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Cosmic microwave background polarization data and galactic foregrounds: estimation of cosmological parameters

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ABSTRACT

We estimate the accuracy with which various cosmological parameters can be determined from the cosmic microwave background (CMB) temperature and polarization data when various galactic unpolarized and polarized foregrounds are included and marginalized using the multi-frequency Wiener filtering technique. We use the specifications of the future CMB missions *MAP* and *Planck* for our study. Our results are in qualitative agreement with earlier results obtained without foregrounds, though the errors in most parameters are higher because of degradation of the extraction of polarization signal in the presence of foregrounds.

Key words: cosmic microwave background.

1 INTRODUCTION

One of the primary goals of cosmology is to accurately determine various cosmological parameters associated with the background Friedmann–Robertson–Walker (FRW) universe and the structure formation in the universe (Ω, Ω_Λ, Ω_B, h_0, etc.). In recent years compelling theoretical arguments have emerged which suggest that the study of cosmic microwave background (CMB) anisotropies is the best hope to achieve this goal (Knox 1995; Bond 1996; Jungman et al. 1996; Bond, Efstathiou & Tegmark 1997; Zaldarriaga, Spergel & Seljak 1997; Lineweaver & Barbosa 1998). On the experimental front, two forthcoming satellite experiments *MAP* and *Planck*[1] along with a series of ground-based and balloon-borne experiments on degree to sub-arcmin scales plan to unravel the angular power spectrum of the CMB to angular scales $\gtrsim$1 arcmin (for details of interferometric ground-based experiments see White et al. 1999 and references therein; for a recent update on balloon-borne experiments see Lee et al. 1999). It has been shown that an accurate determination of the CMB temperature fluctuations down to sub-degree scales could fix the values of nearly 10 cosmological parameters with unprecedented precision (Jungman et al. 1996). In addition, the future satellite missions might detect the small, hitherto elusive signal from CMB polarization fluctuations (Bouchet, Prunet & Sethi 1999 – hereafter Paper I – and references therein). The polarization data can be used to break degeneracy between a few parameters that are determined only in a combination using the temperature data alone (Zaldarriaga et al. 1997; Kamionkowski & Kosowsky 1998).

One of the major difficulties in extracting the power spectrum of temperature and polarization fluctuations of the CMB is the presence of galactic and extragalactic foregrounds. The extragalactic foregrounds (radio and infrared point sources, clusters, etc.) will only affect small angular scales ($\lesssim$10 arcmin) at frequencies dominated by the CMB signal (Toffolatti et al. 1998). The galactic foregrounds, on the other hand, are present at all angular scales and are strongest on the largest scales. They will therefore have to be cleaned from the future data before any definitive statements about the primary CMB signal can be made. A multi-frequency Wiener filtering approach was developed to study the implications of the presence of foregrounds for the performance of future CMB missions (Bouchet, Gispert & Puget 1996; Tegmark & Efstathiou 1996). It was shown that the primary CMB temperature signal is much larger than the contaminating foreground for all the angular scales relevant for future satellite missions. And therefore the performance of future all-sky satellite missions in extracting the CMB temperature power spectra is unlikely to be hindered by galactic foregrounds (Bouchet et al. 1996; Tegmark & Efstathiou 1996; Gispert & Bouchet 1996; Bouchet & Gispert 1999).

In a previous paper (Paper I) we extended this technique to include polarization and temperature-polarization cross-correlation of foregrounds to estimate their effect on the extraction of CMB polarization power spectra. Our analysis showed that the presence of foregrounds should not seriously deter the detection of *E*-mode CMB polarization and *ET* cross-correlation by *Planck*. However, while the detection of CMB polarization will be easiest at the Doppler peaks of polarization fluctuations $\ell \geqslant 100$, where it should help reducing the errors on the measurement of parameters that will already be well constrained by temperature data alone, the truly new information from polarization data in

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[1] For details see http://map.gsfc.nasa.gov/ (*MAP*) and http://astro.estec.esa.nl/SA-general/Projects/Planck/ (*Planck*).

the determination of cosmological parameters is contained in angular scales corresponding to $\ell \leqslant 30$ (Zaldarriaga et al. 1997).

The polarization data helps break degeneracy between C_2, the quadruple moment of CMB temperature fluctuation and τ, the optical depth to the last scattering surface (Zaldarriaga et al. 1997). The former gives the overall normalization of the CMB fluctuations and is fixed at the epoch of inflation in inflationary paradigm. The breaking of this degeneracy also results in a better determination of other inflationary parameters *T*/*S*, the ratio of tensor to scalar quadruple, and the tensor index n_T. The optical depth to the last scattering surface is crucial to understanding the epoch of reionization in the universe. Even a value of τ so small as 0.05 leave a tell-tale signature in the CMB polarization fluctuations that is potentially detectable (Zaldarriaga 1997). The main difficulty in using polarization data to break the C_2–τ degeneracy is that one needs to use information on large angular scales. The power spectra at small ℓ is not only badly determined because of cosmic variance but also because of largely unknown level of polarized foregrounds.

Prunet et al. (1998) attempted to model the dust polarized emission from the galaxy – which is the dominant foreground for *Planck* HFI – for scales between 30 arcmin to a few degrees. They showed that though one might obtain meaningful estimates for degree scales, there can be large uncertainties in the large-scale ($\ell \leqslant 50$) polarized dust emission in the galaxy. The polarized synchrotron is the other major galactic foreground – and it is likely to undermine the performance of *MAP* and *Planck* LFI. For the lack of reliable data, we assumed the power spectra of polarized synchrotron to mimic that of the unpolarized component in Paper I. Though there remain large uncertainties on the polarization foregrounds, these assumed levels of foregrounds combined with the Wiener filtering methods developed in Paper I allow us to quantify the effect of foregrounds on the extraction of CMB signal. In this paper, we use the methods developed in Paper I to ascertain the errors in the cosmological parameter estimation.

In the next section, we briefly review the Fisher matrix approach used in determining the errors on the extraction of cosmological parameters. We take three underlying theoretical models for our study, the rationale for which is briefly described in the next section. The results are presented and discussed in Section 3. In Section 4 we give our conclusions, and discuss the various shortcomings of our approach.

2 FISHER MATRIX AND PARAMETER ESTIMATION

Future CMB missions *MAP* and *Planck* will reach pixel sensitivities of $\simeq 30\,\mu$K and $\simeq 1.5\,\mu$K, respectively. This should allow a very precise determination of temperature power spectrum and a possible detection of the polarization fluctuations (see Paper I). Given the noise level and the underlying theoretical model, the Fisher matrix approach allows one to get an estimate of the errors in the estimation of the parameters of the underlying model. It is defined as an average value of the second derivatives of the logarithm of the Likelihood function with respect to the cosmological parameters, at the true parameters value (for details see Kendall & Stuart 1969):

$$F_{ij} = \left\langle \frac{\partial^2 \mathcal{L}}{\partial \theta_i \partial \theta_j} \right\rangle_{\Theta = \Theta_0}. \qquad (1)$$

For CMB temperature and polarization data, the Fisher matrix can be expressed as (Tegmark, Taylor & Heavens 1997):

$$F_{ij} = \frac{1}{2} Tr \left[\mathcal{C}^{-1} \frac{\partial \mathcal{C}}{\partial \theta_i} \mathcal{C}^{-1} \frac{\partial \mathcal{C}}{\partial \theta_j} \right] \qquad (2)$$

where $\mathcal{C}$ stands for the covariance matrix of the data and θ_i correspond to cosmological parameters. The details of derivation of the covariance matrix and its derivatives in the presence of foregrounds are given in Appendix A. The error in the estimation of parameters is given by:

$$\Delta \theta_i = [F^{-1}]_{ii}^{-1/2}. \qquad (3)$$

2.1 Underlying models

The estimated errors on parameters will depend on the choice of the underlying model. We consider three models for our study. Although these models do not exhaust all the possible models and their variants, our aim is to understand the errors in parameter estimation for sCDM model and its popular variants, within the framework of generic inflationary models. We are interested in the standard parameters of flat FRW cosmology, h, $\Omega_{\rm B}$, Ω_ν, Ω_Λ, the reionization parameter τ, and the inflationary parameters, C_2, n_s, *T*/*S*, and n_T. It is of course possible to consider a more general class of inflationary models which leads to a further proliferation of inflationary parameters (Liddle 1998; Souradeep et al. 1998; Kinney, Dodelson & Kolb 1998; Lesgourgues, Prunet & Polarski 1999). We also do not consider open/closed universes because, as shown in Zaldarriaga et al. (1997), in such universes the shift in the angular size of the horizon at the last scattering surface leaves a very significant sign in the CMB fluctuations which cannot be mimicked by a change in any other parameter, and therefore $\Omega_{\rm total}$ is extremely well determined for open/closed universes. It is possible for the universe to be flat (or nearly flat) with contribution from both matter and cosmological constant, and one could attempt to measure both these parameters from CMB data. However, the degeneracy between these two parameters cannot be broken by CMB data alone and one will have to resort to other measurements like observation of supernova at high z to lift this degeneracy (Tegmark, Eisenstein & Hu 1998; Efstathiou & Bond 1999).

In addition, it is possible to include parameters like n_ν, the number of massless neutrinos, and $Y_{\rm He}$, the helium fraction. However, these parameters can be better determined by particle physics or local observations (Jungman et al. 1996; Bond et al. 1997). Parameters like Ω_ν, the contribution of massive neutrinos to the rest mass density in the universe, can be determined to a comparable accuracy by the data of future galaxy surveys like SDSS (Hu, Eisenstein & Tegmark 1998).

In this paper, we take only CMB data for our study and do not include priors from other measurements like future Galaxy surveys or high-z supernova results. The three models we consider are the following.

(i) An sCDM model with $\tau = 0.1$. The rather large value of τ is taken to bring out the effects of polarization data.

(ii) Tilted CDM model with $\tau = 0.1$, $n_S = 0.9$, $n_T = -0.1$, and $T/S = 0.7$. Note that $T/S = -7 n_T$, which is one of the predictions of slow-roll inflation (Starobinsky 1985; Liddle & Lyth 1992).

(iii) Model 2 with $\Omega_\nu = 0.3$ with two light, massive neutrinos.

3 RESULTS

We use the results of Paper I (the values of various terms in the covariance matrix as defined in Appendix A) for the specification of the future satellite missions. Our results are shown in Tables 1, 2, 3 for the three underlying models.

It should be noted that we fix the value of $C_2 = C_2^S + C_2^T = 796(\mu K)^2$ for all the models. Only for the sCDM model does it correspond to *COBE* normalization. In the models with tensor contribution, the *COBE*-normalized CMB signal is larger than the signal for our normalization by a factor of $\simeq 1.5$.

To assess the reliability of our code we computed the errors in the six-parameter sCDM model of Zaldarriaga et al. (1997) with their instrumental specifications and without foregrounds, and compared our results with both theirs and those obtained by Eisenstein, Hu & Tegmark (1999). Our results are comparable to those of Eisenstein et al. (1999) for most parameters, with the exception of τ where the error we find is noticeably bigger. We think that this discrepency is related to the way we compute the derivatives of the spectra with respect to the parameters (see Appendix A).

The results for sCDM model are shown in Table 1. For comparison, results for the best channel of each experiment without foregrounds are also shown. As is clearly seen, the performance of Wiener filtering matches the best channel case for all the experiments. In Paper I we showed that the Wiener filtering

Table 1. Errors on parameters for sCDM model with $\tau = 0.1$. Also shown are the corresponding errors for the best channel of each experiment.

Parameters	C_2	h	Ω_b	Ω_Λ	τ	n_S
Model	$796(\mu K)^2$	0.5	0.05	0.0	0.1	1.0
Wiener (*Planck*)	2.4%	1.36%	2.3%	0.039	4.6%	0.34%
Best channel (*Planck*)	2.1%	1.06%	1.82%	0.03	3.74%	0.3%
Wiener (HFI)	2.48%	1.39%	2.37%	0.04	5.75%	0.35%
Best channel (HFI)	2.1%	1.06%	1.83%	0.03	3.75%	0.3%
Wiener (LFI)	3.81%	2.26%	3.72%	0.067	10.3%	0.54%
Best channel (LFI)	3.6%	2%	3.3%	0.057	6.5%	0.51%
Wiener (*MAP*)	4.9%	4%	8.9%	0.12	45.6%	1.65%
Best channel (*MAP*)	6.3%	4.5%	10.7%	0.13	43.5%	1.76%

Table 2. Errors on parameters for a model with tensor contribution with or without the inclusion of *B*-mode polarization.

Parameters	C_2	h	Ω_b	Ω_Λ	τ	n_S	n_T	*T/S*
Model	$796(\mu K)^2$	0.5	0.05	0.0	0.1	0.9	−0.1	0.7
Wiener (*Planck*)	8.7%	1.6%	2.7%	0.045	5.5%	0.46%	81%	22.4%
+B-modes (*Planck*)	6.5%	1.55%	2.64%	0.044	4.8%	0.43%	57.1%	17.5%
Wiener (HFI)	9.4%	1.63%	2.8%	0.05	7%	0.47%	87%	24.1%
+B-modes (HFI)	7.7%	1.6%	2.7%	0.05	6%	0.45%	70%	20.6%
Wiener (LFI)	9.8%	5.3%	8.6%	0.15	11.3%	1.65%	91.6%	32.4%
+B-modes (LFI)	9%	4.6%	7.5%	0.13	9.6%	1.42%	83%	28.2%
Wiener (*MAP*)	12.3%	22.3%	40%	0.67	52.5%	7.5%	91%	91%
+B-modes (*MAP*)	12%	20%	36.5%	0.60	46%	7%	90.4%	81.6%

Table 3. Parameter estimation with Ω_ν.

Parameters	C_2	h	Ω_b	Ω_Λ	τ	n_S	n_T	*T/S*	Ω_ν
Model	$796(\mu K)^2$	0.5	0.05	0.0	0.1	0.9	−0.1	0.7	0.3
Wiener (*Planck*)	8.8%	1.6%	3%	0.045	4.6%	0.95%	82%	24%	10.5%
+B-modes (*Planck*)	6.4%	1.45%	2.65%	0.04	4.3%	0.81%	55%	18%	9.45%
Wiener (HFI)	9.6%	1.66%	3.1%	0.045	6%	0.97%	89%	26%	10.8%
+B-modes (HFI)	7.75%	1.55%	2.9%	0.42	5.5%	0.9%	71%	22.6%	10%
Wiener (LFI)	9.2%	3.6%	6%	0.098	11%	2%	84%	31%	26.5%
+B-modes (LFI)	9.2%	3.6%	6%	0.097	10.8%	2%	84.8%	31%	26.5%
Wiener (*MAP*)	12.3%	18.4%	27%	0.69	67.1%	7.5%	115%	75%	201%
+B-modes (*MAP*)	12%	17%	25.4%	0.65	62%	7.1%	115%	70%	200%

performance in extracting the temperature power spectra lies between the expected performances of the best channel and the combined sensitivity of all channel for each experiment, at least for the specific foreground models considered. As the temperature data alone gives a fair idea of the errors on most of the parameters our results could be anticipated from conclusions of Paper I. However, the errors of C_2 and τ are mostly determined by the polarization data. In Paper I we showed that the extraction of polarization power spectra is degraded as compared to the cases with no foreground. Our results in this paper suggest that it should not be too much of a deterrent in determining cosmological parameters. It is also important to note that the present results for the best channel case are comparable to Wiener filtering case. This means that (i) the other channels can effectively be used to clean the best channel, and (ii) the presence of foregrounds do not introduce additional degeneracies that are absent when the data is assumed to contain only CMB and pixel noise.

In Table 2, the expected errors are shown for a model which includes contribution from tensor modes. One of the aims of studying this model is to establish how well the inflationary parameters can be determined. In comparison with the sCDM case, the errors on all the standard parameters are bigger. This is because the additional parameter *T/S* allows one to fix the normalization more freely, thereby introducing additional degeneracies (Zaldarriaga et al. 1997). The errors on parameters like C_2, h and Ω_B are higher than for similar models considered by Zaldarriaga et al. (1997). This is partly owing to our choice of normalization which gives smaller signal. However, it also reflects the degradation of the polarization power spectra extraction in the presence of foregrounds. Other parameters like *T/S*, τ, and n_T are better determined than the results of Zaldarriaga et al. (1997), but it is mostly owing to the fact that we take larger input values for τ and *T/S*. We also show the effect of including very small signal from *B*-mode polarization. As is seen, it results in a better determination of most parameters, especially the inflationary parameters. Though the *B*-mode signal is much smaller compared to *E*-mode signal, and is generally below the pixel noise except for a small range of modes for $\ell \lesssim 100$, its very presence indicates tensor modes in inflationary paradigm. Also, the degradation of the extraction of this signal in the presence of foreground is smaller than for the *E*-mode signal (Paper I). Therefore, it can make a difference in the estimation of parameters. Our results show that the consistency condition of slow-roll inflation, $T/S = -7n_T$, can be checked by future missions (it should be noted here that this relation was imposed only in the fiducial model, but excursions of both parameters were considered independently). *Planck* can extract both these parameters with 1σ errors $\lesssim$50 per cent. However, it should be kept in mind that our results are more optimistic than the results of Zaldarriaga et al. (1997) because of our choice of input model.

The results of adding another parameter Ω_ν in the model above are shown in Table 3. Ω_ν can be determined to an accuracy $\lesssim$10 per cent with *Planck*, though it will be very difficult for *MAP* to determine it. Note that errors on other parameters have not changed much by the addition of this parameter, which suggests that no new degeneracies have cropped up. However, degeneracies between various parameters depend very sensitively on the choice of input model. For instance, if the new parameter Ω_ν was added with the input value $\Omega_\nu = 0$, it would have substantially worsened the estimation of almost all parameters, especially the inflationary parameters. For all the three models considered here, we took $\Omega_\Lambda = 0$. A finite value of Ω_Λ results in a better estimation of all the parameters of FRW model as well as Ω_Λ (Zaldarriaga et al. 1997).

4 CONCLUSIONS AND SUMMARY

In this paper we estimated the effect of foregrounds on the determination of cosmological parameters. The most important result is that although the presence of foregrounds somewhat worsens the parameter estimation by degrading the detection of polarization signal, it does not give rise to any severe degeneracies not already present in the CMB data (CMB signal and pixel noise) without foregrounds. It needs to be further confirmed with a detailed study of the Likelihood function in the multiple parameter space.

Any analysis such as ours can only give a qualitative idea on the accuracy of parameter estimation. This is largely because of its strong dependence on the input model (Zaldarriaga et al. 1997). In addition, there are great uncertainties in the assumed level of foregrounds which we take in the Wiener filtering analysis of Paper I. Moreover, the foreground characteristics (power spectra, frequency dependence) should be determined from the data as well, and this adds uncertainty to the determination of the cosmological parameters (see Knox 1999). It should be noticed at this point that, if Wiener filtering assumes some frequency dependences as well as power spectra for the CMB and the foregrounds, it also gives a measure of the error on the estimation of the power spectra of foregrounds from the filtered data, see Paper I.[2]

Still our results suggest that the primary obstacles for high precision CMB measurements will rather stem from systematic errors and inaccuracies in calibration, baseline drifts, determinations of far side lobes or estimates of filter transmissions, etc., all of which are of course not included in this analysis. Furthermore, a Fisher matrix analysis leads to the smallest possible error bars, which can only be degraded by inaccuracies in devising the Wiener filters (by using approximate power spectra and spectral behaviours).

The next step will be to directly analyse simulated mega-pixel multi-frequency CMB maps relevant to future experiments. However, such an analysis is an extremely difficult (if not intractable) numerical problem (for recent attempts see Muciaccia et al. 1997; Oh et al. 1999; Borrill 1999). In light of this, our results should be regarded as a first attempt on the problem of parameter estimation in the presence of foregrounds, which give a qualitative idea of the expected accuracy in parameter estimation till the analyses of multi-frequency CMB data sets become possible. Since the submission of this work, more detailed analysis of the effect of foregrounds have been investigated by Tegmark et al. (2000). Their results are similar to ours, with maybe slightly higher foreground residuals as they allowed some scatter in the frequency dependence of foregrounds.

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[2] It should also be noticed that any spatial change of the frequency indexes (for dust or synchrotron) should correspond to special *astrophysical* regions (molecular clouds, supernovae remnants, etc.) and that an analysis where any such spatial change of index is simply incorporated as an additional ‘noise’ term would lead to pessimistic results. This would rather point out that a global analysis (in ‘Fourier’ modes) is insufficient to take this effect properly into account.

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APPENDIX A: THE COVARIANCE MATRIX OF CMB DATA

In this Appendix we briefly recapitulate the discussion of Paper I, and derive the covariance matrix of CMB data and its derivatives. The observed CMB data at multiple frequencies can be expressed in multipole space as

$$y^i_\nu(l,m) = A^{ij}_{\nu p}(l,m) x^j_p(l,m) + b^i_\nu(l,m), \tag{A1}$$

where x^j_p is the underlying signal for process p and 'field' j (i.e. temperature or (E,B) polarization modes), and ν is a frequency channel index. In the Wiener filtering method, one considers a linear relation between the true, underlying signal, x^j_p and the linearly optimal estimator of the signal, $\hat{x}^j_p$.

$$\hat{x}^i_p = W^{ij}_{p\nu} y^j_\nu. \tag{A2}$$

Equations (A1) and (A2) can be used to write the estimated power spectrum as

$$\begin{aligned}\langle \hat{x}^i_p \hat{x}^j_{p'}\rangle &= (\mathbf{WA})^{im}_{pp''}(\mathbf{WA})^{jq}_{p'p'''}\langle x^m_{p''} x^q_{p'''}\rangle + W^{il}_{p\nu} W^{jn}_{p'\nu'}\langle b^l_\nu b^n_{\nu'}\rangle \\ &\equiv Q^{ij}_{pp'}\langle x^i_p x^j_{p'}\rangle\end{aligned} \tag{A3}$$

where the last equality comes from the equation defining the Wiener filter (see equation 6 of Paper I). The covariance of the filtered data can then be written as

$$\mathcal{C}_\ell = \begin{pmatrix} Q^{11}_\ell C_{T\ell} & Q^{12}_\ell C_{TE\ell} & 0 \\ Q^{12}_\ell C_{TE\ell} & Q^{22}_\ell C_{E\ell} & 0 \\ 0 & 0 & Q^{33}_\ell C_{B\ell} \end{pmatrix}. \tag{A4}$$

For computing the Fisher matrix we also need to compute the derivative of the covariance with respect to cosmological parameters

$$\frac{\partial \mathcal{C}_\ell}{\partial \theta_i} = \sum_{X=T,E,ET,B} \frac{\partial \mathcal{C}_\ell}{\partial C_\ell(X)} \frac{\partial C_\ell(X)}{\partial \theta_i}. \tag{A5}$$

The derivative of the covariance matrix with respect to various power spectra can be written using equation (A3). These derivatives, it should be borne in mind, are with respect to the *input power spectra* used in estimating the Fisher matrix and not the power spectra used in constructing the Wiener filters, which, therefore, are invariant under this change. These derivatives can be readily calculated as follows.

$$\frac{\partial \langle \hat{x}^T_p x^T_p\rangle}{\partial C^T_p} = (W^{11}_{p\nu} A^{11}_{\nu p})^2, \tag{A6}$$

$$\frac{\partial \langle \hat{x}^T_p x^T_p\rangle}{\partial C^{TE}_p} = 2\times(W^{11}_{p\nu} A^{11}_{\nu p} W^{12}_{p\nu} A^{22}_{\nu p}), \tag{A7}$$

$$\frac{\partial \langle \hat{x}^T_p x^T_p\rangle}{\partial C^E_p} = (W^{12}_{p\nu} A^{22}_{\nu p})^2, \tag{A8}$$

$$\frac{\partial \langle \hat{x}^E_p x^E_p\rangle}{\partial C^E_p} = (W^{22}_{p\nu} A^{22}_{\nu p})^2, \tag{A9}$$

$$\frac{\partial \langle \hat{x}^E_p x^E_p\rangle}{\partial C^{TE}_p} = 2\times(W^{22}_{p\nu} A^{22}_{\nu p} W^{21}_{p\nu} A^{11}_{\nu p}), \tag{A10}$$

$$\frac{\partial \langle \hat{x}^E_p x^E_p\rangle}{\partial C^T_p} = (W^{21}_{p\nu} A^{11}_{\nu p})^2, \tag{A11}$$

$$\frac{\partial \langle \hat{x}^T_p x^E_p\rangle}{\partial C^T_p} = (W^{11}_{p\nu} A^{11}_{\nu p} W^{21}_{p\nu} A^{11}_{\nu p}), \tag{A12}$$

$$\frac{\partial \langle \hat{x}^T_p x^E_p\rangle}{\partial C^{TE}_p} = (W^{11}_{p\nu} A^{11}_{\nu p} W^{22}_{p\nu} A^{22}_{\nu p}) + (W^{12}_{p\nu} A^{22}_{\nu p} W^{21}_{p\nu} A^{11}_{\nu p}), \tag{A13}$$

$$\frac{\partial \langle \hat{x}^T_p x^E_p\rangle}{\partial C^E_p} = (W^{22}_{p\nu} A^{22}_{\nu p} W^{12}_{p\nu} A^{22}_{\nu p}). \tag{A14}$$

Theoretical power spectra are calculated using the CMB Boltzmann code CMBFAST (Seljak & Zaldarriaga 1996). Derivatives with respect to cosmological parameters are calculated numerically using a variant of DFRIDR routine of numerical recipes (Press et al. 1992). We notice that a 5 per cent step in most parameters gives stable results. The only exception is derivative of *E*-mode power spectra with respect to τ when $\tau \leqslant 0.05$ for $\ell \leqslant 20$. This numerical instability is expected as a small change in this parameter when the input value of τ is very small can cause appreciable change in the *E*-mode power spectra at small ℓ. However, the numerical differentiation is quite stable for $\tau \gtrsim 0.05$.

This paper has been typeset from a TEX file prepared by the author.