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# Correlation of the second virial coefficient in the model of steam bubble collapse

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## Abstract

In many previous mathematical models of steam bubble collapse a perfect gas in the bubble was taken into consideration. It might lead to substantial mistakes, for example, wrong calculation of heat capacity $c_V$ of vapour.

A new mathematical model of steam bubble collapse in subcooled liquid has been proposed. It uses the equation of state for a real gas and the Pitzer-Abbott correlation for calculating the second virial coefficient B. It takes also into account the ratio of vapour to liquid and a change in temperature of liquid during the process.

The results of our own experimental and calculated data have been compared with those obtained by other authors. The model has been applied to investigation of the influence of various parameters, namely: subcooling of the liquid, bubble diameter and vapour to liquid ratio, on the bubble collapse time.

## 1. Introduction

Heating of a liquid by means of steam bubbles injection has been applied for a long time. At present, this model of batch injection, carried out to obtain a single bubble, is used to interpret the process of a continuous injection of steam bubbles.

## 2. Models of bubble growth and collapse

The existing models of bubble growth and collapse can be classified into five groups[1]:

1) Rayleigh equation,
2) modified Rayleigh equation,

3) models described by the Rayleigh equation and additional formulae,
4) models related to thermal resistance of liquid or of both bubble and the liquid,
5) models taking into account non-spherical shape of the bubble.

The simplest model of the bubble dynamics is the Rayleigh equation:

$$P_b - P_\infty = \rho_L \left[ R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 \right] \tag{1}$$

Modified Rayleigh equations cover, for example, forces of viscosity or surface tension[2]

$$P_b - P_\infty = \rho_L \left[ R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 \right] + \frac{2\sigma_L}{R} + \frac{4\eta_L}{R}\frac{dR}{dt} \tag{2}$$

In the next group of models[3,10], differential and algebraic equations related to important aspects of the process are added to eq. (1).

In other studies[4,5] attempts have been made to simplify the mechanism of heat transfer to the transfer in a liquid layer only. Thermal resistance on both sides of the bubble wall has been considered by Nigmatulin and Beylich[6,7].

Modelling of non-sphericity is a problem which has been studied recently. Such shapes of the bubble are considered as a spherical cap and a cylinder. From the literature survey it follows that there is no uniform model of this process.

## 3. The proposed mathematical description of the process

The proposed model should be assigned to group III:
1) the momentum balance equation (2)
2) mass balance

$$-d(V_b\rho_b) = \frac{-\lambda_L\left(\frac{\partial T}{\partial r}\right)_{r=R} 4\pi R^2}{h_{LV}}dt - 4\pi R^2 k_b\left(\rho_b - \rho_b^*\right)dt \tag{3}$$

3) inner energy balance

$$c_V d\left(V_b\rho_b T_b\right) = 4\pi R^2\lambda_L\left(\frac{\partial T}{\partial r}\right)_{r=R} dt - \left(P_b - P_\infty\right)dV_b \tag{4}$$

4) the state equation with the second virial coefficient

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$$\frac{P_b V_b}{R_g \theta_b} = 1 + \frac{B}{V_b} \tag{5}$$

where the virial coefficient B is determined from the Pitzer-Abbott correlation:

$$\frac{B P_k}{R_g \theta_k} = B^0 + \omega B^1 \tag{6}$$

at

$$B^0 = 0.083 - \frac{0.422}{\theta_r^{1.6}} \tag{7}$$

$$B^1 = 0.139 - \frac{0.172}{\theta_r^{4.2}} \tag{8}$$

5) molar heat of steam at a constant volume

$$C_p - C_v = -\theta_b \left(\frac{\partial V_b}{\partial \theta_b}\right)^2_{P_b} \left(\frac{\partial P_b}{\partial V_b}\right)_{\theta_b} \tag{9}$$

6) dynamics of a boundary layer.

The boundary layer thickness was determined from the Kirchoff-Fourier equation:

$$\frac{\partial T_L}{\partial t} + u \frac{\partial T_L}{\partial r} = \frac{a_L}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial T_L}{\partial r}\right) \tag{10}$$

where

$$u = \frac{R^2}{r^2} \frac{dR}{dt} \tag{11}$$

assuming a parabolic temperature distribution in the boundary layer given by the equation:

$$T_L - T_b = \left(T_\infty - T_b\right)\left[\frac{2(r-R)}{\delta_L} - \left(\frac{r-R}{\delta_L}\right)^2\right] \tag{12}$$

with the initial conditions

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$$P(r,0) = P_\infty$$
$$T_L(r,0) = T_\infty$$

and boundary conditions

$$T_L(R,t) = T_b$$
$$T_L(R+\delta_L,t) = T_\infty$$
$$P(\infty,t) = P_\infty$$

7) heat balance between the condensing steam and liquid

$$(h_{LV} + c_L T_b)\, dm_{bn} = - c_L d(m_L T_\infty) \tag{13}$$

8) mass balance of the liquid and condensing steam

$$\frac{d\left(\frac{m_L}{m_{bn0}}\right)}{d\tau} = - \frac{1}{\rho_{b0}}\left(3\rho_b \beta^2 \frac{d\beta}{d\tau} + \beta^3 \frac{d\rho_b}{d\tau}\right) \tag{14}$$

The proposed mathematical description can be used to analyze the collapse of many bubbles, without taking into account the coalescence. A change in the liquid temperature and water mass increment induced by bubble condensation can be also calculated.

## 4. Experimental

Experiments were carried out in the apparatus designed by the authors, using two nozzles 0.6 and 1.4 mm in diameter. The subcooling range was from 1 K to 3.5 K. The initial bubble diameter ranged from 2.66 mm to 3.5 mm and its flow velocity was between 15.3 mm/s and 52.4 mm/s. Results were compared with the proposed model and the data obtained by other authors[8,9] (Fig. 1).

## 5. Computer simulation of the process

The effect of some parameters on bubble collapse rate is presented in Figs. 2, 3 and 4. The main results of the simulation cover:

1) predicted time of collapse of bubbles with different diameters for various initial conditions,
2) paying attention to significant elongation of the bubble collapse time with a change in $m_L/m_{bn0}$ ratio below $10^2$,
3) confirmation and explanation of the oscillatory, fading character of pressure and temperature variations with time in the collapsing bubble,
4) settling of a possibility of a momentary steam temperature decrease in the bubble below the liquid temperature in the initial stage of its collapse.

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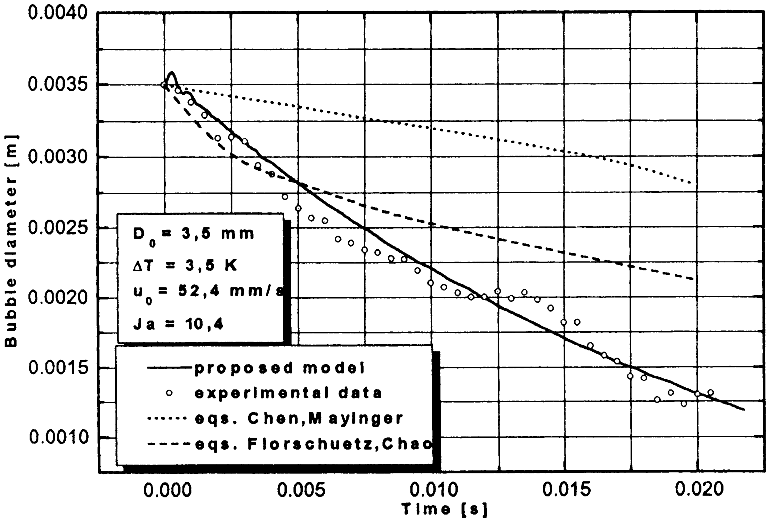


Figure 1: Comparison of the proposed and experimental model with equtions given by other authors.

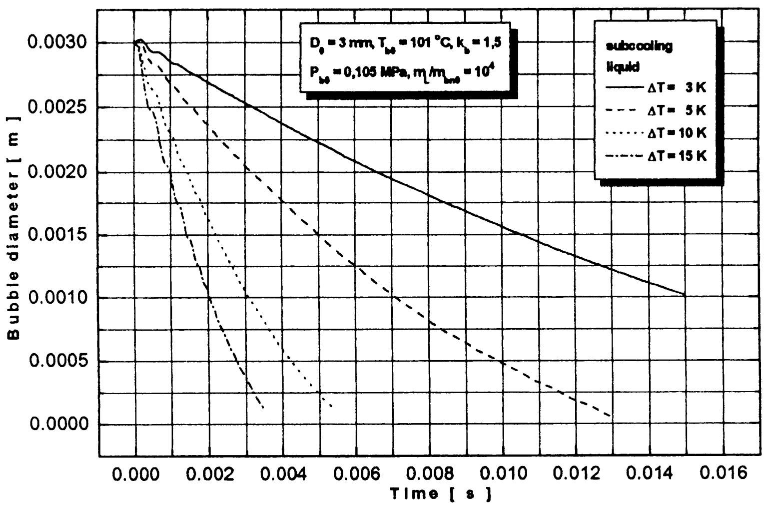


Figure 2: Effect of liquid subcooling on bubble collapse rate.

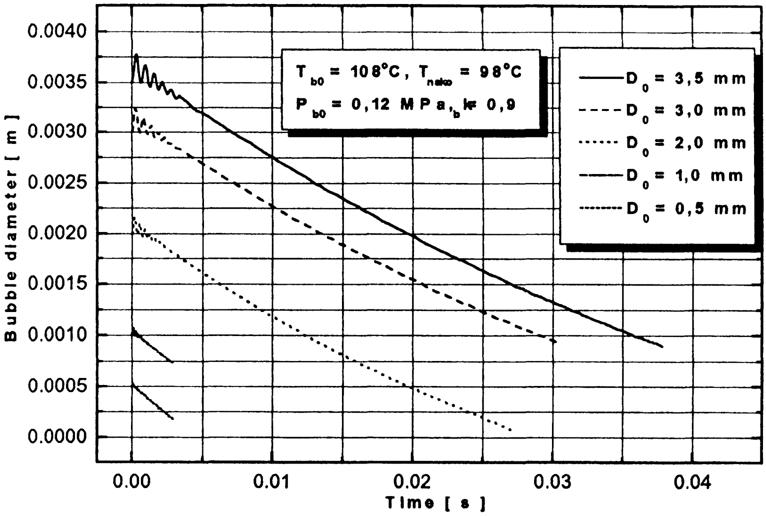


Figure 3: Effect of initial bubble diameter on its collapse rate.

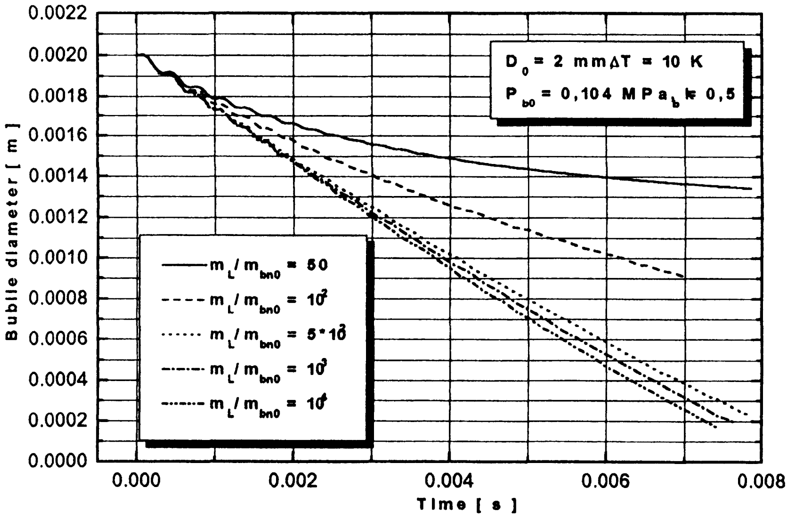


Figure 4: Effect of liquid to vapour mass ratio on bubble collapse rate.

## 6. Nomenclature

$a$ thermal diffusivity $m^2/s$
$C$ molar heat, J/mol K
$c$ specific heat, J/kg K
$D$ bubble diameter, m
$h_{LV}$ heat of condensation, J/kg
$k_1$ adjusting parameter
$m$ mass, kg
$P$ pressure, Pa
$R$ bubble radius, m
$R_g$ universal gas constant, J/mol K
$T$ temperature, °C
$t$ time, s
$u$ bubble velocity, m/s
$V$ volume, $m^3$, $m^3/mol$
$\beta$ relative bubble radius
$\gamma$ boundary layer thickness, m
$\eta$ dynamic viscosity, Pa s
$\theta$ temperature, K
$\lambda$ heat transfer coefficient, W/mK
$\rho$ density, $kg/m^3$
$\sigma$ surface tension, N/m
$\tau$ dimensionless time, –

Subscripts
b bubble inside
K critical
L liquid
h number of bubbles
r reduced
$\propto$ inside the liquid
0 initial state

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