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CORRECTION

Correction to: Cylindrical Martingale Problems Associated with Lévy Generators

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1 Corrections

In this note, we correct claims made in [2]:

(i) It is claimed that the generalized martingale problem introduced in [2] allows explosion in a continuous manner. However, because the cemetery Δ is added to $\mathbb{B}$ as an isolated point, explosion can only happen by a jump and is excluded by [2, Lemma 4.3]. In Sect. 2, we explain how the setup can be adjusted to include the possibility of explosion.

(ii) In the proof of [2, Proposition 4.8], it is needed that the operator A has a non-empty resolvent set $\rho(A)$, i.e., that

$$\rho(A) \triangleq \left\{\lambda \in \mathbb{R}\colon (\lambda - A)^{-1} \text{ exists in } L(\mathbb{B}, \mathbb{B})\right\} \neq \emptyset.$$

This assumption is missing in [2]. It is, e.g., satisfied in case A is the generator of a C_0-semigroup; see [4, Remark 1.1.3, Proposition 1.2.1].

2 A Setup Including Explosion

2.1 Modified Setup

In the following, we explain how Ω, τ_n and τ_Δ have to be redefined such that the setting includes the possibility of explosion.

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For a function $\omega\colon \mathbb{R}_+ \to \mathbb{B}_\Delta$, we define

$$\tau_\Delta(\omega) \triangleq \inf(t \in \mathbb{R}_+ : \omega(t) = \Delta),$$

where, as always, $\inf(\emptyset) \triangleq \infty$. Let Ω to be the space of all right continuous functions $\omega\colon \mathbb{R}_+ \to \mathbb{B}_\Delta$ which are càdlàg on $[0, \tau_\Delta(\omega))$ and satisfy $\omega(t) = \Delta$ for all $t \geq \tau_\Delta(\omega)$. The difference in comparison with the setting in [2] is that $\omega \in \{\tau_\Delta < \infty\}$ might not have a left limit at $\tau_\Delta(\omega)$.

Denote by X the coordinate process, i.e., $X_t(\omega) = \omega(t)$ for all $\omega \in \Omega$ and $t \in \mathbb{R}_+$, and denote by $\mathcal{F} \triangleq \sigma(X_t, t \in \mathbb{R}_+)$ the σ-field generated by X. The proof of the following is given in Sect. 2.2.

Lemma 1 *There exists a metric d_Ω on Ω such that (Ω, d_Ω) is separable and complete and $\mathcal{F}$ is the corresponding Borel σ-field.*

Let $\mathbf{F} = (\mathcal{F}_t)_{t\geq 0}$ be the filtration generated by X, i.e. $\mathcal{F}_t \triangleq \sigma(X_s, s \in [0,t])$ for $t \in \mathbb{R}_+$. Note that τ_Δ is an $\mathbf{F}$-stopping time, because $\{\tau_\Delta \leq t\} = \{X_t = \Delta\} \in \mathcal{F}_t$. For $\Gamma \subseteq \mathbb{B}$, we define

$$\tau(\Gamma) \triangleq \inf\big(t < \tau_\Delta : X_t \in \Gamma \text{ or } X_{t-} \in \Gamma\big) \wedge \tau_\Delta.$$

The proof of the following is given in Sect. 2.3.

Lemma 2 (i) *If $\Gamma \subseteq \mathbb{B}$ is closed, then $\tau(\Gamma)$ is an $\mathbf{F}$-stopping time.*
(ii) *If $\Gamma_1 \subseteq \Gamma_2 \subseteq \Gamma_3 \subseteq \cdots$ is an increasing sequence of open sets in $\mathbb{B}$ such that $\bigcup_{n\in\mathbb{N}} \Gamma_n = \mathbb{B}$, then $\tau(\mathbb{B}\backslash\Gamma_n) \nearrow \tau_\Delta$ as $n \to \infty$.*

We define

$$\tau_n \triangleq \inf\big(t < \tau_\Delta : \|X_t\| \geq n \text{ or } \|X_{t-}\| \geq n\big) \wedge \tau_\Delta \wedge n, \quad n \in \mathbb{N}.$$

By Lemma 2, $(\tau_n)_{n\in\mathbb{N}}$ is a sequence of $\mathbf{F}$-stopping times satisfying $\tau_n \nearrow \tau_\Delta$ as $n \to \infty$. In this modified setting, the GMP can be defined as in [2] and all results from [2] hold. In Sect. 3, we comment on necessary changes in the proofs.

2.2 Proof of Lemma 1

We adapt the proof of [1, Lemma A.7]. Define

$$\Omega^\star \triangleq \big(D(\mathbb{R}_+, \mathbb{B}) \times (0, \infty]\big) \cup \big(\{\omega_\Delta\} \times \{0\}\big),$$

where $\omega_\Delta(t) = \Delta$ for all $t \in \mathbb{R}_+$. For $z \in [0, \infty]$ and $t \in \mathbb{R}_+$, we define

$$\phi_z(t) \triangleq \begin{cases} t, & z = \infty, \\ z\big(1 - e^{-t}\big), & z \in (0, \infty), \\ 0, & z = 0, \end{cases}$$

$$\phi_z^{-1}(t) \triangleq \begin{cases} t, & z = \infty, \\ -\log\left(1 - \frac{t}{z}\right)\mathbf{1}_{\{t<z\}}, & z \in (0, \infty), \\ 0, & z = 0. \end{cases}$$

Moreover, we define $\iota\colon \Omega \to \Omega^\star$ by

$$\iota(\omega) \triangleq (\omega \circ \phi_{\tau_\Delta(\omega)}, \tau_\Delta(\omega)).$$

Lemma 3 *ι is a bijection.*

Proof To check the injectivity, let $\omega, \alpha \in \Omega$ be such that $\iota(\omega) = \iota(\alpha)$. In case $\tau_\Delta(\omega) = \tau_\Delta(\alpha) \in \{0, \infty\}$, we clearly have $\omega = \alpha$. In case $0 < \tau_\Delta(\omega) = \tau_\Delta(\alpha) < \infty$, we can deduce from the first coordinates of $\iota(\omega)$ and $\iota(\alpha)$ that $\omega = \alpha$ on $[0, \tau_\Delta(\omega)) = [0, \tau_\Delta(\alpha))$, which implies $\omega = \alpha$.

To check the surjectivity, note that $\iota(\omega_\Delta) = (\omega_\Delta, 0)$ and that $\iota(\omega \circ \phi_t^{-1}\mathbf{1}_{[0,t)} + \Delta\mathbf{1}_{[t,\infty)}) = (\omega, t)$ for all $(\omega, t) \in D(\mathbb{R}_+, \mathbb{B}) \times (0, \infty]$. □

Let d_D be the Skorokhod metric on $D(\mathbb{R}_+, \mathbb{B}_\Delta)$ and let $d_{[0,\infty]}$ be the arctan metric on $[0, \infty]$. We define

$$d_{D\times[0,\infty]}((\omega, t), (\alpha, s)) \triangleq d_D(\omega, \alpha) + d_{[0,\infty]}(t, s)$$

for $(\omega, t), (\alpha, s) \in D(\mathbb{R}_+, \mathbb{B}_\Delta) \times [0, \infty]$, and set

$$d_{\Omega^\star} \triangleq d_{D\times[0,\infty]}\big|_{\Omega^\star\times\Omega^\star}.$$

We note that $(\Omega^\star, d_{\Omega^\star})$ is separable and complete, because it is a G_δ subspace of $(D(\mathbb{R}_+, \mathbb{B}_\Delta) \times [0, \infty], d_{D\times[0,\infty]})$. Due to Lemma 3, we can equip Ω with the metric

$$\begin{aligned} d_\Omega(\omega, \alpha) &\triangleq d_{\Omega^\star}(\iota(\omega), \iota(\alpha)) \\ &= d_D(\omega \circ \phi_{\tau_\Delta(\omega)}, \alpha \circ \phi_{\tau_\Delta(\alpha)}) + d_{[0,\infty]}(\tau_\Delta(\omega), \tau_\Delta(\alpha)) \end{aligned}$$

for $\omega, \alpha \in \Omega$. In this case, ι is an isometry and (Ω, d_Ω) is separable and complete. In the following, we equip Ω with the topology induced by the metric d_Ω.

We now prove that $\mathcal{F} = \mathcal{B}(\Omega)$. By the definition of the metric d_Ω, the maps

$$\Omega \ni \omega \mapsto \omega \circ \phi_{\tau_\Delta(\omega)} \in D(\mathbb{R}_+, \mathbb{B}_\Delta), \quad \Omega \ni \omega \mapsto \tau_\Delta(\omega) \in [0, \infty]$$

are continuous. For fixed $t \in \mathbb{R}_+$, the map $[0,\infty] \ni z \mapsto \phi_z^{-1}(t) \in \mathbb{R}_+$ is Borel and, consequently, also

$$\Omega \ni \omega \mapsto \phi^{-1}_{\tau_\Delta(\omega)}(t) \in \mathbb{R}_+$$

is Borel. Because right continuous adapted processes are progressively measurable, the map

$$D(\mathbb{R}_+, \mathbb{B}_\Delta) \times \mathbb{R}_+ \ni (\omega, t) \mapsto \omega(t) \triangleq Y(\omega, t) \in \mathbb{B}_\Delta$$

is Borel. We conclude that for every $t \in \mathbb{R}_+$ the map

$$\Omega \ni \omega \mapsto \omega(t) = Y(\omega \circ \phi_{\tau_\Delta(\omega)}, \phi^{-1}_{\tau_\Delta(\omega)}(t))\mathbf{1}_{\{t<\tau_\Delta(\omega)\}} + \Delta\mathbf{1}_{\{t\geq\tau_\Delta(\omega)\}} \in \Omega$$

is Borel. This implies that $\mathcal{F} \subseteq \mathcal{B}(\Omega)$.

Note that ι is $\mathcal{F}/\mathcal{B}(\Omega^\star)$ measurable. Let $f: \Omega \to \mathbb{R}$ be a Borel function. Because ι is an isometry, the inverse map $\iota^{-1}: \Omega^\star \to \Omega$ is continuous and therefore Borel. We conclude that

$$\Omega \ni \omega \mapsto f(\omega) = ((f \circ \iota^{-1}) \circ \iota)(\omega) \in \mathbb{R}$$

is $\mathcal{F}/\mathcal{B}(\mathbb{R})$ measurable as composition of the $\mathcal{B}(\Omega^\star)/\mathcal{B}(\mathbb{R})$ measurable map $f \circ \iota^{-1}$ and the $\mathcal{F}/\mathcal{B}(\Omega^\star)$ measurable map ι. This implies $\mathcal{B}(\Omega) \subseteq \mathcal{F}$ and the proof is complete. □

2.3 Proof of Lemma 2

(i). We have to show that $\{\tau(\Gamma) \leq t\} \in \mathcal{F}_t$ for all $t \in \mathbb{R}_+$. For $x \in \mathbb{B}$, we define $d(x, \Gamma) \triangleq \inf_{y\in\Gamma} \|x - y\|$ and set

$$\Gamma_n \triangleq \big\{x \in \mathbb{B}: d(x, \Gamma) < \tfrac{1}{n}\big\}.$$

Moreover, on $\{t < \tau_\Delta\}$ we set

$$F_t \triangleq \mathrm{cl}_{\mathbb{B}}(\{X_s : s \in [0, t]\}) = \{X_s, X_{s-} : s \in [0, t]\} \subseteq \mathbb{B}.$$

Because $x \mapsto d(x, \Gamma)$ is Lipschitz continuous, the set Γ_n is open, and because Γ is closed, $\Gamma = \{x \in \mathbb{B}: d(x, \Gamma) = 0\}$. Define $\tau \triangleq \sup_{n\in\mathbb{N}} \tau(\Gamma_n)$. Because $\Gamma \subseteq \Gamma_n$, it is clear that $\tau \leq \tau(\Gamma)$. Next, we show that $\tau \geq \tau(\Gamma)$. We claim that this inequality follows if we show that

$$\forall t \in \mathbb{R}_+: \bigcap_{n\in\mathbb{N}} \{F_t \cap \Gamma_n \neq \emptyset\} \subseteq \{F_t \cap \Gamma \neq \emptyset\} \text{ on } \{t < \tau_\Delta\}. \tag{2.1}$$

We explain this: In case $\tau \geq \tau_\Delta$, we have $\tau = \tau(\Gamma) = \tau_\Delta$. Take $\omega \in \{\tau < \tau_\Delta\}$ and let $\varepsilon > 0$ be such that $\varepsilon < \tau_\Delta(\omega) - \tau(\omega)$ in case $\tau_\Delta(\omega) < \infty$. For each $n \in \mathbb{N}$, we find a $t_n \in [\tau(\Gamma_n)(\omega), \tau(\Gamma_n)(\omega) + \varepsilon)$ such that $F_{t_n}(\omega) \cap \Gamma_n \neq \emptyset$. Note that $t \triangleq \sup_{n\in\mathbb{N}} t_n \leq$

$\tau(\omega) + \varepsilon < \tau_\Delta(\omega)$ and that $F_t(\omega) \cap \Gamma_n \neq \emptyset$ for all $n \in \mathbb{N}$. Consequently, in case (2.1) holds we have $F_t(\omega) \cap \Gamma \neq \emptyset$, which implies $\tau(\Gamma)(\omega) \leq t \leq \tau(\omega) + \varepsilon$. We conclude that $\tau \geq \tau(\Gamma)$ as claimed. We proceed showing (2.1). Fix $t \in \mathbb{R}_+$. Because on $\{t < \tau_\Delta\}$

$$\bigcap_{n \in \mathbb{N}} \{F_t \cap \Gamma_n \neq \emptyset\} \subseteq \Big\{ \inf_{x \in F_t} d(x, \Gamma) = 0 \Big\},$$

it suffices to show that on $\{t < \tau_\Delta\}$

$$\Big\{ \inf_{x \in F_t} d(x, \Gamma) = 0 \Big\} \subseteq \{F_t \cap \Gamma \neq \emptyset\}.$$

Take $\omega \in \{t < \tau_\Delta\}$. Because $\{\omega(\cdot \wedge t)\}$ is compact in $D(\mathbb{R}_+, \mathbb{B})$, $F_t(\omega)$ is compact in $\mathbb{B}$ by [4, Problem 16, p. 152]. Consequently, due to its continuity, the function $x \mapsto d(x, \Gamma)$ attains its infimum on $F_t(\omega)$. Thus, because $\Gamma = \{x \in \mathbb{B} : d(x, \Gamma) = 0\}$, if $\inf_{x \in F_t(\omega)} d(x, \Gamma) = 0$, we have $F_t(\omega) \cap \Gamma \neq \emptyset$. We conclude that (2.1) holds and hence that $\tau = \tau(\Gamma)$.

From the equality $\tau = \tau(\Gamma)$, we deduce that for all $t \in \mathbb{R}_+$

$$\{\tau(\Gamma) \leq t\} = \bigcap_{n \in \mathbb{N}} \{\tau(\Gamma_n) \leq t\}. \tag{2.2}$$

Fix $t \in \mathbb{R}_+$ and set $\mathbb{Q}^t_+ \triangleq ([0, t) \cap \mathbb{Q}_+) \cup \{t\}$. We note that

$$\begin{aligned} \{\tau(\Gamma_{n+1}) \leq t < \tau_\Delta\} &= \bigcap_{m \in \mathbb{N}} \{\tau(\Gamma_{n+1}) < t + \tfrac{1}{m} \leq \tau_\Delta\} \\ &\supseteq \Big(\bigcup_{s \in \mathbb{Q}^t_+} \{X_s \in \Gamma_{n+1}\} \Big) \cap \{t < \tau_\Delta\}. \end{aligned} \tag{2.3}$$

Because Γ_{n+1} is open, we have

$$\tau(\Gamma_{n+1}) = \inf \big(t < \tau_\Delta : X_t \in \Gamma_{n+1} \big) \wedge \tau_\Delta.$$

Thus, in case $\tau(\Gamma_{n+1}) \leq t < \tau_\Delta$, the right continuity of X yields that $X_{\tau(\Gamma_{n+1})} \in \mathrm{cl}_{\mathbb{B}}(\Gamma_{n+1}) \subseteq \Gamma_n$. We conclude that on $\{t < \tau_\Delta\}$

$$\{\tau(\Gamma_{n+1}) \leq t\} \subseteq \bigcup_{s \in [0,t]} \{X_s \in \mathrm{cl}_{\mathbb{B}}(\Gamma_{n+1})\} \subseteq \bigcup_{s \in \mathbb{Q}^t_+} \{X_s \in \Gamma_n\}. \tag{2.4}$$

Now, (2.2), (2.3) and (2.4) imply that

$$\{\tau(\Gamma) \leq t < \tau_\Delta\} = \Big(\bigcap_{n \in \mathbb{N}} \bigcup_{s \in \mathbb{Q}^t_+} \{X_s \in \Gamma_n\} \Big) \cap \{X_t \neq \Delta\} \in \mathcal{F}_t.$$

Because

$$\{\tau(\Gamma) \le t, \tau_\Delta \le t\} = \{\tau_\Delta \le t\} = \{X_t = \Delta\} \in \mathcal{F}_t,$$

we conclude that $\tau(\Gamma)$ is a stopping time. The proof of (i) is complete.

(ii). Because $n \mapsto \tau(\mathbb{B}\backslash\Gamma_n)$ is increasing, $\tau(\mathbb{B}\backslash\Gamma_n) \nearrow \tau \triangleq \sup_{n\in\mathbb{N}} \tau(\mathbb{B}\backslash\Gamma_n)$ as $n \to \infty$. Because $\tau \le \tau_\Delta$, it suffices to show that $\tau \ge \tau_\Delta$. For contradiction, suppose that there exists an $\omega \in \{\tau < \tau_\Delta\}$ and set $\omega' \triangleq \omega(\cdot \wedge \tau(\omega)) \in D(\mathbb{R}_+, \mathbb{B})$. Then,

$$\tau(\mathbb{B}\backslash\Gamma_n)(\omega') = \inf\big(t \in \mathbb{R}_+ : \omega'(t) \notin \Gamma_n \text{ or } \omega'(t-) \notin \Gamma_n\big) \nearrow \infty \text{ as } n \to \infty.$$

Because $\tau(\mathbb{B}\backslash\Gamma_n)$ is an $\mathbf{F}$-stopping time by (i), so is τ and Galmarino's test (see [6, Lemma III.2.43]) implies that $\tau(\omega) = \tau(\omega') = \infty$. This is a contradiction and $\tau = \tau_\Delta$ follows. The proof of (ii) is complete. □

3 Modifications, Corrections and Comments on Proofs

3.1 [2, Lemma 4.3]

The last conclusion in [2, Lemma 4.3] is empty: In the setting of [2], it cannot happen that $X_{\tau_\Delta -} = \Delta$.

3.2 [2, Lemmata 4.3, 4.5]

Due to the initial value and the possibility that X has no left limit at τ_n, some bounds in the proofs of [2, Lemmata 4.3, 4.5] are only valid on the open stochastic interval $]\!]0, \tau_n[\![$. Because singletons have Lebesgue measure zero, the arguments require no further changes.

The last conclusion in the proof of [2, Lemma 4.5] follows from the dominated convergence theorem.

3.3 [2, Proposition 4.8]

In the proof, it has been used that $\rho(A^*) \neq \emptyset$, see [8, Lemma 4.1]. Because $\mathbb{B}$ is separable and reflexive, its dual $\mathbb{B}^*$ is separable and D in the proof of [2, Proposition 4.8] can be constructed more directly: The assumption $\rho(A) \neq \emptyset$ implies that $\rho(A^*) \neq \emptyset$, see [7, Theorem 5.30, p. 169]. Let $D' \subset \mathbb{B}^*$ be a countable dense subset of $\mathbb{B}^*$ and take $\lambda \in \rho(A^*)$. Now, set $R(\lambda, A^*) \triangleq (\lambda - A^*)^{-1}$ and define $D \triangleq \{R(\lambda, A^*)x : x \in D'\} \subseteq D(A^*)$. We claim that for each $x \in D(A^*)$ there exists a sequence $(x_n)_{n\in\mathbb{N}} \subset D$ such that $x_n \to x$ and $A^*x_n \to A^*x$ in the operator norm as $n \to \infty$. To see this, take $x \in D(A^*)$ and set $y \triangleq \lambda x + A^*x$. There exists a sequence $(y_n)_{n\in\mathbb{N}} \subset D'$ such that $y_n \to y$ as $n \to \infty$. Finally, set $x_n \triangleq R(\lambda, A^*)y_n \in D$. Because $R(\lambda, A^*) \in L(\mathbb{B}^*, \mathbb{B}^*)$, we have $x_n \to R(\lambda, A^*)y = x$ as $n \to \infty$. Moreover, the triangle inequality yields that

$$\|A^*x_n - A^*x\| \leq \|y_n - y\| + |\lambda|\|x_n - x\| \to 0 \text{ as } n \to \infty.$$

The claim is shown.

3.4 [2, Lemma 4.10]

Due to Lemma 1, it is not necessary to pass to $D(\mathbb{R}_+, \mathbb{B}_\Delta)$. Moreover, it can be seen more easily that Φ is Borel. Indeed, Φ is continuous.

3.5 [2, Lemma 4.11]

In the proof of P-a.s.

$$E^P\big[\big(M^f_{t\wedge\tau_n} - M^f_{s\wedge\tau_n}\big) \circ \theta_\xi \mathbf{1}_{\{\xi<\tau_\Delta\}}\big|\mathcal{F}_{s+\xi}\big] = 0,$$

the variable n is used twice, which results in a conflict of notation. We correct the argument: Note that $\tau_{n+k} \circ \theta_\xi + \xi \leq \tau_{2(n+k)}$ on $\{\xi < \tau_{n+k}\}$ for all $k \in \mathbb{N}$. Set $\sigma_r \triangleq r \wedge \tau_n \circ \theta_\xi + \xi$. We obtain that P-a.s.

$$\begin{aligned} &E^P\big[\big(M^f_{t\wedge\tau_n} - M^f_{s\wedge\tau_n}\big) \circ \theta_\xi \mathbf{1}_{\{\xi<\tau_\Delta\}}\big|\mathcal{F}_{s+\xi}\big] \\ &= \lim_{k\to\infty} E^P\big[\big(M^f_{\sigma_t} - M^f_{\sigma_s}\big)\mathbf{1}_{\{\xi<\tau_{n+k}\}}\big|\mathcal{F}_{s+\xi}\big] \\ &= \lim_{k\to\infty} E^P\big[\big(M^f_{\sigma_t\wedge\tau_{2(n+k)}} - M^f_{\sigma_s\wedge\tau_{2(n+k)}}\big)\mathbf{1}_{\{\xi<\tau_{n+k}\}}\big|\mathcal{F}_{s+\xi}\big] \\ &= \lim_{k\to\infty} \big(M^f_{\sigma_t\wedge\tau_{2(n+k)}\wedge(s+\xi)} - M^f_{\sigma_s\wedge\tau_{2(n+k)}\wedge(s+\xi)}\big)\mathbf{1}_{\{\xi<\tau_{n+k}\}} = 0, \end{aligned}$$

by the optional stopping theorem.

3.6 [2, Section 4.3.2]

Because X has no left limit at τ_Δ, the random measure μ^X cannot be defined as in [2, Eq. 4.20]. We pass to a stopped version: Let $\widehat{X}$ be defined as in Eq. 4.11 in [2] and set $X^n \triangleq \widehat{X}_{\cdot\wedge\tau_n}$ and

$$\begin{aligned} \mu^n(\omega; dt, dx) &\triangleq \sum_{s>0} \mathbf{1}_{\{\Delta X^n_s(\omega)\neq 0\}} \varepsilon_{(s,\Delta X^n_s(\omega))}(dt, dx), \\ \nu^n(\omega; dt, dx) &\triangleq \mathbf{1}_{\{t\leq\tau_n(\omega)\}} K(X^n_t(\omega), dx)dt. \end{aligned}$$

We have the following version of [2, Lemmata 4.17, 4.18, 4.19]:

Lemma 4 *For all $n \in \mathbb{N}$ the random measure μ^n is $(\mathbf{F}^P, P)$-optional with $\mathscr{P}^P$-σ-finite Doléans measure and $(\mathbf{F}^P, P)$-predictable compensator ν^n.*

Because the proofs of [2, Lemmata 4.17, 4.18] contain typos and the proof of [2, Lemma 4.19] requires some minor modification, as the set $\mathcal{Z}_1 \times \mathcal{Z}_2$ has not all claimed properties, we give a proof:

Proof Due to [3, Theorem IV.88B, Remark below], the set $\{\Delta X^n \neq 0\}$ is $\mathbf{F}^P$-thin. Hence, [6, II.1.15] yields that μ^n is $\mathbf{F}^P$-optional. It follows as in [9, Example 2, pp. 160] that $M^P_{\mu^n}$ is $\mathscr{P}^P$-σ-finite. Next, we show that ν^n is $\mathbf{F}^P$-predictable with $\mathscr{P}^P$-σ-finite Doléons measure $M^P_{\nu^n}$. For $m \in \mathbb{N}$ we set $G_m \triangleq \{x \in \mathbb{B}\colon \|x\| \geq \frac{1}{m}\} \cup \{0\}$. Let W be a nonnegative $\mathscr{P}^P \otimes \mathscr{B}(\mathbb{B})$-measurable function which is bounded by a constant $c > 0$. Because P-a.s.

$$W\mathbf{1}_{[\![0,\tau_m]\!]}\mathbf{1}_{G_m}\star\nu^n_\infty \leq cm \sup_{\|x\|\leq m} K(x, \{z \in \mathbb{B}\colon \|z\| \geq \tfrac{1}{m}\}) < \infty,$$

we conclude that $M^P_{\nu^n}$ is $\mathscr{P}^P$-σ-finite. Furthermore, the process

$$W\star\nu^n = \lim_{m\to\infty} W\mathbf{1}_{[\![0,\tau_m]\!]}\mathbf{1}_{G_m}\star\nu^n$$

is $\mathbf{F}^P$-predictable as the pointwise limit of an $\mathbf{F}^P$-predictable process. We conclude that ν^n is an $\mathbf{F}^P$-predictable random measure.

It remains to show that ν^n is the $(\mathbf{F}^P, P)$-predictable compensator of μ^n. Let $\mathcal{Z}_1$ be the collection of sets $A \times \{0\}$ for $A \in \mathcal{F}^P_0$ and $[\![0, \xi]\!]$ for all $\mathbf{F}^P$-stopping times ξ, and let $\mathcal{Z}_2$ be the collection of all sets

$$G \triangleq \{x \in \mathbb{B}\colon (\langle x, y^*_1\rangle, \dots, \langle x, y^*_d\rangle) \in A\} \in \mathcal{B}(\mathbb{B}), \tag{3.1}$$

for $A \in \mathcal{B}(\mathbb{R}^d)$, $y^*_1, \dots, y^*_d \in D(A^*)$ and $d \in \mathbb{N}$. Note that $M^P_{\mu^n}(A \times \{0\} \times G) = M^P_{\nu^n}(A \times \{0\} \times G) = 0$ for all $A \in \mathcal{F}^P_0$ and $G \in \mathcal{B}(\mathbb{B})$. Fix an $\mathbf{F}^P$-stopping time ξ and the cylindrical set G given by (3.1). Denote $Y^n \triangleq (\langle X^n, y^*_1\rangle, \dots, \langle X^n, y^*_d\rangle)$. By [2, Lemma 4.7], we obtain

$$E^P\Big[\mathbf{1}_{[\![0,\xi]\!]\times G}\star\mu^n_\infty\Big] = E^P\Big[\mathbf{1}_{[\![0,\xi]\!]\times A}\star\mu^{Y^n}_\infty\Big] = E^P\Big[\mathbf{1}_{[\![0,\xi]\!]\times G}\star\nu^n_\infty\Big],$$

which implies $M^P_{\mu^n} = M^P_{\nu^n}$ on $\mathcal{Z}_1 \times \mathcal{Z}_2$. Take a norming sequence $(x^*_m)_{m\in\mathbb{N}} \subset \mathbb{B}^*$ of unit vectors, see p. 522 in [5] for a definition, and note that

$$B_m \triangleq \big\{x \in \mathbb{B}\colon \|x\| > \tfrac{1}{m}\big\} = \bigcup_{k\in\mathbb{N}} \big\{x \in \mathbb{B}\colon |\langle x, x^*_k\rangle| > \tfrac{1}{m}\big\}.$$

For $m, k \in \mathbb{N}$ set

$$\gamma(m, k) \triangleq \inf(t \in \mathbb{R}_+\colon \mu^n([0, t] \times B_m) > k) \wedge m.$$

The dominated convergence theorem yields that

$$M^P_{\mu^n}((A \times B) \cap ([\![0, \gamma(m, k)]\!] \times B_m)) = M^P_{\nu^n}((A \times B) \cap ([\![0, \gamma(m, k)]\!] \times B_m))$$

for all $A \times B \in \mathcal{Z}_1 \times \mathcal{Z}_2$. Now, we conclude from the uniqueness theorem for measures that $M^P_{\mu^n} = M^P_{\nu^n}$ on the trace σ-field $(\mathscr{P}^P \otimes \mathcal{B}(\mathbb{B})) \cap ([\![0, \gamma(m,k)]\!] \times (B_m \cup \{0\}))$. Finally, taking $k, m \to \infty$ and using the monotone convergence theorem show that $M^P_{\mu^n} = M^P_{\nu^n}$ on $\mathscr{P}^P \otimes \mathcal{B}(\mathbb{B})$. The proof is complete. □

The candidate density process Z can be defined as in [2, Lemma 4.21] with μ^X and ν^X replaced by μ^n and ν^n.

3.7 [2, Lemmata 4.21, 4.22]

In the proofs, the process X should be replaced by $\widehat{X}$.

3.8 [2, Proposition 3.7]

The representation of the CMG densities and the function V^k in [2, Lemma 4.23] should be multiplied by $\mathbf{1}_{\{\tau_n < \tau_\Delta\}}$. Moreover, in all Lebesgue integrals X_- should be replaced by X.

3.9 [2, Lemma 3.16]

Instead of the Yamada–Watanabe argument, the uniqueness also follows from the observation that for a pseudo-contraction semigroup $(S_t)_{t\geq 0}$ and a square integrable Lévy process L the law of $\int_0^\cdot S_{\cdot - s} dL_s$ is completely determined by L. This can be seen with the approximation argument used in the proof of [11, Theorem 9.20].

4 Final Comment

Above [2, Proposition 3.9] it is noted that "in a non-conservative setting, one can try to conclude existence from an extension argument in a larger path space, [but] in this case one has to prove that the extension is supported on $(\Omega, \mathcal{F})$" as defined in [2]. The larger path space, to which this comment refers, is the path space defined in this correction note. In our modified setting, it follows from Parthasarathy's extension theorem (see [10]) that under the assumptions imposed in [2] the GMP $(A, b', a, K', \eta, \tau_\Delta -)$ has a solution whenever the GMP $(A, b, a, K, \eta, \tau_\Delta -)$ has a solution. This observation extends [2, Theorem 3.6].

References

1. Biagini, S., Bouchard, B., Kardaras, C., Nutz, M.: Robust fundamental theorem for continuous processes. Math. Finance **27**(4), 963–987 (2017)
2. Criens, D.: Cylindrical martingale problems associated with Lévy generators. J. Theor. Probab. **32**(3), 1306–1359 (2019)
3. Dellacherie, C., Meyer, P.: Probabilities and Potential. North-Holland, Amsterdam (1978)
4. Ethier, S., Kurtz, T.: Markov Processes: Characterization and Convergence. Wiley, Hoboken (2005)

5. Hytönen, T., van Neerven, J., Veraar, M., Weis, L.: Analysis in Banach Spaces: Volume I: Martingales and Littlewood–Paley Theory. Springer, Berlin (2016)
6. Jacod, J., Shiryaev, A.: Limit Theorems for Stochastic Processes, 2nd edn. Springer, Berlin (2003)
7. Kato, T.: Perturbation Theory for Linear Operators, 2nd edn. Springer, Berlin (1980)
8. Kunze, M.: On a class of martingale problems on Banach spaces. Electron. J. Probab. **18**(104), 1–30 (2013)
9. Liptser, R., Shiryaev, A.N.: Theory of Martingales. Springer, Berlin (1989)
10. Parthasarathy, K.: Probability Measures on Metric Spaces. Academic Press, New York (1967)
11. Peszat, S., Zabczyk, J.: Stochastic Partial Differential Equations with Lévy Noise: An Evolution Equation Approach. Cambridge University Press, Cambridge (2007)