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COSINE SIMILARITY MEASURE FOR ROUGH INTUITIONISTIC FUZZY SETS AND ITS APPLICATION IN MEDICAL DIAGNOSIS

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Abstract: Similarity measure is an important mathematical tool used in medical diagnosis, pattern recognition etc. In this paper a cosine similarity for rough intuitionistic fuzzy set is proposed. Also we apply the notion of weighted rough intuitionistic fuzzy approach to cosine similarity measure for further investigation. Finally a medical diagnosis problem is defined to verify the proposed similarity measure.

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1. Introduction

Similarity measure is an imperative topic in fuzzy set theory. It is a vital tool for determining the degree of similarity between two objects. It has been rigorously

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investigated by researchers to deal with different types of problems. Many measures of similarity between fuzzy sets had been proposed based on their use in areas such as pattern recognition, machine learning, decision making, etc.

An intuitionistic fuzzy set is an extension of fuzzy set. After the proposal of fuzzy sets by Zadeh [8] lot of researches were made in the generalization of the concept and later the concept of an intuitionistic fuzzy set was given by Atanassov [1] in 1986.

Rough set theory proposed by Pawlak [5] 1982 is an extension of classical set theory. It deals with uncertainty or imprecision information.The concept of rough intuitionistic fuzzy sets in a lattice was proposed by K.V. Thomas et al.The notion of intuitionistic fuzzy set is a powerful tool to handle vagueness while the theory of rough intuitionistic fuzzy set is a powerful tool to handle incompleteness.

In this paper we prresent cosine similarity measure under rough intuitionistic fuzzy environment and give an example of medical diagnosis a model to demonstrate the applicability and effectiveness of the proposed approach

2. Preliminaries

Definition 2.1. [5] Let U be a non-empty universe of discourse, R an equivalence relation on U called an indistinguishable relation, $[X]_R$(briefly written [X]) is a R-equivalent class. $W = (U, R)$ iscalled an approximation space. For all $X \subseteq U$, suppose $X_L = \{x \in U | [x] \subseteq X\}$, $X_U = \{x \in U | [x] \cap X \neq \phi\}$,are called rough sets written as $X = (X_L, X_U)$ where X_L and X_U are the lower approximation and the upper approximation of X on W respectively.

Definition 2.2. [6] Let X be a non-null set and R be an equivalence relation X. Let F be an intuitionistic fuzzy set in X with the membership function μ_F and non-membership function ν_F. The lower and the upper approximations R1(F)and R2(F) respectively of the intuitionistic fuzzy sets of the quotient set X/R with

(1) Membership function defined by

$$\mu_{R1(F)}(X_i) = inf\{\mu_F(x) : x \in X_i\},$$
$$\mu_{R2(F)}(X_i) = sup\{\mu_F(x) : x \in X_i\},$$

(2) and non-membership function defined by

$$\nu_{R1(F)}(X_i) = sup\{\nu_F(x) : x \in X_i\},$$

$$\nu_{R2(F)}(X_i) = inf\{\nu_F(x) : x \in X_i\}.$$

For $x \in X_i, \mu_F(x) + \nu_F(x) \leq 1, \mu_F(x) \leq 1 - \nu_F(x)$. The rough intuitionistic fuzzy set of F is given by the pair $R(F) = (R1(F), R2(F))$ with

$$sup\{\mu_F(x) : x \in X_i\} + inf\{\nu_F(x) : x \in X_i\} \leq 1.$$

Definition 2.3. Cosine similarity is a measure of similarity between two vectors of an inner product space that measures the cosine of angle between them.

Cosine similarity is particularly used in a positive space, where the outcome is neatly bounded in [0,1].

It is nothing but the cosine of the angle between vector representation of two fuzzy sets.

Given two attribute vectors $X = \{x_1, x_2, , ...x_n, \}$ and $Y = \{y_1, y_2, , ...y_n, \}$ the cosine similarity, $cos\theta$ is presented as follows:

$$cos\theta = \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}}.$$

Let $A = (\mu_A(x_1), \mu_A(x_2), ..\mu_A(x_n))$ and $B = (\mu_B(x_1), \mu_B(x_2), ..\mu_B(x_n))$ are two fuzzy sets in the universe of discourse $X = \{x_1, x_2 ....x_n, x_i\} \in X$. A cosine similarity measure based on Bhattacharya's[2] distance between $\mu_A(x_i)$ and $\mu_B(x_i)$ can be defined as follows:

$$C_F(A, B) = \frac{\sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i)}{\sqrt{\sum_{i=1}^{n} \mu_A^2(x_i)}\sqrt{\sum_{i=1}^{n} \mu_B^2(x_i)}}.$$

Assume that there are two IFSs A and B in $X = \{x_1, x_2 ....x_n\}$. Based on the extension of the cosine measure for fuzzy sets a cosine similarity measure between intuitionistic fuzzy sets A and B is proposed as [4]:

$$C_{IFS}(A, B) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i) + \nu_A^2(x_i)}\sqrt{\mu_A^2(x_i) + \nu_A^2(x_i)}}.$$

3. Cosine Similarity Measure for Rough Intuitionistic Fuzzy Sets

Definition 3.1. Assume that there are two rough intuitionistic fuzzy sets A and B in $X = \{x_1, x_2.....x_n\}$. A cosine similarity measure between rough intuitionistic fuzzy sets A and B is given as:

$$C_{RIFS} = \frac{1}{n}\sum_{i=1}^{n} \frac{\phi\mu_A(x_i)\phi\mu_B(x_i) + \phi\nu_A(x_i)\phi\nu_B(x_i)}{\sqrt{(\phi\mu_A(x_i))^2 + (\phi\nu_A(x_i))^2}\sqrt{(\phi\mu_A(x_i))^2 + (\phi\nu_A(x_i))^2}}, \tag{1}$$

where

$$\phi\mu_A(x_i) = \frac{\underline{\mu}_A(x_i) + \overline{\mu}_A(x_i)}{2}, \quad \phi\nu_A(x_i) = \frac{\underline{\nu}_A(x_i) + \overline{\nu}_A(x_i)}{2},$$
$$\phi\mu_B(x_i) = \frac{\underline{\mu}_B(x_i) + \overline{\mu}_B(x_i)}{2}, \quad \phi\nu_B(x_i) = \frac{\underline{\nu}_B(x_i) + \overline{\nu}_B(x_i)}{2}.$$

Proposition 3.2. *The cosine similarity measure between two rough intuitionistic fuzzy sets should satisfy the following rules*

(1) $0 \le C_{RIFS} \le 1$.

(2) $C_{RIFS}(A, B) = C_{RIFS}(B, A)$

(3) $C_{RIFS}(A, B) = 1$ *if and only if* $A = B$.

(4) If $A \subseteq B \subseteq C$ *then* $C_{RIFS}(A, C) \le C_{RIFS}(A, B)$ *and* $C_{RIFS}(A, C) \le C_{RIFS}(B, C)$.

Proof. (1) It is noticeable that (1) is true by the value of cosine.

(2) Obvious.

(3) When $A = B$ then $C_{RIFS} = 1$. Conversely if $C_{RIFS(A,B)} = 1$ then $\phi\mu_A(x_i) = \phi\mu_B(x_i), \phi\nu_A(x_i) = \phi\nu_B(x_i)$. (i.e)$\underline{\mu}_A = \underline{\mu}_B, \overline{\mu}_A = \overline{\mu}_B$ and $\underline{\nu}_A = \underline{\nu}_B, \overline{\nu}_A = \overline{\nu}_B \Rightarrow A = B$.

(4) If $A \subseteq B \subseteq C$ then $\underline{\mu}_A(x) \subseteq \underline{\mu}_B(x) \subseteq \underline{\mu}_C(x), \overline{\mu}_A(x) \subseteq \overline{\mu}_B(x) \subseteq \overline{\mu}_C(x)$ and $\underline{\nu}_A(x) \supseteq \underline{\nu}_B(x) \supseteq \underline{\nu}_C(x), \overline{\nu}_A(x) \supseteq \overline{\nu}_B(x) \supseteq \overline{\nu}_C(x)$.

Then $C_{RIFS}(A, C) \subseteq C_{RIFS}(A, B)$ and $C_{RIFS}(A, C) \subseteq C_{RIFS}(B, C)$ since in the interval $[0, \frac{\pi}{2}]$ the cosine function is a decreasing function. □

The weighted cosine similarity measure between rough intuitionistic fuzzy set A and B for a considered weight w_i is projected as follows:

$$W_{RIFS}(A,B) = \sum_{i=1}^{n} w_i \frac{\phi\mu_A(x_i)\phi\mu_B(x_i) + \phi\nu_A(x_i)\phi\nu_B(x_i)}{\sqrt{\phi\mu_A(x_i)^2 + \phi\nu_A(x_i)^2}\sqrt{\phi\mu_A(x_i)^2 + \phi\nu_A(x_i)^2}},$$

where $w_i \in [0,1], i = 1,2....n$ and $\sum_{i=1}^{n} w_i = 1$. If $w_i = \frac{1}{n}, i = 1,2...n$ then $W_{RIFS}(A,B) = C_{RIFS}(A,B)$.

Perceptibly the weighted cosine similarity measure of two rough intuitionistic fuzzy sets A and B satisfies the following properties:

(1) $0 \leq W_{RIFS} \leq 1$.

(2) $W_{RIFS}(A,B) = W_{RIFS}(B,A)$.

(3) $W_{RIFS}(A,B) = 1$ if and only if $A = B$.

(4) If $A \subseteq B \subseteq C$ then $W_{RIFS}(A,C) \leq W_{RIFS}(A,B)$ and $W_{RIFS}(A,C) \leq W_{RIFS}(B,C)$.

Proof. Similar to Proposition 4.2. □

4. Application

Similarity measure for rough intuitionistic fuzzy sets are applied in Medical diagnosis to demonstrate their effectiveness. Since in some practical situation there exists a possiblility for each element to be classified with a upper and lower approximations of intuitionstic fuzzy sets we use the following similarity measure to demonstrate the effect of the medical diagnosis using rough intuitionistic fuzzy cosine similarity measure

Let $P = \{P_1, P_2, P_3, P_4, P_5, P_6\}$ be the set of patients,D= (viral fever, stomach problem, malaria) be a set of disease and S = (Temperature, Vomiting, Stomach pain) be a set of symptoms. Let the rough intuitionistic fuzzy relation between patients and symptoms be given by R_1 and the relation between symptoms and disease are given R_2

From formula (1) we can compute the cosine similarity between R_1 and R_2 is given in the tables.

From the above table we conclude that patient 1, patient 4, patient 6, patient 2 suffers from Viral fever and patient 3, patient 5 suffers from malaria.

R_1	Temperature	Vomiting	Stomach pain
P_1	((0.3, 0.5),(0.6, 0.4))	((0.2, 0.6),(0.3, 0.3))	((0.4, 0.4),(0.5, 0.3))
P_2	((0.2, 0.6),(0.3, 0.1))	((0, 0.8),(0.1, 0.4))	((0.4, 0.4),(0.6, 0.3))
P_3	((0.1, 0.6),(0.7, 0.2))	((0.6, 0.2),(0.7, 0.1))	((0.3, 0.5),(0.4, 0.5))
P_4	((0.2, 0.7),(0.3, 0.5))	((0.3, 0.6),(0.5, 0.4))	((0.1, 0.7),(0.2, 0.5))
P_5	((0.1, 0.8),(0.7, 0.4))	((0.7, 0.2),(0.9, 0))	((0.3, 0.6),(0.4, 0.4))
P_1	((0.5, 0.3),(0.6, 0.2))	((0.3, 0.4),(0.5, 0.4))	((0.1, 0.6),(0.2, 0.4))

Table 1

R_2	Viral fever	Stomach problem	Malaria
P_1	((0.3, 0.5),(0.4, 0.2))	((0.5, 0.4),(0.6, 0.3))	((0.3, 0.6),(0.4, 0.5))
P_2	((0.2, 0.7),(0.3, 0.6))	((0.3, 0.6),(0.6, 0.3))	((0.7, 0.2),(0.8, 0.1))
P_3	((0.4, 0.5),(0.5, 0.4))	((0.8, 0.1),(0.9, 01))	((0.4, 0.5),(0.5, 0.4))

Table 2

R_3	Viral fever	Stomach problem	Malaria
P_1	**0.9941**	0.9311	0.8720
P_2	**0.9836**	0.8738	0.7794
P_3	0.8482	0.82982	**0.9869**
P_4	**0.9117**	0.7227	0.8695
P_5	0.8125	0.7898	**0.9940**
P_6	**0.9102**	0.7958	0.8502

Table 3

5. Conclusion

This paper proposed the cosine similarity measure for rough intuitionistic fuzzy set based on a cosine function. Then the weighted cosine similarity measure for rough intutionistic fuzzy sets are proposed by considering the importance of each element. Finaly a medical diagnosis problem with simplified rough intuitionistic fuzzy information are provided to demonstrate the application and effectiveness of medical diagnosis method using the provided cosine similarity measure.

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