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Correlation of Viscosity, Thermal Conductivity and Prandtl Number for Water and Steam as a Function of Temperature and Pressure*

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In this report equations of viscosity, thermal conductivity and Prandtl number for water and steam were correlated as a function of temperature and pressure, while most of equations in previous investigations had been expressed as a function of temperature and density.

Directly from values of temperature and pressure without knowing the value of density, property values can be calculated using equations in this report. Whole regions of up to 1 000°C, 3 500 bar for viscosity, up to 700°C, 500 bar for thermal conductivity and up to 600°C, 500 bar for Prandtl number are covered by three equations each, namely equations for the water, the steam and the critical regions. In the water and the steam regions those properties are expressed as the polynomials of pressure in two or three terms, while the narrow region near the critical point is expressed in a slightly more complicated form. Values calculated by the correlated equations were compared with the recent experimental values.

1. Introduction

Most of previously published equations of viscosity and thermal conductivity for water and steam are expressed as a function of temperature and density, while few of them choose temperature and pressure as independent variables. Although it is quite difficult to express those properties in terms of temperature and pressure, this type of equation is often convenient in practical calculations of industrial purposes. In this report, the viscosity, the thermal conductivity and the Prandtl number for water and steam are correlated into equations as a function of temperature and pressure.

After a critical survey of the latest experimental investigations including the authors' experiments, sets of basic data for correlations were constructed from the data which were believed to be the most reliable at the present time. Regions covered by equations of this investigation are; temperature range of 0～1 000°C and pressure range of 1～3 500 bar for the viscosity, 0～700°C and 1～500 bar for the thermal conductivity and 0～600°C and 1～500 bar for the Prandtl number. These whole regions are divided into three subregions each, namely (1) the water and high density steam region (stated hereinafter as the compressed water region), (2) the steam region and (3) the critical region. Mostly, a simple polynomial form in pressure terms was used but in the narrow region near the critical point it was tried to express properties as implicit functions.

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2. Notations

c_p: specific heat at constant pressure J/g°K
P: pressure bar
t: temperature °C
η: viscosity μP
λ: thermal conductivity milliwatt/m°K
P_r: Prandtl number $\eta c_p/\lambda$

Subscripts and primes

1: atmospheric pressure
c: critical point
cal: calculated
E: experimental
B: basic data
$'$: saturated water
$''$: saturated steam

3. Viscosity

3·1 Basic data

One of the standard sets of the viscosity data for water and steam is that compiled by the International Conference on the Properties of Steam (ICPS) in 1964[1]. This was established after a critical survey of all the experimental data available in 1963, and has been believed to be the most reliable until several years ago. However, in addition to this table having a vacant region and comparatively large tolerances, it was revealed that there exist deviations from recent experimental values by the authors[2], Rivkin and co-workers[3] and so on. Details of these comparisons were reported in the previous report (2)

The basic data for this investigation are values by authors' empirical equation (independent variables are temperature and density) for up to 1 000°C and 1 000 bar, and smoothed values of Dudziak and Franck[4] and of others for up to 3 500 bar. Experimental values by Dudziak and Franck were those measured using an oscillating disk type apparatus in the region up to 560°C and 3 500 bar. This is not an absolute measurement but is relative to ICPS values at 800 bar on each isotherms. References for the basic data at high pressures below 100°C are values by Bridgeman[5], Horne and Johnson[6] and some others.

3·2 Equations of viscosity

As a viscosity equation with independent variables of temperature and pressure, there is a Rivkin's equation[7]. Rivkin correlated viscosities of compressed water and of high density steam in the region up to 550°C and 800 bar into an implicit equation, where pressure was expressed as a function of temperature and the residual viscosity, $\eta-\eta_1$. He used the ICPS values (1964) as the basic data and they fitted very well, although most of steam region is not included in the equation.

Present authors once correlated equations of viscosity as a function of temperature and pressure[8] but the study was based on the ICPS values (1964). This time, more recent experimental values are included in the basic data and the form of equations for the critical region was revised.

In the critical region, isotherms are almost vertical on $\eta-P$ diagram as shown in Fig. 1 and there is a region where the viscosity can hardly be expressed as a function of temperature and pressure. On the other hand, isotherms in the steam and the compressed water regions have a simple form and moreover these two regions are inportant in practical industrial calculations. Then dividing the whole region into three sub-regions as shown in Fig. 2, the compressed water and the steam regions are covered by polynomial equations of up to the second or the third order in pressure, while the coefficient of each pressure term is expressed as a function of temperature. Both regions were determined as wide as possible and the remaining narrow region was covered by the equation of the critical region, which denotes pressure as a function of temperature and the residual viscosity. In the critical region, the

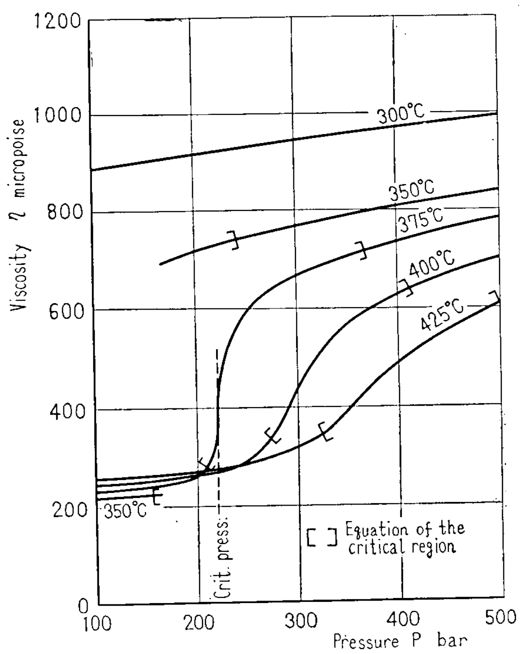

Fig. 1 Viscosity in the critical region

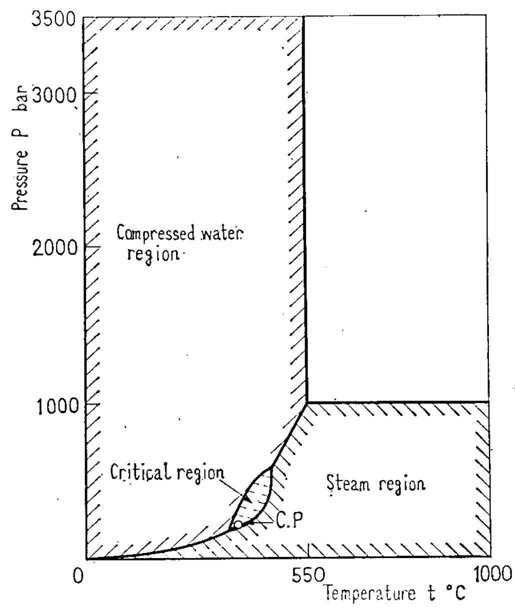

Fig. 2 Sub-regions (Viscosity)

curve of the residual viscosity, $\eta-\eta_1=$ constant can be approximated by a straight line. For the case of implicit functions, calculations can easily be carried out on an electronic computer. For viscosity values of atmospheric steam, Shifrin's equation[9] is used.

Three equations are made to overlap each other on boundaries and effective regions of each equation are shown in Tables 2 and 3 of calculated values. Only a very narrow region near the critical point (approximately within $t_c\pm1$°C and $P_c\pm1$ bar) was excluded from present correlations, because large variation of viscosity in this region can hardly be followed by an equation without having density as an explicit independent variable, and also because this region is not so important for industrial purposes. Correlated three equations are as follows.

3·2·1 Compressed water region

$$\eta=\sum_{i=0}^{3} A_i P^i \qquad (1)$$

$$A_0=1/\sum_{i=0}^{4} A_{0i}t^i+A_{05}/\{(t-20)^2+A_{06}\}+A_{07}$$

$$A_1=A_{10}+A_{11}/\{(t-500)^2+A_{12}\}+A_{13}/(t+100)^{10}$$

$$A_2=A_{20}+A_{21}/\{(t-490)^2+A_{22}\}+A_{23}/(t+100)^{10}$$

$$A_3=\sum_{i=0}^{1} A_{3i}t^i+A_{32}/\{(t-475)^2+A_{33}\}+A_{34}/(t+100)^{10}$$

3·2·2 Steam region

$$\eta=\sum_{i=0}^{2} B_i P^i \qquad (2)$$

$$B_0=\sum_{i=0}^{1} B_{0i}t^i$$

$$B_1=\sum_{i=0}^{2} B_{1i}t^i+B_{13}/\{(t-250)^4+B_{14}\}$$

$$B_2=B_{20}+B_{21}/\{(t-300)^4+B_{22}\}$$

3·2·3 Critical region

$$P=\sum_{i=0}^{1} C_i t^i \qquad (3)$$

$$C_0=\sum_{i=0}^{1} C_{0i}\Delta\eta^i$$

$$C_1=\sum_{i=0}^{1} C_{1i}\Delta\eta^i+C_{12}\Delta\eta^6+C_{13}/\Delta\eta^2$$

$$\Delta\eta=\eta-\eta_1$$

Constants in Eqs. (1)~(3) are shown in Table 1. When only rough estimation is needed, approximate viscosity value can easily be calculated by Eqs. (1)~(3) using coefficients read on Fig. 3. To get the viscosity of steam at atmospheric pressure, it is enough to calculate only the term of B_0 in Eq. (2).

3·3 Calculated results

Viscosity of water and steam calculated by Eqs. (1)~(3) is shown in Tables 2 and 3. Solid lines in the tables give boundaries between subregions. Isotherms on $\eta-P$ diagram are shown in Fig. 4. Calculated values by Eqs. (1)~(3) agree with the basic data within 3% deviations in most of the region, although there is a point of 5.5% deviation in the steam region at high temperature and pressure.

Comparisons of calculated values with recent experimental investigations are shown in Fig. 5. Equation (1) was compared with some values by the authors[2] and by Agaev[10] in the compressed water region, while in the steam region as well as in the critical region Eqs. (1)~(3) are compared with some experimental values by the authors, Rivkin[3] and Tanaka[11]. As seen in Fig. 5, calculated values represent those recent experimental values reasonably well within

Table 1 Constants in Eqs. (1)~(3)

A_{00}	0.444444×10^{-4}	A_{33}	1.31758×10^{4}
A_{01}	0.126158×10^{-5}	A_{34}	1.450×10^{-2}
A_{02}	-0.522934×10^{-8}	B_{00}	80.4×10^{0}
A_{03}	0.103065×10^{-10}	B_{01}	0.4070×10^{0}
A_{04}	-0.738169×10^{-14}	B_{10}	-7.26111×10^{-1}
A_{05}	2.521281×10^{4}	B_{11}	1.97222×10^{-3}
A_{06}	1.044441×10^{2}	B_{12}	-1.2111×10^{-6}
A_{07}	-5.0×10^{3}	B_{13}	0.766827×10^{6}
A_{10}	0.10×10^{0}	B_{14}	0.697115×10^{7}
A_{11}	1.22230×10^{4}	B_{20}	3.7×10^{-5}
A_{12}	1.82432×10^{4}	B_{21}	1.42313×10^{6}
A_{13}	-8.80×10^{19}	B_{22}	1.105570×10^{9}
A_{20}	0.35×10^{-4}	C_{00}	-55.3×10^{0}
A_{21}	-5.37016×10^{0}	C_{01}	-4.55×10^{0}
A_{22}	1.74356×10^{4}	C_{10}	0.753×10^{0}
A_{23}	2.20×10^{16}	C_{11}	1.2100×10^{-2}
A_{30}	-5.83×10^{-9}	C_{12}	2.922848×10^{-17}
A_{31}	1.014×10^{-11}	C_{13}	-7.67625×10^{1}
A_{32}	4.89614×10^{-4}		

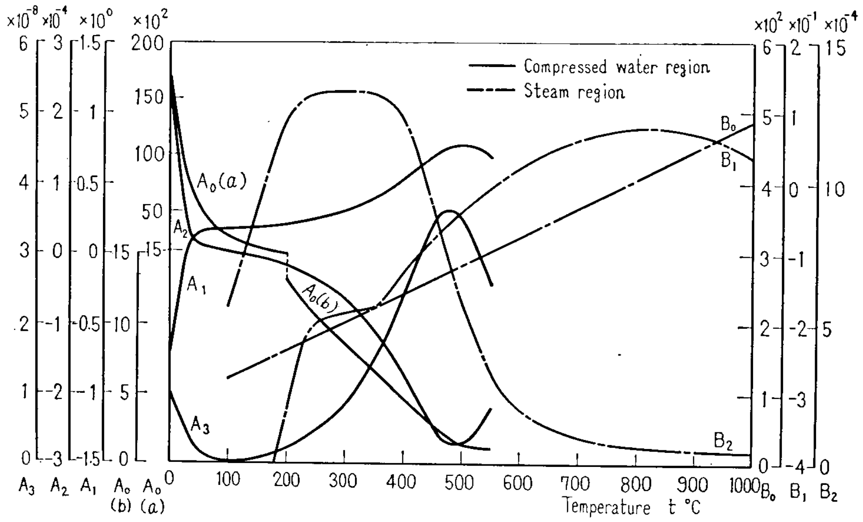

Fig. 3 Coefficients in Eq. (1) and Eq. (2) (Viscosity)

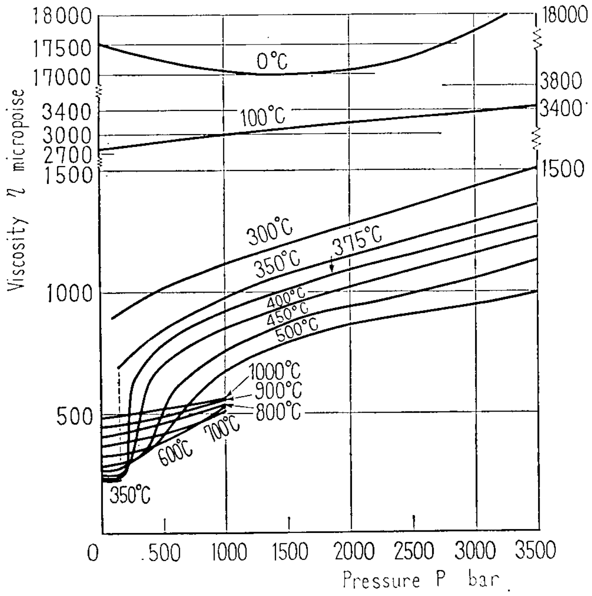

Fig. 4 Viscosity of water and steam

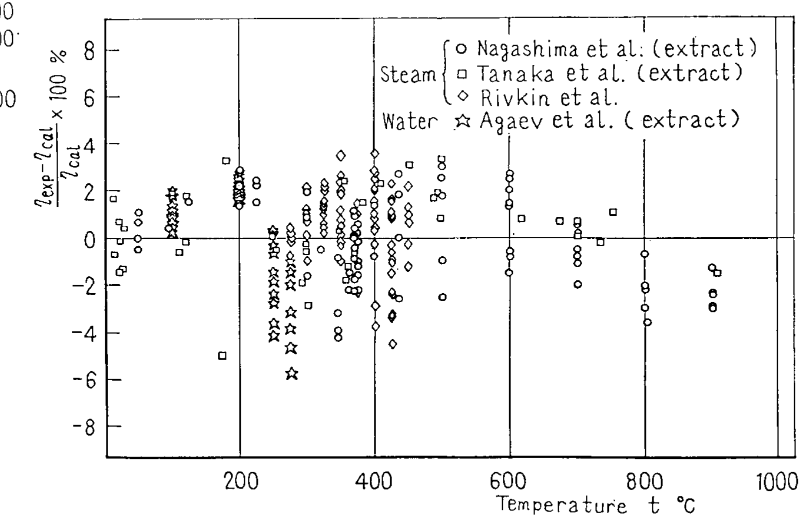

Fig. 5 Comparisons of viscosity values

Table 2 Viscosity calculated by Eqs. (1)∼(3)

Pressure (bar)	Temperature (°C) 0	50	100	150	200	250	300	350	375	400	425
1	17 550	5 475	120.6	141.0	161.5	182.0	202.3	222.7	232.9	243.1	253.3
5	17 547	5 476	2 825	1 801	160.2	181.2	201.6	222.0	232.3	242.6	252.9
10	17 543	5 477	2 826	1 802	158.7	180.3	200.8	221.2	231.6	442.0	252.4
25	17 532	5 479	2 828	1 805	1 325	178.0	198.7	219.2	230.0	240.7	251.4
50	17 514	5 482	2 832	1 810	1 330	1 065	196.5	217.3	228.6	239.8	250.8
75	17 497	5 486	2 837	1 815	1 336	1 072	196.0	217.0	228.7	240.3	251.6
100	17 479	5 489	2 841	1 819	1 341	1 078	886.5	218.3	230.5	242.4	253.7
125	17 462	5 492	2 845	1 824	1 346	1 084	893.9	221.3	233.9	246.0	257.2
150	17 446	5 497	2 849	1 828	1 352	1 090	901.2	225.9	238.9	251.2	262.1
175	17 429	5 500	2 853	1 833	1 357	1 096	908.4	697.0	245.6	257.8	268.3
200	17 413	5 504	2 858	1 838	1 362	1 102	915.6	716.0	253.8	266.0	275.9
225	17 397	5 507	2 862	1 842	1 367	1 108	922.6	731.5	454.0	275.7	284.8
250	17 382	5 511	2 866	1 847	1 372	1 114	929.6	758.4	602.2	286.9	295.1
275	17 366	5 515	2 870	1 852	1 377	1 119	936.6	767.0	644.6	352.4	306.8
300	17 351	5 518	2 875	1 856	1 382	1 125	943.4	775.5	672.1	431.0	319.8
350	17 323	5 525	2 883	1 865	1 392	1 137	956.9	792.2	710.4	561.4	400.0
400	17 294	5 533	2 892	1 874	1 402	1 148	970.2	808.4	728.3	633.3	479.9
450	17 268	5 541	2 900	1 884	1 412	1 159	983.1	829.1	745.9	670.2	551.9
500	17 243	5 549	2 909	1 893	1 422	1 170	995.8	839.4	762.9	689.3	608.7
550	17 219	5 558	2 917	1 902	1 432	1 181	1 008	854.2	779.4	707.8	641.4
600	17 196	5 564	2 926	1 911	1 442	1 192	1 020	868.6	795.3	735.5	661.3
650	17 165	5 572	2 935	1 920	1 451	1 202	1 032	882.6	810.7	742.6	680.3
700	17 155	5 580	2 943	1 929	1 461	1 213	1 044	896.2	825.6	759.1	698.6
750	17 136	5 588	2 952	1 938	1 471	1 223	1 055	909.4	840.0	775.0	716.1
800	17 118	5 597	2 961	1 947	1 480	1 233	1 067	922.2	854.0	790.2	732.9
850	17 102	5 605	2 969	1 956	1 489	1 243	1 078	934.7	867.4	804.8	749.0
900	17 087	5 613	2 978	1 965	1 499	1 253	1 088	946.7	880.4	819.0	764.4
950	17 073	5 522	2 987	1 974	1 508	1 263	1 099	958.5	893.0	832.5	779.1
1 000	17 060	5 630	2 995	1 983	1 517	1 272	1 109	969.9	905.1	845.6	793.2
1 500	17 012	5 721	3 084	2 071	1 607	1 364	1 203	1 068	1 006	950.7	903.9
2 000	17 104	5 821	3 175	2 159	1 692	1 447	1 284	1 144	1 080	1 023	975.2
2 500	17 345	5 932	3 268	2 246	1 776	1 527	1 357	1 210	1 142	1 081	1 030
3 000	17 744	6 056	3 364	2 333	1 858	1 605	1 431	1 277	1 206	1 143	1 089
3 500	18 308	6 194	3 462	2 420	1 942	1 687	1 510	1 355	1 286	1 226	1 177

estimated experimental errors, although there exists little difference between trends of the water and the steam regions. Only values on 200°C isotherm in the water region systematically deviate by about 2%, which is smaller than the tolerance of ICPS (1964) but slightly exceed the estimated experimental error. In the steam region at around critical pressure in temperature range 350～600°C, there exists a deviation of up to 10% between the ICPS values (1964) and recent experimental values. As stated in detail in the reference (2), the latter was used as the basic data also in this correlation.

4. Thermal conductivity

4・1 Basic data

Until several years ago, there were no other reliable experimental investigations of the thermal conductivity of water and steam in high temperature and high pressure region except a series of investigations by Vargaftik. The 6th ICPS established a Skeleton Table (1964) of the standard values[(1)], which were determined such as to be close to values by Vargaftik taking also those by Vukalovich and Cherneeva[(12)] into consideration[(13)]. The ICPS Table (1964) covers the region of up to 700°C and 500 bar. Since 1964, several investigations have been performed in USSR, UK and France, and most of them showed reasonable agreement with Vargaftik's. So the ICPS values were used as the basic data in the present investigation.

4・2 Equations of thermal conductivity

Fundamental ideas and procedures for equations of thermal conductivity are similar to those for equations of viscosity. The widest region in the water and the steam regions was covered by simple polynomial equations in pressure, while

Table 2 (continued)

Pressure (bar)	Temperature (°C) 450	475	500	550	600	650	700	750	800	900	1000
1	263.5	273.7	283.9	304.2	324.6	345.0	365.4	385.7	406.1	446.8	487.4
5	263.2	273.4	283.7	304.2	324.7	345.2	365.6	386.0	406.4	447.0	487.6
10	262.8	273.2	283.5	304.2	324.8	345.4	365.9	386.4	406.8	447.4	487.8
25	262.0	272.6	283.2	304.3	325.3	346.1	366.9	387.5	408.0	448.4	488.3
50	261.7	272.4	283.2	304.7	326.1	347.5	368.6	389.4	410.0	450.2	489.3
75	262.5	273.2	283.9	305.5	327.3	348.0	370.4	391.4	412.1	452.1	490.3
100	264.4	274.8	285.3	306.7	328.6	350.6	372.3	393.5	414.3	454.0	491.3
125	267.5	277.4	287.4	308.3	330.3	352.4	374.3	395.7	416.5	455.9	492.4
150	271.1	270.9	290.2	310.3	332.1	354.4	376.5	398.0	418.8	458.0	493.6
175	277.1	285.3	293.7	312.7	334.2	356.5	378.7	400.4	421.2	460.0	494.8
200	283.7	290.6	297.9	315.6	336.5	358.8	381.1	402.8	423.7	462.2	496.1
225	291.4	296.8	302.8	318.8	339.1	361.2	383.6	405.4	426.2	464.4	497.4
250	300.2	304.0	308.4	322.4	341.9	363.8	386.2	408.0	428.9	466.7	498.8
275	310.2	312.1	314.8	326.4	344.9	366.6	388.9	410.8	431.5	469.0	500.3
300	321.3	321.1	321.8	330.8	348.2	369.5	391.7	413.6	434.3	471.4	501.8
350	347.1	341.8	338.0	340.9	355.5	378.5	397.7	419.5	440.1	476.3	504.9
400	377.4	366.2	356.9	352.5	363.8	382.8	404.2	425.7	446.1	481.5	508.3
450	450.2	394.2	378.7	365.8	373.0	390.2	411.0	432.3	452.5	487.0	511.8
500	503.3	426.0	403.4	380.7	383.1	398.4	418.4	439.3	459.1	492.6	515.6
550	553.1	461.3	430.8	397.1	394.3	407.2	426.2	446.6	466.1	498.5	519.6
600	608.1	500.4	461.0	415.2	406.3	416.6	434.4	454.2	473.3	504.7	523.9
650	629.2	543.1	494.1	434.9	419.4	426.6	443.1	462.2	480.9	511.1	528.3
700	649.3	589.5	530.0	456.2	433.4	437.3	452.3	470.6	488.7	517.7	532.9
750	668.6	619.4	568.7	479.0	448.3	448.6	461.9	479.3	496.8	524.6	537.8
800	687.0	639.2	592.6	503.5	464.2	460.5	471.9	488.4	505.3	531.7	542.9
850	704.5	658.1	612.3	529.6	481.1	473.0	482.4	497.8	514.0	539.0	548.2
900	721.3	676.1	631.1	557.3	498.9	486.2	493.4	507.6	523.0	546.6	553.7
950	737.3	693.2	649.0	580.2	517.7	500.5	504.8	517.8	532.3	554.4	559.4
1000	752.6	709.6	666.0	596.8	537.5	514.5	516.6	528.3	541.9	562.5	565.3
1500	869.8	833.1	794.5	727.5							
2000	940.6	903.7	865.7	806.1							
2500	990.7	948.4	905.7	852.1							
3000	1 046	994.4	940.4	884.9							
3500	1 132	1 069	996.1	923.8							

only the narrow region near the critical point was expressed by an implicit equation of the thermal conductivity. Boundaries of these three equations are shown in Tables 3 and 5. A very narrow region near the critical point (approximately within $t_c \pm 1$°C and $P_c \pm 1$ bar) was excluded from the formulation. The equation for the critical region was correlated considering the quasi-linear behavior of $\lambda - \lambda_1$ as seen in Fig. 6, which is similar to the case of the viscosity.

4・2・1 Compressed water region

$$\lambda = \sum_{i=0}^{2} D_i P^i \quad \cdots\cdots(4)$$

$$D_0 = \sum_{i=0}^{4} D_{0i} t^i \qquad D_1 = \sum_{i=0}^{3} D_{1i} t^i$$

$$D_2 = \sum_{i=0}^{2} D_{2i} t^i$$

4・2・2 Steam region

$$\lambda = \sum_{i=0}^{3} E_i P^i \quad \cdots\cdots(5)$$

$$E_0 = \sum_{i=0}^{2} E_{0i} t^i \qquad E_1 = \sum_{i=0}^{3} E_{1i} t^i$$

$$E_2 = \sum_{i=0}^{2} E_{2i} t^i \qquad E_3 = \left(\sum_{i=0}^{3} E_{3i} t^i \right) / t^{10}$$

4・2・3 Critical region

$$P = \sum_{i=0}^{1} F_i t^i \quad \cdots\cdots(6)$$

$$F_0 = \sum_{i=0}^{1} F_{0i} \Delta\lambda^i - \exp\left(\sum_{i=2}^{5} F_{0i} \Delta\lambda^{i-2} \right)$$

$$F_1 = \sum_{i=0}^{2} F_{1i} \Delta\lambda^i$$

$$\Delta\lambda = \lambda - \lambda_1$$

Constants in Eqs. (4)～(6) are shown in Table 4. Variations of constants in Eqs. (4) and (5) against temperature are shown in Fig. 7, which can conveniently be used for approximate calculation of the thermal conductivity. When the thermal conductivity of steam at atmospheric pressure is needed, it is enough to calculate the term E_0 in Eq. (5).

4・3 Culculated results

Thermal conductivity values of water and steam calculated by Eqs. (4)～(6) are given in Tables 3 and 5 as well as in Fig. 8. Solid lines

Table 3 Calculated properties at saturation points

t_s °C	P_s bar	$\eta \times 10^{-6}$P		$\lambda \times 10^{-3}$ WATT/M°K		P_r	
		η'	η''	λ'	λ''	P_r'	P''_r
100	1.013 25	2 824	120.6	680.7	24.4	1.740	0.985
110	1.432 7	2 546	124.4	684.1	25.3	1.571	1.035
120	1.985 4	2 312	128.2	686.4	26.2	1.433	1.071
130	2.701 1	2 114	132.0	687.5	27.1	1.320	1.098
140	3.613 6	1 946	135.7	687.5	28.1	1.227	1.119
150	4.759 7	1 802	139.3	686.4	29.2	1.150	1.136
160	6.180 4	1 678	142.9	684.3	30.4	1.088	1.150
170	7.920 2	1 571	146.4	681.0	31.7	1.036	1.163
180	10.027	1 477	149.9	676.6	33.1	0.995	1.176
190	12.553	1 395	153.4	671.2	34.7	0.962	1.190
200	15.550	1 324	157.1	664.7	36.4	0.937	1.205
210	19.080	1 260	160.9	657.1	38.4	0.919	1.224
220	23.202	1 204	164.8	648.3	40.6	0.906	1.246
230	27.979	1 153	168.6	638.5	43.1	0.899	1.273
240	33.480	1 106	172.4	627.5	45.9	0.897	1.304
250	39.776	1 063	176.3	615.4	49.1	0.900	1.343
260	46.941	1 023	180.4	602.2	52.6	0.906	1.386
270	55.052	985.9	184.7	587.9	56.7	0.917	1.438
280	64.191	950.2	188.9	572.6	61.3	0.933	1.500
290	74.449	915.8	192.7	556.3	66.5	0.953	1.573
300	85.917	882.6	196.3	539.0	72.3	0.980	*1.664
310	98.694	850.2	200.5	520.8	78.9	1.016	*1.781
320	112.89	818.6	205.6	501.8	86.2	1.064	*1.936
330	128.64	787.7	212.0	482.2	94.4	1.127	*2.157
340	146.08	757.6	219.9	462.0	103.7	1.210	*2.483
350	165.37	688.1	250.9	441.6	126.5	1.512	*2.990
360	186.74	638.2	262.2	397.3	146.2		
370	210.53	545.6	288.6	344.8	210.9		

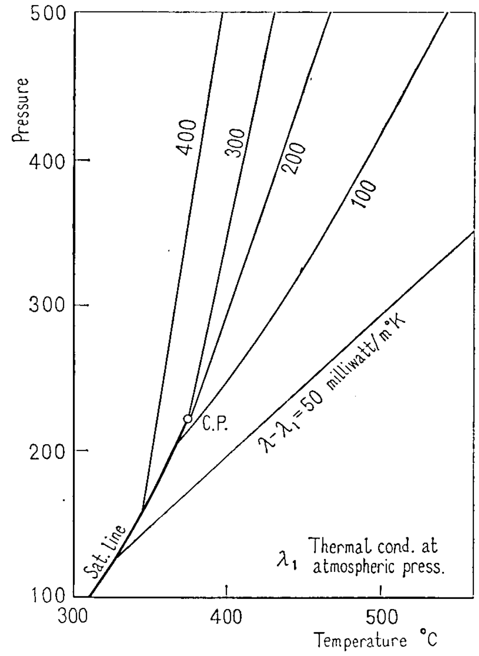

Fig. 6 Residual thermal conductivity $\lambda - \lambda_1$

Table 4 Constants in Eqs. (4)～(6)

D_{00}	$0.568\ 102 \times 10^{3}$	E_{13}	$-0.383\ 33 \times 10^{-8}$
D_{01}	$0.199\ 947 \times 10^{1}$	E_{20}	$4.314\ 375 \times 10^{-3}$
D_{02}	$-0.106\ 453 \times 10^{-1}$	E_{21}	-1.180×10^{-5}
D_{03}	$0.229\ 461 \times 10^{-4}$	E_{22}	$0.806\ 25 \times 10^{-8}$
D_{04}	$-0.391\ 562 \times 10^{-7}$	E_{30}	$-0.390\ 267 \times 10^{20}$
D_{10}	$0.946\ 043 \times 10^{-1}$	E_{31}	$-1.332\ 959 \times 10^{18}$
D_{11}	$-0.310\ 186 \times 10^{-3}$	E_{32}	$1.073\ 091 \times 10^{16}$
D_{12}	$-0.205\ 224 \times 10^{-5}$	E_{33}	$-0.149\ 865 \times 10^{14}$
D_{13}	$0.189\ 662 \times 10^{-7}$	F_{00}	702.62×10^{0}
D_{20}	$-0.453\ 755 \times 10^{-4}$	F_{01}	-7.38×10^{0}
D_{21}	$0.108\ 352 \times 10^{-5}$	F_{02}	$6.952\ 65 \times 10^{0}$
D_{22}	$-0.544\ 899 \times 10^{-8}$	F_{03}	$-1.625\ 385 \times 10^{-2}$
E_{00}	$1.749\ 54 \times 10^{1}$	F_{04}	$0.593\ 922 \times 10^{-4}$
E_{01}	$6.770\ 77 \times 10^{-2}$	F_{05}	$-0.241\ 224 \times 10^{-6}$
E_{02}	$6.082\ 39 \times 10^{-5}$	F_{10}	$0.476\ 192 \times 10^{0}$
E_{10}	$0.604\ 0 \times 10^{0}$	F_{11}	$0.737\ 856 \times 10^{-2}$
E_{11}	$-0.346\ 17 \times 10^{-2}$	F_{12}	$0.219\ 524 \times 10^{-4}$
E_{12}	$0.066\ 0 \times 10^{-4}$		

in the tables are regional boundaries of three sub-regions.

Calculated values of this report agree with ICPS values (1964) within their tolerances as seen in Fig. 9. At 375°C and 225 bar the deviation amounts to about 8%, although the ICPS tolerance at this point is 10%. This point is the closest to the critical point among the given grid points in the ICPS table and it is very difficult to express the viscosity as a function of temperature and pressure.

Calculated values are also compared with recent experimental values. At atmospheric pressure, comparisons with values by Brain[14], Tarzimanoff[15], Vargaftik[16] and Geier[17] are shown in Fig. 10. In Fig. 10 also shown are those by Desmond[18] who calculated the thermal conductivity value from the experimentally obtained Prandtl number. Brain's values agree with Vargaftik's, although the former show bigger scattering. It is quite interesting that Desmond's values obtained by the procedure different from ordinary measuring methods fall within their estimated errors from the values of present report. Desmond with his co-worker calculated the Prandtl number from their measurement of the

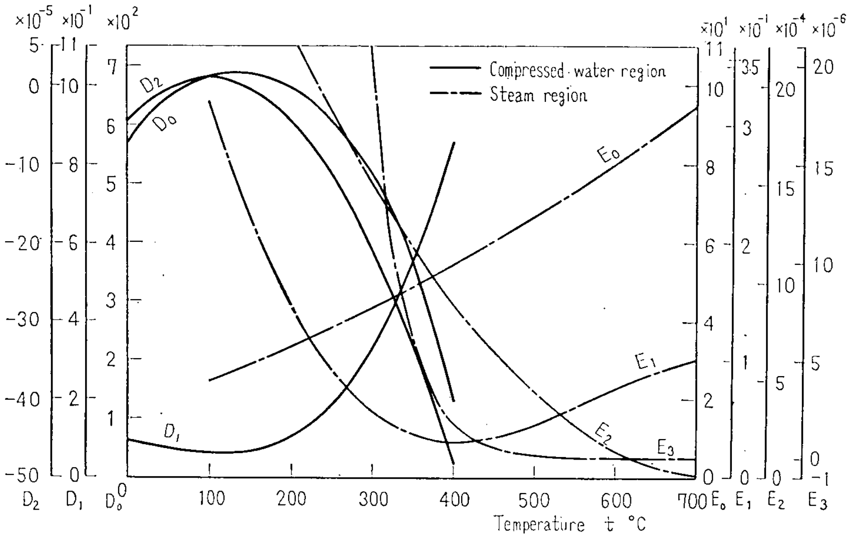

Fig. 7 Coefficients in Eq. (4) and Eq. (5) (Thermal conductivity)

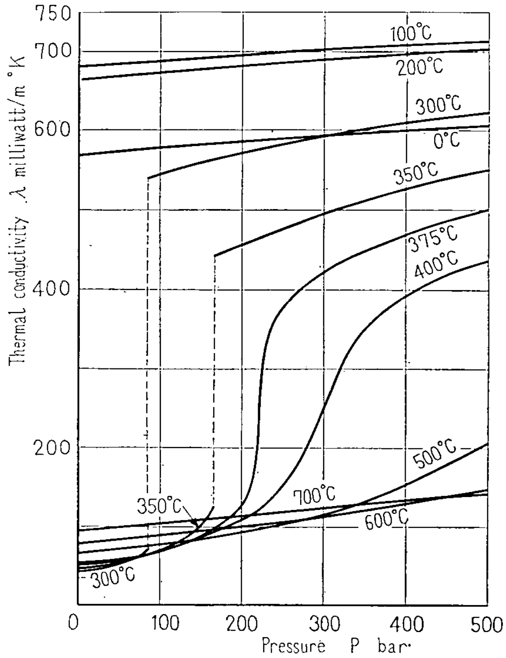

Fig. 8 Thermal conductivity of water and steam

Table 5 Thermal conductivity calculated by Eqs. (4)～(6)

Pressure (bar)	Temperature (°C) 0	50	100	150	200	250	300	350	375	400	425	450	475	500	550	600	650	700
1	568	644	24.4	29.3	33.6	38.3	43.3	48.7	51.5	54.3	57.3	60.3	63.4	66.6	73.2	80.1	87.3	94.8
5	569	644	681	686	34.2	38.7	43.6	48.9	51.6	54.5	57.4	60.5	63.6	66.8	73.4	80.4	87.7	95.2
10	569	645	681	687	35.2	39.4	44.0	49.1	51.9	54.7	57.6	60.7	63.8	67.0	73.7	80.8	88.1	95.7
25	570	646	682	688	666	42.7	45.9	50.4	52.9	55.6	58.5	61.5	64.7	67.9	74.8	82.0	89.5	97.2
50	573	648	684	689	668	617	52.4	54.2	56.0	58.2	60.8	63.6	66.6	69.8	76.7	84.1	91.8	99.7
75	575	650	685	691	671	621	64.7	60.5	61.0	62.3	64.2	66.6	69.3	72.3	79.0	86.4	94.2	102
100	577	652	687	693	673	626	543	70.0	68.1	67.9	68.8	70.5	72.7	75.4	81.6	88.9	96.7	105
125	579	654	689	694	675	630	550	83.3	77.8	75.3	74.7	75.4	76.9	79.0	84.5	91.1	99.1	107
150	581	655	690	696	677	633	557	101	90.3	84.7	82.1	81.3	81.8	83.2	87.6	94.2	102	110
175	583	657	692	697	680	637	563	446	106	96.1	90.8	88.3	87.5	88.0	91.1	97.1	104	112
200	585	659	693	699	682	641	570	456	125	110	101	96.4	94.1	93.4	94.8	100	107	115
225	587	661	695	701	684	644	576	466	271	126	113	106	101	99.4	98.8	103	110	117
250	589	663	697	702	686	647	581	476	375	161	127	116	110	106	103	107	112	120
275	591	665	698	704	688	650	587	486	402	210	143	128	119	113	108	110	115	122
300	592	667	700	705	690	653	592	494	421	265	178	141	129	121	112	114	118	124
350	596	670	703	709	693	659	602	511	450	352	238	188	152	139	123	122	124	129
400	597	674	707	712	697	664	610	526	468	393	296	230	179	160	134	130	129	134
450	601	677	710	715	700	668	618	539	485	418	343	271	210	183	146	138	135	138
500	604	681	714	718	703	672	624	551	500	438	377	309	245	209	159	147	141	142

recovery factor, and then obtained the thermal conductivity value using the experimental viscosity value by Kestin and co-workers and the specific heat value in the NBS table (1955). Comparisons with values by Vargaftik[19], Cherneeva[20] and Amirkhanov[21] at high pressure are shown in Fig. 11. As a whole, agreement is satisfactory.

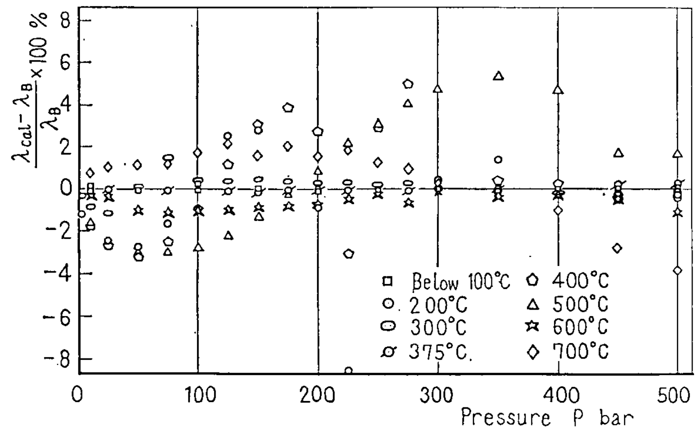

Fig. 9 Comparisons of calculated thermal conductivity values with basic data (ICPS standard values)

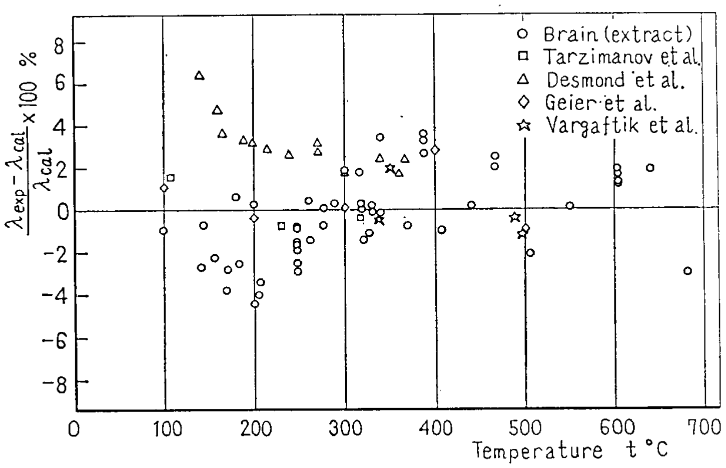

Fig. 10 Thermal conductivity of steam at atmospheric pressure

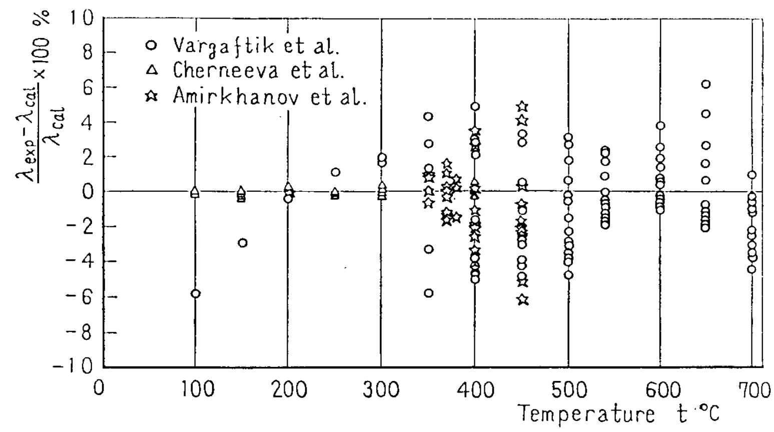

Fig. 11 Comparisons of thermal conductivity values at high pressures

Cherneeva's values seem to scatter less because they are smoothed out from experimental values. At high temperature and pressure, tolerances of ICPS values as well as estimated errors of experimental investigations are quite large and it seems to be meaningless at present to make more precise formulations without knowing more reliable experimental data.

5. Prandtl number

5・1 Basic data

Basic data of the Prandtl number are determined by selecting the data of the viscosity, the thermal conductivity and the specific heat. In the present investigation, basic data described in the previous section 3 and section 4 were used for the Prandtl number calculation. As the data for the specific heat, Sirota's skeleton table of 1963[22] was used. The effective region of present formulation was determined as up to 600°C and 500 bar, where Sirota's table is given. There might be a way to proceed with this correlation, for consistency, using the viscosity and the thermal conductivity values calculated by previous equations rather than previous basic data. However, in order to avoid accumulation of errors in the viscosity, the thermal conductivity and the Prandtl number correlations, the basic data of the former two properties were used also in compilation of the basic data of the Prandtl number.

5・2 Equations of Prandtl number

In the compressed water and the steam region, simple polynomial forms of the second or the third order in pressure were adopted. In the critical region, the specific haet shows a very big change with temperature and pressure, and it is more difficult to express the Prandtl number as a function of temperature and pressure than in the case of other properties. In the present investigation, the narrow region near the critical point was excluded from the effective region of the equation, and the explicit function was used instead of the implicit form. The Prandtl number was expressed as a function of $P-P_m$, considering the fact that the isotherm of Prandtl number has symmetrical form on both sides of P_m. P_m is the pressure where the specific heat shows a maximum value on each isotherm.

5・2・1 Compressed water region

$$P_r=\sum_{i=0}^{3} G_i p^i \qquad (7)$$

$$G_0=1/\left\{\left(\sum_{i=0}^{3} G_{0i}t^i\right)(t+100)^2\right\}$$
$$G_1=\left(\sum_{i=0}^{1} G_{1i}t^i\right)t^8+G_{12}/(t+50)^3$$
$$G_2=\left(\sum_{i=0}^{3} G_{2i}t^i\right)(t^8+100)$$
$$G_3=\left(\sum_{i=0}^{3} G_{3i}t^i\right)(t^8+100)$$

5・2・2 Steam region

$$P_r=\sum_{i=0}^{2} H_i P^i \quad\cdots\cdots(8)$$
$$H_0=\sum_{i=0}^{1} H_{0i}t^i+H_{02}/t^3$$
$$H_1=\left(\sum_{i=0}^{3} H_{1i}t^i\right)/t^5$$
$$H_2=\left(\sum_{i=0}^{3} H_{2i}t^i\right)/t^5$$

In the case of the calculation in the narrow region near the critical point below 400°C (around the grid points marked with* in Tables 3 and 7), the next term should be added:

$$P_{r1}=H_3P^{16} \quad\cdots\cdots(8\cdot a)$$
$$H_3=H_{31}(t/100)^{-50}$$

5・2・3 Critical region

$$P_r=1/\{K_1+K_2(P-P_m)^2+K_3(P-P_m)^4\} \quad\cdots\cdots(9)$$
$$K_1=(1/t^7)\sum_{i=0}^{3} K_{1i}t^i+K_{14}/\{(t-t_c)^2+K_{15}\}$$
$$K_2=(1/t^6)\sum_{i=0}^{3} K_{2i}t^i+K_{24}/\{(t-t_c)^4+K_{25}\}$$
$$K_3=(1/t^5)\sum_{i=0}^{3} K_{3i}t^i+K_{34}/\{(t-t_c)^4+K_{35}\}$$

P_m is the pressure where C_p has a maximum value on each isotherm and is expressed as:

$$P_m=\sum_{i=0}^{1} K_{0i}(t-t_c)^i$$

Constants in Eqs. (7)~(9) are shown in Table 6. They are also shown in Fig. 12 for approximate calculation.

5・3 Calculated results

The Prandtl numbers of water and steam calculated by Eqs. (7)~(9) are given in Tables 3 and 7. Solid lines in the tables show boundaries of three sub-regions, while values marked with* show those calculated by Eq. (8) accompanied by Eq. (8・a). Calculated Prandtl number is also shown in Fig. 13.

Comparisons of atmospheric values with those by Grigull[23], 1968 JSME Steam Tables[24],

Table 6 Constants in Eqs. (7)~(9)

G_{00}	7.6923×10^{-6}	H_{22}	-4.69341×10^{4}
G_{01}	1.31156×10^{-7}	H_{23}	3.46433×10^{1}
G_{02}	-7.35446×10^{-10}	H_{31}	5.362×10^{-9}
G_{03}	9.14321×10^{-13}	K_{00}	221.2×10^{0}
G_{10}	-1.2736×10^{-22}	K_{01}	2.69822×10^{0}
G_{11}	3.1840×10^{-25}	K_{10}	4.960269×10^{19}
G_{12}	-4.0075×10^{2}	K_{11}	-2.625608×10^{17}
G_{20}	-2.29386×10^{-24}	K_{12}	3.487276×10^{14}
G_{21}	2.18664×10^{-26}	K_{13}	-9.6×10^{7}
G_{22}	-6.45544×10^{-29}	K_{14}	-1.48379×10^{-1}
G_{23}	5.9776×10^{-32}	K_{15}	8.57683×10^{-1}
G_{30}	4.17501×10^{-27}	K_{20}	-1.797694×10^{14}
G_{31}	-3.80277×10^{-29}	K_{21}	1.289022×10^{12}
G_{32}	1.10881×10^{-31}	K_{22}	-3.043666×10^{9}
G_{33}	-1.03749×10^{-34}	K_{23}	2.369637×10^{6}
H_{00}	0.98122×10^{0}	K_{24}	3.21904×10^{-3}
H_{01}	-1.622×10^{-4}	K_{25}	6.43808×10^{0}
H_{02}	-1.64×10^{5}	K_{30}	5.61575×10^{7}
H_{10}	6.01559×10^{3}	K_{31}	-4.05695×10^{5}
H_{11}	-3.98124×10^{1}	K_{32}	9.65706×10^{2}
H_{12}	8.78715×10^{-2}	K_{33}	-7.57896×10^{-1}
H_{13}	-6.33450×10^{-5}	K_{34}	-3.57244×10^{-7}
H_{20}	-2.58632×10^{9}	K_{35}	1.78622×10^{0}
H_{21}	2.20037×10^{7}		

Table 7 Prandtl number calculated by Eqs. (7)~(9)

Pressure (bar)	Temperature (°C) 0	50	100	150	200	250	300	350	375	400	425	450	475	500	550	600
1	13.0	3.55*	0.984	0.982	0.954	0.939	0.930	0.922	0.918	0.914	0.911	0.907	0.903	0.899	0.892	0.883
25	12.9	3.54	1.74	1.15	0.935	1.18	1.04	0.966	0.949	0.937	0.930	0.924	0.920	0.915	0.904	0.889
50	12.8	3.53	1.74	1.15	0.931	0.893	1.22	1.05	1.01	0.977	0.959	0.948	0.940	0.933	0.918	0.895
75	12.8	3.52	1.73	1.15	0.926	0.878	1.47	1.17	1.09	1.03	0.999	0.977	0.963	0.952	0.931	0.901
100	12.7	3.51	1.73	1.14	0.921	0.864	0.962	1.34	1.20	1.11	1.05	1.01	0.988	0.972	0.944	0.908
125	12.6	3.50	1.73	1.14	0.916	0.851	0.932	*1.55	1.33	1.20	1.11	1.05	1.02	0.993	0.958	0.915
150	12.5	3.49	1.72	1.14	0.912	0.839	0.906	*2.00	1.50	1.30	1.18	1.10	1.05	1.02	0.971	0.923
175	12.4	3.48	1.72	1.14	0.908	0.829	0.883	1.47	*1.76	1.43	1.26	1.15	1.08	1.04	0.984	0.931
200	12.4	3.47	1.72	1.14	0.903	0.820	0.864	1.29	*2.59	1.57	1.35	1.21	1.12	1.06	0.998	0.939
225	12.3	3.46	1.71	1.13	0.900	0.812	0.849	1.18	12.9	1.72	1.45	1.27	1.16	1.09	1.01	0.947
250	12.2	3.45	1.71	1.13	0.896	0.805	0.835	1.13	2.19	2.48	1.56	1.34	1.20	1.12	1.02	0.956
275	12.1	3.44	1.71	1.13	0.893	0.799	0.825	1.08	1.63	3.91	1.68	1.41	1.25	1.14	1.04	0.966
300	12.0	3.43	1.70	1.13	0.890	0.794	0.816	1.04	1.41	4.26	1.97	1.49	1.29	1.17	1.05	0.975
350	11.9	3.41	1.70	1.12	0.886	0.787	0.803	0.964	1.23	1.91	2.50	1.67	1.40	1.24	1.08	0.996
400	11.7	3.39	1.69	1.12	0.883	0.784	0.794	0.906	1.08	1.44	2.19	1.87	1.52	1.30	1.10	1.02
450	11.6	3.37	1.69	1.12	0.883	0.784	0.786	0.864	0.981	1.20	1.58	1.84	1.64	1.37	1.13	1.04
500	11.4	3.35	1.68	1.12	0.886	0.787	0.777	0.838	0.938	1.07	1.32	1.63	1.54	1.45	1.16	1.07

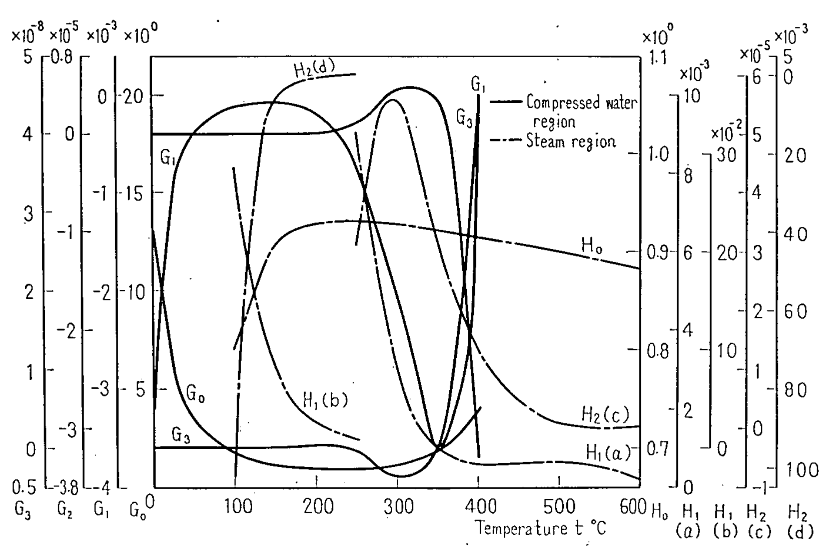

Fig. 12 Coefficients in Eq. (7) and Eq. (8) (Prandtl number)

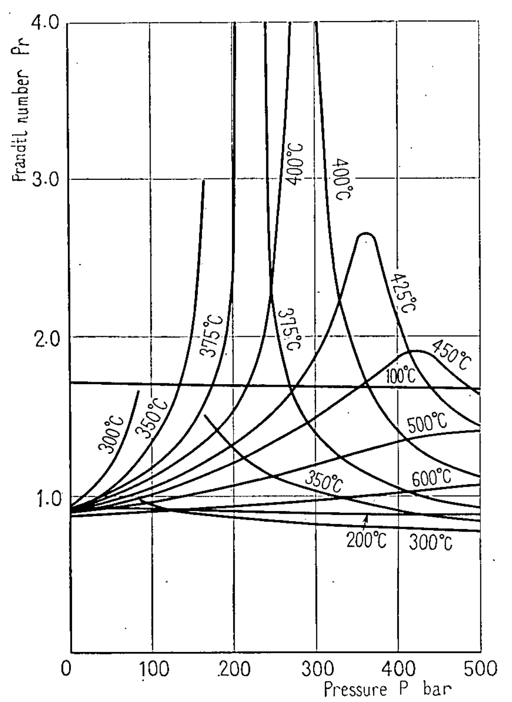

Fig. 13 Prandtl number of water and steam

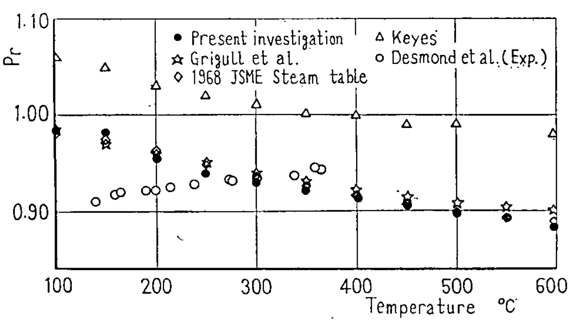

Fig. 14 Prandtl number of steam at atmospheric pressure

Keyes[25] and Desmond[18] are shown in Fig. 14. Higher tendency of Keyes' seems to be due to their basic data. Desmond's trend against temperature differs from those of other investigations and this is open to future examination.

6. Conclusions

Correlations of the viscosity, the thermal conductivity and the Prandtl number as a function of temperature and pressure were established and tables of calculated properties are given in a wide range of temperatures and pressures. Calculated results were compared with some of recent experimental investigations. Calculation by the equation having the density as an explicit variable needs the density calculation prior to the intended properties calculation when temperature and pressure are given. But if the explicit independent variables are temperature and pressure as in equations of present correlation, property values can conveniently be calculated without knowing the density value. Comparisons and examinations of recent experimental investigations reveal the necessity for more reliable experimental data especially on the thermal conductivity.

7. Aknowledgement

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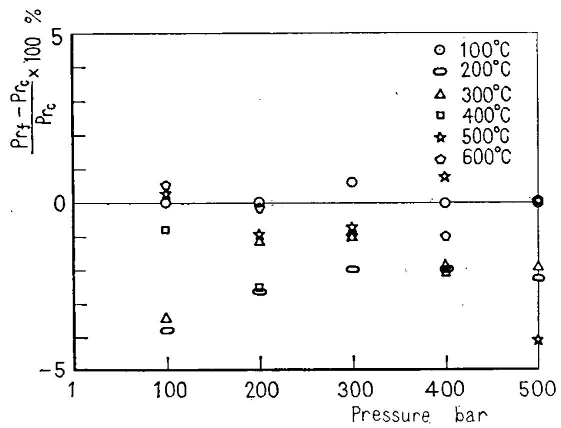
Discussion

K. Miyabe (Kyushu Institute of Techonlogy):
Correlating the Prandtl number, the authors used the values by the authors' empirical equations for the viscosity, the ICPS values for the thermal conductivity and the values by Sirota[22] for the spesific heat, and stated the reason for their choice as avoiding the accumulation of deviations in each preceding correlation. However, considering the consistency of correlated equations and applicability to heat transfer calculations, it is preferable to use the values of the viscosity and the thermal conductivity by authors' equations and also to use the specific heat values by an equation correlated from Sirota's values.

In Fig. 14, comparisons of the Prandtl numbers are given only at 1 bar. But additional comparisons at higher pressures and temperatures will be convenient to judge the quality of present correlations.



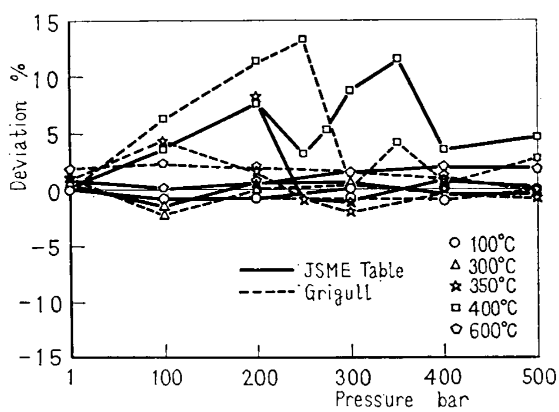
Pr_c: value by Eqs. (7)～(9).
Pr_f: value by η in Table 2, λ in Table 5 and c_p in Sirota's table.

Append.- Fig. 1 Prandtl number

Authors' closure

The authors appreciate Dr Miyabe's comments on the correlation of the Prandtl number. Concerning the choice of the basic data for the Prandtl number, the "basic data" were not used as the solid standard values but rather as guess values only for the correlation techniques in present investigation. Equations obtained were adjusted for better consistency among different correlations. As shown in Append.- Fig. 1, the resulting difference between the values calculated by Eqs. (7)～(9) and those calculated from the viscosity by Eqs. (1)～(3), the thermal conductivity by Eqs. (4)～(6) and the specific heat of Sirota's table, is well less than 4% except in the narrow region near the critical point.

The correlation of the specific heat was not included in the present investigation because it



Append.- Fig. 2 Comparison of Prandtl number (high pressure) could be calculated by existing equations of state for water and steam. The simple equation of the specific heat in the industrially important region of temperature and pressure, as well as of other properties, may be one of the future tasks.

Comparisons of the Prandtl number in the high temperature and pressure region are shown in Append.- Fig. 2. Deviations at around 400°C came mainly from the choice of different viscosity sources. Review of existing viscosity values will be found in reference (2). Recent experimental values, which were used in this investigation, are lower by about 10% than ICPS values in this region. In other regions, discrepancies are less than 5% up to high pressures.