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Correction to: "On Operators Derived from Extensions of the Fourier Transform"

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1. Introduction

In this note we make a number of comments which correct various mistatements in the paper [1] concerning the operator defined by

$$F_\sigma^{(\nu)}(f)(x)=(2\pi)^{-\frac{1}{2}}|x|^{\nu+\sigma}\int_{\to-\infty}^{\to\infty}|t|^\nu e^{ixt}f(t)\,dt,$$

whenever the integral exists for almost all x.

Our first proof of the boundedness of $F_\sigma^{(\nu)}$ from L^p into L^q, $0<1/p\leqq 1$, $1/q=1-(1/p)-\sigma>0$, $q\geqq p$, $q>1$, was incomplete. Hence the validity of the results for all ν in the range $(1/p)-1<\nu<(1/p)$ is incorrect except when $F_\sigma^{(\nu)}$ is restricted to certain subsets of L^p.

A full discussion of these conclusions is given in [2].

We state below the results which we have so far proved concerning cases in which $F_\sigma^{(\nu)}$ is bounded on L^p.

2. The Main Theorems Involving Boundedness of $F_\sigma^{(\nu)}$

(2.1) (Theorem 6.1 and Note 6.1.1 of [2].) *Let*

$$1<p\leqq 2,\qquad 1/q=1-(1/p)-\sigma>0,\qquad q\geqq 2,$$
$$(1/p)-1<\nu\leqq\min(0,-\sigma).$$

Then $F_\sigma^{(\nu)}$ can be extended to a bounded operator on L^p, there is a finite constant $k(p,\nu,\sigma)\equiv k$ such that

$$\|F_\sigma^{(\nu)}(f)\|_q\leqq k\,\|f\|_p,\qquad (f\in L^p),$$

and

$$\text{(2.1.1)}\qquad \lim_{a,b\to\infty}\int_{-\infty}^{\infty}\left|F_\sigma^{(\nu)}(f)(x)-(2\pi)^{-\frac{1}{2}}|x|^{\nu+\sigma}\int_{-a}^{b}|t|^\nu e^{ixt}f(t)\,dt\right|^q dx=0.$$

Also for g in $L^{q'}$, *we have*

$$\|F_{-\sigma}^{(\nu+\sigma)}(g)\|_{p'}\leqq k\,\|g\|_{q'}.$$

Further, for f in L^p and g in $L^{q'}$,

(2.1.2) $$\int_{-\infty}^{\infty} g(x) F_\sigma^{(\nu)}(f)(x)\,dx = \int_{-\infty}^{\infty} f(t) F_{-\sigma}^{(\nu+\sigma)}(g)(t)\,dt,$$

(2.1.3) $$F_\sigma^{(\nu)}(f)(x) = (2\pi)^{-\frac{1}{2}} |x|^{\nu+\sigma} (d/dx) \int_{-\infty}^{\infty} |t|^\nu (e^{ixt}-1)(it)^{-1} f(t)\,dt,$$

(2.1.4) $$f(x) = (2\pi)^{-\frac{1}{2}} |x|^{-\nu} (d/dx) \int_{-\infty}^{\infty} |t|^{-\nu-\sigma} (e^{-ixt}-1)(-it)^{-1} F_\sigma^{(\nu)}(f)(t)\,dt,$$

for almost all x.

(2.2) (Theorem 7.6 of [2].) *Suppose that either of the following sets of conditions hold:*

(i) $1<p<\infty$, $1/q = 1-(1/p)-\sigma$, $\max(2,p) \leqq q < 2p$, $(1/p)-1<\nu \leqq (2/q)-1$,

(ii) $1<q<\infty$, $1/q = 1-(1/p)-\sigma$, $\max(\frac{1}{2}, 1/q) \leqq 1/p < \frac{1}{2}(1+1/q)$, $(1/p)-1<\nu \leqq (2/q)-1+\sigma$

then the same conclusions as in (2.1) *hold.*

(2.2.1) *Note.* The conditions (ii) of (2.2) are not given explicitly in [2]. They may easily be obtained by writing

$$p \text{ for } q', \qquad q \text{ for } p', \qquad -\sigma \text{ for } \sigma, \quad \text{and} \quad \nu \text{ for } \nu+\sigma$$

in the conjugate inequality involving $F_{-\sigma}^{(\nu+\sigma)}$.

(2.3) (Theorems 8.1 and 8.2 of [2].)

(i) *Let* $1<p<\infty$, $\tau = 1-(2/p)$, $(1/p)-1<\nu \leqq \min(0,(2/p)-1)$. *Then the conclusions of* (2.1) *hold with* $\sigma=\tau$, $q=p$.

(ii) *Let* $f \in L^p$, $1<p<\infty$, $\tau = 1-(2/p)$, $\max(0,(2/p)-1) \leqq \nu < 1/p$, *and suppose that* $F_\tau^{(\nu)}(f) \in L^p$. *Then there is a constant* $k(p,\nu) \equiv k$ *such that*

$$\|f\|_p \leqq k \|F_\tau^{(\nu)}(f)\|_p.$$

(2.4) *Remark.* We have shown in [2] by considering the example

$$f(t) = t^{-\nu}(1-t)^{-\nu}, \quad (0<t<1); \qquad f(t)=0, \quad t \notin (0,1),$$

that $F_\tau^{(\nu)}$ is not bounded in L^p for $1/(2p) < \nu < (1/p)$. Further details of results so far proved can be seen in [2] and [4].

3. The Other Results of [1]

The conclusions given in Section 5 and 6 remain valid as they are, although the proofs given there need modification since $F_\tau^{(\nu)}$ is not bounded in L^p for all ν in $((1/p)-1, (1/p))$. Alternative proofs of those in Section 5 are contained in [3], and the conclusions of Section 6 may be justified similarly.

See remarks (9) of [2].

References

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3. – On certain extensions of the Hilbert operator. Math. Ann. **169**, 315–327 (1967).
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