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Author
Simard, Richard

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Richard Simard and Mahiko Suzuki

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CORRECTIONS TO THE GELL-MANN-OKUBO FORMULA DUE TO SECOND ORDER SU(4) BREAKING

Richard Simard* and Mahiko Suzuki†

Lawrence Berkeley Laboratory and Department of Physics
University of California, Berkeley, California 94720

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Abstract: It is shown by a current algebra technique that second order SU(4) breaking effects shift the charmed vector meson masses upwards from the values predicted by the quadratic Gell-Mann-Okubo formula. The magnitude of the shift is estimated in a simple quark model of meson coupling constants.

1. Introduction

If the narrow resonance $\psi(3.1)$ (1-3) fits in a hexadecimet $(1 \oplus 15)$ of SU(4) together with ρ, ω, ϕ, and K^*, the Gell-Mann-Okubo formula predicts that nonstrange charmed mesons (D^0, D^-, D^+, and $\bar{D}^0$) and strange charmed mesons (F^- and F^+) should be found around 2.2 ~ 2.3 GeV, provided that $\underset{\sim}{1}$ and $\underset{\sim}{15}$ are degenerate in the symmetry limit. The huge mass difference between ψ and (ρ, ω, ϕ, and K^*), indicates, however, that SU(4) is so badly broken that the G-M-O formula derived to the first order breaking is likely to be subject to large corrections due to higher order SU(4) breaking.

In the e^+e^- annihilation experiment at SPEAR (4), a marked threshold behavior has been observed in the ratio R over

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$\sqrt{s}$ = 3.8 ~ 4.0 GeV. This behavior could be interpreted as pair production of charmed mesons whose masses are 1.9 ~ 2.0 GeV, lower than the predicted values of the charmed vector mesons. The pair-produced mesons may be pseudoscalar mesons. But there has so far been no evidence for a charmonium of positive charge conjugation below the second $\psi(3.7)$ in a measurement of monochromatic γ rays from $\psi(3.7)$.*

In the present paper we examine whether or not second order SU(4) breaking effects can really lower the masses of the charmed vector mesons from the G-M-O values to ~1.9 GeV. We estimate with the help of current algebra at equal times the sign and the magnitude of second order SU(4) corrections to the G-M-O formulae. We find that it is far more likely for the charmed vector meson masses to be raised by the second order corrections than to be lowered. A quantitative estimate is given for the magnitude of the shift, assuming a simple quark model or a spin-independent coupling model of three-body meson interactions. The same estimate is done for the pseudoscalar mesons, and a brief comment is made on the baryon multiplets.

2. Corrections to the Gell-Mann-Okubo formula

We use the equal-time algebra of the SU(4) vector charges. Assuming that the strong interaction Hamiltonian transforms like $\underset{\sim}{1} + c_8\lambda_8 + c_{15}\lambda_{15}$ under SU(4), one can derive independently of dynamical structure of H

$$\left[Q^{D+}(t), \frac{d}{dt} Q^{D+}(t)\right] = 0 \quad \text{and} \quad \left[Q^{F+}(t), \frac{d}{dt} Q^{F+}(t)\right] = 0 , \tag{2.1}$$

* An experiment currently undertaken by the Magnetic Detector Group of SLAC-LBL collaboration.

where

$$Q^{D+}(t) = \int d^3x \left(V_0^{(11)} + iV_0^{(12)}\right) \quad \text{and}$$

$$Q^{F+}(t) = \int d^3x \left(V_0^{(13)} + iV_0^{(14)}\right) \quad , \tag{2.2}$$

$$V_\mu^{(k)} = \frac{1}{2}\,\bar{q}\gamma_\mu\lambda_k q \qquad (k = 1,\cdots,15) \tag{2.3}$$

Taking the matrix element of the first relation in (2.1) between the one-particle vector-meson states $\langle D^+|$ and $|D^-\rangle$, and separating the one-particle intermediate states belonging to the same adjoint representation as ψ and D's, one obtains through the infinite momentum technique

$$\frac{1}{2}\left(m_\rho^{\ 2} + m_\omega^{\ 2}\right) + m_\psi^{\ 2} - 2m_D^{\ 2} = \int ds \frac{1}{s - m_D^{\ 2}} W^D(s) \quad , \tag{2.4a}$$

where

$$W^D(s) = \sum_{n\neq(\rho,\omega,\psi)} (2\pi)^3\, \delta^{(4)}(p + q - p_n)$$

$$\times \left\langle D^+(\underset{\sim}{p})|\partial^\mu V_\mu^{(D+)}|n\right\rangle \left\langle n|\partial^\nu V_\nu^{(D+)}|D^-(\underset{\sim}{p})\right\rangle\Big|_{q^2=0} \quad , \tag{2.5}$$

with the relativistic normalization

$$\langle \underset{\sim}{p}|\underset{\sim}{q}\rangle = \sqrt{4E_pE_q}\,(2\pi)^3\,\delta(\underset{\sim}{p} - \underset{\sim}{q})$$

Similarly from the second relation in (2.1), one obtains

$$m_\phi^{\ 2} + m_\psi^{\ 2} - 2m_F^{\ 2} = \int ds \frac{1}{s - m_F^{\ 2}} W^F(s) \tag{2.4b}$$

The integrals in (2.4a) and (2.4b), sometimes referred to as the leakage terms of current algebra, give the second order symmetry breaking corrections to the G-M-O formula. It should be remarked here that the SU(4) symmetric values have been used for $\langle M_i|Q^{(k)}|M_j\rangle$ when the one-particle intermediate states are singled out. Since the SU(4) breaking effects in $\langle M_i|Q^{(k)}|M_j\rangle$ are of second order, as proven by the Ademollo-Gatto theorem, (5) and they appear in product with $(m_i^{\ 2} - m_j^{\ 2})$ in (2.4), it is only the third order SU(4) breaking effects that have been ignored in (2.4a) and (2.4b).

We first examine eq. (2.4a). The integral probably converges very fast since the double charm exchange amplitude W^D is expected to diminish very rapidly in the Regge asymptotic region. It appears at first sight as if the integral might have large contribution from near $s = m_D^{\ 2}$, but this is not the case: $W^D(s)$ has a factor $(s - m_D^{\ 2})^{2\ell+2}$ for the $\underline{\ell}$th partial wave of (D^+D^-), and therefore the contribution of the intermediate states near $s = m_D^{\ 2}$ is, on the contrary, suppressed. The crucial problem is where the integral is to be cut off in the approximation of resonance saturation. If we include all components of each multiplet symmetrically, the SU(4) symmetric limit would be the only consistent solution. SU(4) breaking occurs when some components contribute more effectively than others in a single multiplet because of mass splitting and symmetry violation in coupling constants. In view of the fast convergence of the integral, we postulate here that low mass states dominate, thus leading to SU(4) breaking. Since W^D vanishes at $s = m_D^{\ 2}$, it is reasonable to assume that the integral in

(2.4a) should be saturated sufficiently well with the intermediate states of mass smaller than m_D. For the convenience of discussion on the sign of the integral, we split the integral into two parts according to intermediate states of positive and negative charge conjugation;

$$\int ds \frac{1}{s - m_D^{\,2}} W^D(s) \simeq \int ds \frac{1}{s - m_D^{\,2}} W^D(s)\, \theta(m_D^{\,2} - s)$$

$$= \int^{m_D^{\,2}} ds \frac{1}{s - m_D^{\,2}} W_+^D(s) - \int^{m_D^{\,2}} ds \frac{1}{s - m_D^{\,2}} W_-^D(s), \tag{2.6}$$

where

$$W_\pm^D(s) \equiv \pm \sum_{n \neq (\rho,\omega,\psi)} (2\pi)^3\, \delta^{(4)}(p + q - p_n)$$

$$\left\langle D^+ \right| \partial^\mu V_\mu^{(D+)} \left| n(C = \pm 1) \right\rangle \left\langle n(C = \pm 1) \right| \partial^\nu V_\nu^{(D+)} \left| D^- \right\rangle \Big|_{q^2=0} \quad . \tag{2.7}$$

Note that $W_\pm^D$ are both positive definite.

We saturate, as usual, the summation over intermediate states with resonances. The approximation of resonance saturation is probably acceptable since the double charm exchange amplitude is free not only from ordinary Regge poles, but also from the pomeron dual to the direct-channel nonresonant continuum. We tabulate the relevant resonances from the Particle Data Table (6):

$$\begin{aligned} &C = +1;\quad S(993),\quad A_1^{\,0}(1100),\quad f(1270),\quad A_2^{\,0}(1310)\cdots\ , \\ &C = -1;\quad B^0(1235),\quad \rho'^{\,0}(1600),\quad \omega'(1675),\quad g^0(1680)\cdots\ . \end{aligned} \tag{2.8}$$

Among these resonances, the f and A_2 are by far the most conspicuous resonances. As will be shown by the numerical estimate in the following section, the other resonances are less important, in particular, ρ', ω', and g are almost negligible. We thus draw a qualitative conclusion on the sign of the continuum integral,

$$\tfrac{1}{2}(m_\rho^{\,2} + m_\omega^{\,2}) + m_\psi^{\,2} - 2m_D^{\,2} < 0 \quad . \tag{2.9a}$$

The same argument leads us to

$$m_\phi^{\,2} + m_\psi^{\,2} - 2m_F^{\,2} < 0 \quad . \tag{2.9b}$$

We have to introduce a few dynamical assumptions to relate coupling constants when we estimate the numerical values of (2.9a) and (2.9b).

3. Duality in the Current Amplitude

Duality has been assumed in the preceding section between Regge singularities in t-channel and intermediate states in s-channel after the Born terms (ρ, ω, ψ) are subtracted out. In hadron-hadron scattering, however, duality holds between t-channel Regge singularities and s-channel intermediate states including Born terms. There is an important difference between hadron-hadron amplitudes and current amplitudes. We clarify this point here, since it is crucial to the conclusion of the present paper.

Let us consider the symmetry limit of the nonexotic current amplitude at $\underset{\sim}{q} = 0$ defined as

$$\lim_{\underset{\sim}{p}\to\infty} \int d^3x \left\langle D^+(\underset{\sim}{p}) | V^{(D+)}(x)\, V^{(D-)}(0) | D^+(\underset{\sim}{p}) \right\rangle \qquad (3.1)$$

Only the ρ, ω, and ψ states, which would be degenerate in mass, can contribute in the intermediate states. There is no continuum at all, and no Regge pole appears in t-channel. Although Regge poles are allowed by quantum numbers, their residues vanish in the symmetry limit, as demanded by the current conservation $\partial_\mu V^\mu = 0$. When a small symmetry breaking ε is introduced, the contribution of the Born terms decreases slightly by $O(\varepsilon^2)$ according to the Ademollo-Gatto theorem. Accordingly a continuum appears proportionally to $O(\varepsilon^2)$, and therefore the Regge residues are no longer equal to zero, but of the order of ε^2. This indicates that in the current amplitude the Regge singularities are dual to the continuum and a tiny portion $(\sim O(\varepsilon^2))$ of the Born terms. In our amplitude (2.5), the Born terms enter the sum with an extra SU(4) breaking factor $m_i^{\,2} - m_j^{\,2}$, though it may not be very small numerically, while the continuum is still of $O(\varepsilon^2)$. Therefore it is a consistent picture to relate t-channel Regge singularities only with the continuum in the duality relation.

4. A Numerical Estimate

We give an estimate of the contribution of each resonant state. The charm changing vector currents are replaced by the charmed vector meson fields through the vector dominance relation

$$V_\mu^{(k)} = m_k^{\ \alpha} f\, \mathcal{V}_\mu^{(k)} \quad , \qquad (k = 1,2,\cdots,15)\ , \qquad (4.1)$$

where $\mathcal{V}_\mu^{(k)}$ are the vector fields and α is some power, customarily chosen to be two. We follow this common practice of choosing $\alpha = 2$, but to obtain the best fit to the experimental value of $\Gamma(\psi \to \ell^+\ell^-)/\Gamma(\rho^0 \to \ell^+\ell^-)$ within the SU(4) charm model, $\alpha = 3/2$ is more appropriate. The next problem is to evaluate three-body meson coupling constants. Without knowing experimental values of D^+D^--meson coupling constants, we have to resort to simple dynamical models to relate them to those already measured experimentally. The simple rules that we use here are spin independence of meson couplings to relate vector coupling to pseudoscalar coupling, and SU(4) relations to relate the charm sector to the noncharm sector. The former rule may be stated as a "simple quark model", (7) often equivalent to SU(6) symmetry. We briefly explain for each meson how we estimate the D^+D^--meson coupling.

<u>fD^+D^-</u>: From the observed decay widths of $f \to \pi^+\pi^-$, $f \to K\bar{K}$, $A_2 \to K\bar{K}$, and $f' \to K\bar{K}$, we have found that the coupling constants defined through

$$H_{int} = (G/m^*)\, f^{\mu\nu} \phi^\dagger \partial_\mu \partial_\nu \phi \qquad (4.2)$$

satisfy the SU(3) relations most accurately. The parameter m^* is a common mass scale independent of the 2^+ meson mass and the pseudoscalar meson masses. This eliminates the annoying ambiguity in mass dependence of coupling constants in a broken symmetry. We assume the coupling of, say, $f\rho^+\rho^-$ to be of the form of

$$H_{int} = (G/m^*)\, f^{\mu\nu} \rho^{\dagger\lambda} \partial_\mu \partial_\nu \rho_\lambda \qquad (4.3)$$

with the same G as $f\pi^+\pi^-$. This specific form of coupling is a consequence of the assumption of spin independence. Then SU(4) leads us to

$$H_{int} = \tfrac{1}{2}(G/m^*)\, f^{\mu\nu} D^{\dagger\lambda} \partial_\mu \partial_\nu D_\lambda \quad . \qquad (4.4)$$

The $A_2{}^0 D^+ D^-$ coupling is obtained through SU(4). No extrapolation effect is taken into account from mass shell of D to a light-like point, in other words, the strong form of vector-meson dominance is assumed throughout our calculation. The same procedure is followed to obtain the coupling constants of D^+D^- with $S(0^+)$, $\rho'(1^-)$, $\omega'(3^-)$, and $g(3^-)$.

$\underline{A_1{}^0 D^+ D^-}$: The $A_1\rho\pi$ coupling is estimated from its width through the coupling

$$\begin{aligned} H_{int} &= m^* G\, A_1^\mu \rho_\mu \pi \quad , \\ &= m^* G\, \underset{\sim}{A_1} \cdot \underset{\sim}{\rho} \pi \qquad (\text{in the rest frame of } A_1). \end{aligned} \qquad (4.5)$$

The $A_1{}^0 D^+ D^-$ coupling is assumed to be of the form

$$\begin{aligned} H_{int} &= (m^*/m_{A1}) G'\, \varepsilon_{\mu\nu\lambda\kappa}\, \partial^\mu A_1{}^\nu D^{\dagger\lambda} D^\kappa \quad , \\ &= m^* G'\, \underset{\sim}{A_1} \cdot (\underset{\sim}{D}^\dagger \times \underset{\sim}{D}) \qquad (\text{in the rest frame of } A_1) \quad . \end{aligned} \qquad (4.6)$$

Then G and G' are related through the Clebsch-Gordan coefficient of SU(4), or equivalently by counting quark lines. The $B^0 D^+ D^-$ coupling constant is evaluated in the same way.

The numerical values for the resonance contribution to the integral (2.4a) are given in the table to show typical magnitudes. The coupling constants estimated above are admittedly crude and subject to large errors, but we hope that the figures given there are correct semiquantitatively. A remark is in order. The excited ψ states at 3.7 GeV and 4.1 GeV have been ignored here in line with our assumption on the resonance saturation. If $\psi(3.7)$ and $\psi(4.1)$ couple far more weakly with D^+D^- than $\psi(3.1)$ does, this is a good approximation. The similarity in the cascade decay between $\psi(3.7)$ and $\psi(3.1)$ with that between $\rho'(1600)$ and ρ may be considered as a support of this conjecture. Beyond this plausibility argument, however, no strong justification may be given for ignoring the excited ψ states. From the form of the integral one easily finds that the two ψ's, states of negative charge conjugation, contribute to the integral with a negative sign, just as states of positive charge conjugation below $\sqrt{s} = m_D$ do. Therefore, they would make more negative the number in the sum total appearing at the bottom of the table, in case that our assumption on the cutoff of the integral should fail.

The numerical estimate for the F^+ charge algebra is the same within small SU(3) breaking as that for the D^+ current in the present approximation.

5. Discussion

According to the preceding estimate, the G-M-O formula for the vector multiplet is modified as

$$\tfrac{1}{2}(m_\rho{}^2 + m_\omega{}^2) + m_\psi{}^2 - 2\, m_D{}^2 \simeq -0.59\ \text{GeV}^2 \quad , \qquad (5.1)$$

which shifts m_D upwards from 2.26 GeV (the first order G-M-O value) to 2.39 GeV. It appears that the second order SU(4) correction is surprisingly small, but the right-hand side of (5.1) in its absolute magnitude is one order of magnitude larger than the second order SU(3) correction to the G-M-O formula for the pseudoscalar mesons ($4\, m_K{}^2 - 3\, m_\eta{}^2 - m_\pi{}^2 \simeq 0.06\ \text{GeV}^2$). It may, therefore, not be regarded as unduly small. It is the large mass of D that makes the mass shift smaller. The large D mass has also been used to justify the fast convergence of the continuum integral. Considering our drastic approximation, we feel that the figures in the table may easily change by a factor of two or three, but the sign of the sum total may be fairly reliable.

The same calculation can be done for the pseudoscalar meson multiplet. Since no pseudoscalar $c\bar{c}$ has yet been found, a resulting relation tells only whether $2\,m_{D'}^{\,2} - m_X^{\,2}$ ($X = {}^1S_0$ of $c\bar{c}$) should turn out to be larger or smaller than what is predicted by the G-M-O. The relevant intermediate states are those of $0^-, 1^+, 2^-, \cdots$. Aside from an obscure resonance at 1640 MeV, seen only in the $(f\pi)$ mode, only $A_1(1100)$, $B(1235)$, and $E(1420)$ can enter the sum. They shift $2\,m_{D'}^{\,2} - m_X^{\,2}$ downwards from the G-M-O value if we carry out the same numerical analysis with the experimental decay widths for $A_1 \to \rho\pi$, $B \to \omega\pi$, and $E \to K\bar{K}^* + K^*\bar{K}$,

$$2\,m_{D'}^{\,2} - m_X^{\,2} - \frac{1}{2}\,m_\pi^{\,2} - \frac{1}{6}\,m_\eta^{\,2} - \frac{1}{3}\,m_{\eta'}^{\,2} = -0.39 + 0.12 - 0.53\,(\mathrm{GeV}^2), \tag{5.2}$$

where the three terms in the right-hand side come from A_1, B, and E, respectively. Possible excited states of X would contribute with a positive sign. However, it is probably safe to take the sign of the sum in (5.2) seriously.

Finally, a brief comment is made on the baryons. If we make the same approximation of ignoring the intermediate states above the noncharm resonance region as we did for the mesons, there will be no or little correction to the linear mass formula provided that the lowest charmed baryons are around 3 GeV or heavier. This results from a different structure of correction formulae for multiplets which are not self-charge-conjugate. It happens that the linear mass formula of

SU(3) fits the $\frac{1}{2}^+$ octet and the $\frac{3}{2}^+$ decimet perfectly within the electromagnetic mass splitting. The agreement of the first order G-M-O is far better for the baryons than for the mesons. Our dynamical assumption on resonance saturation in the present paper suggests that a similar situation will emerge for the charmed hadrons in SU(4), too. It is important and also interesting in its own right to investigate further whether the resonance saturation only in the noncharm region is really justifiable or not.

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Table 1. Contribution from each resonant intermediate state to the integral (2.5), which is equal to $\frac{1}{2}(m_\rho^2 + m_\omega^2) + m_\psi^2 - 2\, m_D^2$. The power α [(4.1) in the text] has been chosen to be two. For $\alpha = 3/2$, multiply the figures by 1/3 $(= m_\rho/m_D)$. Additional negative contributions are expected from $\varepsilon(700)$, D(1285), and $A_3(1640)$.

Resonance	J^P	C	$\int ds(s - m_D^2)^{-1}\, W^D(s)$ (in GeV^2)
S(993)	0^+	+	-0.09
$A_1(1100)$	1^+	+	-0.12
f(1270)	2^+	+	-0.25
$A_2(1310)$	2^+	+	-0.23
$\psi(3.7)$	1^-	-	-?
$\psi(4.1)$	1^-	-	-?
B(1235)	1^+	-	+0.04
$\rho'(1600)$	1^-	-	~0
$\omega'(1675)$	$3^-(?)$	-	+0.02
g(1680)	3^-	-	+0.02
Total			-0.59

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