Correlation Matrix Distance, a Meaningful Measure for Evaluation of Non-Stationary MIMO Channels

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***Abstract*— The *Correlation Matrix Distance* (CMD), an earlier introduced measure for characterization of non-stationary MIMO channels, is analyzed regarding its capability to predict performance degradation in MIMO transmission schemes. For that purpose we consider the performance reduction that a prefiltering MIMO transmission scheme faces due to non-stationary changes of the MIMO channel. We show that changes in the spatial structure of the channel corresponding to high values in the CMD also show up as a significant reduction in performance of the considered MIMO transmission scheme. Such significant changes in the spatial structure of the mobile radio channel are shown to appear also for small movements within an indoor environment. Stationarity can therefore not always be assumed for indoor MIMO radio channels.**

Keywords: ***MIMO; Stationarity; Indoor***

I. Introduction

Wide-sense stationarity and uncorrelated scattering (WSSUS) is often assumed for mobile radio channels. If these assumptions are valid, hence the second order statistics stays constant over time and frequency, transmission schemes can be applied that take advantage from estimating the channel statistics and adapting to it. In case of MIMO channels, channel correlation has to be taken into account for stationarity considerations. Since the spatial structure (angles of arrival/departure of impinging/departing waves) directly corresponds to the statistics of the channel, it plays a dominant role for the performance of MIMO transmission schemes. Hence, it makes sense, to consider stationarity regarding the time- and frequency-variation of the spatial structure only. Up to now, different approaches where made to measure the non-stationarity of the mobile radio channel (e.g. [1]–[3]), however, all of them have shortcomings when applied to MIMO channels. In [4], the *Correlation Matrix Distance* (CMD) was introduced as useful measure to characterize non-stationary MIMO channels. There, measurements where analyzed solely regarding the change in the spatial structure. In this paper, we want to show that changes in the spatial structure that lead to significantly large values of the CMD also show up as significant performance reduction in MIMO transmission schemes. For that purpose, we consider a transmit prefiltering transmission scheme that makes use of the channel statistics by adapting to the current transmit correlation matrix.

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II. Definition

A. System model

We consider the transmit prefiltering scheme that was proposed by Kiessling et al. [5]. The system model for this transmission scheme is given by

$$\mathbf{y} = \mathbf{H}\mathbf{F}\mathbf{s} + \mathbf{n}, \quad (1)$$

where $\mathbf{y}$ is the $n_R \times 1$ receive signal vector, $\mathbf{H}$ the $n_R \times n_T$ MIMO channel matrix, $\mathbf{F}$ the $n_T \times L$ transmit prefilter, $\mathbf{s}$ the $L \times 1$ transmit symbol vector, $\mathbf{n}$ the $n_R \times 1$ additive white Gaussian noise vector, n_t the number of transmit and n_r the number of receive antennas. We consider L independent data streams that are transmitted via the transmit prefilter over n_T transmit antennas where each data stream is QPSK modulated. At receive side an MMSE detector is used. Based on the channel matrix we define the transmit correlation matrix as[2] $\mathbf{R}_{\mathrm{Tx}} = \mathrm{E}\{\mathbf{H}^T\mathbf{H}^*\}$ and the receive correlation matrix as $\mathbf{R}_{\mathrm{Rx}} = \mathrm{E}\{\mathbf{H}\mathbf{H}^H\}$. The transmit prefilter is designed such that the overall bit error ratio is approximately minimized which is achieved by transmitting only over the strongest L transmit eigenmodes where an appropriate eigenmode weighting is applied and additionally, each data stream is distributed over all L selected eigenmodes via a discrete Fourier transform (DFT) matrix. Thus, the transmit prefilter matrix is given by

$$\mathbf{F} = \mathbf{V}_{\mathrm{Tx}}\mathbf{\Phi}_{\mathbf{F}}\mathbf{D}_L. \quad (2)$$

Here, $\mathbf{V}_{\mathrm{Tx}}$ contains the first L eigenvectors of the transmit correlation matrix, $\mathbf{\Phi}_{\mathbf{F}}$ is the diagonal weighting matrix and $\mathbf{D}_L$ an $L \times L$ DFT matrix. For details on the prefilter design we refer to [5].

[2]$(\cdot)^T$ denotes transpose, $(\cdot)^H$ hermitian transpose, $(\cdot)^*$ conjugation, $\mathrm{tr}\{\cdot\}$ matrix trace, $||\cdot||_f$ the Frobenius norm and $E\{\cdot\}$ the expectation operator.

B. Correlation Matrix Distance (CMD)

The CMD is the distance between two correlation matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ as defined by

$$d_{\text{corr}}(\mathbf{R}_1, \mathbf{R}_2) = 1 - \frac{\text{tr}\{\mathbf{R}_1\mathbf{R}_2\}}{\|\mathbf{R}_1\|_f \|\mathbf{R}_2\|_f} \quad \in [0, 1]. \tag{3}$$

It becomes zero if the correlation matrices are equal up to a scaling factor and one if they differ to a maximum extent. There are two justifications for this metric. First, it can be reformulated as inner product between the vectorized correlation matrices:

$$d_{\text{corr}}(\mathbf{R}_1, \mathbf{R}_2) = 1 - \frac{\langle \text{vec}\{\mathbf{R}_1\}, \text{vec}\{\mathbf{R}_2\} \rangle}{\big|\big|\text{vec}\{\mathbf{R_1}\}\big|\big|_2 \big|\big|\text{vec}\{\mathbf{R_2}\}\big|\big|_2}. \tag{4}$$

This makes clear that it measures the 'orthogonality' between the considered correlation matrices in the $n \times n$ dimensional space. This can be shown even better when using the eigenvalue decomposition. The product of the correlation matrices can be written as

$$\mathbf{R}_1\mathbf{R}_2 = \mathbf{U}_1\mathbf{\Lambda}_1\mathbf{U}_1^H\mathbf{U}_2\mathbf{\Lambda}_2\mathbf{U}_2^H = \mathbf{U}_1\mathbf{D}\mathbf{U}_2^H \tag{5}$$

with

$$(\mathbf{D})_{ij} = \mathbf{u}_{(1),i}^H \mathbf{u}_{(2),j} \lambda_{(1),i} \lambda_{(2),j}. \tag{6}$$

Here, $(\mathbf{D})_{ij}$ denotes element (i, j) of $\mathbf{D}$, $\mathbf{u}_{(1),i}$ and $\mathbf{u}_{(2),j}$ the ith and jth column of $\mathbf{U}_1$ and $\mathbf{U}_2$, repectively and $\lambda_{(1),i}$ and $\lambda_{(2),j}$ the corresponding eigenvalue. This means, $\mathbf{D}$ becomes zero if for every pair of eigenvectors either $\mathbf{u}_{(1),i}$ and $\mathbf{u}_{(2),j}$ are orthogonal or either $\lambda_{(1),i}$ or $\lambda_{(2),j}$ is zero. If $\mathbf{D}$ is the zero matrix, then $\text{tr}\{\mathbf{R}_1\mathbf{R}_2\}$ becomes zero, too and the correlation matrix therefore one. However, the more the signal spaces of $\mathbf{R}_1$ and $\mathbf{R}_2$ overlap, the higher becomes the trace of the product and therefore the CMD decreases.

This property of the CMD makes it a useful measure to evaluate whether the spatial structure of the channel, hence, the channel statistics have changed to a significant amount.

In this paper we consider non-stationarity due to a moving mobile. To measure the change of the spatial second-order statistics, we therefore investigate the CMD between the correlation matrices measured at starting position and after having moved by a certain distance. The CMD between these correlation matrices reflects the variation of the channel statistics for the considered time interval.

III. Measurement description

Measurements were performed with an Elektrobit[3] [6] PropSound channel sounder at 2.45 GHz center frequency where we used 120 MHz of the measurement bandwidth for the evaluations. At the transmit side, a 7+1 element monopole circular array in the horizontal plane was used. Each antenna element was omni-directional in the horizontal plane. At the receive side, a 16-element (4x4) dual-polarized rectangular patch array was utilized. The patch elements were arranged in a vertical plane. This means that the antenna is both horizontally and vertically directional with the main lobe into broadside direction of the array. For the evaluation, all transmit antennas and the two upper rows of the receiver array were used, where only one polarization was considered. The transmit antennas was mounted at a height of about 2m, the receive antenna at about 1.6m.

We measured in an indoor office environment. The transmitter was positioned in a corridor and the receiver was *moved* along several routes in different office rooms connected to the corridor having mostly non-line-of-sight (NLOS) to the transmitter (see Fig. 1). During one measurement, the MIMO channel was recorded at a rate of about 25 measurements per second. We considered four different broadside directions of the receive antenna (D1, D2, D3 and D4). A measurement was then specified by the TX position and the movement route, the orientation of the receive antenna and the movement direction. For instant the measurement with transmitter at Tx1, receiver movement route 6, receiver orientation D3 and movement direction v2 is labeled by 'Tx1, Route 6, D3, v2'.

During the measurement, the receiver was moved with constant speed. This was ensured by using a laser distance meter that logged the distance to a reference position and the movement speed. To have accurate estimates of the correlation matrices also at the beginning and the end of the measurements, we used only that part of the measurements where the average movement speed of typically 0.3-0.5m/s was already reached, i.e. we removed the starting and ending phase of the measurement with increasing and decreasing speed (phases of acceleration and deceleration). Based on the measured distance we consider in the following the evolution of the correlation matrices and the performance of the transmit prefiltering scheme with distance. For more details on the measurements we refer to [7].

IV. Evaluations

For correlation matrix estimation, a set of 10 subsequent temporal snapshots and all frequency samples of the MIMO channel within the considered bandwidth were used. This window of 10 temporal snapshots was then moved over the whole measurement interval to estimate the transmit and receive correlation matrices for each time instant. Given a set of N MIMO channel snapshots, the correlation matrices were estimated by

$$\hat{\mathbf{R}}_{\text{Tx}} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{H}^T \mathbf{H}^*, \tag{7}$$

and

$$\hat{\mathbf{R}}_{\text{Rx}} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{H}\mathbf{H}^H. \tag{8}$$

Based on the estimated time/distance-varying correlation matrices we created a large number of channel realizations $\mathbf{H}$ for each time/distance point using the Kronecker model

[3]www.elektrobit.com

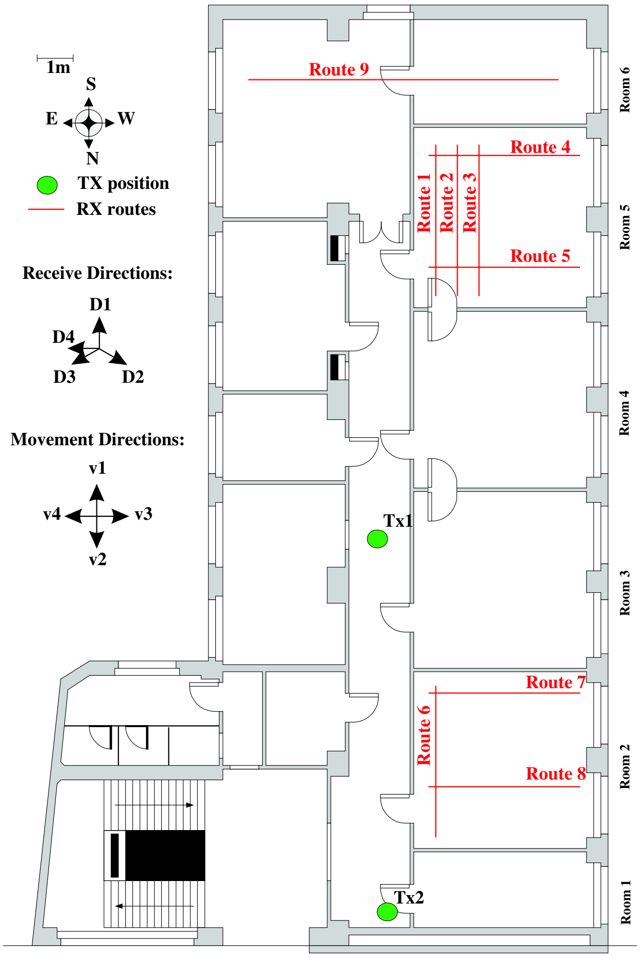

Fig. 1. Measurement scenario with TX positions and RX movement routes, Institut für Nachrichtentechnik und Hochfrequenztechnik, Technische Universität Wien.

[8]–[11]:

$$\mathbf{H} = \frac{1}{\sqrt{\mathrm{tr}\{\mathbf{R}_{\mathrm{RX}}\}}} \mathbf{R}_{\mathrm{RX}}^{1/2} \mathbf{G} \left(\mathbf{R}_{\mathrm{TX}}^{1/2} \right)^{T} . \tag{9}$$

Here, $\mathbf{G}$ is an i.i.d. complex Gaussian fading matrix with zero mean and variance one and $(\cdot)^{1/2}$ denotes the matrix square root. Note, that although the Kronecker channel model has some important deficiencies in modeling the MIMO channel [12], [13] it can be applied here since we only consider the transmit correlation matrix for the prefilter design.

For each time/distance point we simulated the BER performance for an 8×8 MIMO system with the previously described transmit prefiltering scheme for different signal-to-noise ratios (SNRs). We used 6 data streams, QPSK modulation, and an MMSE detector. Results were compared with blind transmission, i.e. no channel knowledge at transmit side but full channel knowledge at receive side, where only antennas 1–3 and 6–8 were used on transmit side for a fair comparison.

We simulate the performance of the transmit prefiltering scheme when using either the correct transmit correlation

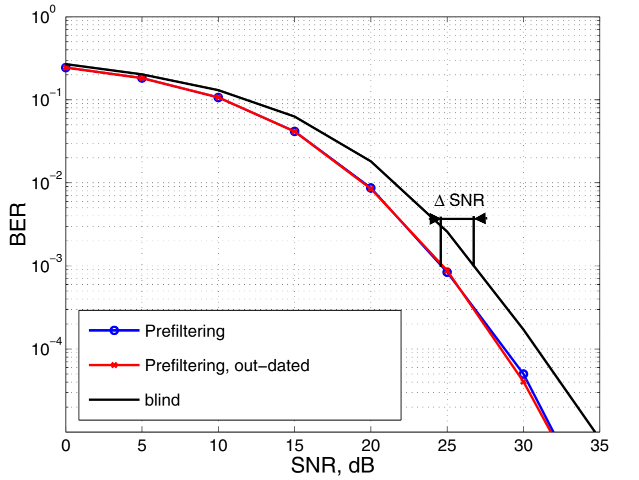

Fig. 2. Prefiltering gain with correct and out-dated transmit prefiltering: Start of movement (Tx1, Route 1, D3, v2).

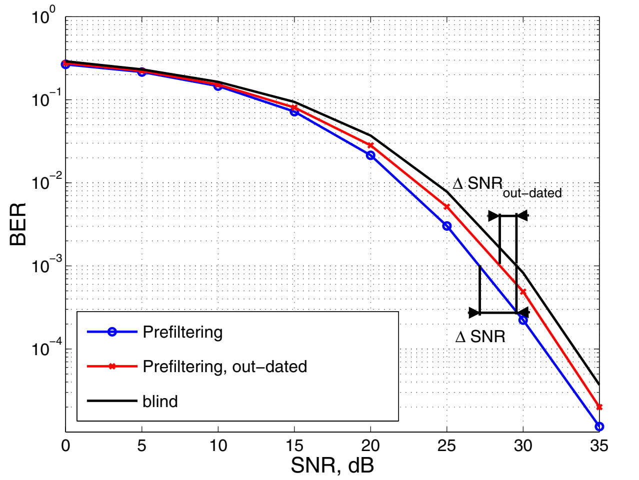

Fig. 3. Prefiltering gain with correct and out-dated transmit prefiltering: End of movement (Tx1, Route 1, D3, v2).

matrix or an out-dated version of it. As out-dated transmit correlation matrix we always take the estimate from the start of the measurement, i.e. from the beginning of the receiver movement. From the BER result we then obtained the SNR gain that was achieved over blind transmission. This was done for the correct and the out-dated transmit correlation matrix. The SNR gain was always obtained at a target BER of 10^{-3}.

Figures 2 and 3 show the BER result for an exemplary scenario when the receiver was at the beginning of the movement route and at the end of the movement route, respectively. Additionally, the SNR gain is shown. When starting the movement, the correct transmit correlation matrix equals the out-dated one, therefore the performance is equal for both. However, after having moved the receiver by a certain distance, in this case about 3.5m, they differ and therefore the resulting performance differs also. In this example we can see that both schemes achieve an SNR gain over blind transmission of about 2.2dB in the beginning. At the end of the movement route, the gain is about 2.3dB for correct prefiltering but has reduced to about 1dB when using out-dated prefiltering. Note, that the absolute performance has changed, the SNR required for the target BER is about 24dB for correct prefiltering at the beginning of the movement and 27.5dB at the end of the movement, the gain by correct prefiltering, however, does not change here. This is a typical result for the considered measurements, but there are scenarios where the performance loss due to out-dated prefiltering is much higher.

In the following, we compare the achieved SNR gain to the correlation matrix distance between the correct transmit correlation matrix and the out-dated one.

V. Results

The evaluations were done for a number of movement routes, here we show two exemplary scenarios with relatively high variation of the spatial structure.

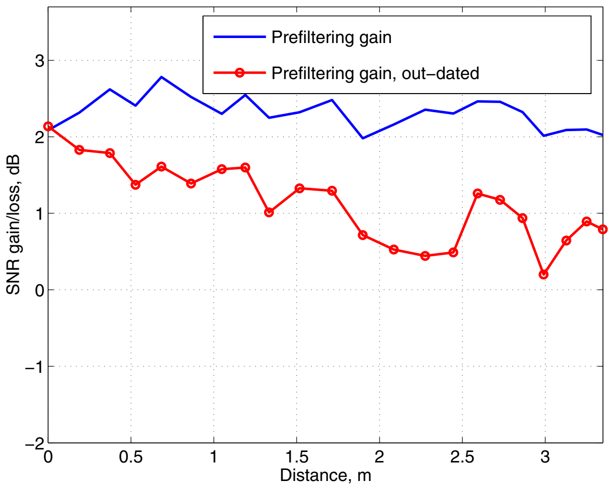

Fig. 4. Temporal evolution of the SNR gain due to prefiltering at a target BER of 10^{-3}, Tx1, Route 5, D3, v3 (Scenario 1).

Figure 4 shows the prefiltering gain for correct and out-dated transmit prefiltering for Scenario 1 (Tx1, Route 5, D3, v3) and Fig. 5 for Scenario 2 (Tx2, Route 6, D3, v2). In both cases we can see a relatively constant gain for correct prefiltering and a significant performance reduction in prefiltering when using out-dated channel state information. Actually, the SNR gain reduces to marginal values in Scenario 1 and to zero in Scenario 2. When comparing this to the corresponding Correlation Matrix Distance (Fig. 6), we find a good accordance. Especially, when considering Scenario 2, we see that the temporary decrease in the CMD at a distance of 0.5 to 1m from starting position, is directly reflected in an increase of out-dated prefiltering gain. This means the CMD appears as a good indication for the performance loss due to out-dated prefiltering.

Note that the CMD that we measure and the SNR gain due to correct or out-dated prefiltering relates only to changes of the transmit correlation matrix. This means that even though

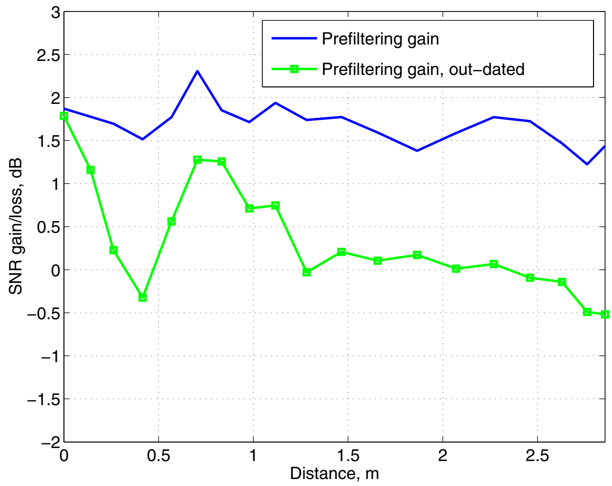

Fig. 5. Temporal evolution of the SNR gain due to prefiltering at a target BER of 10^{-3}, Tx2, Route 6, D3, v2 (Scenario 2).

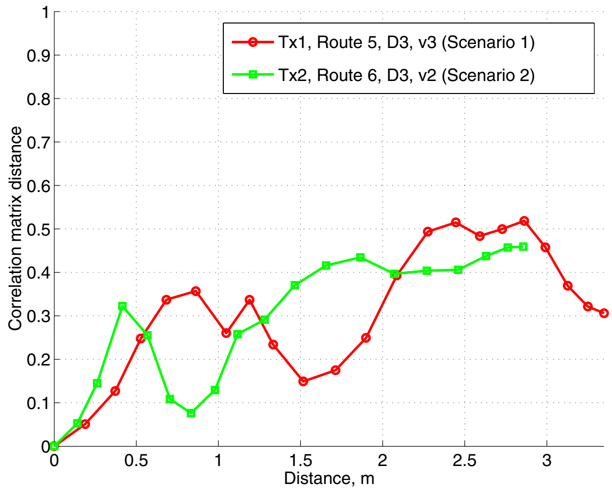

Fig. 6. Correlation Matrix Distance between transmit correlation matrices at distance on abscissa and distance 0, Scenario 1 and 2.

the transmitter is fixed, the spatial structure at transmit side changes significantly in the considered scenarios when the receiver is moved. Comparing the results to the scenario, it becomes clear what is happening. In Scenario 1 (Fig. 4), the receiver is moved from the door to the corridor towards the other side of the room. At the starting position, a significant amount of power is propagated through the door and impinging at the receiver. When the receiver is moved, other paths become more dominant. This of course leads to a changed spatial structure.

Considering Scenario 2 (Fig. 5), we have a similar situation. Here, the receiver is moved parallel to the corridor into movement direction v2. The receive antenna looks towards the corridor wall and passes the door to the corridor. This again leads to a significantly changed wave propagation and therefore significant change in the spatial structure.

Up to now, we only considered the downlink, i.e. trans-

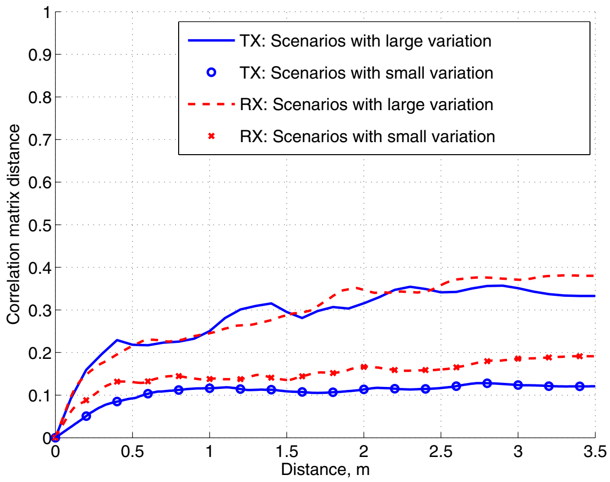

Fig. 7. Correlation Matrix Distance between transmit (receive) correlation matrices at distance on abscissa and distance 0, averaged over scenarios with high spatial variation and small spatial variation.

mission from a fixed access point or base station on the corridor to a moving mobile. But what happens for the uplink, where the mobile becomes the transmitter? In this case the receive correlation matrix for the downlink becomes the transmit correlation matrix for the uplink. Hence, we compared the CMD for both transmit and receive correlation matrices of the measurements. For that purpose we categorized the measurements into two classes, measurements with a temporal average of the CMD of below 0.2 and above 0.2. We found that from a total number of 41 scenarios, at transmit (BS) side 8 scenarios and at receive (mobile) side 26 scenarios show a high variation in the spatial structure regarding this criteria. Figure 7 shows the average CMD vs time, where the average is now taken over the CMDs of each class. It becomes clear that the spatial variation at receive side is typically much higher than at transmit side, which is consistent with expectations since the surroundings of the receiver are changing but the surroundings of the transmitter are not. We emphasize that the problem of having accurate knowledge of the channel statistics at transmit side is therefore more severe for the uplink than for the downlink (in case of a fixed base station and a moving receiver, which is the typical case).

VI. Conclusions

To which extent the mobile radio channel can be considered as stationary is crucial for the performance of advanced MIMO transmission schemes that rely on estimates of the channel statistics. We showed that the usage of out-dated channel statistics for a transmit prefiltering scheme in an indoor office environment leads to significantly reduced performance, in extreme cases to zero gain over blind transmission. Furthermore, we showed that the *Correlation Matrix Distance* (CMD) reflects clearly the performance reduction that the considered transmit prefiltering scheme faces. This suggests that the CMD is a meaningful measure to check the validity of stationarity assumptions. As a last point we discussed the stationarity problem for downlink and uplink. We came to the conclusion that in a typical scenario with a fixed base station and a moving receiver, an advanced transmission schemes for the uplink will face more severe performance degradations due to non-stationarity of the channel than for the downlink.

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