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Correlation Coefficient Measures of Interval Bipolar Neutrosophic Sets for Solving Multi-Attribute Decision Making Problems

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Abstract. Interval bipolar neutrosophic set is a significant extension of interval neutrosophic set where every element of the set comprises of three independent positive membership functions and three independent negative membership functions. In this study, we first define correlation coefficient, and weighted correlation coefficient measures of interval bipolar neutrosophic sets and prove their basic properties. Then, we develop a new multi-attribute decision making strategy based on the proposed weighted correlation coefficient measure. Finally, we solve an investment problem with interval bipolar neutrosophic information and comparison is given to demonstrate the applicability and effectiveness of the proposed strategy.

Keywords: Interval bipolar neutrosophic set, multi-attribute decision making, correlation coefficient measure.

1 Introduction

Correlation coefficient is an important decision making apparatus in statistics to evaluate the relation between two sets. In neutrosophic environment [1], Hanafy et al. [2] derived a formula for correlation coefficient between two neutrosophic sets (NSs). Hanafy et al. [3] obtained the correlation coefficient of NSs by using centroid strategy which lies in [-1, 1]. The correlation coefficient obtained from [3] provides the information about the degree of the relationship between two NSs and also informs us whether the NSs are positive or negatively related. In 2013, Ye [4] defined correlation, correlation coefficient, weighted correlation coefficient in single valued neutrosophic set (SVNS) [5] environment and established a multi-criteria decision making (MCDM) based on the proposed weighted correlation coefficient measure. Broumi and Smarandache [6] introduced the concept of correlation coefficient and weighted correlation coefficient between two interval neutrosophic sets (INSs) [7] and established some of their basic properties. Hanafy et al. [8] studied the notion of correlation and correlation coefficient of neutrosophic data under probability spaces. Ye [9] suggested an improved correlation coefficient between two SVNSs in order to overcome the drawbacks of the correlation coefficient discussed in [4] and investigated its properties. In the same study, Ye [9] extended the concept of correlation coefficient measure of SVNS to correlation coefficient measure of INS environment. Furthermore, Ye [9] developed strategies for solving multi-attribute decision making (MADM) problems with single valued neutrosophic and interval neutrosophic environments based on the proposed correlation coefficient measures. Broumi and Deli [10] defined correlation measure of two neutrosophic refined (multi) sets [11] by extending the correlation measure of two intuitionistic fuzzy multi-sets proposed by Rajarajeswari and Uma [12] and proved some of its basis properties. Zhang et al. [13] defined an improved weighted correlation coefficient on the basis of integrated weight for INSs and a decision making strategy is developed. Karaaslan [14] proposed a strategy to compute correlation coefficient between possibility neutrosophic soft sets and presented several properties related to the proposed strategy. Karaaslan [15] defined a new mathematical structure called single-valued neutrosophic refined soft sets (SNRSSs) and presented its set theoretical operations such as union, intersection and complement and proved some of their basic properties. In the same study [15], two formulas to determine correlation coefficient between two SNRSSs are proposed and the developed strategy is used to solve a clustering analysis problem. Şahin and Liu [16] defined single valued neutrosophic hesitant fuzzy sets (SVNHFSs) and established some basic properties and finally proposed a decision making strategy. Liu and Luo [17] defined correlation coefficient and weighted correlation coefficient for interval-valued neutrosophic hesitant fuzzy sets (INHFSs) due to Liu and Shi [18] and studied their properties. Then, Liu and Luo [17] developed a MADM strategy within the framework of INHFSs based on weighted correlation coefficient. Ye [19] suggested a dynamic single valued neutrosophic multiset (DSVNM) based on dynamic information obtained from different time intervals in several practical situations in order to express dynamical data and operational relations of DSVNMs. In the same study [19], correlation coefficient and weighted correlation coefficient measures between DSVNMs are proposed and a MADM strategy is developed on the basis of the proposed weighted correlation coefficient under DSVNM setting. Recently, Ye [20] proposed two correlation coefficient between normal neutrosophic sets (NNSs) based on the score functions of normal neutrosophic numbers and investigated their essential properties. In the same study, Ye [20] formulated a MADM strategy by employing correlation coefficient of NNSs in normal neutrosophic environment. Pramanik et al. [21] defined correlation coefficient and weighted correlation coefficient between two rough neutrosophic sets and proved their basic properties. In the same study, Pramanik et al. [21] developed a multi-criteria decision making strategy based on the proposed correlation coefficient measure and solved an illustrative example in medical diagnosis.

In 2015, Deli et al. [22] introduced a novel concept called bipolar neutrosophic sets (BNSs) by generalizing the concepts of bipolar fuzzy sets [23, 24] and bipolar intuitionistic fuzzy sets [25]. In the same study, Deli et al. [22] defined score, accuracy and certainty functions to compare BNSs and formulated a MCDM approach based on the score, accuracy and certainty functions and bipolar neutrosophic weighted average operator (A_w) and bipolar neutrosophic weighted geometric operator (G_w). In bipolar neutrosophic environment, Dey et al. [26] developed a MADM approach based on technique for order of preference by similarity to ideal solution (TOPSIS) strategy. Deli and Subas [27] and Şahin et al. [28] developed MCDM strategies based on correlation coefficient and Jaccard similarity measures, respectively in BNS environment. Uluçay et al. [29] defined Dice, weighted Dice similarity measures, hybrid and weighted hybrid similarity measures for MCDM problems with bipolar neutrosophic information. Pramanik et al. [30] defined projection, bidirectional projection and hybrid projection measures between BNSs and proved their basic properties and then, three new MADM models are developed based on proposed measures.

Mahmood et al. [31] and Deli et al. [32] incorporated the notion of interval bipolar neutrosophic sets (IBNSs) and defined some operations and operators for IBNSs. Recently, Pramanik et al. [33] defined new cross entropy and weighted cross entropy measures in BNS and IBNS environment and discussed some of their essential properties. In the same study, Pramanik et al. [33] developed two novel MADM strategies on the basis of the proposed weighted cross entropy measures.

Research gap:

MADM strategy based on correlation coefficient under IBNSs environment.

This paper answers the following research questions:

i. Is it possible to introduce a novel correlation coefficient measure for IBNSs?
ii. Is it possible to introduce a novel weighted correlation coefficient measure for IBNSs?
iii. Is it feasible to formulate a novel MADM strategy based on the proposed correlation coefficient measure in IBNS environment?
iv. Is it feasible to formulate a novel MADM strategy based on the proposed weighted correlation coefficient measure in IBNS environment?

Motivation:

The aforementioned analysis presents the motivation behind developing correlation coefficient -based strategy for handling MADM problems with IBNS information.

The objectives of the paper are as follows:

1. To define a new correlation coefficient measure and a new weighted correlation coefficient measure in IBNS environment and prove their basic properties.
2. To develop a new MADM strategy based on weighted correlation coefficient measure in IBNS environment.

In order to fill the research gap, we propose correlation coefficient-based MADM strategy in IBNS environment.

Rest of the article is organized as follows. Section 2 provides the preliminaries of bipolar fuzzy sets, bipolar intuitionistic fuzzy sets, BNSs and IBNSs. Section 3 defines the correlation coefficient and weighted correlation coefficient measures in IBNS environment and establishes their basic properties. In section 4, a new MADM strategy based on the proposed weighted correlation coefficient measure is developed. In section 5, we solve a numerical example and comparison analysis is given. Finally, in the last section, conclusions are presented.

2 Preliminaries

2.1 Bipolar fuzzy sets

A bipolar fuzzy set [23, 24] B in X is characterized by a positive membership function $\alpha_B^+(x)$ and a negative membership function $\alpha_B^-(x)$. A bipolar fuzzy set B is expressed in the following way.

$B = \{x, \langle \alpha_B^+(x), \alpha_B^-(x) \rangle \mid x \in X\}$

where $\alpha_B^+(x)$: $X \to [0, 1]$ and $\alpha_B^-(x)$: $X \to [-1, 0]$ for each point $x \in X$.

2.2 Bipolar intuitionistic fuzzy sets

Consider X be a non-empty set, then a BIFS [25] E is expressed in the following way.

$E = \{x, \langle \alpha_E^+(x), \alpha_E^-(x), \beta_E^+(x), \beta_E^-(x) \rangle \mid x \in X\}$

where $\alpha_E^+(x), \beta_E^+(x)$: $X \to [0, 1]$ and $\alpha_E^-(x), \beta_E^-(x)$: $X \to [-1, 0]$ for each point $x \in X$ such that $0 \le \alpha_E^+(x) + \beta_E^+(x) \le 1$ and $-1 \le \alpha_E^-(x) + \beta_E^-(x) \le 0$.

2.3 Bipolar neutrosophic sets

A BNS [22] M in X is presented as follows:

$M = \{x, \langle \alpha_M^+(x), \beta_M^+(x), \gamma_M^+(x), \alpha_M^-(x), \beta_M^-(x), \gamma_M^-(x) \rangle \mid x \in X\}$

where $\alpha_M^+(x)$, $\beta_M^+(x)$, $\gamma_M^+(x)$: $X \to [0, 1]$ and $\alpha_M^-(x)$, $\beta_M^-(x)$, $\gamma_M^-(x)$: $X \to [-1, 0]$. The positive membership degrees $\alpha_M^+(x)$, $\beta_M^+(x)$, $\gamma_M^+(x)$ denote the truth membership, indeterminate membership, and false membership functions of an object $x \in X$ corresponding to a BNS M and the negative membership degrees $\alpha_M^-(x)$, $\beta_M^-(x)$, $\gamma_M^-(x)$ denote the truth membership, indeterminate membership, and false membership of an object $x \in X$ to several implicit counter property associated with a BNS M.

Definition 2.3.1

Let, $M_1 = \{x, \langle \alpha_{M_1}^+(x), \beta_{M_1}^+(x), \gamma_{M_1}^+(x), \alpha_{M_1}^-(x), \beta_{M_1}^-(x), \gamma_{M_1}^-(x) \rangle \mid x \in X\}$ and $M_2 = \{x, \langle \alpha_{M_2}^+(x), \beta_{M_2}^+(x), \gamma_{M_2}^+(x), \alpha_{M_2}^-(x), \beta_{M_2}^-(x), \gamma_{M_2}^-(x) \rangle \mid x \in X\}$ be any two BNSs. Then, a BNS M_1 is contained in another BNS M_2, represented by $M_1 \subseteq M_2$ if and only if

$\alpha_{M_1}^+(x) \le \alpha_{M_2}^+(x)$, $\beta_{M_1}^+(x) \ge \beta_{M_2}^+(x)$, $\gamma_{M_1}^+(x) \ge \gamma_{M_2}^+(x)$;

$\alpha_{M_1}^-(x) \ge \alpha_{M_2}^-(x)$, $\beta_{M_1}^-(x) \le \beta_{M_2}^-(x)$, $\gamma_{M_1}^-(x) \le \gamma_{M_2}^-(x)$ for all $x \in X$.

Definition 2.3.2

Let, $M_1 = \{x, \langle \alpha_{M_1}^+(x), \beta_{M_1}^+(x), \gamma_{M_1}^+(x), \alpha_{M_1}^-(x), \beta_{M_1}^-(x), \gamma_{M_1}^-(x) \rangle \mid x \in X\}$ and $M_2 = \{x, \langle \alpha_{M_2}^+(x), \beta_{M_2}^+(x), \gamma_{M_2}^+(x), \alpha_{M_2}^-(x), \beta_{M_2}^-(x), \gamma_{M_2}^-(x) \rangle \mid x \in X\}$ be any two BNSs [22] , then $M_1 = M_2$ if and only if

$\alpha_{M_1}^+(x) = \alpha_{M_2}^+(x)$, $\beta_{M_1}^+(x) = \beta_{M_2}^+(x)$, $\gamma_{M_1}^+(x) = \gamma_{M_2}^+(x)$,

$\alpha_{M_1}^-(x) = \alpha_{M_2}^-(x)$, $\beta_{M_1}^-(x) = \beta_{M_2}^-(x)$, $\gamma_{M_1}^-(x) = \gamma_{M_2}^-(x)$ for all $x \in X$.

Definition 2.3.3

The complement of a BNS [33] M is M^c == $\{x, \langle \alpha_{M^C}^+(x), \beta_{M^C}^+(x), \gamma_{M^C}^+(x), \alpha_{M^C}^-(x), \beta_{M^C}^-(x), \gamma_{M^C}^-(x) \rangle \mid x \in X\}$

where

$\alpha_{M^c}^+(x) = \gamma_M^+(x)$, $\beta_{M^c}^+(x) = 1 - \beta_M^+(x)$, $\gamma_{M^c}^+(x) = \alpha_M^+(x)$;

$\alpha_{M^c}^-(x) = \gamma_M^-(x)$, $\beta_{M^c}^-(x) = -1 - \beta_M^-(x)$, $\gamma_{M^c}^-(x) = \alpha_M^-(x)$.

Definition 2.3.4

The union [30]of two BNSs M_1 and M_2 represented by $M_1 \cup M_2$ is defined as follows:

$M_1 \cup M_2$ = {Max ($T_{M_1}^+(x)$, $T_{M_2}^+(x)$), Min ($I_{M_1}^+(x)$, $I_{M_2}^+(x)$), Min ($F_{M_1}^+(x)$, $F_{M_2}^+(x)$), Min ($T_{M_1}^-(x)$, $T_{M_2}^-(x)$), Max ($I_{M_1}^-(x)$, $I_{M_2}^-(x)$), Max ($F_{M_1}^-(x)$, $F_{M_2}^-(x)$)}, $\forall$ $x \in X$.

Definition 2.3.5

The intersection [30] of two BNSs M_1 and M_2 denoted by $M_1 \cap M_2$ is defined as follows:

$M_1 \cap M_2$ = {Min ($T_{M_1}^+(x)$, $T_{M_2}^+(x)$), Max ($I_{M_1}^+(x)$, $I_{M_2}^+(x)$), Max ($F_{M_1}^+(x)$, $F_{M_2}^+(x)$), Max ($T_{M_1}^-(x)$, $T_{M_2}^-(x)$), Min ($I_{M_1}^-(x)$, $I_{M_2}^-(x)$), Min ($F_{M_1}^-(x)$, $F_{M_2}^-(x)$)}, $\forall$ $x \in X$.

2.4 Interval bipolar neutrosophic sets

Consider X be the space of objects, then an IBNS [31, 32] L in X is is represented as follows:

$$L=\{x,\left\langle\begin{matrix}[\inf\alpha_L^+(x),\sup\alpha_L^+(x)],[\inf\beta_L^+(x),\sup\beta_L^+(x)],\\ [\inf\gamma_L^+(x),\sup\gamma_L^+(x)],[\inf\alpha_L^-(x),\sup\alpha_L^-(x)],\\ [\inf\beta_L^-(x),\sup\beta_L^-(x)],[\inf\gamma_L^-(x),\sup\gamma_L^-(x)]\end{matrix}\right\rangle \mid x\in X\}$$

where L is characterized by positive and negative truth-membership $\alpha_L^+(x)$, $\alpha_L^-(x)$; inderterminacy-membership $\beta_L^+(x)$, $\beta_L^-(x)$; falsity-membership $\gamma_L^+(x)$, $\gamma_L^-(x)$ functions respectively. Here, $\alpha_L^+(x)$, $\beta_L^+(x)$, $\gamma_L^+(x)\subseteq[0,1]$; $\alpha_L^-(x)$, $\beta_L^-(x)$, $\gamma_L^-(x)\subseteq[-1, 0]$ for all $x\in X$ with the conditions $0\le\sup\alpha_L^+(x)+\sup\beta_L^+(x)+\sup\gamma_L^+(x)\le 3$, and $-3\le\sup\alpha_L^-(x)+\sup\beta_L^-(x)+\sup\gamma_L^-(x)\le 0$.

Definition 2.4.1 : Let $L_1=\{x, <[\inf\alpha_{L_1}^+(x), \sup\alpha_{L_1}^+(x)]; [\inf\beta_{L_1}^+(x), \sup\beta_{L_1}^+(x)]; [\inf\gamma_{L_1}^+(x), \sup\gamma_{L_1}^+(x)]; [\inf\alpha_{L_1}^-(x), \sup\alpha_{L_1}^-(x)]; [\inf\beta_{L_1}^-(x), \sup\beta_{L_1}^-(x)]; [\inf\gamma_{L_1}^-(x), \sup\gamma_{L_1}^-(x)]>\mid x\in X\}$ and $L_2==\{x, <[\inf\alpha_{L_2}^+(x), \sup\alpha_{L_2}^+(x)]; [\inf\beta_{L_2}^+(x), \sup\beta_{L_2}^+(x)]; [\inf\gamma_{L_2}^+(x), \sup\gamma_{L_2}^+(x)]; [\inf\alpha_{L_2}^-(x), \sup\alpha_{L_2}^-(x)]; [\inf\beta_{L_2}^-(x), \sup\beta_{L_2}^-(x)]; [\inf\gamma_{L_2}^-(x), \sup\gamma_{L_2}^-(x)]>\mid x\in X\}$ be two IBNSs [31] . Then $L_1\subseteq L_2$ if and only if

$\inf\alpha_{L_1}^+(x)\le\inf\alpha_{L_2}^+(x)$, $\sup\alpha_{L_1}^+(x)\le\sup\alpha_{L_2}^+(x)$, $\inf\beta_{L_1}^+(x)\ge\inf\beta_{L_2}^+(x)$, $\sup\beta_{L_1}^+(x)\ge\sup\beta_{L_2}^+(x)$, $\inf\gamma_{L_1}^+(x)\ge\inf\gamma_{L_2}^+(x)$, $\sup\gamma_{L_1}^+(x)\ge\sup\gamma_{L_2}^+(x)$, $\inf\alpha_{L_1}^-(x)\ge\inf\alpha_{L_2}^-(x)$, $\sup\alpha_{L_1}^-(x)\ge\sup\alpha_{L_2}^-(x)$, $\inf\beta_{L_1}^-(x)\le\inf\beta_{L_2}^-(x)$, $\sup\beta_{L_1}^-(x)\le\sup\beta_{L_2}^-(x)$, $\inf\gamma_{L_1}^-(x)\le\inf\gamma_{L_2}^-(x)$, $\sup\gamma_{L_1}^-(x)\le\sup\gamma_{L_2}^-(x)$, for all $x\in X$.

Definition 2.4.2: Consider $L_1=\{x, <[\inf\alpha_{L_1}^+(x), \sup\alpha_{L_1}^+(x)]; [\inf\beta_{L_1}^+(x), \sup\beta_{L_1}^+(x)]; [\inf\gamma_{L_1}^+(x), \sup\gamma_{L_1}^+(x)]; [\inf\alpha_{L_1}^-(x), \sup\alpha_{L_1}^-(x)]; [\inf\beta_{L_1}^-(x), \sup\beta_{L_1}^-(x)]; [\inf\gamma_{L_1}^-(x), \sup\gamma_{L_1}^-(x)]>\mid x\in X\}$ and $L_2=\{x, <[\inf\alpha_{L_2}^+(x), \sup\alpha_{L_2}^+(x)]; [\inf\beta_{L_2}^+(x), \sup\beta_{L_2}^+(x)]; [\inf\gamma_{L_2}^+(x), \sup\gamma_{L_2}^+(x)]; [\inf\alpha_{L_2}^-(x), \sup\alpha_{L_2}^-(x)]; [\inf\beta_{L_2}^-(x), \sup\beta_{L_2}^-(x)]; [\inf\gamma_{L_2}^-(x), \sup\gamma_{L_2}^-(x)]>\mid x\in X\}$ be two IBNSs [31] . Then $L_1=L_2$ if and only if

$\inf\alpha_{L_1}^+(x)=\inf\alpha_{L_2}^+(x)$, $\sup\alpha_{L_1}^+(x)=\sup\alpha_{L_2}^+(x)$, $\inf\beta_{L_1}^+(x)=\inf\beta_{L_2}^+(x)$, $\sup\beta_{L_1}^+(x)=\sup\beta_{L_2}^+(x)$, $\inf\gamma_{L_1}^+(x)=\inf\gamma_{L_2}^+(x)$, $\sup\gamma_{L_1}^+(x)=\sup\gamma_{L_2}^+(x)$, $\inf\alpha_{L_1}^-(x)=\inf\alpha_{L_2}^-(x)$, $\sup\alpha_{L_1}^-(x)=\sup\alpha_{L_2}^-(x)$, $\inf\beta_{L_1}^-(x)=\inf\beta_{L_2}^-(x)$, $\sup\beta_{L_1}^-(x)=\sup\beta_{L_2}^-(x)$, $\inf\gamma_{L_1}^-(x)=\inf\gamma_{L_2}^-(x)$, $\sup\gamma_{L_1}^-(x)=\sup\gamma_{L_2}^-(x)$, for all $x\in X$.

Definition 2.4.3: The complement [33]of $L=\{x, <[\inf\alpha_L^+(x), \sup\alpha_L^+(x)]; [\inf\beta_L^+(x), \sup\beta_L^+(x)]; [\inf\gamma_L^+(x), \sup\gamma_L^+(x)]; [\inf\alpha_L^-(x), \sup\alpha_L^-(x)]; [\inf\beta_L^-(x), \sup\beta_L^-(x)]; [\inf\gamma_L^-(x), \sup\gamma_L^-(x)]>\mid x\in X\}$ is defined as $L^C=\{x, <[\inf\alpha_{L^C}^+(x), \sup\alpha_{L^C}^+(x)]; [\inf\beta_{L^C}^+(x), \sup\beta_{L^C}^+(x)]; [\inf\gamma_{L^C}^+(x), \sup\gamma_{L^C}^+(x)]; [\inf\alpha_{L^C}^-(x), \sup\alpha_{L^C}^-(x)]; [\inf\beta_{L^C}^-(x), \sup\beta_{L^C}^-(x)]; [\inf\gamma_{L^C}^-(x), \sup\gamma_{L^C}^-(x)]>\mid x\in X\}$ where

$\inf\alpha_{L^C}^+(x)=\inf\gamma_L^+(x)$, $\sup\alpha_{L^C}^+(x)=\sup\gamma_L^+(x)$, $\inf\beta_{L^C}^+(x)=1-\sup\beta_L^+(x)$, $\sup\beta_{L^C}^+(x)=1-\inf\beta_L^+(x)$, $\inf\gamma_{L^C}^+(x)=\inf\alpha_L^+$, $\sup\gamma_{L^C}^+(x)=\sup\alpha_L^+$, $\inf\alpha_{L^C}^-(x)=\inf\gamma_L^-$, $\sup\alpha_{L^C}^-(x)=\sup\gamma_L^-$, $\inf\beta_{L^C}^-(x)=-1-\sup\beta_L^-(x)$, $\sup\beta_{L^C}^-(x)=-1-\inf\beta_L^-(x)$, $\inf\gamma_{L^C}^-(x)=\inf\alpha_L^-(x)$, $\sup\gamma_{L^C}^-(x)=\sup\alpha_L^-(x)$ for all $x\in X$.

3 Correlation coefficient measures under IBNSs setting

Definition 3.1: Let L_1 and L_2 be two IBNSs in $X=\{x_1, x_2, \ldots, x_n\}$, then the correlation between L_1 and L_2 is defined as follows:

$$R(L_1, L_2)=\sum_{i=1}^{n}\begin{pmatrix}\inf\alpha_{L_1}^+(x_i).\inf\alpha_{L_2}^+(x_i)+\sup\alpha_{L_1}^+(x_i).\sup\alpha_{L_2}^+(x_i)+\\ \inf\beta_{L_1}^+(x_i).\inf\beta_{L_2}^+(x_i)+\sup\beta_{L_1}^+(x_i).\sup\beta_{L_2}^+(x_i)+\\ \inf\gamma_{L_1}^+(x_i).\inf\gamma_{L_2}^+(x_i)+\sup\gamma_{L_1}^+(x_i).\sup\gamma_{L_2}^+(x_i)+\\ \inf\alpha_{L_1}^-(x_i).\inf\alpha_{L_2}^-(x_i)+\sup\alpha_{L_1}^-(x_i).\sup\alpha_{L_2}^-(x_i)+\\ \inf\beta_{L_1}^-(x_i).\inf\beta_{L_2}^-(x_i)+\sup\beta_{L_1}^-(x_i).\sup\beta_{L_2}^-(x_i)+\\ \inf\gamma_{L_1}^-(x_i).\inf\gamma_{L_2}^-(x_i)+\sup\gamma_{L_1}^-(x_i).\sup\gamma_{L_2}^-(x_i)\end{pmatrix}$$

Definition 3.2: Consider L_1 and L_2 be two IBNSs in $X=\{x_1, x_2, \ldots, x_n\}$, then the correlation coefficient between L_1 and L_2 is defined as follows:

$$Cor\ (L_1, L_2) = \frac{R(L_1, L_2)}{[R(L_1, L_1).R(L_2, L_2)]^{1/2}} \quad (1)$$

where

$$R(L_1, L_2) = \sum_{i=1}^{n} \begin{pmatrix} \inf \alpha^+_{L_1}(x_i).\inf \alpha^+_{L_2}(x_i) + \sup\alpha^+_{L_1}(x_i).\sup\alpha^+_{L_2}(x_i) + \\ \inf \beta^+_{L_1}(x_i).\inf \beta^+_{L_2}(x_i) + \sup\beta^+_{L_1}(x_i).\sup\beta^+_{L_2}(x_i) + \\ \inf \gamma^+_{L_1}(x_i).\inf \gamma^+_{L_2}(x_i) + \sup\gamma^+_{L_1}(x_i).\sup\gamma^+_{L_2}(x_i) + \\ \inf \alpha^-_{L_1}(x_i).\inf \alpha^-_{L_2}(x_i) + \sup\alpha^-_{L_1}(x_i).\sup\alpha^-_{L_2}(x_i) + \\ \inf \beta^-_{L_1}(x_i).\inf \beta^-_{L_2}(x_i) + \sup\beta^-_{L_1}(x_i).\sup\beta^-_{L_2}(x_i) + \\ \inf \gamma^-_{L_1}(x_i).\inf \gamma^-_{L_2}(x_i) + \sup\gamma^-_{L_1}(x_i).\sup\gamma^-_{L_2}(x_i) \end{pmatrix},$$

$$R(L_1, L_1) = \sum_{i=1}^{n} \begin{pmatrix} (\inf \alpha^+_{L_1}(x_i))^2 + (\sup\alpha^+_{L_1}(x_i))^2 + (\inf \beta^+_{L_1}(x_i))^2 + \\ (\sup\beta^+_{L_1}(x_i))^2 + (\inf \gamma^+_{L_1}(x_i))^2 + (\sup\gamma^+_{L_1}(x_i))^2 + \\ (\inf \alpha^-_{L_1}(x_i))^2 + (\sup\alpha^-_{L_1}(x_i))^2 + (\inf \beta^-_{L_1}(x_i))^2 + \\ (\sup\beta^-_{L_1}(x_i))^2 + (\inf \gamma^-_{L_1}(x_i))^2 + (\sup\gamma^-_{L_1}(x_i))^2 \end{pmatrix}$$

$$R(L_2, L_2) = \sum_{i=1}^{n} \begin{pmatrix} (\inf \alpha^+_{L_2}(x_i))^2 + (\sup\alpha^+_{L_2}(x_i))^2 + (\inf \beta^+_{L_2}(x_i))^2 + \\ (\sup\beta^+_{L_2}(x_i))^2 + (\inf \gamma^+_{L_2}(x_i))^2 + (\sup\gamma^+_{L_2}(x_i))^2 + \\ (\inf \alpha^-_{L_2}(x_i))^2 + (\sup\alpha^-_{L_2}(x_i))^2 + (\inf \beta^-_{L_2}(x_i))^2 + \\ (\sup\beta^-_{L_2}(x_i))^2 + (\inf \gamma^-_{L_2}(x_i))^2 + (\sup\gamma^-_{L_2}(x_i))^2 \end{pmatrix}$$

Theorem 1. The correlation coefficient measure *Cor* (L_1, L_2) between two IBNSs L_1, L_2 satisfies the following properties:

(C1) *Cor* (L_1, L_2) = *Cor* (L_2, L_1) ;
(C2) $0 \le$ *Cor* (L_1, L_2) ≤ 1;
(C3) *Cor* (L_1, L_2) = 1, if $L_1 = L_2$.

Proof:

(1) $Cor\ (L_1, L_2) = \frac{R(L_1, L_2)}{[R(L_1, L_1) \times R(L_2, L_2)]^{1/2}}$

$= \frac{R(L_2, L_1)}{[R(L_2, L_2) \times R(L_1, L_1)]^{1/2}} = Cor\ (L_2, L_1)$.

(2) Since, R (L_1, L_2) ≥ 0, R (L_1, L_1) ≥ 0, R (L_2, L_2) ≥ 0 and using Cauchy-Schwarz inequality we can easily prove that *Cor* (L_1, L_2) ≤ 1, therefore, $0 \le$ *Cor* (L_1, L_2) ≤ 1.

(3) If $L_1 = L_2$, then $\inf\ \alpha^+_{L_1}(x) = \inf \alpha^+_{L_2}(x)$, $\sup \alpha^+_{L_1}(x) = \sup \alpha^+_{L_2}(x)$, $\inf\ \beta^+_{L_1}(x) = \inf \beta^+_{L_2}(x)$, $\sup \beta^+_{L_1}(x) = \sup \beta^+_{L_2}(x)$, $\inf \gamma^+_{L_1}(x) = \inf \gamma^+_{L_2}(x)$, $\sup \gamma^+_{L_1}(x) = \sup \gamma^+_{L_2}(x)$, $\inf \alpha^-_{L_1}(x) = \inf \alpha^-_{L_2}(x)$, $\sup \alpha^-_{L_1}(x) = \sup \alpha^-_{L_2}(x)$, $\inf \beta^-_{L_1}(x) = \inf \beta^-_{L_2}(x)$, $\sup \beta^-_{L_1}(x) = \sup \beta^-_{L_2}(x)$, $\inf\ \alpha^-_{L_1}(x) = \inf\ \alpha^-_{L_2}(x)$, $\sup \gamma^-_{L_1}(x) = \sup \gamma^-_{L_2}(x)$ for any $x \in X$ and therefore, *Cor* (L_1, L_2) = 1.

Definition 3.3: Let $w_i = (w_1, w_2, ..., w_n) \in [0, 1]$ be the weight vector of the elements x_j (j = 1, 2, ..., *n*), the weighted correlation coefficient between two IBNSs L_1, L_2 can be defined by the following formula

$$Cor_w\ (L_1, L_2) = \frac{R_w(L_1, L_2)}{[R_w(L_1, L_1).R_w(L_2, L_2)]^{1/2}} \quad (2)$$

where

$$R_w(L_1, L_2) = \sum_{i=1}^{n} w_i \begin{pmatrix} \inf \alpha^+_{L_1}(x_i).\inf \alpha^+_{L_2}(x_i) + \sup\alpha^+_{L_1}(x_i).\sup\alpha^+_{L_2}(x_i) + \\ \inf \beta^+_{L_1}(x_i).\inf \beta^+_{L_2}(x_i) + \sup\beta^+_{L_1}(x_i).\sup\beta^+_{L_2}(x_i) + \\ \inf \gamma^+_{L_1}(x_i).\inf \gamma^+_{L_2}(x_i) + \sup\gamma^+_{L_1}(x_i).\sup\gamma^+_{L_2}(x_i) + \\ \inf \alpha^-_{L_1}(x_i)\inf \alpha^-_{L_2}(x_i) + \sup\alpha^-_{L_1}(x_i).\sup\alpha^-_{L_2}(x_i) + \\ \inf \beta^-_{L_1}(x_i).\inf \beta^-_{L_2}(x_i) + \sup\beta^-_{L_1}(x_i).\sup\beta^-_{L_2}(x_i) + \\ \inf \gamma^-_{L_1}(x_i).\inf \gamma^-_{L_2}(x_i) + \sup\gamma^-_{L_1}(x_i).\sup\gamma^-_{L_2}(x_i) \end{pmatrix},$$

$$R_w(L_1, L_1) = \sum_{i=1}^{n} w_i \begin{pmatrix} (\inf \alpha^+_{L_1}(x_i))^2 + (\sup\alpha^+_{L_1}(x_i))^2 + (\inf \beta^+_{L_1}(x_i))^2 + \\ (\sup\beta^+_{L_1}(x_i))^2 + (\inf \gamma^+_{L_1}(x_i))^2 + (\sup\gamma^+_{L_1}(x_i))^2 + \\ (\inf \alpha^-_{L_1}(x_i))^2 + (\sup\alpha^-_{L_1}(x_i))^2 + (\inf \beta^-_{L_1}(x_i))^2 + \\ (\sup\beta^-_{L_1}(x_i))^2 + (\inf \gamma^-_{L_1}(x_i))^2 + (\sup\gamma^-_{L_1}(x_i))^2 \end{pmatrix}$$

$$R(L_2, L_2) = \sum_{i=1}^{n} w_i \begin{pmatrix} (\inf \alpha^+_{L_2}(x_i))^2 + (\sup\alpha^+_{L_2}(x_i))^2 + (\inf \beta^+_{L_2}(x_i))^2 + \\ (\sup\beta^+_{L_2}(x_i))^2 + (\inf \gamma^+_{L_2}(x_i))^2 + (\sup\gamma^+_{L_2}(x_i))^2 + \\ (\inf \alpha^-_{L_2}(x_i))^2 + (\sup\alpha^-_{L_2}(x_i))^2 + (\inf \beta^-_{L_2}(x_i))^2 + \\ (\sup\beta^-_{L_2}(x_i))^2 + (\inf \gamma^-_{L_2}(x_i))^2 + (\sup\gamma^-_{L_2}(x_i))^2 \end{pmatrix}$$

If $w = (1/n, 1/n, ..., 1/n)^T$, the Eq. (2) is reduced to Eq. (1).

Theorem 2. The weighted correlation coefficient measure Cor_w (L_1, L_2) between two IBNSs L_1, L_2 *also* satisfies the following properties:

(C1) Cor_w (L_1, L_2) = Cor_w (L_2, L_1);
(C2) $0 \le Cor_w$ (L_1, L_2) ≤ 1;
(C3) Cor_w (L_1, L_2) = 1, if $L_1 = L_2$.

Proof:

(1) $Cor_w\ (L_1, L_2) = \frac{R_w(L_1, L_2)}{[R_w(L_1, L_1).R_w(L_2, L_2)]^{1/2}}$

$= \frac{R_w(L_2, L_1)}{[R_w(L_2, L_2).R_w(L_1, L_1)]^{1/2}} = Cor_w\ (L_2, L_1)$.

(2) Since, R_w (L_1, L_2) ≥ 0, R_w (L_1, L_1) ≥ 0, R_w (L_2, L_2) ≥ 0 and using Cauchy-Schwarz inequality we can easily prove that Cor_w (L_1, L_2) ≤ 1, so, $0 \le Cor_w$ (L_1, L_2) ≤ 1.

(3) If $L_1 = L_2$, then $\inf \alpha^+_{L_1}(x) = \inf \alpha^+_{L_2}(x)$, $\sup \alpha^+_{L_1}(x) = \sup \alpha^+_{L_2}(x)$, $\inf \beta^+_{L_1}(x) = \inf \beta^+_{L_2}(x)$, $\sup \beta^+_{L_1}(x) = \sup \beta^+_{L_2}(x)$, $\inf \gamma^+_{L_1}(x) = \inf \gamma^+_{L_2}(x)$, $\sup \gamma^+_{L_1}(x) = \sup \gamma^+_{L_2}(x)$, $\inf \alpha^-_{L_1}(x) = \inf \alpha^-_{L_2}(x)$, $\sup \alpha^-_{L_1}(x) = \sup \alpha^-_{L_2}(x)$, $\inf \beta^-_{L_1}(x) = \inf \beta^-_{L_2}(x)$, $\sup \beta^-_{L_1}(x) = \sup \beta^-_{L_2}(x)$, $\inf \alpha^-_{L_1}(x) = \inf \alpha^-_{L_2}(x)$, $\sup \gamma^-_{L_1}(x) = \sup \gamma^-_{L_2}(x)$ for any $x \in X$ and hence, $Cor_w(L_1, L_2) = 1$.

Example 1. Suppose that L_1 = < [0.3, 0.7], [0.3, 0.8], [0.5, 0.9], [-0.9, -0.3], [-0.6, -0.2], [-0.8, -0.4] > and L_2 = < [0.1, 0.6], [0.2, 0.7], [0.3, 0.5], [-0.8, -0.2], [-0.8, -0.3], [-0.7, -0.4] > be two IBNSs, then correlation coefficient between L_1 and L_2 is obtain using Eq. (1) as follows:

$Cor(L_1, L_2) == 0.4870391$.

Example 2. If $w = 0.4$, then the weighted correlation coefficient between L_1 = < [0.3, 0.7], [0.3, 0.8], [0.5, 0.9], [-0.9, -0.3], [-0.6, -0.2], [-0.8, -0.4] > and L_2 = < [0.1, 0.6], [0.2, 0.7], [0.3, 0.5], [-0.8, -0.2], [-0.8, -0.3], [-0.7, -0.4] > is calculated by using Eq. (2) as follows.

$Cor_w(L_1, L_2) = 0.5689123$.

4. MADM strategy based on weighted correlation coefficient measure in IBNS environment

In this section, we have developed a novel MADM strategy based on weighted correlation coefficient measure in interval bipolar neutrosophic environment. Let, $F = \{F_1, F_2, \ldots, F_m\}$, $(m \geq 2)$ be a discrete set of m feasible alternatives, $G = \{G_1, G_2, \ldots, G_n\}$, $(n \geq 2)$ be a set of n predefined attributes and w_j be the weight vector of the attributes such that $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$. The steps for solving MADM problems in IBNS environment are presented as follows.

Step 1. The evaluation of the performance value of alternative F_i (i = 1, 2, …, m) with regard to the predefined attribute G_j (j = 1, 2, …, n) provided by the decision maker or expert can be presented in terms of interval bipolar neutrosophic values q_{ij} = < $[\inf \alpha^+_{ij}, \sup \alpha^+_{ij}]$, $[\inf \beta^+_{ij}, \sup \beta^+_{ij}]$, $[\inf \gamma^+_{ij}, \sup \gamma^+_{ij}]$, $[\inf \alpha^-_{ij}, \sup \alpha^-_{ij}]$, $[\inf \beta^-_{ij}, \sup \beta^-_{ij}]$, $[\inf \gamma^-_{ij}, \sup \gamma^-_{ij}]$ > = < c_{ij}, d_{ij}, e_{ij}, f_{ij}, g_{ij}, h_{ij}, r_{ij}, s_{ij}, t_{ij}, u_{ij}, v_{ij}, w_{ij}>, i = 1, 2, ..., m; j = 1, 2, ..., n. The interval bipolar neutrosophic decision matrix $[\tilde{R}_{ij}]_{m\times n}$ is presented as given below.

$$[\tilde{R}_{ij}]_{m\times n} = \begin{array}{c} \\ F_1 \\ F_2 \\ . \\ . \\ F_m \end{array} \begin{array}{c} \begin{array}{cccc} G_1 & G_2 & \ldots & G_n \end{array} \\ \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ q_{m1} & q_{m2} & \cdots & q_{mn} \end{pmatrix} \end{array}$$

Step 2. The interval bipolar neutrosophic positive ideal solution (IBN-PIS) can be defined as follows: $Q^* = < c^+_j, d^+_j, e^+_j, f^+_j, g^+_j, h^+_j, r^+_j, s^+_j, t^+_j, u^+_j, v^+_j, w^+_j >$ = < [{ $\underset{i}{\text{Max}}(c_{ij})$ | j ∈ J^+; $\underset{i}{\text{Min}}(c_{ij})$ | j ∈ J^-}, { $\underset{i}{\text{Max}}(d_{ij})$ | j ∈ J^+}; $\underset{i}{\text{Min}}(d_{ij})$ | j ∈ J^-}], [{ $\underset{i}{\text{Min}}(e_{ij})$ | j ∈ J^+; $\underset{i}{\text{Max}}(e_{ij})$ | j ∈ J^-}, { $\underset{i}{\text{Min}}(f_{ij})$ | j ∈ J^+}; $\underset{i}{\text{Max}}(f_{ij})$ | j ∈ J^-}], [{ $\underset{i}{\text{Min}}(g_{ij})$ | j ∈ J^+; $\underset{i}{\text{Max}}(g_{ij})$ | j ∈ J^-}, { $\underset{i}{\text{Min}}(h_{ij})$ | j ∈ J^+}; $\underset{i}{\text{Max}}(h_{ij})$ | j ∈ J^-}], [{ $\underset{i}{\text{Min}}(r_{ij})$ | j ∈ J^+; $\underset{i}{\text{Max}}(r_{ij})$ | j ∈ J^-}, { $\underset{i}{\text{Min}}(s_{ij})$ | j ∈ J^+; $\underset{i}{\text{Max}}(s_{ij})$ | j ∈ J^-}], [{ $\underset{i}{\text{Max}}(t_{ij})$ | j ∈ J^+; { $\underset{i}{\text{Min}}(t_{ij})$ | j ∈ J^-}, { $\underset{i}{\text{Max}}(u_{ij})$ | j ∈ J^+}; $\underset{i}{\text{Min}}(u_{ij})$ | j ∈ J^-}], [{ $\underset{i}{\text{Max}}(v_{ij})$ | j ∈ J^+; { $\underset{i}{\text{Min}}(v_{ij})$ | j ∈ J^-}, { $\underset{i}{\text{Max}}(w_{ij})$ | j ∈ J^+}; $\underset{i}{\text{Min}}(w_{ij})$ | j ∈ J^-}] >, j = 1, 2, …, n, where J^+, J^- denote the benefit and cost type attributes, respectively.

Step 3. The weighted correlation coefficient of IBNS between alternative F_i (i = 1, 2, ..., m) and the ideal alternative Q^* can be derived as follows:

$$Cor_w(F_i, Q^*) = \frac{R_w(F_i, Q^*)}{[R_w(F_i, F_i).\, R_w(Q^*, Q^*)]^{1/2}}$$

where,

$$R_w(F_i, Q^*) = \sum_{j}^{n} w_j[c_{ij}.c^+_j + d_{ij}.d^+_j + e_{ij}.e^+_j + f_{ij}.f^+_j + g_{ij}.g^+_j + h_{ij}.h^+_j + r_{ij}.r^+_j + s_{ij}.s^+_j + t_{ij}.t^+_j + u_{ij}.u^+_j + v_{ij}.v^+_j + w_{ij}.w^+_j]$$

$$R_w(F_i, F_i) = \sum_{j=1}^{n} w_j[(c_{ij})^2 + (d_{ij})^2 + (e_{ij})^2 + (f_{ij})^2 + (g_{ij})^2 + (h_{ij})^2 + (r_{ij})^2 + (s_{ij})^2 + (t_{ij})^2 + (u_{ij})^2 + (v_{ij})^2 + (w_{ij})^2]$$

$$R_w(Q^*, Q^*) = \sum_{i=1}^{n} w_j[(c^+_j)^2 + (d^+_j)^2 + (e^+_j)^2 + (f^+_j)^2 + (g^+_j)^2 + (h^+_j)^2 + (r^+_j)^2 + (s^+_j)^2 + (t^+_j)^2 + (u^+_j)^2 + (v^+_j)^2 + (w^+_j)^2].$$

Step 4: The biggest value of $Cor_w(F_i, Q^*)$, i = 1, 2, ..., m implies F_i, (i = 1, 2, ..., m) is the better alternative.

In Fig 1. we represent the steps for solving MADM problems based on weighted correlation coefficient measure in IBNS environment.

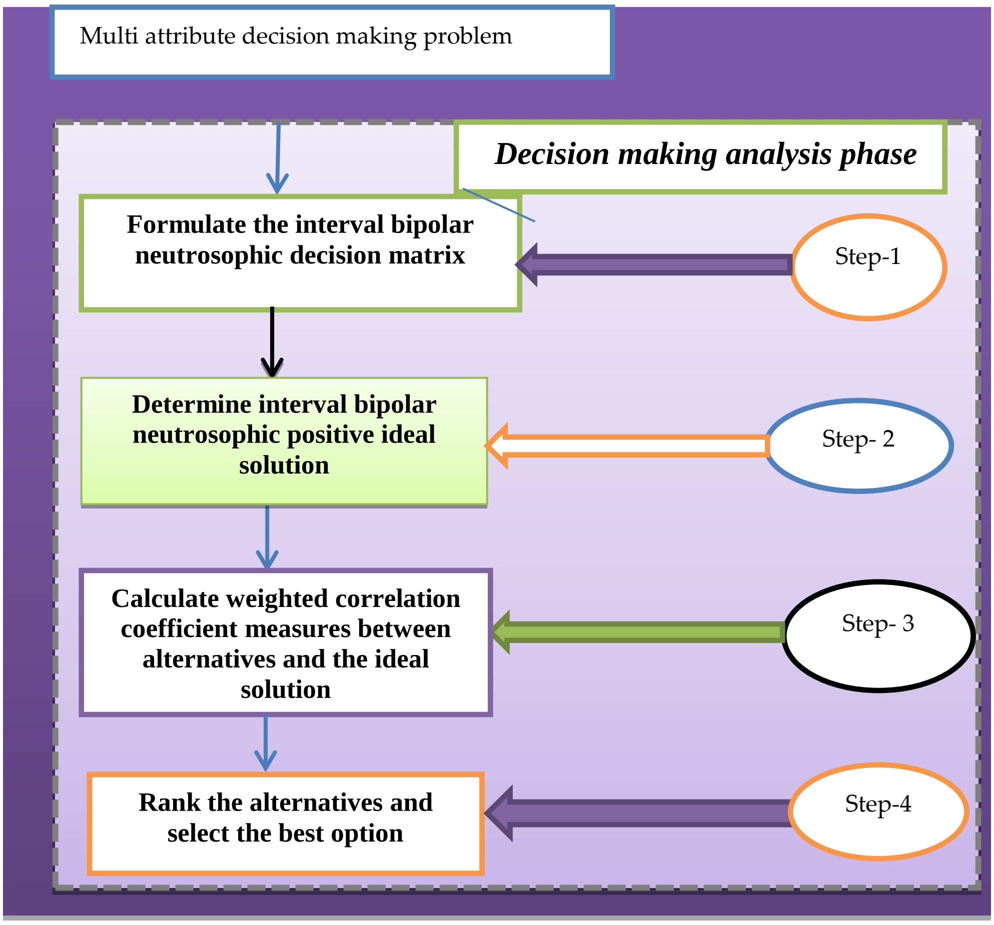

Figure.1 Decision making procedure of proposed MADM strategy

5. Numerical example

In this section, an illustrative numerical problem is solved to illustrate the proposed strategy. We consider an MADM studied in [31, 33] where there are four possible alternatives to invest money namely, a food company (F_1), a car company (F_2), a arm company (F_3), and a computer company (F_4). The investment company must take a decision based on the three predefined attributes namely growth analysis (G_1), risk analysis (G_2), and environment analysis (G_3) where G_1, G_2 are the benefit type and G_3 is the cost type attribute [34] and the weight vector of G_1, G_2, and G_3 is given by $w = (w_1, w_2, w_3) = (0.35, 0.25, 0.4)$ [31].

The proposed strategy consisting of the following steps:

Step 1. The evaluation of performance value of the alternatives with respect to the attributes provided by the decision maker can be expressed by interval bipolar neutrosophic values and the decision matrix is presented as follows:

Interval bipolar neutrosophic decision matrix

$$G_1$$

$$\begin{pmatrix} F_1 & [[0.4,0.5],[0.2,0.3],[0.3,0.4],[-0.3,-0.2],[-0.4,-0.3],[-0.5,-0.4]] \\ F_2 & [[0.6,0.7],[0.1,0.2],[0.2,0.3],[-0.2,-0.1],[-0.3,-0.2],[-0.7,-0.6]] \\ F_3 & [[0.3,0.6],[0.2,0.3],[0.3,0.4],[-0.3,-0.2],[-0.4,-0.3],[-0.6,-0.3]] \\ F_4 & [[0.7,0.8],[0.0,0.1],[0.1,0.2],[-0.1,-0.0],[-0.2,-0.1],[-0.8,-0.7]] \end{pmatrix}$$

$$G_2$$

$$\begin{pmatrix} F_1 & [[0.4,0.6],[0.1,0.3],[0.2,0.4],[-0.3,-0.1],[-0.4,-0.2],[-0.6,-0.4]] \\ F_2 & [[0.6,0.7],[0.1,0.2],[0.2,0.3],[-0.2,-0.1],[-0.3,-0.2],[-0.7,-0.6]] \\ F_3 & [[0.5,0.6],[0.2,0.3],[0.3,0.4],[-0.3,-0.2],[-0.4,-0.3],[-0.6,-0.5]] \\ F_4 & [[0.6,0.7],[0.1,0.2],[0.1,0.3],[-0.2-0.1],[-0.3,-0.1],[-0.7,-0.6]] \end{pmatrix}$$

$$G_3$$

$$\begin{pmatrix} F_1 & [[0.7, 0.9], [0.2, 0.3], [0.4, 0.5], [-0.3, -0.2], [-0.5, -0.4], [-0.9, -0.7]] \\ F_2 & [[0.3, 0.6], [0.3, 0.5], [0.8, 0.9], [-0.5, -0.3], [-0.9, -0.8], [-0.6, -0.3]] \\ F_3 & [[0.4, 0.5], [0.2, 0.4], [0.7, 0.9], [-0.4, -0.2], [-0.9, -0.7], [-0.5, -0.4]] \\ F_4 & [[0.6, 0.7], [0.3, 0.4], [0.8, 0.9], [-0.4, -0.3], [-0.9, -0.8], [-0.7, -0.6]] \end{pmatrix}$$

.

Step 2. Determine the IBN-PIS (Q^*) from interval bipolar neutrosophic decision matrix as follows:

$\left\langle [c_1^+, d_1^+], [e_1^+, f_1^+], [g_1^+, h_1^+], [r_1^-, s_1^-], [t_1^-, u_1^-], [v_1^-, w_1^-] \right\rangle =$ < [0.7, 0.8], [0.0, 0.1], [0.1, 0.2], [-0.3, -0.2], [-0.2, -0.1], [-0.5, -0.3];

$\left\langle [c_2^+, d_2^+], [e_2^+, f_2^+], [g_2^+, h_2^+], [r_2^-, s_2^-], [t_2^-, u_2^-], [v_2^-, w_2^-] \right\rangle =$ < [0.6, 0.7], [0.1, 0.2], [0.1, 0.3], [-0.3, -0.2], [-0.3, -0.1], [-0.6, -0.4];

$\left\langle [c_3^+, d_3^+], [e_3^+, f_3^+], [g_3^+, h_3^+], [r_3^-, s_3^-], [t_3^-, u_3^-], [v_3^-, w_3^-] \right\rangle =$ < [0.3, 0.5], [0.3, 0.5], [0.8, 0.9], [-0.3, -0.2], [-0.9, -0.8], [-0.9, -0.7].

Step 3. The weighted correlation coefficient $Cor_w(F_i, Q^*)$ between alternative F_i (i = 1, 2, ..., m) and IBN-PIS Q^* is obtained as given below.

$R_w(F_1, Q^*) = 2.4465$, $R_w(F_1, F_1,) = 2.585351$, $R_w(Q^*, Q^*) = 2.850693$, $Cor_w(F_1, Q^*) = 0.331952$,

$R_w(F_2, Q^*) = 2.9205$, $R_w(F_2, F_2) = 2.905408$, $Cor_w(F_2, Q^*) = 0.3526141$,

$R_w(F_3, Q^*) = 2.6625$, $Q_w(F_3, F_3) = 2.701919$, $Cor_w(F_3, Q^*) = 0.3456741$,

$R_w(F_4, Q^*) = 3.098$, $Q_w(F_4, F_4) = 3.048081$, $Cor_w(F_4, Q^*) = 0.3565369$.

We observe that $Cor_w(F_4, Q^*) > Cor_w(F_2, Q^*) > Cor_w(F_3, Q^*) > Cor_w(F_1, Q^*)$.

Step 4. According to the weighted correlation coefficient values, the ranking order of the companies is presented as: $F_4 > F_2 > F_3 > F_1$.

Hence, the most desirable investment company is F_4.

In Fig 2. we represent the graphical representation of alternatives versus weighted correlation coefficient values.

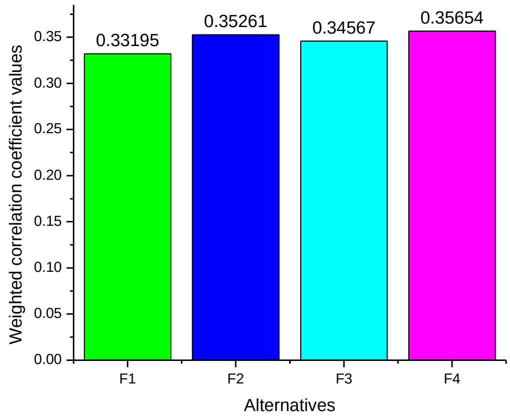

Fig 2. Graphical representation of alternatives versus weighted correlation coefficient values.

Next, we compare the obtained results with the results of Mahmood et al. [31] and Pramanik et al. [33] in Table 1 where the weight vector of the attributes is w = (0.35, 0.25, 0.4) [31]. We see that ranking orders of alternatives derived by the proposed strategy and the strategies discussed by Mahmood et al. [31] and Pramanik et al. [33] are different. We also observe that F_4 is the best option obtained by the proposed strategy as well as the strategy discussed by Mahmood et al. [31] . However, Pramanik et al. [33] found that F_2 is the most desirable alternative based on weighted cross entropy measure.

Table 1. The results derived from different strategies

strategy	Ranking results	Best choice
The proposed weighted correlation coefficient strategy	$F_4 \succ F_2 \succ F_3 \succ F_1$	F_4
Mahmood et al.'s strategy [31]	$F_4 \succ F_1 \succ F_3 \succ F_2$	F_4
Weighted cross entropy measure [33]	$F_1 \prec F_3 \prec F_4 \prec F_2$	F_2

6 Conclusion

In the study, we have defined correlation coefficient and weighted correlation coefficient measures in interval bipolar neutrosophic environments and prove their basic properties. Using the proposed weighted correlation coefficient measure, we have developed a novel MADM strategy in interval bipolar neutrosophic environment. We have solved an investment problem with interval bipolar neutrosophic information. Comparison analysis with other existing strategies is presented to demonstrate the feasibility and applicability of the proposed strategy. We hope that the proposed correlation coefficient measures can be employed to tackle realistic multi attribute decision making problems such as clustering analysis [15], medical diagnosis [21], weaver selection [35-37], fault diagnosis [38], brick selection [39- 40], data mining [41], logistic centre location selection [42- 43], school selection [44], teacher selection [45-47], image processing, information fusion, etc. in interval bipolar neutrosophic environment. Using aggregation operators, the proposed strategy can be extended to multi attribute group decision making problem in interval bipolar neutrosophic set environment.

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