Correlation of Rates of Intramolecular Tunneling Processes, with Application to Some Group V Compounds

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A method is presented whereby the WKB expression for tunneling frequencies permits correlation of these frequencies for members of homologous series of molecules. The necessary data required are the tunneling rate for one member of the series, and structural parameters and vibrational frequencies for all members of interest. The method is applied to ND3, NF3, PH3, and AsH3, using NH3 as the molecule whose frequency is known. The method also is used to support the hypothesis that pseudorotation occurs in the trigonal bipyramids PF5 and PCl5.

I. INTRODUCTION

THE study of intramolecular tunneling motions has A focused on the evaluation of two characteristic quantities: the energy level splittings or tunneling frequencies, on one hand, and the heights of energy barriers inhibiting internal motions, on the other. Barrier heights have been examined extensively¹ because they are sensitive parameters and easily understood whenever the shape of a barrier is well established. And in the class of molecules for which barriers are best known, the barrier shape is well represented by a cosine function.2 In contrast, there are many systems which can exhibit intramolecular tunneling for which the barrier shape is not well established, and, in fact, for which tunneling may have been observed for but one or two members of a large series. When one is faced with such a situation, it is more useful to be able to estimate the tunneling frequency than to try to evaluate a barrier height. First, the tunneling frequency or energylevel splitting is often directly observable; second, the barrier height is meaningful only if the form of the potential-energy surface is known. With relatively exact data regarding the energy-level separations, one can try to investigate the shape of the potential in detail. Without such information, the barrier height itself gives very little information about the observable properties and dynamical behavior of a molecule. It therefore becomes useful to be able to estimate tunneling frequencies by some process which does not depend in any critical way on exact shapes or heights of potential barriers.

The ensuing discussion develops a method for estimating tunneling frequencies for members of a homologous series of compounds. The motivation for this treatment is twofold. First, hopefully, it will provide a semiquantitative method whose accuracy is sufficient to allow one to select the appropriate experimental technique for determining any desired tunneling frequency in the laboratory. It should enable one, for example, to choose between infrared, microwave, and

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mining a given tunneling rate. Second, the method is used to develop a possible interpretation of the observed behavior of PF5 and PCl5, an interpretation based on the hypothesis that internal tunneling occurs in these molecules. In Sec. II, a general treatment is developed in which

radio-frequency spectroscopy as possible tools for deter-

one obtains tunneling frequencies for any member of a homologous series if the appropriate frequency is known for a single member. The method, based on the Dennison-Uhlenbeck formulation of the WKB approach, uses as its primary data the structural parameters and vibration frequencies of the various molecules under consideration. Section III is devoted to the application of the method to the pyramidal molecules ND₃, NF₃, PH₃, and AsH₃; NH₃ is taken as the molecule whose tunneling frequency is known. This application provides a basis from which one can assess the accuracy of the method and determine which of several possible approximations is most appropriate. Finally, Sec. IV contains a discussion of the possibility of internal motion in PF5 and PCl5, with an attempt to reconcile the available experimental results for these two molecules.

II. GENERAL FORMULATION

An expression for the tunneling frequency for an internal motion, like the inversion of ammonia, has been derived by Dennison and Uhlenbeck,3 using the WKB method. They find that if the tunneling motion consists predominantly of one normal vibrational mode, and if $\nu = \text{tunneling frequency}$, $\nu_0 = \text{vibrational fre-}$ quency, μ =reduced mass for tunneling coordinate, $s = \text{displacement along the tunneling coordinate}, s_0 = s$ at which E=V(s), measured from the barrier maximum, V(s) = potential energy, and ϵ = total energy, then

$$\nu = (\nu_0/\pi) e^{\tau}, \tag{1}$$

where

$$I = -\left(1/\hbar\right) \int_{-s_0}^{s_0} \left\{ 2\mu \left[V(s) - \epsilon\right] \right\}^{\frac{1}{2}} ds. \tag{2}$$

¹ For a review of recent results, see E. B. Wilson, Proc. Natl. Acad. Sci. U. S. 43, 816 (1957).

² D. R. Herschbach and J. D. Swalen, J. Chem. Phys. 29, 761

³ D. M. Dennison and G. E. Uhlenbeck, Phys. Rev. 41, 313 (1932).

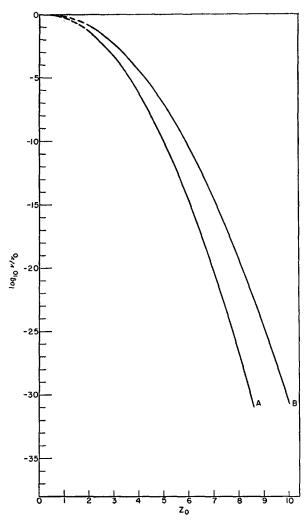


Fig. 1. Logarithmic curves of the ratio of the tunneling frequency ν to the principal vibrational frequency ν_0 along the tunneling coordinate, ν_0 the reduced distance Z_0 (number of classical amplitudes for oscillator of frequency ν_0) from the potential minimum to the potential barrier maximum. Curve A represents the quadratic approximation, Curve B the cosine form.

The expression for I can be integrated for a variety of potentials V. With the barrier height V(s=0) sufficiently high, so that two or more vibrational levels lie below the barrier, it is convenient for us to change the variable of integration. When V is high, the lowest vibrational state is approximately that of a harmonic oscillator with frequency $\omega_0 = 2\pi\nu_0$; we introduce the variable

$$Z = (\mu \omega_0 / \hbar)^{\frac{1}{2}} r, \tag{3a}$$

where r is the displacement along the tunneling coordinate with the origin at a potential minimum. Then if R is the distance from the origin to the potential maximum, we let

$$Z_0 = (\mu \omega_0 / \hbar)^{\frac{1}{2}} R. \tag{3b}$$

The dimensionless variable Z has as its unit the classical amplitude of oscillation for an oscillator of frequency ω_0 , in its lowest quantum state. Therefore at s_0 , Z=1. The expression for I becomes

$$I = -2(2/\hbar\omega_0)^{\frac{1}{2}} \int_1^{z_0} [V(Z) - \epsilon]^{\frac{1}{2}} dZ.$$
 (4)

If we let $\epsilon = \frac{1}{2}\hbar\omega_0$, then

$$I = -2 \int_{1}^{z_0} [V(Z)/\epsilon - 1]^{\frac{\epsilon}{2}} dZ, \qquad (5)$$

which is a particularly useful form to use for the integration. For most reasonable potentials, V has a quadratic leading term, so that the tunneling frequency will vary roughly as

$$\nu \sim \exp{-Z_0^2 f(Z_0)},\tag{6}$$

where $f(Z_0)$ is constant or slowly varying compared with Z_0^2 . If V is formed from two intersecting parabolas, then the integrated expression gives, for the nth vibrational level,

$$I_{q} = -\left[Z_{0}(Z_{0}^{2} - 2n - 1)^{\frac{1}{2}} - (2n + 1)\log Z_{0} + (Z_{0}^{2} - 2n - 1)^{\frac{1}{2}}\right].$$
 (7)

Alternatively, if V is represented as a cosine function,

$$V_{\cos} = \frac{1}{2} V_0 [1 - \cos(\pi Z/Z_0)],$$
 (8)

then for a state of energy ϵ ,

$$I_{\text{pos}} = (16\sqrt{2}/\pi^3) Z_0^2 \lceil E(k) - (\epsilon/V_0) K(k) \rceil, \qquad (9)$$

where $k = [(V_0 - \epsilon)/V_0]^{\frac{1}{2}}$ and E(k) and K(k) are the complete elliptic integrals of the second and first kinds, respectively. Approximating ϵ as $\hbar\omega_0/2$ as we have done, with ω_0 the circular frequency for a harmonic oscillator

TABLE I. Data used for treatment of NH3-like molecules.

Molecule	<i>r</i> _{X-Y} (A)	θа	$\nu_2^{\rm b}~{ m (cm^{-1})}$	μ (amu)
NH_3	1.014°	25° 2′°	1055c,d	2.47°
$\mathrm{ND_3}$	1.014°	25° 2′°	805ª	4.200
NF_3	1.371°	26° 3′°	505f	11.24
$\mathrm{PH_{3}}$	1.428	32° 45′s	992ь	2.73
AsH_3	1.528	33° 50′¤	974 i	2.88

 $^{^{\}rm a}\,\theta$ is the angle between the X—Y bond and the plane perpendicular to the molecular symmetry axis.

b Frequencies are observed values except for NH2, ND3, and AsH2, where harmonic frequencies were used (see reference 11 and d. below).

^e See reference 12.

^d D. M. Dennison, Revs. Modern Phys. 12, 175 (1940).

e P. Kisliuk, J. Chem. Phys. 22, 86 (1954).

f Bailey, Hale, and Thompson, J. Chem. Phys. 5, 274 (1937).

² C. C. Loomis and M. W. P. Strandberg, Phys. Rev. 81, 798 (1951); L. S. Bartell and R. C. Hirst, J. Chem. Phys. 31, 449 (1959).

h V. M. McConaghie and H. H. Nielsen, J. Chem. Phys. 21, 1836 (1953).

i V. M. McConaghie and H. H. Nielsen, Phys. Rev. 75, 633 (1949).

Molecule	Quadratic approximation		Cosine approximation			
	Z_0	$\nu(\mathrm{sec}^{-1})$	$\tau(sec)$	Z_0	$\nu(\mathrm{sec}^{-1})$	$ au(\mathrm{sec})$
NH ₃	2.88			3.42	2.3786×10 ¹⁰ (obs.) ^a	
ND_3	3.29	1.84×10°	2.7×10 ⁻¹⁰	3.91	1.43×10 ⁹ 1.6×10 ⁹ (obs.) ^a 1.25×10 ⁹ (calc.) ^b	3.5×10 ⁻¹⁰
NF3	5.96 5.98	$0.031 \\ 0.023$	16 22	7.08 7.10	0.015 0.012	33 42
$^{ m PH_3}$	5.26 5.38	156. 45.2	3.2×10^{-3} 1.1×10^{-2}	6.25 6.39	78.5 21.4	6.4×10^{-3} 2.3×10^{-2}
AsH₃	5.90 6.06	0.130 0.020	$\begin{array}{c} 3.9 \\ 25 \end{array}$	7.00 7.19	5.9×10^{-2} 1.5×10^{-3}	8.5 53

Table II. Parameters, tunneling frequencies and times for NH₃-like molecules. Upper values are taken with Y₃ moving as a rigid triangle, lower values with the X-Y bond length held constant.

whose force constant is the same as that of V_{cos} , then Eq. (9) can be simplified by the substitution

$$\epsilon/V_0 = \pi^2/4Z_0^2$$
. (10)

In both these cases, the tunneling frequency is indeed roughly dependent on $\exp(-AZ_0^2)$. This suggests that one might correlate tunneling frequencies for similar molecules by assuming any reasonable potential, providing (a) one knows the tunneling frequency for one molecule of the set, and (b) the tunneling coordinate is dominated by one normal mode, or at least by modes of comparable frequency. Then the quantity Z_0 becomes a parameter of correlation only, and depends on the potential form chosen; at the same time the form of Z_0 demands that for any two substances A and B, their Z_0 parameters be related by the condition that

$$Z_0(A)/Z_0(B) = (\mu_A \omega_{0A}/\mu_B \omega_{0B})^{\frac{1}{2}} (R_A/R_B).$$
 (11)

The masses μ_A and μ_B , the frequencies ω_{0A} and ω_{0B} , and the distances R_A and R_B are determined in a great number of cases by the symmetry, the structural parameters, and the normal vibration frequencies. Consequently one can pick a likely analytical form for V, and reading values from a plot of $\log_{10}(\nu/\nu_0)$ vs Z_{0} determine the approximate tunneling rate for any member of a homologous series, knowing previously the rate for one member only.

In Fig. 1 are plotted the curves of $\log_{10}(\nu/\nu_0)$ vs Z_0 for the cosine and parabolic potentials. The two curves are nearly parallel, and the similarity of results they give is demonstrated in Sec. III.

III. APPLICATION TO SOME XY3 PYRAMIDAL MOLECULES

The tunneling process corresponding to the inversion of NH₃ and similar molecules provides a straightforward test of the method developed in Sec. II. Inversion splittings are well known for NH₃ and ND₃, but have not been observed spectroscopically for any heavier homologs.4 The problem of inversion frequencies has been treated by several authors.5-11 Of principal interest have been the barrier shape and height although there has been one method developed to correlate the NH₃, PH₃, and AsH₃ barriers and tunneling frequencies.11 The most accurate potential for which both NH₃ and ND₃ tunneling frequencies are known is the one developed by Manning⁸; this, together with the observed results, will serve as a standard for our approximation method.

The tunneling frequencies of ND₃, NF₃, PH₃, and AsH₃ were calculated four ways; the two potentials were each tried with the assumptions that in XY₃, the XY bond lengths remain constant, or alternatively, that Y₃ moves as a rigid triangle. The tunneling coordinate of NH3 almost certainly is dominated by one normal coordinate, the ν_2 vibration. This vibration in turn, expressed in valence force-field coordinates, consists principally of the S_2 angle-bending vibration¹²:

$$S_1/S_2=1/10.96$$
,

based on the data contained in Table I. (This value is

⁸ See reference 4.

b See reference 4 or 8.

⁴C. H. Townes and A. L. Schawlow, Microwave Spectroscopy (McGraw-Hill Book Company, Inc., New York, 1955).

F. Hund, Z. Physik 43, 805 (1927).

P. M. Morse and E. C. G. Stueckelberg, Helv. Phys. Acta 4,

^{337 (1931).}

⁷ N. Rosen and P. M. Morse, Phys. Rev. **42**, 210 (1932).

M. F. Manning, J. Chem. Phys. 3, 136 (1935).
 F. T. Wall and G. Glockler, J. Chem. Phys. 5, 314 (1937).
 R. R. Newton and L. H. Thomas, J. Chem. Phys. 16, 310

^{(1948).} ¹¹ C. C. Costain and G. B. B. M. Sutherland, J. Phys. Chem. **56,** 321 (1952).

¹² G. Herzberg, Infrared and Raman Spectra (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1945). See especially, pp. 154-157, 175-177.

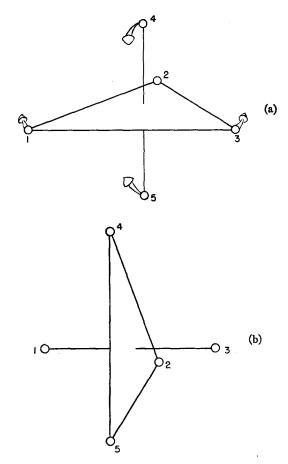


Fig. 2. Schematic representation of pseudorotation in the trigonal bipyramid.

not in exact agreement with that of reference 11; by using this value for S_1/S_2 , the potential computed in the Costain-Sutherland treatment is about 100 cm⁻¹ less than the value reported.) We therefore take the ν_2 vibration as the one to be substituted for ν_0 in Eq. (1) and (11); as Costain and Sutherland¹¹ point out, it is more appropriate to use the normal vibration frequency of the state in question than the observed infrared frequency. This has been done whenever normal frequencies were available. The effective masses have been taken as constants, with

$$\mu = 3m_x m_y / (m_x + 3m_y)$$
.

The expression is almost unaffected by the angledependent contribution which would appear in a rigorous treatment. Structural parameters, effective masses, and vibration frequencies are given in Table I.

Table II gives the results obtained with the four sets of assumptions. Several points should be noted. First, the frequencies found for ND3, with quadratic and cosine potentials of 1840 and 1430 mc, respectively, should be compared with the observed value of 1600 mc and with the value computed by Manning as 1250 mc. Second, the tunneling frequencies for PH3 and AsH3 are much closer to each other than the frequency of NH₃ to either. This seems quite reasonable in light of the similarities in bond lengths, angles, effective masses, and frequencies. It also is interesting that NF₃, with its rather flat structure, tunnels at a rate very close to AsH₃; in the former, the effective mass is the principal cause of the low frequency; in the latter, the actual distance which the hydrogens must move is the major source of the slow rate. Finally, the most important of all, we see that the various approximations all lead to the same order of magnitude in their predictions, so that one might have enough confidence in the method to estimate rates for the purpose of designing experiments.

IV. PENTAHALIDES OF PHOSPHORUS

The molecules PF₅ and PCl₅ have been the subject of a variety of studies which have suggested the possibility that they might exhibit internal motions. Both molecules are trigonal bipyramids so that, at any instant, the halogen atoms occupy two kinds of sites; three are assigned to equatorial positions, and the other two to polar positions. The F19-resonance spectrum of PF5 has been observed by Gutowsky, McCall, and Slichter.¹³ They report only a single fluorine resonance peak (split into a doublet due to P-F coupling), so that either the five fluorines are equivalent to a measurement as slow as NMR, or the chemical shifts and P-F couplings are accidentally identical for fluorines at both kinds of sites. The latter possibility is rather unlikely, particularly in light of the data to be cited regarding PCl₅, so it will not be considered further.

The infrared spectrum of PF₅ has been observed by Gutowsky and Liehr, 14 who find an apparently normal spectrum for a trigonal bipyramid. This suggests that the fluorine atoms may exchange at a rate slow compared with the time necessary to establish sharp vibrational levels, and fast compared with the inverse line widths associated with nuclear resonance measurements. Hence any exchange process which the fluorines

TABLE III. Data used for treatment of PF5 and PCl5.

Molecule	r _{X-Y} of XY ₂ (A)	rxy of XY ₃ (A)	ν ₀ (cm ⁻¹)
$PF_{\mathfrak{s}}$	1.57ª	1.57*	534° or ~180 (assumed)
PCl ₅	2.04b	2.18 ^b	335 ^d or 100 ^d

⁸ L. O. Brockway and J. Y. Beach, J. Am. Chem. Soc. 60, 1836 (1938).

b See reference 16. c See reference 14.

d See reference 15.

¹⁸ Gutowsky, McCall, and Slichter, J. Chem. Phys. 21, 279

<sup>(1953).

14</sup> H. S. Gutowsky and A. D. Liehr, J. Chem. Phys. 20, 1652

	of Table III, lower values using the lower	er frequencies.
^	. 1 . 4ta	Control of the time

TABLE IV Parameters, tunneling frequencies and times for PE, and PCI. Upper values are derived using the high frequency

	Quadratic approximation		Cosine approximation			
Molecule	Z_0	ν (sec-1)	τ	Z_0	$\nu (\mathrm{sec}^{-1})$	au
PF_{δ}	4.37	10 ⁵ (assumed)		5.17	10 ⁵ (assumed)	
PCl_5	6.41	2.4×10^{-5} 1.2×10^{-4}	5.8 hr 41 sec	7.59	$9.6 \times 10^{-6} \\ 6 \times 10^{-4}$	12.8 hr 1.4 min

undergo should occur at a frequency between about 10⁻⁸ and 10-2 sec.

Now we turn to the data regarding PCl₅. Infrared and Raman spectra have been taken and analyzed by Wilmshurst and Bernstein. 15 The structure is known from electron diffraction work of Rouault.16 The possibility of internal exchange is suggested in the isotopic exchange studies of Downs and Johnson. 17,18 They report that three chlorines are capable of very rapid exchange, giving complete equilibration in 5 sec or less; the other two chlorine nuclei do exchange, but attain 90% of their equilibrium isotopic concentration only after 100 min. Presumably the two slowly exchanging chlorines are those occupying the polar positions, and the rate difference truly is due to a difference in reactivity of the two kinds of sites. By inference, then, it is likely that the two kinds of sites in PF5 are also different chemically, so that instantaneous determination of electron density around the two positions would yield two different results. The reactivity of chlorines in PCl₅ thus lends credence to the hypothesis that some kind of exchange process establishes the F¹⁹ equivalence in PF₅ for NMR observations.

One particularly attractive possible mechanism for the exchange process in PF5 and PCl5 is a purely internal pseudorotation as indicated in Fig. 2. The motion leaves the molecule in a rotated and permuted form of its original state.19 If the internal exchange or pseudorotation hypothesis is tenable, the disparity between the necessary PF5 and PCl5 exchange rates must be rationalized. For this we may use the method developed in Sec. II.

The data, including the vibration frequencies of possible importance, are collected in Table III. It would be desirable to be able to attribute the tunneling coordinate with a single normal mode, but unfortunately this cannot be done necessarily. In addition to the problem of finding normal coordinates for PF₅, whose spectrum is not entirely known, there seems to be some doubt about the assignments of the ν_6 and ν_7 e' vibrations. The tentative assignment of Gutowsky and Liehr¹⁴ suggested that the PX₃ deformation is ν_6 , at 534 cm⁻¹ for PF₅. The normal coordinate analysis of Wilmshurst and Bernstein¹⁵ gives ν_6 as the PX₂ polar bend in PCl₅ at 335 cm⁻¹, and ν_7 as the PX₃ deformation at 100 cm⁻¹. The assignments of Wilmshurst and Bernstein however, may be questioned because they found it necessary to call the 394-cm⁻¹ band degenerate, assigning it to two vibrations of the same symmetry, the ν_1 and ν_2 a_1' modes. It is conceivable but surprising that such a combination shows no significant splitting. In any case, we can compute the data of interest using both the 534 and 335-cm⁻¹ bands and the assumed frequency of 180 cm⁻¹ for PF₅ with the observed 100-cm⁻¹ PCl₅ band. These two combinations give results which are all relatively satisfactory for reconciling the tunneling rates.

We assume that the tunneling frequency of PF₅ is halfway between the limits set by its normal infrared spectrum and its single NMR peak; namely, we assume that $\nu(PF_5)\sim 10^5 \text{ sec}^{-1}$. Then with the assistance of Fig. 1, we compute the corresponding Z_0 . To calculate the PCl₅ tunneling frequency, we need to know the distance traveled by the point representing the tunneling particle. We supposed that the halide moves along a straight line from one kind of position to the other, so that the P-X distance varies linearly with displacement. Then the average P-X distance along this coordinate is used to calculate Z_0 for PCl₅. Note that in contrast to the NH3-like molecules, the moving nuclei in trigonal bipyramids all must travel through the same angle.

The results of the calculations are shown in Table IV. It is rather surprising and somewhat fortuitous that the various approximations do give tunneling times which bracket the characteristic time which would obtain if pseudorotation governed the slow exchange rate in PCl₅. The important conclusion to be drawn is simply that the hypothesis of pseudorotation in PF5 and PCl5 is consistent with the experimental data now available. Further experimentation is clearly necessary to confirm or reject the hypothesis.

Other related systems such as SF₄ and ClF₃ have been studied by Muetterties and Phillips.^{20,21} It has been

¹⁶ J. K. Wilmshurst and H. J. Bernstein, J. Chem. Phys. 27, 661 (1957).

¹⁶ M. Rouault, Ann. phys. 14, 78 (1940).

¹⁷ J. J. Downs and R. E. Johnson, J. Chem. Phys. 22, 143

¹⁸ J. J. Downs and R. E. Johnson, J. Am. Chem. Soc. 77, 2098 (1955).

¹⁹ The author is indebted to Dr. F. T. Smith who first pointed out to him the existence of the pseudorotation operation of a trigonal bipyramid.

²⁰ E. L. Muetterties and W. D. Phillips, J. Am. Chem. Soc. 79, 322 (1957).

21 E. L. Muetterties and W. D. Phillips, J. Am. Chem. Soc. 81,

^{1084 (1959).}

pointed out that these compounds are capable of exhibiting pseudorotation,²² and the results of NMR studies by Muetterties and Phillips^{20,21} are consistent with this hypothesis. Chemical exchange is, however, at least as likely for these molecules.23 Therefore we do not suggest that pseudorotation is necessarily the explanation for the observed line narrowings which appear as sample temperatures are increased. It is important, nonetheless, to note that the results of Muetterties and Phillips require that the pseudorotation times for SF₄ and ClF₃ be considerably longer than for PF5. This is rationalized easily by the following argument. In PF5, the phosphorus atom lies at the center of gravity and, therefore, need not move during the course of pseudorotation. Since SF₄^{24,25} and ClF₃^{26,27} are not symmetrical structures, the heavy atoms do not lie at the molecular centers of gravity and therefore must participate in pseudorotation. The large effective masses for motion along the tunneling coordinates would depress the tunneling rates, presumably by several orders of magnitude.

V. SUMMARY

A method has been presented whereby the tunneling frequencies ν for members of a homologous series of molecules may be estimated. The method depends on knowledge of the tunneling rate for one member of the series, and the vibrational frequencies and structure for this molecule and for any molecule whose tunneling frequency is desired. The method, useful only for finding orders of magnitude of ν , depends on the insensitivity of the tunneling frequency to barrier shape, and on the assumption that barrier shapes for the various members of the series are similar.

The method is applied to the NH₃-like molecules ND₃, NF₃, PH₃, and AsH₃. The ND₃ rate is close to the observed value if either a quadratic or cosine potential is assumed. The PH₃ rate is calculated to be of the order of 10 sec⁻¹, while the NF₃ and AsH₃ rates are of the order of 1 min⁻¹.

The method also is applied to correlate the rates of the hypothesized pseudorotation of PF₅ and PCl₅. If a characteristic time of 10^{-5} sec is assumed for PF₅, then the computed time for PCl₅ lies between $\frac{1}{2}$ min and 12 hrs. The hypothesis of pseudorotation therefore is consistent with the spectra and exchange experiments which have been performed on these two molecules.

ACKNOWLEDGMENT

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²² R. S. Berry, Revs. Modern Phys. (to be published).

²³ M. T. Rogers and J. J. Katz, J. Am. Chem. Soc. 74, 1375 (1952).

²⁴ Dodd, Woodward, and Roberts, Trans. Faraday Soc. 52, 1052 (1955).

 ²⁵ Cotton, George, and Waugh, J. Chem. Phys. 28, 994 (1958).
 ²⁶ R. D. Burbank and F. N. Bensey, J. Chem. Phys. 21, 602

²⁷ D. F. Smith, J. Chem. Phys. 21, 609 (1953).