# CORRELATION MEASURE FOR INTUITIONISTIC FUZZY MULTI SETS 

P. Rajarajeswari ${ }^{1}$, N. Uma ${ }^{2}$<br>${ }^{I}$ Department of Mathematics, Chikkanna Arts College, Tirupur, Tamil Nadu, India<br>${ }^{2}$ Department of Mathematics, SNR Sons College, Coimbatore, Tamil Nadu, India


#### Abstract

In this paper, the Correlation measure of Intuitionistic Fuzzy Multi sets (IFMS) is proposed. The concept of this Correlation measure of IFMS is the extension of Correlation measure of IFS. Using the Correlation of IFMS measure, the application of medical diagnosis and pattern recognition are presented. The new method also shows that the correlation measure of any two IFMS equals one if and only if the two IFMS are the same.


Keywords: Intuitionistic fuzzy set, Intuitionistic Fuzzy Multi sets, Correlation measure.

## 1. INTRODUCTION

The Intuitionistic Fuzzy sets (IFS) introduced by Krasssimir T. Atanassov [1, 2] is the generalisation of the Fuzzy set (FS). The Fuzzy set ( $F S$ ) proposed by Lofti A. Zadeh [3] allows the uncertainty belong to a set with a membership degree $(\mu)$ between 0 and 1. That is, the one and only membership function $(\mu \in[0,1])$ and the non membership function equals one minus the membership degree. Whereas IFS represent the uncertainty with respect to both membership ( $\mu \in[0,1]$ ) and non membership $(\vartheta \in[0,1])$ such that $\mu+\vartheta \leq 1$. The number $\pi=1-\mu-\vartheta$ is called the hesitiation degree or intuitionistic index.

Several authors like Murthy and Pal [4] investigated the correlation between two fuzzy membership functions, Chiang and Lin [5] studied the correlation of fuzzy sets and Chaudhuri and Bhattacharya [6] discussed the correlation between two fuzzy sets on same universal discourse. As the Intuitionistic fuzzy sets is widely used in various fields like pattern recognition, medical diagnosis, logic programming, decision making, market prediction, etc. Correlation Analysis of IFS plays a vital role in recent research area. Gerstenkorn and Manko [7] defined and examined the properties the correlation measure of IFS for finite universe of discourse. Later the concepts of correlation and the correlation coefficient of IFS in probability spaces were derived by Hong, Hwang [8] for the infinite universe of discourse. Hung and $\mathrm{Wu}[9,10]$ proposed a centroid method to calculate the correlation coefficient of $I F S$ s, using the positively and negatively correlated values. The correlation coefficient of IFS in terms of statistical values, using mean aggregation functions was presented by Mitchell [11]. Based on geometrical representation of IFSs and three parameters, a correlation coefficient of IFSs was defined by Wenyi Zeng and Hongxing Li [12].

The Multi set [13] repeats the occurrences of any element. And the Fuzzy Multi set (FMS) introduced by R. R. Yager
[14] can occur more than once with the possibly of the same or the different membership values. Recently, the new concept Intuitionistic Fuzzy Multi sets (IFMS) was proposed by T.K Shinoj and Sunil Jacob John [15].

As various distance and similarity methods of IFS are extended for IFMS distance and similarity measures [16, 17, 18 and 19], this paper is an extension of the correlation measure of IFS to IFMS. The numerical results of the examples show that the developed similarity measures are well suited to use any linguistic variables.

## 2. PRELIMINARIES

### 2.1 Definition:

Let X be a nonempty set. A fuzzy set $A$ in $X$ is given by
$A=\left\{\left\langle x, \mu_{A}(x)\right\rangle / x \in X\right\}$
where $\mu_{A}: \mathrm{X} \rightarrow[0,1]$ is the membership function of the fuzzy set $A$ (i.e.) $\mu_{A}(x) \in[0,1]$ is the membership of $x \in X$ in $A$. The generalizations of fuzzy sets are the Intuitionistic fuzzy (IFS) set proposed by Atanassov [1, 2] is with independent memberships and non memberships.

### 2.2 Definition:

An Intuitionistic fuzzy set (IFS), $A$ in $X$ is given by

$$
\begin{equation*}
\mathrm{A}=\left\{\left\langle x, \mu_{A}(x), \vartheta_{A}(x)\right\rangle / x \in X\right\} \tag{2.2}
\end{equation*}
$$

where $\mu_{A}: \mathrm{X} \rightarrow[0,1]$ and $\vartheta_{A}: \mathrm{X} \rightarrow[0,1]$ with the condition $0 \leq \mu_{A}(x)+\vartheta_{A}(x) \leq 1, \forall x \in X \quad$ Here $\mu_{A}(x)$ and $\vartheta_{A}(x) \in[0,1]$ denote the membership and the non membership functions of the fuzzy set $A$;
For each Intuitionistic fuzzy set in $X, \pi_{A}(x)=1-\mu_{A}(x)-$ $\left[1-\mu_{A}(x)\right]=0$ for all $x \in X$ that is
$\pi_{A}(x)=1-\mu_{A}(x)-\vartheta_{A}(x)$ is the hesitancy degree of $x \in X$ in $A$. Always $0 \leq \pi_{A}(x) \leq 1, \forall x \in X$.

The complementary set $A^{c}$ of $A$ is defined as

$$
\begin{equation*}
A^{c}=\left\{\left\langle x, \vartheta_{A}(x), \mu_{A}(x)\right\rangle / x \in X\right\} \tag{2.3}
\end{equation*}
$$

### 2.3 Definition:

Let X be a nonempty set. A Fuzzy Multi set (FMS) A in X is characterized by the count membership function Mc such that $\mathrm{Mc}: \mathrm{X} \rightarrow \mathrm{Q}$ where Q is the set of all crisp multi sets in $[0,1]$. Hence, for any $x \in X, \operatorname{Mc}(\mathrm{x})$ is the crisp multi set from $[0,1]$. The membership sequence is defined as

$$
\begin{aligned}
& \quad\left(\mu_{A}^{1}(x), \mu_{A}^{2}(x), \ldots \ldots \ldots \mu_{A}^{p}(x)\right) \text { where } \mu_{A}^{1}(x) \geq \mu_{A}^{2}(x) \geq \\
& \cdots \geq \mu_{A}^{p}(x) .
\end{aligned}
$$

Therefore, A $F M S A$ is given by

$$
A=\left\{\left\langle x,\left(\mu_{A}^{1}(x), \mu_{A}^{2}(x), \ldots \ldots \ldots \mu_{A}^{p}(x)\right)\right\rangle / x \in X\right\}
$$

### 2.4 Definition:

Let X be a nonempty set. A Intuitionistic Fuzzy Multi set (IFMS) $A$ in X is characterized by two functions namely count membership function Mc and count non membership function NMc such that Mc: $\mathrm{X} \rightarrow \mathrm{Q}$ and $\mathrm{NMc}: \mathrm{X} \rightarrow \mathrm{Q}$ where Q is the set of all crisp multi sets in $[0,1]$. Hence, for any $x \in X, \operatorname{Mc}(\mathrm{x})$ is the crisp multi set from $[0,1]$ whose membership sequence is defined as

$$
\left(\mu_{A}^{1}(x), \mu_{A}^{2}(x), \ldots \ldots \ldots \mu_{A}^{p}(x)\right)
$$

Where $\mu_{A}^{1}(x) \geq \mu_{A}^{2}(x) \geq \cdots \geq \mu_{A}^{p}(x)$ and the corresponding non membership sequence NMc (x) is defined as ( $\left.\vartheta_{A}^{1}(x), \vartheta_{A}^{2}(x), \ldots \ldots \ldots \vartheta_{A}^{p}(x)\right)$ where the non membership can be either decreasing or increasing function. such that $0 \leq \mu_{A}^{i}(x)+\vartheta_{A}^{i}(x) \leq 1, \forall x \in X$ and $i=1,2, \ldots p$.

Therefore, An IFMS $A$ is given by
$A=$
$\{\langle x$,
$\mu A 1 x, \mu A 2 x, \ldots \ldots \ldots \mu A p x,(\vartheta A 1 x, \vartheta A 2 x, \ldots \ldots \ldots \vartheta A p x)$
$/ x \in X$
Where $\mu_{A}^{1}(x) \geq \mu_{A}^{2}(x) \geq \cdots \geq \mu_{A}^{p}(x)$
The complementary set $A^{c}$ of $A$ is defined as

$$
\begin{gathered}
A^{c}=\left\{\left\langlex, \quad\left(\vartheta_{A}^{1}(x), \vartheta_{A}^{2}(x), \ldots \ldots \ldots \vartheta_{A}^{p}(x)\right)\right.\right. \\
\\
\left.\left(\mu_{A}^{1}(x), \mu_{A}^{2}(x), \ldots \ldots \mu_{A}^{p}(x)\right),\right\rangle / x \\
\in X\}
\end{gathered}
$$

where $\vartheta_{A}^{1}(x) \geq \vartheta_{A}^{2}(x) \geq \cdots \geq \vartheta_{A}^{p}(x)$

### 2.5 Definition:

The Cardinality of the membership function $\operatorname{Mc}(\mathrm{x})$ and the non membership function NMc (x) is the length of an element x in an IFMS A denoted as $\eta$, defined as $\eta=|\operatorname{Mc}(\mathrm{x})|=$ $|\operatorname{NMc}(\mathrm{x})|$

If A, B and C are the $I F M S$ defined on $X$, then their cardinality $\eta=\operatorname{Max}\{\eta(A), \eta(B), \eta(C)\}$.

### 2.6 Definition:

$S(\boldsymbol{A}, \boldsymbol{B})$ is said to be the similarity measure between A and B , where $\mathrm{A}, \mathrm{B} \in \mathrm{X}$ and X is an $I F M S$, as $S(\boldsymbol{A}, \boldsymbol{B})$ satisfies the following properties

1. $S(\boldsymbol{A}, \boldsymbol{B}) \in[0,1]$
2. $S(\boldsymbol{A}, \boldsymbol{B})=1$ if and only if $\mathrm{A}=\mathrm{B}$
3. $S(\boldsymbol{A}, \boldsymbol{B})=S(\boldsymbol{B}, \boldsymbol{A})$

## 3. CORRELATION MEASURE

### 3.1 Fuzzy Correlation Measure

Let $\mathrm{A}=\left\{\left\langle x_{i}, \mu_{A}\left(x_{i}\right)\right\rangle / x_{i} \in X\right\}$ and $\mathrm{B}=\left\{\left\langle x_{i}, \mu_{B}\left(x_{i}\right)\right\rangle /\right.$ $\left.x_{i} \in X\right\}$ be two $F S$ s on the finite universe of discourse $\mathrm{X}=\{$ $\left.x_{1}, x_{2}, \ldots, x_{n}\right\}$, then the correlation coefficient of A and B $[5,6]$ is

$$
\rho_{F S}(A, B)=\frac{C_{F S}(A, B)}{\sqrt{C_{F S}(A, A) * C_{F S}(B, B)}}
$$

Where $C_{F S}(A, B)=\sum_{i=1}^{n} \mu_{A}\left(x_{i}\right) \mu_{B}\left(x_{i}\right)$ and $C_{F S}(A, A)=$ $\sum_{i=1}^{n} \mu_{A}\left(x_{i}\right) \mu_{A}\left(x_{i}\right)$

### 3.2 Intuitionistic Fuzzy Correlation Measure

Let $\mathrm{X}=\left\{x_{1}, x_{2}, \ldots, x_{n}\right\}$ be the finite universe of discourse and $\mathrm{A}=\left\{\left\langle x_{i}, \mu_{A}\left(x_{i}\right), \vartheta_{A}\left(x_{i}\right)\right\rangle / x_{i} \in X\right\}, \quad \mathrm{B}=\left\{\left\langle x_{i}\right.\right.$, $\mu B x i, \vartheta B x i /$ xi $\in X\}$ be two IFSs then the correlation coefficient of A and B introduced by Gerstenkorn and Manko [7] was

$$
\rho_{I F S}(A, B)=\frac{C_{I F S}(A, B)}{\sqrt{C_{I F S}(A, A) * C_{I F S}(B, B)}}
$$

Where
$C_{I F S}(A, B)=\sum_{i=1}^{n}\left\{\mu_{A}\left(x_{i}\right) \mu_{B}\left(x_{i}\right)+\vartheta_{A}\left(x_{i}\right) \vartheta_{B}\left(x_{i}\right)\right.$ and
$C_{I F S}(A, B)=\sum_{i=1}^{n}\left\{\mu_{A}\left(x_{i}\right) \mu_{B}\left(x_{i}\right)+\vartheta_{A}\left(x_{i}\right) \vartheta_{B}\left(x_{i}\right)\right\}$
The correlation coefficient of A and B in X, the infinite universe of discourse defined by Hong and Hwang [8] is

$$
\rho_{I F S}(A, B)=\frac{C_{I F S}(A, B)}{\sqrt{C_{I F S}(A, A) * C_{I F S}(B, B)}}
$$

Where

$$
\begin{gathered}
C_{I F S}(A, B)=\int\left(\mu_{A}\left(x_{i}\right) \mu_{B}\left(x_{i}\right)+\vartheta_{A}\left(x_{i}\right) \vartheta_{B}\left(x_{i}\right)\right) d x \text { and } \\
C_{I F S}(A, A)=\int\left(\mu_{A}\left(x_{i}\right) \mu_{A}\left(x_{i}\right)+\vartheta_{A}\left(x_{i}\right) \vartheta_{A}\left(x_{i}\right)\right) d x
\end{gathered}
$$

### 3.3 Proposed Correlation Similarity Measure for IFMS

### 3.3.1 Intuitionistic Fuzzy Multi Correlation Measure

Let $\mathrm{X}=\left\{x_{1}, x_{2}, \ldots, x_{n}\right\}$ be the finite universe of discourse and $\mathrm{A}=\left\{\left\langle x_{i}, \mu_{\mathrm{A}}^{\mathrm{j}}\left(x_{i}\right), \vartheta_{\mathrm{A}}^{j}\left(x_{i}\right)\right\rangle / x_{i} \in X\right\}, \quad \mathrm{B}=\{$ $\left.\left\langle x_{i}, \mu_{\mathrm{B}}^{\mathrm{j}}\left(x_{i}\right), \vartheta_{\mathrm{B}}^{\mathrm{j}}\left(x_{i}\right)\right\rangle / x_{i} \in X\right\}$ be two IFMSs consisting of the membership and non membership functions, then the correlation coefficient of A and B

$$
\rho_{I F M S}(A, B)=\frac{C_{I F M S}(A, B)}{\sqrt{C_{I F M S}(A, A) * C_{I F M S}(B, B)}}
$$

Where $C_{I F M S}(A, B)=\frac{1}{\eta} \sum_{j=1}^{\eta}\left\{\sum_{i=1}^{n}\left(\mu_{A}^{j}\left(x_{i}\right) \mu_{B}^{j}\left(x_{i}\right)+\right.\right.$ vAjxi vBjxi and

$$
C_{I F M S}(A, A)=\frac{1}{\eta} \sum_{j=1}^{\eta}\left\{\sum _ { i = 1 } ^ { n } \left(\mu_{A}^{j}\left(x_{i}\right) \mu_{A}^{j}\left(x_{i}\right)+\right.\right.
$$ vAjxi vAjxi

Let $\mathrm{A}=\left\{\left\langle x_{i}, \mu_{\mathrm{A}}^{\mathrm{j}}\left(x_{i}\right), \vartheta_{\mathrm{A}}^{\mathrm{j}}\left(x_{i}\right), \pi_{\mathrm{A}}^{\mathrm{j}}\left(x_{i}\right)\right\rangle / x_{i} \in X\right\}$ and $\mathrm{B}=\{$ $\left.\left\langle x_{i}, \mu_{\mathrm{B}}^{\mathrm{j}}\left(x_{i}\right), \vartheta_{\mathrm{B}}^{\mathrm{j}}\left(x_{i}\right), \pi_{\mathrm{A}}^{\mathrm{j}}\left(x_{i}\right)\right\rangle / x_{i} \in X\right\}$ be two IFMSs consisting of the membership, non membership functions and the hesitation functions, then the correlation coefficient of A and B

$$
\rho_{I F M S}(A, B)=\frac{C_{I F M S}(A, B)}{\sqrt{C_{I F M S}(A, A) * C_{I F M S}(B, B)}}
$$

Where $C_{I F M S}(A, B)=\frac{1}{\eta} \sum_{j=1}^{\eta}\left\{\sum_{i=1}^{n}\left(\mu_{A}^{j}\left(x_{i}\right) \mu_{B}^{j}\left(x_{i}\right)+\right.\right.$ vAjxi vBjxi+ $\pi A j x i \pi B j x i$
and $\quad C_{I F M S}(A, A)=\frac{1}{\eta} \sum_{j=1}^{\eta}\left\{\sum_{i=1}^{n}\left(\mu_{A}^{j}\left(x_{i}\right) \mu_{A}^{j}\left(x_{i}\right)+\right.\right.$ ӨAjxi vAjxi $+\pi A j x i \pi B j x i$

### 3.4 Proposition

The defined Similarity measure $\boldsymbol{\rho}_{\text {IFMS }}(\boldsymbol{A}, \boldsymbol{B})$ between IFMS A and $B$ satisfies the following properties
D1. $0 \leq \rho_{I F M S}(A, B) \leq 1$
D2. $\quad \rho_{I F M S}(A, B)=1$ if and only if $A=B$
D3. $\rho_{I F M S}(A, B)=\rho_{I F M S}(B, A)$
Proof
D1. $\quad \mathbf{0} \leq \rho_{I F M S}(A, B) \leq \mathbf{1}$
As the membership and the non membership functions of the IFMSs lies between 0 and $1, \rho_{\text {IFMS }}(A, B)$ also lies between 0 and 1.

D2. $\quad \rho_{I F M S}(A, B)=1$ if and only if $A=B$
(i) Let the two $I F M S \mathrm{~A}$ and B be equal (i.e.) $\mathbf{A}=\mathbf{B}$. Hence for any $\mu_{A}^{j}\left(x_{i}\right)=\mu_{B}^{j}\left(x_{i}\right)$ and $\vartheta_{A}^{j}\left(x_{i}\right)=\vartheta_{B}^{j}\left(x_{i}\right)$ then $C_{I F M S}(A, A)=C_{I F M S}(B, B)=$ $\frac{1}{\eta} \sum_{j=1}^{\eta}\left\{\sum_{i=1}^{n}\left(\mu_{A}^{j}\left(x_{i}\right) \mu_{A}^{j}\left(x_{i}\right)+\vartheta_{A}^{j}\left(x_{i}\right) \vartheta_{A}^{j}\left(x_{i}\right)\right)\right\}$
and $\quad C_{I F M S}(A, B)=\frac{1}{\eta} \sum_{j=1}^{\eta}\left\{\sum_{i=1}^{n}\left(\mu_{A}^{j}\left(x_{i}\right) \mu_{B}^{j}\left(x_{i}\right)+\right.\right.$ $\left.\left.\vartheta_{A}^{j}\left(x_{i}\right) \vartheta_{B}^{j}\left(x_{i}\right)\right)\right\} \quad=$
$\frac{1}{\eta} \sum_{j=1}^{\eta}\left\{\sum_{i=1}^{n}\left(\mu_{A}^{j}\left(x_{i}\right) \mu_{A}^{j}\left(x_{i}\right)+\vartheta_{A}^{j}\left(x_{i}\right) \vartheta_{A}^{j}\left(x_{i}\right)\right)\right\}=$ $C_{\text {IFMS }}(A, A)$

Hence $\boldsymbol{\rho}_{\text {IFMS }}(\boldsymbol{A}, \boldsymbol{B})=$

$$
\frac{C_{I F M S}(A, B)}{\sqrt{C_{I F M S}(A, A) * C_{I F M S}(B, B)}}=\frac{C_{I F M S}(A, A)}{\sqrt{C_{\text {IFMS }}(A, A) * C_{I F M S}(A, A)}}=\mathbf{1}
$$

(ii) Let the $\boldsymbol{\rho}_{\text {IFMS }}(\boldsymbol{A}, \boldsymbol{B})=1$

The unit measure is possible only if

$$
\frac{C_{I F M S}(A, B)}{\sqrt{C_{I F M S}(A, A) * C_{I F M S}(B, B)}}=\mathbf{1}
$$

this refers that $\mu_{A}^{j}\left(x_{i}\right)=\mu_{B}^{j}\left(x_{i}\right)$ and $\vartheta_{A}^{j}\left(x_{i}\right)=\vartheta_{B}^{j}\left(x_{i}\right)$ for all $\mathrm{i}, \mathrm{j}$ values. Hence $\mathrm{A}=\mathrm{B}$.

D3. $\quad \rho_{I F M S}(A, B)=\rho_{I F M S}(B, A)$
It is obvious that $\rho_{\text {IFMS }}(A, B)=\frac{C_{I F M S}(A, B)}{\sqrt{C_{I F M S}(A, A) * C_{I F M S}(B, B)}}=$ $\frac{C_{\text {IFMS }}(B, A)}{\sqrt{C_{\text {IFMS }}(A, A) * C_{\text {IFMS }}(A, A)}}=\rho_{\text {IFMS }}(B, A)$ as
$C_{\text {IFMS }}(A, B)=\frac{1}{\eta} \sum_{j=1}^{\eta}\left\{\sum_{i=1}^{n}\left(\mu_{A}^{j}\left(x_{i}\right) \mu_{B}^{j}\left(x_{i}\right)+\right.\right.$ vAjxi vBjxi
$=\quad \frac{1}{\eta} \sum_{j=1}^{\eta}\left\{\sum_{i=1}^{n}\left(\mu_{B}^{j}\left(x_{i}\right) \mu_{A}^{j}\left(x_{i}\right)+\vartheta_{B}^{j}\left(x_{i}\right) \vartheta_{A}^{j}\left(x_{i}\right)\right)\right\}=$ $C_{I F M S}(B, A)$

### 3.5 Numerical Evaluation

### 3.5.1 Example:

Let $X=\left\{A_{1}, A_{2}, A_{3}, A_{4} \ldots \ldots . . A_{n}\right\}$ with $A=\left\{A_{1}, A_{2}, A_{3}, A_{4}\right.$, $\left.\mathrm{A}_{5}\right\}$ and $\mathrm{B}=\left\{\mathrm{A}_{6}, \mathrm{~A}_{7}, \mathrm{~A}_{8}, \mathrm{~A}_{9}, \mathrm{~A}_{10}\right\}$ are the IFMS defined as

## $A=\{$

$\left\langle A_{1}:(0.6,0.4),(0.5,0.5)\right\rangle,\left\langle A_{2}:(0.5,0.3),(0.4,0.5)\right\rangle$,
$\left\langle A_{3},(0.5,0.2),(0.4,0.4)\right\rangle,\left\langle A_{4}:(0.3,0.2),(0.3,0.2)\right\rangle$,
$\left.\left\langle A_{5}:(0.2,0.1),(0.2,0.2)\right\rangle\right\}$
$B=\{$
$\left\langle A_{6}:(0.8,0.1),(0.4,0.6)\right\rangle$,
$\left\langle A_{7}:(0.7,0.3),(0.4,0.2)\right\rangle,\left\langle A_{8},(0.4,0.5),(0.3,0.3)\right\rangle$
$\left.\left\langle A_{9}:(0.2,0.7),(0.1,0.8)\right\rangle,\left\langle A_{10}:(0.2,0.6),(0,0.6)\right\rangle\right\}$
Here, the cardinality $\eta=5$ as $|\operatorname{Mc}(A)|=|\operatorname{Ncc}(A)|=5$ and $|\operatorname{Mc}(\mathrm{B})|=|\mathrm{NMc}(\mathrm{B})|=5$ and the Correlation IFMS similarity measure is $=\mathbf{0 . 8 1 4 7}$

### 3.5.2 Example:

Let $X=\left\{A_{1}, A_{2}, A_{3}, A_{4} \ldots \ldots . A_{n}\right\}$ with $A=\left\{A_{1}, A_{2}\right\}$ and $B$ $=\left\{\mathrm{A}_{9}, \mathrm{~A}_{10}\right\}$ are the IFMS defined as
$\mathrm{A}=\left\{\left\langle A_{1}:(0.1,0.2)\right\rangle,\left\langle A_{2}:(0.3,0.3)\right\rangle\right\}, \mathrm{B}=\left\{\left\langle A_{9}:\right.\right.$ $: 0.1,0.2, A 10: 0.2,0.3\}$

Here, the cardinality $\eta=2$ as $|\operatorname{Mc}(A)|=|\operatorname{NMc}(A)|=2$ and $|\operatorname{Mc}(\mathrm{B})|=|\mathrm{NMc}(\mathrm{B})|=2$ and the Correlation IFMS similarity measure is $\mathbf{0 . 9 8 2 7}$

### 3.5.3 Example:

Let $\mathrm{X}=\left\{\mathrm{A}_{1}, \mathrm{~A}_{2}, \mathrm{~A}_{3}, \mathrm{~A}_{4} \ldots \ldots . \mathrm{A}_{\mathrm{n}}\right\}$ with $\mathrm{A}=\left\{\mathrm{A}_{1}, \mathrm{~A}_{2}\right\}$ and B $=\left\{\mathrm{A}_{3}, \mathrm{~A}_{4}\right\}$ are the IFMS defined as

$$
\begin{aligned}
& \mathrm{A}=\left\{\left\langle A_{1}(0.4,0.2,0.1),(0.3,0.1,0.2),(0.2,0.1,0.2),(0.1,0.4,0.3)\right\rangle,\right. \\
& \left.\left\langle A_{2}(0.6,0.3,0),(0.4,0.5,0.1),(0.4,0.3,0.2),(0.2,0.6,0.2)\right\rangle\right\} \\
& \mathrm{B}=\left\{\left\langle A_{3}:(0.5,0.2,0.3),(0.4,0.2,0.3),(0.4,0.1,0.2),(0.1,0.1,0.6)\right\rangle\right. \\
& \left.\left\langle A_{4}:(0.4,0.6,0.2),(0.4,0.5,0),(0.3,0.4,0.2),(0.2,0.4,0.1)\right\rangle\right\}
\end{aligned}
$$

The cardinality $\eta=2$ as $|\mathrm{Mc}(\mathrm{A})|=|\mathrm{NMc}(\mathrm{A})|=|\mathrm{Hc}(\mathrm{A})|=2$ and $|\operatorname{Mc}(B)|=|\operatorname{NMc}(B)|=|\operatorname{Hc}(B)|=2$. Here, the Correlation IFMS similarity is $\mathbf{0 . 8 9 3 9}$

### 3.5.4 Example:

Let $X=\left\{A_{1}, A_{2}, A_{3}, A_{4} \ldots . A_{n}\right\}$ with $A=\left\{A_{1}, A_{2}\right\}$ and $B=$ $\left\{\mathrm{A}_{6}\right\}$ such that the IFMS A and B are

$$
\begin{aligned}
\mathrm{A}= & \left\{\left\langle A_{1}:(0.6,0.2,0.2),(0.4,0.3,0.3),(0.1,0.7,0.2)\right\rangle,\right. \\
& \left.\left\langle A_{2}:(0.7,0.1,0.2),(0.3,0.6,0.1),(0.2,0.7,0.1)\right\rangle\right\} \\
\mathrm{B}=\{ & \left.\left\langle A_{6}:(0.8,0.1,0.1),(0.2,0.7,0.1),(0.3,0.5,0.2)\right\rangle\right\}
\end{aligned}
$$

As $|\operatorname{Mc}(\mathrm{A})|=|\operatorname{NMc}(\mathrm{A})|=|\operatorname{Hc}(\mathrm{A})|=2$ and $|\operatorname{Mc}(\mathrm{B})|=$ $|\mathrm{NMc}(\mathrm{B})|=|\mathrm{Hc}(\mathrm{B})|=1$,
their cardinality $\eta=\operatorname{Max}\{\eta(A), \eta(B)\}=\max \{2,1\}=2$. The Correlation IFMS measure is $\mathbf{0 . 9 2 1 4}$

## 4. MEDICAL DIAGNOSIS USING IFMS CORRELATION MEASURE

As Medical diagnosis contains lots of uncertainties, they are the most interesting and fruitful areas of application for fuzzy set theory. A symptom is an uncertain indication of a disease and hence the uncertainty characterizes a relation between symptoms and diseases. Thus working with the uncertainties leads us to accurate decision making in medicine. In most of the medical diagnosis problems, there exist some patterns, and the experts make decision based on the similarity between unknown sample and the base patterns.

In some situations, terms of membership function (fuzzy set theory) alone is not adequate. Hence, the terms like membership and non membership function (Intuitionistic fuzzy set theory) is considered to be the better one. Due to the increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of disease becomes difficult. Recently, there are various models of medical diagnosis under the general framework of fuzzy sets are proposed. In some practical situations, there is the possibility of each element having different membership and non membership functions. The distance and similarity measure among the Patients Vs Symptoms and Symptoms Vs diseases gives the proper medical diagnosis. Here, the proposed correlation measure point out the proper diagnosis by the highest similarity measure.

The unique feature of this proposed method is that it considers multi membership and non membership. By taking one time inspection, there may be error in diagnosis. Hence, this multi time inspection, by taking the samples of the same patient at different times gives best diagnosis.

Let $P=\left\{P_{1}, P_{2}, P_{3}, P_{4}\right\}$ be a set of Patients.
$D=\{$ Fever, Tuberculosis, Typhoid, Throat disease $\}$ be the set of diseases
and $S=\{$ Temperature, Cough, Throat pain, Headache, Body pain \} be the set of symptoms.

Our solution is to examine the patient at different time intervals (three times a day), which in turn give arise to different membership and non membership function for each patient.

Table: $\mathbf{1}$ - IFMs Q : The Relation between Patient and Symptoms

| Q | Temperature | Cough | Throat Pain | Head Ache | Body Pain |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}_{1}$ | $(0.6,0.2)$ | $(0.4,0.3)$ | $(0.1,0.7)$ | $(0.5,0.4)$ | $(0.2,0.6)$ |
|  | $(0.7, .1)$ | $(0.3,0.6)$ | $(0.2,0.7)$ | $(0.6,0.3)$ | $(0.3,0.4)$ |
|  | $(0.5,0.4)$ | $(0.4,0.4)$ | $(0,0.8)$ | $(0.7,0.2)$ | $(0.4,0.4)$ |
| $\mathrm{P}_{2}$ | $(0.4,0.5)$ | $(0.7,0.2)$ | $(0.6,0.3)$ | $(0.3,0.7)$ | $(0.8,0.1)$ |
|  | $(0.3,0.4)$ | $(0.6,0.2)$ | $(0.5,0.3)$ | $(0.6,0.3)$ | $(0.7,0.2)$ |
|  | $(0.5,0.4)$ | $(0.8,0.1)$ | $(0.4,0.4)$ | $(0.2,0.7)$ | $(0.5,0.3)$ |
| $\mathrm{P}_{3}$ | $(0.1,0.7)$ | $(0.3,0.6)$ | $(0.8,0)$ | $(0.3,0.6)$ | $(0.4,0.4)$ |
|  | $(0.2,0.6)$ | $(0.2,0)$ | $(0.7,0.1)$ | $(0.2,0.7)$ | $(0.3,0.7)$ |
|  | $(0.1,0.9)$ | $(0.1,0.7)$ | $(0.8,0.1)$ | $(0.2,0.6)$ | $(0.2,0.7)$ |
| $\mathrm{P}_{4}$ | $(0.5,0.4)$ | $(0.4,0.5)$ | $(0.2,0.7)$ | $(0.5,0.4)$ | $(0.4,0.6)$ |
|  | $(0.4,0.4)$ | $(0.3,0.3)$ | $(0.1,0.6)$ | $(0.6,0.3)$ | $(0.5,0.4)$ |
|  | $(0.5,0.3)$ | $(0.1,0.7)$ | $(0,0.7)$ | $(0.3,0.6)$ | $(0.4,0.3)$ |

Let the samples be taken at three different timings in a day (morning, noon and night)

Table: 2 - IFMs R: The Relation among Symptoms and Diseases

| R | Viral Fever | Tuberculosis | Typhoid | Throat disease |
| :---: | :---: | :---: | :---: | :---: |
| Temperature | $(0.8,0.1)$ | $(0.2,0.7)$ | $(0.5,0.3)$ | $(0.1,0.7)$ |
| Cough | $(0.2,0,7)$ | $(0.9,0)$ | $(0.3,0,5)$ | $(0.3,0,6)$ |
| Throat Pain | $(0.3,0.5)$ | $(0.7,0.2)$ | $(0.2,0.7)$ | $(0.8,0.1)$ |
| Head ache | $(0.5,0.3)$ | $(0.6,0.3)$ | $(0.2,0.6)$ | $(0.1,0.8)$ |
| Body ache | $(0.5,0.4)$ | $(0.7,0.2)$ | $(0.4,0.4)$ | $(0.1,0.8)$ |

Table: 3 - The Correlation Measure between IFMs Q and R :

| Correlation <br> Measure | Viral Fever | Tuberculosis | Typhoid | Throat disease |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}_{1}$ | $\mathbf{0 . 9 1 1 7}$ | 0.6835 | 0.9096 | 0.5767 |
| $\mathrm{P}_{2}$ | 0.7824 | $\mathbf{0 . 9 1 4 0}$ | 0.8199 | 0.7000 |
| $\mathrm{P}_{3}$ | 0.6289 | 0.7319 | 0.7143 | $\mathbf{0 . 9 3 4 9}$ |
| $\mathrm{P}_{4}$ | 0.8795 | 0.6638 | $\mathbf{0 . 9 4 3 7}$ | 0.6663 |

The highest similarity measure from the table 4.3 gives the proper medical diagnosis.
Patient P1 suffers from Viral Fever, Patient P2 suffers from Tuberculosis, Patient P3 suffers from Throat disease and the Patient P4 suffers from Typhoid.

## 5. PATTERN RECOGNISION OF IFMS

## CORRELATION SIMILARITY MEASURE

### 5.1 Example

Let $X=\left\{A_{1}, A_{2}, A_{3}, A_{4} \ldots \ldots . A_{n}\right\}$ with $A=\left\{A_{1}, A_{2}, A_{3}, A_{4}\right.$, $\left.\mathrm{A}_{5}\right\}$ and $\mathrm{B}=\left\{\mathrm{A}_{2}, \mathrm{~A}_{5}, \mathrm{~A}_{7}, \mathrm{~A}_{8}, \mathrm{~A}_{9}\right\}$ are the IFMS defined as

Pattern I $=\left\{\left\langle A_{1}:(0.6,0.4),(0.5,0.5)\right\rangle,\left\langle A_{2}:\right.\right.$
$: 0.5,0.3,0.4,0.5, ~ A 3,0.5,0.2,0.4,0.4$,
$\left.\left\langle A_{4}:(0.3,0.2),(0.3,0.2)\right\rangle,\left\langle A_{5}:(0.2,0.1),(0.2,0.2)\right\rangle\right\}$

Pattern II $=\left\{\left\langle A_{2}:(0.5,0.3),(0.4,0.5)\right\rangle,\left\langle A_{5}:\right.\right.$
(0.2,0.1), (0.2, 0.2) $\rangle$
$\left\langle A_{7}:(0.7,0.3),(0.4,0.2)\right\rangle,\left\langle A_{8},(0.4,0.5),(0.3,0.3)\right\rangle$, $\left.\left\langle A_{9}:(0.2,0.7),(0.1,0.8)\right\rangle\right\}$

Then the testing IFMS Pattern III be $\left\{\mathrm{A}_{6}, \mathrm{~A}_{7}, \mathrm{~A}_{8}, \mathrm{~A}_{9}, \mathrm{~A}_{10}\right\}$ such that $\left\{\left\langle A_{6}:(0.8,0.1),(0.4,0.6)\right\rangle\right.$,
$\left\langle A_{7}:(0.7,0.3),(0.4,0.2)\right\rangle,\left\langle A_{8},(0.4,0.5),(0.3,0.3)\right\rangle,\left\langle A_{9}:\right.$ :0.2,0.7, 0.1, $0.8, A 10: 0.2,0.6,0,0.6\}$

Here, the cardinality $\eta=5$ as $\quad|\mathrm{Mc}(\mathrm{A})|=|\mathrm{NMc}(\mathrm{A})|=$
5 and $|\operatorname{Mc}(\mathrm{B})|=|\operatorname{NMc}(\mathrm{B})|=5$
then the Correlation Similarity measure between Pattern (I, III) is 0.8147 , Pattern (II, III) is $\mathbf{0 . 8 8 5 2}$

## The testing Pattern III belongs to Pattern II type

### 5.2 Example:

Let $X=\left\{A_{1}, A_{2}, A_{3}, A_{4} \ldots \ldots . \mathrm{A}_{n}\right\}$ with $\mathrm{A}=\left\{\mathrm{A}_{1}, \mathrm{~A}_{2}\right\} ; B=\{$ $\left.\mathrm{A}_{4}, \mathrm{~A}_{6}\right\} ; \mathrm{C}=\left\{\mathrm{A}_{1}, \mathrm{~A}_{10}\right\} ; \mathrm{D}=\left\{\mathrm{A}_{4}, \mathrm{~A}_{6}\right\} ; \quad \mathrm{E}=\left\{\mathrm{A}_{4}\right.$,
$\mathrm{A}_{6}$ \}are the IFMS defined as
$\mathrm{A}=\left\{\left\langle A_{1}:(0.1,0.2)\right\rangle,\left\langle A_{2}:(0.3,0.3)\right\rangle\right\} ; \quad \mathrm{B}=\left\{\left\langle A_{4}:\right.\right.$
:0.2,0.2, $A 6: 0.3,0.2\} ;$
$\mathrm{C}=\left\{\left\langle A_{1}:(0.1,0.2)\right\rangle,\left\langle A_{10}:(0.2,0.3)\right\rangle\right\} ; \mathrm{D}=\left\{\left\langle A_{3}:\right.\right.$ :0.2,0.1, A4:0.3, 0.2 \};
$\mathrm{E}=\left\{\left\langle A_{1}:(0.5,0.4)\right\rangle,\left\langle A_{4}:(0.8,0.1)\right\rangle\right\}$
The IFMS Pattern $\mathrm{Y}=\left\{\quad\left\langle A_{1}:(0.1,0.2)\right\rangle,\left\langle A_{10}:\right.\right.$ $: 0.2,0.3$ \}
Here, the cardinality $\eta=2$ as $|\operatorname{Mc}(A)|=|N M c(A)|=2$ and $|\operatorname{Mc}(\mathrm{B})|=|\operatorname{NMc}(\mathrm{B})|=2$,
then the Correlation measure between the Patten $(\mathrm{A}, \mathrm{Y})=$ 0.9829, Patten $(B, Y)=0.9258$, Patten $(C, Y)=1$, Patten $(D$, $\mathrm{Y})=0.8889$, Patten $(\mathrm{E}, \mathrm{Y})=0.7325$ and the testing Pattern $Y$ belongs to Pattern $C$ type

### 5.3 Example:

Let $\mathrm{X}=\left\{\mathrm{A}_{1}, \mathrm{~A}_{2}, \mathrm{~A}_{3}, \mathrm{~A}_{4} \ldots \ldots . . \mathrm{A}_{\mathrm{n}}\right\}$ with $\mathrm{X} 1=\left\{\mathrm{A}_{1}, \mathrm{~A}_{2}\right\} ; \mathrm{X} 2$ $=\left\{\mathrm{A}_{3}, \mathrm{~A}_{4}\right\} ; \mathrm{X} 3=\left\{\mathrm{A}_{1}, \mathrm{~A}_{4}\right\}$ are the IFMS defined as
$\mathrm{A}=\left\{\left\langle A_{1}:(0.4,0.2,0.1),(0.3,0.1,0.2),(0.2,0.1,0.2),(0.1,0.4,0.3)\right\rangle\right.$, $\left.\left\langle A_{2}:(0.6,0.3,0),(0.4,0.5,0.1),(0.4,0.3,0.2),(0.2,0.6,0.2)\right\rangle\right\}$
$\mathrm{B}=\left\{\left\langle A_{3}:(0.5,0.2,0.3),(0.4,0.2,0.3),(0.4,0.1,0.2),(0.1,0.1,0.6)\right\rangle\right.$
$\left.\left\langle A_{4}:(0.4,0.6,0.2),(0.4,0.5,0),(0.3,0.4,0.2),(0.2,0.4,0.1)\right\rangle \quad\right\}$
$\mathrm{C}=\left\{\left\langle A_{1}:(0.4,0.2,0.1),(0.3,0.1,0.2),(0.2,0.1,0.2),(0.1,0.4,0.3)\right\rangle\right.$,
$\left.\left\langle A_{4}:(0.4,0.6,0.2),(0.4,0.5,0),(0.3,0.4,0.2),(0.2,0.4,0.1)\right\rangle\right\}$
then the Pattern D of $I F M S$ referred as $\{$

$$
\left\langle A_{5}:(0.4,0.6,0.2),(0.4,0.5,0),(0.3,0.4,0.2),(0.2,0.4,0.1)\right\rangle,
$$

$$
\left.\left\langle A_{6}:(0.4,0.2,0.2),(0.5,0.5,0),(0.2,0.4,0.2),(0.2,0.5,0.1)\right\rangle\right\}
$$

The cardinality $\eta=2$ as $|\operatorname{Mc}(\mathrm{A})|=|\operatorname{NMc}(\mathrm{A})|=|\operatorname{Hc}(\mathrm{A})|=$ 2 and $|\operatorname{Mc}(B)|=|\operatorname{NMc}(B)|=|\operatorname{Hc}(B)|=2$, then the Proposed Correlation Similarity measure between the Pattern (A, D) is $\mathbf{0 . 8 6 2 8}$; the Pattern (B, D) is 0.8175 and the Pattern (C, D) is 0.8597 .

Hence, the testing Pattern D belongs to Pattern A type

## 6. CONCLUSIONS

The Correlation similarity measure of IFMS from IFS theory is derived in this paper. The prominent characteristic of this method is that the Correlation measure of any two IFMSs equals to one if and only if the two IFMSs are the same, referred in the example 5:2 of pattern recognition. From the numerical evaluation, it is clear that this proposed method can be applied to decision making problems. The example 3.5.1, 3.5.2 of numerical evaluation shows that the new measure perform well in the case of membership and non membership function and example 3.5.3, 3.5.4 of numerical evaluation depicts that the proposed measure is effective with three representatives of IFMS - membership, non membership and hesitation functions. Finally, the medical diagnosis has been given to show the efficiency of the developed Correlation similarity measure of IFMS.

## REFERENCES

[1]. Atanassov K., Intuitionistic fuzzy sets, Fuzzy Sets and System 20 (1986) 87-96.
[2]. Atanassov K., More on Intuitionistic fuzzy sets, Fuzzy Sets and Systems 33 (1989) 37-46.
[3]. Zadeh L. A., Fuzzy sets, Information and Control 8 (1965) 338-353.
[4]. Murthy C.A., Pal S.K., Majumder D. D., Correlation between two fuzzy membership functions, Fuzzy Sets and Systems 17 (1985) 23-38.
[5]. Chiang D.A., Lin N.P., Correlation of fuzzy sets, Fuzzy Sets and Systems 102 (1999) 221-226.
[6]. Chaudhuri B.B., Bhattachary P., On correlation between fuzzy sets, Fuzzy Sets and Systems 118 (2001) 447-456.
[7]. Gerstenkorn T., Manko J., Correlation of Intuitionistic fuzzy sets, Fuzzy Sets and Systems 44 (1991) 39-43.
[8]. Hong D.H., Hwang, S. Y., Correlation of Intuitionistic fuzzy sets in probability spaces, Fuzzy Sets and Systems 75 (1995) 77-81.
[9]. Hung W.L., Wu J.W., Correlation of Intuitionistc fuzzy sets by centroid method, Information Sciences 144 (2002) 219-225.
[10]. Hung W.L., Using statistical viewpoint in developing correlation of Intuitionistic fuzzy sets, International Journal of Uncertainty Fuzziness Knowledge Based Systems 9 (2001) 509-516.
[11]. Mitchell H.B., A Correlation coefficient for Intuitionistic fuzzy sets, International Journal of Intelligent Systems 19 (2004) 483-490.
[12]. Wenyi Zeng, Hongxing Li ., Correlation coefficient of Intuitionistic fuzzy sets, Journal of Industrial Engineering International Vol. 3, No. 5 (2007) 33-40.
[13]. Blizard W. D., Multi set Theory, Notre Dame Journal of Formal Logic, Vol. 30, No. 1 (1989) 36-66.
[14]. Yager R. R., On the theory of bags,(Multi sets), Int. Jou. Of General System, 13 (1986) 23-37.
[15]. Shinoj T.K., Sunil Jacob John , Intuitionistic Fuzzy Multi sets and its Application in Medical Diagnosis, World Academy of Science, Engineering and Technology, Vol. 61 (2012).
[16]. P. Rajarajeswari., N. Uma., On Distance and Similarity Measures of Intuitionistic Fuzzy Multi Set, IOSR Journal of Mathematics (IOSR-JM) Vol. 5, Issue 4 (Jan. Feb. 2013) 19-23.
[17]. P. Rajarajeswari., N. Uma., Hausdroff Similarity measures for Intuitionistic Fuzzy Multi Sets and Its Application in Medical diagnosis, International Journal of Mathematical Archive-4(9),(2013) 106-111.
[18]. P. Rajarajeswari., N. Uma., A Study of Normalized Geometric and Normalized Hamming Distance Measures in Intuitionistic Fuzzy Multi Sets, International Journal of Science and Research (IJSR), Vol. 2, Issue 11, November 2013, 76-80.
[19]. P. Rajarajeswari., N. Uma., Intuitionistic Fuzzy Multi Similarity Measure Based on Cotangent Function, International Journal of Engineering Research \& Technology (IJERT) Vol. 2 Issue 11, (Nov- 2013) 1323-1329.

