\frac{\xi^2}{r}\sigma_{r\theta}(\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_{\theta,r}-\mathbf{A}_{r,r}^{\mathrm{T}}\mathbf{A}_\theta)\end{array} \\
\begin{array}{c}\left(\frac{\xi}{r}\sigma_{\theta s}\hat{\kappa}_2-\sigma_{ss}\hat{\kappa}_2\tau\right)(\mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_{s,\theta} \\ -\mathbf{A}_{\theta,\theta}^{\mathrm{T}}\mathbf{A}_s)-\frac{\xi}{r}\sigma_{\theta s}\hat{\kappa}_1\mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_s \\ +\xi\sigma_{sr}\hat{\kappa}_2(\mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_{s,r}-\mathbf{A}_{\theta,r}^{\mathrm{T}}\mathbf{A}_s)\end{array} &
\begin{array}{c}-\sigma_{ss}\hat{\kappa}_1\hat{\kappa}_2\mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_r+\left(\frac{\xi}{r}\tau\sigma_{\theta s}\right. \\ \left.-\frac{\xi^2}{r^2}\sigma_{\theta\theta}\right)(\mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_{r,\theta}-\mathbf{A}_{\theta,\theta}^{\mathrm{T}}\mathbf{A}_r) \\ +\frac{\xi^2}{r}\sigma_{r\theta}(\mathbf{A}_{\theta,r}^{\mathrm{T}}\mathbf{A}_r-\mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_{r,r})\end{array} &
\begin{array}{c}\left(\sigma_{ss}\hat{\kappa}_2^2+\frac{\xi^2}{r^2}\sigma_{\theta\theta}\right)\mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_\theta \\ +\xi^2\sigma_{rr}\mathbf{A}_{\theta,r}^{\mathrm{T}}\mathbf{A}_{\theta,r}+\left(\frac{\xi^2}{r^2}\sigma_{\theta\theta}\right. \\ \left.+\sigma_{ss}\tau^2-\frac{2\xi}{r}\tau\sigma_{\theta s}\right)\mathbf{A}_{\theta,\theta}^{\mathrm{T}}\mathbf{A}_{\theta,\theta} \\ +\left(\frac{\xi^2}{r}\sigma_{r\theta}-\xi\sigma_{sr}\tau\right)(\mathbf{A}_{\theta,\theta}^{\mathrm{T}}\mathbf{A}_{\theta,r} \\ +\mathbf{A}_{\theta,r}^{\mathrm{T}}\mathbf{A}_{\theta,\theta})\end{array}
\end{bmatrix}
$$

$$
\hat{\mathbf{P}}_2 = \frac{1}{\xi^2}\begin{bmatrix}
\begin{array}{c}\left(\frac{\xi}{r}\sigma_{\theta s}-\tau\sigma_{ss}\right)\mathbf{A}_{s,\theta}^{\mathrm{T}}\mathbf{A}_s \\ +\xi\sigma_{sr}\mathbf{A}_{s,r}^{\mathrm{T}}\mathbf{A}_s\end{array} & \hat{\kappa}_1\sigma_{ss}\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_r & -\hat{\kappa}_2\sigma_{ss}\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_\theta \\
-\hat{\kappa}_1\sigma_{ss}\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_s & \begin{array}{c}\left(\frac{\xi}{r}\sigma_{\theta s}-\tau\sigma_{ss}\right)\mathbf{A}_{r,\theta}^{\mathrm{T}}\mathbf{A}_r \\ +\xi\sigma_{sr}\mathbf{A}_{r,r}^{\mathrm{T}}\mathbf{A}_r\end{array} & \frac{\xi}{r}\sigma_{\theta s}\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_\theta \\
\hat{\kappa}_2\sigma_{ss}\mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_s & -\frac{\xi}{r}\sigma_{\theta s}\mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_r & \begin{array}{c}\left(\frac{\xi}{r}\sigma_{\theta s}-\tau\sigma_{ss}\right)\mathbf{A}_{\theta,\theta}^{\mathrm{T}}\mathbf{A}_\theta \\ +\xi\sigma_{sr}\mathbf{A}_{\theta,r}^{\mathrm{T}}\mathbf{A}_\theta\end{array}
\end{bmatrix}
$$

$$
\hat{\mathbf{P}}_3 = \frac{1}{\xi^2}\begin{bmatrix}
\begin{array}{c}\left(\frac{\xi}{r}\sigma_{\theta s}-\tau\sigma_{ss}\right)\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_{s,\theta} \\ +\xi\sigma_{sr}\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_{s,r}\end{array} & -\hat{\kappa}_1\sigma_{ss}\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_r & \hat{\kappa}_2\sigma_{ss}\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_\theta \\
\hat{\kappa}_1\sigma_{ss}\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_s & \begin{array}{c}-\tau\sigma_{ss}\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_{r,\theta} \\ +\frac{\xi}{r}\sigma_{\theta s}\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_{r,\theta}+\xi\sigma_{sr}\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_{r,r}\end{array} & -\frac{\xi}{r}\sigma_{\theta s}\mathbf{A}_r^{\mathrm{T}}\mathbf{A}_\theta \\
-\hat{\kappa}_2\sigma_{ss}\mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_s & \frac{\xi}{r}\sigma_{\theta s}\mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_r & \begin{array}{c}\left(\frac{\xi}{r}\sigma_{\theta s}-\tau\sigma_{ss}\right)\mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_{\theta,\theta} \\ +\xi\sigma_{sr}\mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_{\theta,r}\end{array}
\end{bmatrix}
$$

$$
\hat{\mathbf{P}}_4 = \frac{\sigma_{ss}}{\xi^2}\begin{bmatrix}
\mathbf{A}_s^{\mathrm{T}}\mathbf{A}_s & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{A}_r^{\mathrm{T}}\mathbf{A}_r & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{A}_\theta^{\mathrm{T}}\mathbf{A}_\theta
\end{bmatrix}
$$

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