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Correlations in economic time series

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Abstract

A financial index of the New York stock exchange, the S&P500, is analyzed at 1 min intervals over the 13 yr period, January 84–December 96. We quantify the correlations of the absolute values of the index increment. We find that these correlations can be described by two different power laws with a crossover time $t_\times \approx 600$ min. Detrended fluctuation analysis gives exponents $\alpha_1 = 0.66$ and $\alpha_2 = 0.93$ for $t < t_\times$ and $t > t_\times$, respectively. Power spectrum analysis gives corresponding exponents $\beta_1 = 0.31$ and $\beta_2 = 0.90$ for $f > f_\times$ and $f < f_\times$, respectively.

A topic of considerable recent interest to both the economics and physics communities is whether there are correlations in economic time series and, if so, how to best quantify these correlations [1–5]. Here we study the S&P500 index of the New York stock exchange over a 13 yr period (Fig. 1a). We calculate the logarithmic increments $g(t) \equiv \ln Z(t+1) - \ln Z(t)$ over a fixed time lag of 1 min, where $Z(t)$ denotes the index at time t (t counts the number of minutes during the opening hours of the stock market), and quantify the correlations as follows:

(i) We find that the correlation function of $g(t)$ decays exponentially with a characteristic time of the order of 1–10 min, but the absolute value $|g(t)|$ does not. This result is consistent with previous studies on several economic series [3–5].

(ii) We calculate the power spectrum of $|g(t)|$ (Fig. 2a), and find that the data fit not one but rather two separate power laws: for $f > f_\times$ the power law exponent is $\beta_1 = 0.31$, while for $f < f_\times$ the exponent $\beta_2 = 0.90$ is three times larger; here $f_\times$ is called the crossover frequency.

(iii) We confirm these results using the DFA (detrended fluctuation analysis) method (see Fig. 2b), which allows accurate estimates of exponents *independent* of local trends

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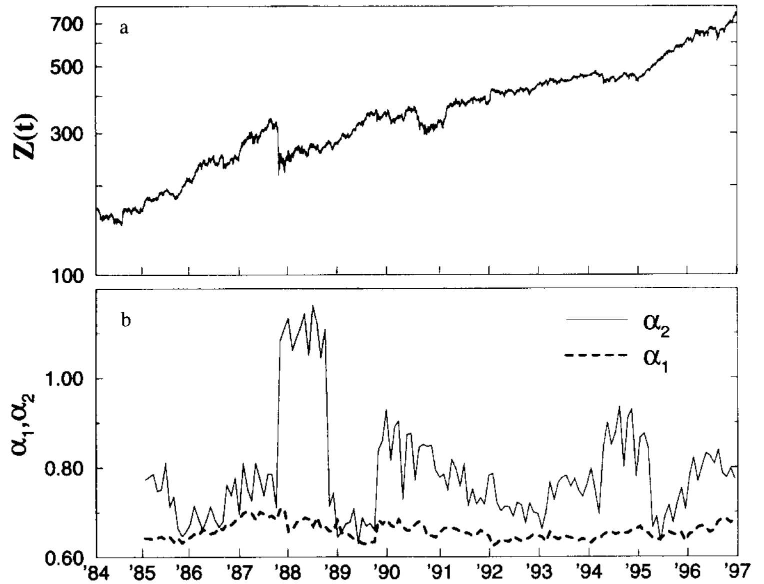

Fig. 1. (a) Raw data analyzed: The S&P500 index $Z(t)$ for the 13 year period 1 January 1984 – 31 December 1996 at intervals of 1 min (these data extend by 7 yr the data set analyzed by Mantegna and Stanley [6–8]). Note the large fluctuations, such as that on 19 October 1987 ("black Monday"). (b) Results of dragging a window of size 1 yr down the same data base, one month at a time, and calculating the best fit exponent α_1 (dashed line) and α_2 (full line) for the time intervals $t < t_\times$ and $t > t_\times$, respectively.

[9]. From the behavior of the power spectrum, we expect that the DFA method will also predict two distinct regions of power law behavior, with exponents $\alpha_1 = 0.66$ and $\alpha_2 = 0.95$ for t less than or greater than a characteristic time scale $t_\times \equiv 1/f_\times$, where we have used the general mathematical result [10] that $\alpha = (1+\beta)/2$. The data of Fig. 2b yield $\alpha_1 = 0.66$, $\alpha_2 = 0.93$, thereby confirming the consistency of the power spectrum and DFA methods. Also the crossover time is very close to the result obtained from the power spectrum, with $t_\times \approx 1/f_\times \approx 600$ min (about 1.5 trading days).

We observed the crossover behavior noted above by considering the entire 13 yr period studied, so it is natural to enquire whether it will still hold for periods smaller than 13 yr. Therefore, we choose a sliding window (with size 1 yr) and calculate both exponents α_1 and α_2 within this window as the window is dragged, down the data set. We find (Fig. 1b) that the value of α_1 is very "stable" (independent of the position of the window) fluctuating around the mean value 2/3. Surprisingly, however, the variation of α_2 is much greater, showing sudden jumps when very volatile periods enter or leave the time window.

We studied several standard mathematical models, such as fractional Brownian motion [10–15] and fractional ARIMA processes [16], commonly used to account for long-range correlation in a time series and found that none of them can reproduce the large fluctuation of α_2.

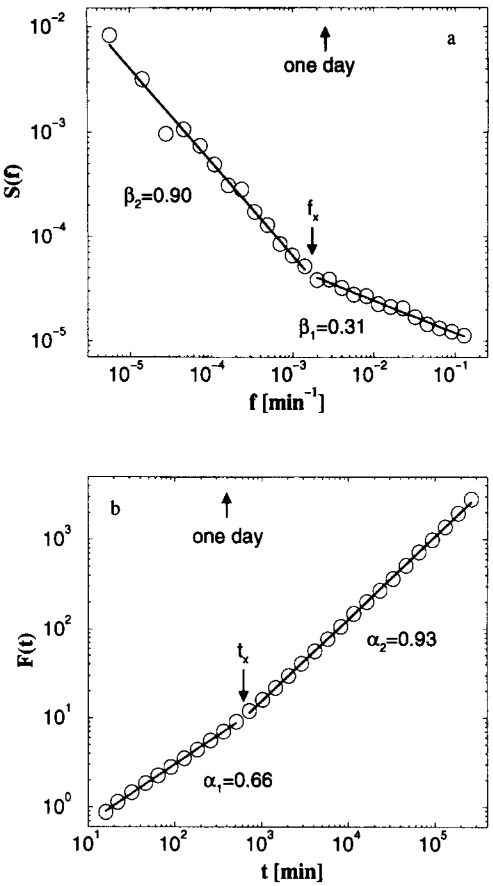

Fig. 2. Plot of (a) the power spectrum $S(f)$ and (b) the detrended fluctuation analysis $F(t)$ of the absolute values of the 1 min increments. The lines show the best power law fits (r values are better than 0.99) to the data above and below the crossover frequency of $f_\times = (1/570)\,\mathrm{min}^{-1}$ in (a) and of the crossover time $t_\times = 600\,\mathrm{min}$ in (b). To remove artificial correlations resulting from the intra-day pattern of the market activity [11–14], we analyze normalized data $|g_n(t)| \equiv |g(t)|/A(t)$, where $A(t)$ is the activity at the same time of the day averaged over all days of the data set. For the DFA method, we integrate $|g_n(t)|$ once; then we determine the fluctuations $F(t)$ of the integrated signal around the best linear fit in a time window of size t.

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