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COSMIC STRINGS IN AN EXPANDING SPACETIME

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ABSTRACT

We investigate the stability of a static, infinitely long and straight vacuum string solution under inhomogeneous axisymmetric time-dependent perturbations. We find it to be perturbatively stable.

We further extend our work by finding a string solutions in an expanding Universe. The back reaction of the string on the gravitational field has been ignored. The background is assumed to be a Friedman-Robertson-Walker (FRW) cosmology. By numerically integrating the field equations in a radiation and matter dominated models, we discover oscillatory solutions. The possible damping of these oscillations is discussed. For late times the solution becomes identical to the static one studied in the first part of the paper.

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I. Introduction

The early Universe probably underwent a number of phase transitions as it cooled down from its hot initial state. During some of these phase transitions it is probable that defects or mismatches in the orientation of say, a Higgs field, in causally disconnected regions of spacetime, could have formed. There are essentially three types of stable topological defect (depending on the homotopy class of the broken symmetry group) that can occur when the symmetry is spontaneously broken and the universe undergoes a phase transition. These are: monopoles, strings and walls[1]. Walls are unacceptable as they would destroy the isotropy of the microwave background radiation[2]. Monopoles could be both a blessing and a curse; we would like to have them present in order to explain charge quantization, but unfortunately most Grand Unified Theories predict a very large number of massive monopoles[3] ($M_m \sim 10^{16}$Gev). This could be catastrophic for the Universe. One of the immediate consequences is that the Universe would become closed and recollapse in a very short time. Of course there exist mechanisms to reduce the number density of monopoles[4], however, most schemes would still leave a substantial number of them, and we still find ourselves in the embarrassing situation of not having detected one either in laboratory or cosmic ray experiment.

One of the best solutions to this so called monopole problem is provided by Inflation[5]. The sudden exponential expansion of the Universe together with a generation of an enormous amount of entropy would dilute the monopoles very effectively, leaving essentially one monopole per horizon volume. Unfortunately, it seems, at least in the simplest models, that by the same process we also get rid of all strings. There have been claims recently that it might be possible to have inflation and string formation at the same time [6]

In this paper we will assume that strings are generated during or after inflation and not worry about how this is achieved.

The interest in retaining cosmic strings in the early universe stems from the fact that they provide one of the most promising scenarios for the generation of large scale structure in the Universe[7]. We do not mean to say that this is the only possible way of doing it, but apart from Inflation[8], this is the best scenario that we have. Furthermore, cosmic strings have some features and make some predictions that are unique, so making the theory falsifiable.

Much work, both analytical as well as numerical[9] has been done in the last five years or so to understand the formation of large scale structure via strings, Nevertheless, when one looks for the formal definition of a string, or asks what type of gravitational field it generates, or wishes to study a string solution in an expanding universe, there is a void in the literature. Some attempts at trying to find the metric for the string have been done by Vachaspati[10] using the kinematical properties of the string trajectories. Little is known about the metric outside or inside a string when the spacetime is curved and/or expanding. Most calculations of string configurations have been done in the weak gravitational field approximation and usually with a static, flat background. The best-known string solution in the cosmological context (apart from the Nielsen-Olesen [11] vortex solution) is that found by Vilenkin[12] . This solution represents an infinitely long static straight string in a non-expanding flat background in the weak field approximation. In this solution, the string is pictured as a one dimensional object embedded in a three dimensional space, it has no internal structure and the energy density ρ is related to the pressure in the direction of the string axis simply by $\rho = -p_z$ assuming the string is on the z axis. This solution was later generalized to the strong field limit by Gott and Hiscock[13] independently. In these two papers, Einstein's equations are solved for an axisymmetric static spacetime, where an exterior vacuum solution is matched to a non-singular, non-vacuum interior spacetime with $\rho = -p_z$. This solution in the weak field approximation becomes the Vilenkin solution. In the same spirit Stein-Schabes[14] further generalized this solution to the non-static case and found a new class of solutions. Recently, a more formal approach to the matching conditions between the interior and exterior solutions for the string has been taken by Matzner and Laguna-Castillo[15] . Since then a plethora of new solutions describing different types of strings have appeared in the literature. Unfortunately, none of these solutions truly solves the problem in its entirety. It is usually assumed that the energy momentum tensor $T_{\mu\nu}$ has a rather ad hoc form. In most cases the density and pressures are chosen more on a phenomenological bases rather than from the Lagrangian for the theory. Recently, Garfinkle[16] has tackled the problem the right way. Starting from a Lagrangian describing a string in a given curved background, he obtains and numerically solves the equations of motion and studies the form and behaviour of the energy momentum tensor. His calculation is done for static backgrounds both when the model is flat and curved. However, the work was not extended to encompass non-static spacetimes nor was the stability of these solutions investigated. This is part of what we shall do in this paper.

The paper will be organized as follows. In Sec. II. we will re-analyse briefly the solutions found by Garfinkle[16] and study their stability under inhomogeneous axisymmetric time-dependent perturbations. In Sec. III we will consider the string solution in an expanding homogeneous and isotropic spacetime, for both a matter and radiation dominated epochs. We will treat the string as a perturbation on spacetime and ignore the back-reaction of the string on the evolution of the geometry. Most of the calculations will be done explicitly for the local $U(1)$ string but the results can easily be reduced to the case of a global string. We will finish with some comments and conclusions.

II. Stability of Cosmic Strings in Minkowski Spacetime

The set of equations describing a string field configuration have not been solved analytically so far. Nevertheless, particular solutions can be found which satisfy the appropriate boundary conditions at infinity to describe an isolated well defined object. In this section we shall study the stability of a static string solution in a Minkowski background under inhomogeneous time-dependent perturbations. We will assume that the perturbations are such that they preserve the symmetries of both the fields and the spacetime, for this reason they will only be functions of the radial coordinate and time. Implicitly we are assuming that the string is infinite in length and straight. It has been argued that for topological reasons the string has to be stable. In fact, the stability of such solutions under time-dependent perturbations which preserve the isometries have not been considered. Nevertheless, using the method of trial functions[17] it has been shown that the string described above is stable under time-independent perturbations and that the configuration is the one of minimum energy. This, however, does not prove that if the string is perturbed in a more general way, say in a time-dependent fashion, it could not find a new stable state. We will show that is not the case. For a very similar discussion regarding monopoles see ref.18. The static string as described in ref. (16) is stable against time-dependent perturbations, and the energy is at its minimum. Our inability to construct the full analytical solution to the string equations makes the process of determining its stability a bit more involved. We will put forward an argument that shows that the asymptotic (large distance) solution to the string equations is stable. We will then extract some generic features of this solution and use them as an initial state for a full numerical integration of the perturbation equations.

We define a *local* string as that obtained when a *local* $U(1)$ gauge group is broken (by analogy we define the *global* string). The Lagrangian for this theory takes the following form,

$$\mathcal{L} = D_\mu \phi^* D^\mu \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}\lambda \left(\phi^* \phi - \eta^2\right)^2 \tag{2.1}$$

where $D_\mu = \partial_\mu - ieA_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and ∂_μ is the covariant derivative. We will now decompose the complex field ϕ into its magnitude and phase and adopt the same notation as in ref.(16). Let $\phi = Re^{i\psi}$, then from (2.1) we can get the field equations

$$\partial^\mu \left[R^2 \left(\partial_\mu \psi + eA_\mu\right)\right] = 0 \tag{2.2}$$

$$\partial_\mu \partial^\mu R - R \left[4\lambda \left(R^2 - \eta^2\right) + \left(\partial_\mu \psi + eA_\mu\right)\left(\partial^\mu \psi + eA^\mu\right)\right] = 0 \tag{2.3}$$

$$\partial^\mu F_{\mu\nu} - 4\pi e R^2 \left(\partial_\nu \psi + eA_\nu\right) = 0 \tag{2.4}$$

We still have the freedom to chose the gauge, so we will impose the Lorentz gauge where $\partial^\mu A_\mu = 0$.

It will be useful to introduce now the total energy per unit length E_t,

$$E_t = 2\pi \int \left[(D_0\phi)^*(D_0\phi) + (D_i\phi)^*(D^i\phi) + \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\lambda(\phi^*\phi - \eta^2)^2\right] r dr \tag{2.5}$$

The background Minkowski metric in cylindrical coordinates is

$$ds^2 = -dt^2 + dz^2 + dr^2 + r^2 d\varphi^2 \tag{2.6}$$

this metric will be used to raise and lower indices.

If we make the following ansatz

$$R = R(r) \tag{2.7a}$$

$$\psi = \varphi \tag{2.7b}$$

$$A_\mu(r) = \frac{1}{e}(P(r) - 1)\delta_\mu^\varphi \tag{2.7c}$$

where δ_μ^φ is the Kronecker delta function. We can see that eq.(2.2) is immediately satisfied and eqs.(2.3) and (2.4) become

$$r\frac{d}{dr}\left(r\frac{dR}{dr}\right) = R\left[4\lambda r^2\left(R^2 - \eta^2\right) + P^2\right] \tag{2.8}$$

$$r\frac{d}{dr}\left(\frac{1}{r}\frac{dP}{dr}\right) = 4\pi e^2 R^2 P \tag{2.9}$$

It has been shown[11,12,16,17] that there exist solutions to these equations that satisfy the appropriate boundary conditions at infinity,

$$\lim_{r\to\infty} R(r) = \eta$$

$$\lim_{r\to\infty} P(r) = 0$$

and are regular at the origin, i.e., $R(0) = 0, P(0) = 1$. The conditions at infinity define an isolated object with finite energy per unit length. The case of a global string can be obtained by setting the gauge field to zero i.e. $P(r) = 1$. In this case the energy is logarithmically divergent so a cutoff is necessary.

With these definitions at hand we will assume that the general solution to (2.8) and (2.9) is known and we will investigate its stability. Let R_0 and P_0 be this solution, and construct a perturbation to this in the following way,

$$R(r,t) = R_0(r) + R_1(r,t) \tag{2.10}$$

$$A_\mu(r,t) = A^0_\mu(r) + S_\mu(r,t) \tag{2.11}$$

with $A^0_\mu(r) = \frac{1}{e}(P_0(r) - 1)\delta^{\varphi}_{\mu}$. By demanding the perturbed gauge field to satisfy the Lorentz gauge condition we immediately get the allowed functional form for it: $S_\mu(r,t) = (S_0(r), S_1(t), S_2(r,t), S_3(r,t))$. Substituting these into (2.8), (2.9) and linearizing around the unperturbed solution we get the following equations,

$$\partial^\mu\partial_\mu S_\nu = 4\pi e R_0^2\left[eR_0 S_\nu + 2P_0 R_1 \delta^{\varphi}_{\nu}\right] \tag{2.12}$$

$$\partial^\mu \partial_\mu R_0 = 4\lambda \left[3R_0^2 - \eta^2\right] + \frac{P_0^2}{r^2} \tag{2.13}$$

with $\partial^\mu \partial_\mu = \partial_r^2 + \frac{1}{r}\partial_r$. The energy difference between the perturbed and unperturbed solutions will be denoted by $\Delta E(t) = E_1(t) - E_0 = 2\pi \int \Delta\mathcal{E}(t) r dr$ where the energy density can be obtained from eq.(2.5),

$$\Delta\mathcal{E} = R_0' R_1' + \frac{1}{2er^2} P_0' S_3' + \frac{e}{r^2} R_0^2 P_0 S_3 + \frac{1}{r^2} R_0 P_0^2 R_1 + \frac{1}{2}\lambda \left(R_0^2 - \eta^2\right) R_0 R_1 \tag{2.14}$$

($' \equiv \partial_r$). To this order neither S_0, S_1 or S_2 contribute to the energy difference, so we will set them to zero. In order to solve eqs.(2.12) and (2.13) we expand both perturbations in Fourier components,

$$\left(S_3(r,t), R_1(r,t)\right) = \sum_\omega e^{-i\omega t} \left(S_r(r;\omega), R_r(r;\omega)\right) \tag{2.15}$$

then the perturbation equations become

$$S_r'' + \frac{1}{r} S_r' + (\omega^2 - 4\pi e^2 R_0^2) S_r = 8\pi e R_0 P_0 R_r \tag{2.16}$$

$$R_r'' + \frac{1}{r} R_r' + \left(\omega^2 - 4\lambda \left(3R_0^2 - \eta^2\right) - \frac{P_0^2}{r^2}\right) R_r = \frac{2e}{r^2} R_0 P_0 S_r \tag{2.17}$$

Clearly, the stability will depend on the sign of $\mathrm{Im}(\omega)$. The criteria is clear, the solution is stable if $\mathrm{Im}(\omega) \leq 0$, unstable otherwise. The only constraint we will impose on the perturbation is that they are small and have finite energy. We cannot analytically solve eqs.(2.16) and (2.17) for obvious reasons, but since the crucial piece of information is the value of ω, we shall study the solution to these equations in the far field limit i.e., $r \to \infty$. In this limit the unperturbed solution takes the following form, $R_0 \simeq \eta$ and $P_0 \simeq 0$. If we input these values, rescale the radial coordinate $\rho = \lambda\eta^2 r$ and introduce a new frequency $\hat{\omega}^2 = \lambda\eta^2\omega^2$ we get,

$$W_i'' + \frac{1}{\rho}W_i' + (\hat{\omega}^2 - \alpha_i^2)W_i = 0 \tag{2.18}$$

with $W_i(\rho) \equiv (R_\rho, S_\rho)$ and $\alpha_i^2 \equiv (8, 4\pi e^2/\lambda)$. The general solution to (2.18) will be given as a linear combination of Bessel functions. In the large ρ limit the solution takes the following form

$$W_i = \frac{W_{i0}}{\sqrt{\rho}}\left[C_1 \cos((\hat{\omega}^2 - \alpha_i^2)^{\frac{1}{2}}\rho) + C_2 \sin((\hat{\omega}^2 - \alpha_i^2)^{\frac{1}{2}}\rho)\right] \tag{2.19}$$

with W_{i0}, C_1 and C_2 constants. Let us decompose $\hat{\omega}$ into its real and imaginary part, $\hat{\omega} = \hat{\omega}_1 + i\hat{\omega}_2$ with $\hat{\omega}_1$ and $\hat{\omega}_2$ real. Then the condition for having a finite energy perturbation becomes, $\mathrm{Im}(\sqrt{\hat{\omega}^2 - \alpha_i^2}) = 0$ which in turn implies that $\hat{\omega}^2$ is real and such that $\hat{\omega}_1^2 \geq \alpha_i^2$ and $\hat{\omega}_2 = 0$. As a consequence of this all the time dependence in (2.15) is oscillatory and there is a continuum spectrum of frequencies with $\hat{\omega}^2 \geq 4\pi e^2/\lambda \simeq (m_{A_\nu}/m_\phi)^2$ (see Vilenkin in ref.1). The smallest frequency of the Fourier spectrum turns out to be given by the ratio of the mass of the Higgs field to the gauge field. Thus we can conclude that in the large ρ limit ($r \to \infty$) the solution is stable. To confirm our result we numerically integrated the system of equations (2.12) and (2.13) and plotted the results for different frequencies. The scalar field and the gauge field are plotted in figs. 1a and 1b respectively as functions of the radial distance for a fixed time t_*. When $\hat{\omega}^2 < 0$ the solutions diverge and have infinite energies (they rapidly escape the linear regime where our approximation is valid). It is only when $\hat{\omega}^2 \geq 0$ that the solutions stay bounded. The solid line represents the unperturbed solution. In the cases shown λ was chosen such that $4\pi e^2/\lambda \sim 1$. The dividing line between having finite and infinite energy perturbations given by $\hat{\omega}^2 = \mathcal{O}(1)$ as predicted.

Finally, if we calculate the energy difference for the allowed range of frequencies, using eq.(2.14) we get $\Delta E \geq 0$ ($\simeq \mathcal{O}(10^{-3})$). The important point is that it is positive, confirming our early claim that the string is stable. Even though the perturbations are not the most general type they seem to indicate that the string is a very stable object.

III. Cosmic Strings in a Friedman-Robertson-Walker Spacetime

In this section we would like to discuss what happens when a cosmic string defined by (2.1) is embedded into an expanding FRW model. The relevance of this questioned seems to us to be undeniable, and some of the answers could dramatically affect the scenarios for the formation of large scale structure. We will consider the string to be nonstatic, infinitely long and straight, and such that it acts only as a perturbation on spacetime. We shall not worry about the back-reaction of the string and gauge fields on the geometry which, as stated earlier, will be that of a flat FRW model. This approximation is valid when the curvature of the string R_{string} is much larger than H_0^{-1}, but the core of the string r_{string} is not negligible compare to H_0^{-1} (the string is not a delta function), i.e., $\frac{1}{\lambda^{\frac{1}{2}}\eta} \sim r_{string} \leq H_0^{-1} << R_{string}$. In this limit the isotropy and homogeneity of the FRW background are not destroyed far from the string and the effect of expansion on the Higgs and Gauge fields is non-negligible. We shall consider the problem in two different regimes: when the universe is radiation dominated and when it is matter dominated. An implicit assumption is that metric perturbations are small so that the metric remains Friedmanian throughout the evolution. By treating the string as a perturbation on the universe, we are effectively looking at length scales which are small compared to the Hubble radius.

The equations of motion for this case are given by eqs.(2.2),(2.3) and (2.4), but the metric will be that of a flat FRW. In cylindrical coordinates it becomes

$$ds^2 = -dt^2 + a(t)^2 \left(dz^2 + dr^2 + r^2 d\varphi^2\right) \tag{3.1}$$

Eq.(2.2) can again be satisfied by the taking $\psi = \varphi$ and

$$A_\mu(r,t) = \frac{1}{e}(P(r,t) - 1)\delta_\mu^\varphi$$

while the equations for $R(r,t)$ and $P(r,t)$ become

$$\frac{\ddot{R}}{R} + 3\frac{\dot{a}}{a}\frac{\dot{R}}{R} - \frac{1}{a^2}\left[\frac{R''}{R} + \frac{1}{r}\frac{R'}{R}\right] = -4\lambda\left(R^2 - \eta^2\right) - \frac{P^2}{a^2 r^2} \tag{3.2}$$

$$\frac{\ddot{P}}{P} + 3\frac{\dot{a}}{a}\frac{\dot{P}}{P} - \frac{1}{a^2}\left[\frac{P''}{P} - \frac{1}{r}\frac{P'}{P}\right] = -4\pi e^2 R^2 \tag{3.3}$$

We have been unable to solve this system of equation analytically, instead we have solved the equations numerically and the results are shown in figs. 2a, 2b, 3a and 3b for the global string in a radiation and matter universe respectively, and in fig.4 for the local string. Before discussing the solutions, we would like to highlight some important points. The first and most obvious one is that, since the universe is expanding like some power law $a \sim t^\alpha$ with $\alpha = \frac{1}{2}$ for radiation and $\alpha = \frac{2}{3}$ for matter, then there are no static solutions for P or R. This is important because it means that a string cannot just sit quietly while the universe expands. We should make clear that we are considering a straight string and not a loop. For a loop it is known that the solution cannot be static (due to the tension), however, in this case the tension acts only in the direction of the string axis.

The integration has been done using the following boundary conditions,

$$R(0,t_0) = 0, \quad \dot{R}(0,t_0) = 0 \tag{3.4a}$$

$$R(r_*,t_0) = \eta, \quad \dot{R}(r_*,t_0) = 0 \tag{3.4b}$$

$$P(0,t_0) = 1, \quad \dot{P}(0,t_0) = 0 \tag{3.4c}$$

$$P(r_*,t_0) = 0, \quad \dot{P}(r_*,t_0) = 0 \tag{3.4d}$$

where $r_* >> 1$. These initial and boundary conditions define the static string and among other things they make the energy finite at infinity (at least for the local string). We will assume that at t_0 the string is identical to the static string, then we *switch on* expansion and study the results. The reason for this choice of initial conditions is twofold, one is that even though no analytical solutions exist of the static case, both numerical and analytical work have established this as a good limiting case and in the limit of slow expansion the static string solution (in comoving coordinates) is recovered. The boundary conditions for the global $U(1)$ string can be obtained by setting $P(r,t) = 1 \quad \forall \quad t, r$.

We shall only discuss the local string as most of what we say can be apply to the global case by taking the appropriate formal limit. One of the features of the solution is the oscillatory character of the string field (similar in nature to the response of the static string found in the previous section). Figs. 2a and 3a show the oscillations of $R(r,t)$ with time. The oscillation frequency ω_0 is of order $m_\phi \simeq \lambda^{\frac{1}{2}}\eta$. Both the string field as well as the gauge field oscillate around some static minimum value and the motion gets damped due to the expansion of the universe, on a timescale of order H_0^{-1}. After some time the radial derivatives of the fields get suppressed by the expansion and the damping term goes to zero like some power of t. In principle, if we assume the string does not couple to anything else it would continue to oscillate (due to the self-coupling it could radiate Higgs particles with masses of order the frequency of oscillation), however, it would seem more natural to speculate that the string will radiate particles and loose energy until it settles down into some static configuration. It is also conceivable that as the string oscillates it emits cylindrical gravitational waves. It is known[19] that cylindrical configurations (like in a cylindrical collapse) emit gravitational waves which resemble the Einstein-Rosen solutions, furthermore, there is a well defined quantity that characterizes this solutions, the so called C-energy introduced in ref.(19). These two possible mechanism for losing energy are now under investigation.

Whatever the mechanism is, once the universe has expanded for a while, the string will effectively decouple from the expansion in the radial direction and will only feel the expansion in the z-direction. We can understand these oscillations in the following way; at $t = t_0$ the string is in the static configuration, expansion is then switched on. The string will try to expand with the background, however, due to the transverse non-zero components of the stress tensor, it will try to return to its original state, resulting in an oscillatory motion. In fact, when $a(t)$ is large, the gradient terms in eqs.(3.2) and (3.3) get suppressed and what is left of the equations resemble a damped harmonic oscillator.

These oscillations could have some observable consequences. An observer close to the string, could measure a time changing field produced by the string, and as mentioned earlier, there might be emission of gravitational radiation. It is interesting to note (fig.5) that the energy, density ϵ does not present the oscillatory behaviour, but rather just decreases as time goes by.

Unfortunately, the damping time is very short $\tau \simeq \mathcal{O}(H_0^{-1})$, which would indicate that this effect is probably not observable today.

We can see the time variation of the energy by calculating $T_{\mu\nu}$. From the Lagrangian we get

$$T_{\mu\nu} = R_{;\mu}R_{;\nu} + R^2P^2\delta_\mu^\varphi\delta_\nu^\varphi + F_\mu^\sigma F_{\nu\sigma} + \mathcal{L}g_{\mu\nu} \tag{3.5}$$

One can immediately see that if $R = R(r,t)$ and $P = P(r,t)$, then the stress tensor will have some non-zero off-diagonal components. If we define a set of basis vectors

$$\hat{t}_\mu = \left(\frac{d}{dt}\right)_\mu \tag{3.6a}$$

$$\hat{r}_\mu = \frac{1}{a}\left(\frac{d}{dr}\right)_\mu \tag{3.6b}$$

$$\hat{z}_\mu = \frac{1}{a}\left(\frac{d}{dz}\right)_\mu \tag{3.6c}$$

$$\hat{\varphi}_\mu = \frac{1}{ar}\left(\frac{d}{d\varphi}\right)_\mu \tag{3.6d}$$

then the stress tensor can be written as

$$T_{\mu\nu} = \epsilon\hat{t}_\mu\hat{t}_\nu + P_r\hat{r}_\mu\hat{r}_\nu + P_z\hat{z}_\mu\hat{z}_\nu + P_\varphi\hat{\varphi}_\mu\hat{\varphi}_\nu + P_{tr}(\hat{t}_\mu\hat{r}_\nu + \hat{r}_\mu\hat{t}_\nu) \tag{3.7}$$

where

$$\epsilon = \dot{R}^2 + \left(\frac{R'}{a}\right)^2 + \frac{R^2P^2}{a^2r^2} + 2\lambda\left(R^2-\eta^2\right)^2 + \frac{1}{4\pi e^2a^2r^2}\left(\dot{P}^2 + \left(\frac{P'}{a}\right)^2\right) \tag{3.8a}$$

$$P_z = \dot{R}^2 - \left(\frac{R'}{a}\right)^2 - \frac{R^2P^2}{a^2r^2} - 2\lambda\left(R^2-\eta^2\right)^2 + \frac{1}{4\pi e^2a^2r^2}\left(\dot{P}^2 - \left(\frac{P'}{a}\right)^2\right) \tag{3.8b}$$

$$P_r = \dot{R}^2 + \left(\frac{R'}{a}\right)^2 - \frac{R^2P^2}{a^2r^2} - 2\lambda\left(R^2-\eta^2\right)^2 + \frac{1}{4\pi e^2a^2r^2}\left(\dot{P}^2 - \left(\frac{P'}{a}\right)^2\right) \tag{3.8c}$$

$$P_\varphi = \dot{R}^2 - \left(\frac{R'}{a}\right)^2 + \frac{R^2P^2}{a^2r^2} - 2\lambda\left(R^2-\eta^2\right)^2 + \frac{1}{4\pi e^2a^2r^2}\left(\dot{P}^2 - \left(\frac{P'}{a}\right)^2\right) \tag{3.8d}$$

$$P_{tr} = \frac{1}{r}\left(\dot{R}R' + \frac{\dot{P}P'}{4\pi e^2a^2r^2}\right) \tag{3.8e}$$

Two important points should be noticed, the first is that the energy momentum tensor is not diagonal and $\epsilon \neq -P_z$. This fact would be of prime importance when considering the back reaction, since we know that the Ricci tensor for a FRW model is diagonal, so making it inconsistent with this energy-momentum tensor. In order to make this tensor compatible with the metric, when considering the back reaction, we would be forced to include some inhomogeneities. The second point is that the off diagonal component of the stress tensor goes to zero like $\frac{1}{r}$ for large r. Furthermore, as $t \to \infty$ both, $\dot{R}(r,t)$ and $\dot{P}(r,t) \to 0$, making the stress tensor diagonal with $\epsilon = -P_z$. The energy density as a function of r for different times is shown in fig.5. However, near the string, the spacetime metric would probably have to be at least anisotropic.

IV Conclusions

In this paper we have addressed two questions. One was the stability of the static string under time-dependent axisymmetric inhomogeneous perturbations. We found it to be stable against these type of perturbations, making it a good candidate for the ground state solution. We also proposed that this could be a good approximation for the behaviour of a string in a slowly expanding universe. The perturbed scalar and gauge fields oscillate around the unperturbed solution. If we define the core radius of the string as the distance from the center at which the Higgs field aquires a predefined value, then the string undergoes radial oscillations when perturbed in this way. We found that the minimum frequency of oscillation is given by the ratio between the mass of the vector field and the Higgs field ($\hat{\omega}_{min} \sim (m_{A_\nu}/m_\phi)^2$. We did not investigate the stability against more general perturbations that would break the axial symmetry, however, we feel that the string would probably just radiate away all the modes which are not compatible with the symmetries in the form of gravitational waves, and if coupled to any other field, it would radiate particles or photons.

We then solved the string equations when embedded in an expanding radiation or matter-dominated flat model. The results do not differ much for these two cases. We chose the background to be flat so as to avoid problems with the embedding and the different symmetries of the two spacetimes. We did not consider the back-reaction of the string on the metric, but by calculating the energy-momentum tensor for this field configuration we found it to be non-diagonal. This is an indication that the metric must be anisotropic and probably inhomogeneous near the string.

The oscillatory behaviour disappears in the limit of small and large r (due to the boundary conditions). Presumably in a more realistic scenario the string would emit particles or radiate gravitational waves, to settle eventually into a static configuration. If the model universe is not expanding too fast, then we can set $a(t) \sim a_0, \quad \dot{a}(t) = 0$ in eqs.(3.2) and (3.3) and recover the static solution. The static solution appears as a special limiting case of the non-static solution. We have not proven that this solution is stable, but it is a tempting possibility. The perturbative study of this case is also underway.

We have found that although interesting consequences can be derived from the oscillatory nature of the solution, the effect is probably not too important from the observational point of view, since damping occurs in a few Hubble times after the formation of the string, and by now the amplitude of the oscillations is very small. One possible observable consequence of the damping could be a gravitational wave background coming from the oscillating strings.

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Figure Captions

1 In fig. 1a we have plotted the magnitude of the scalar field ϕ represented by $R(r, t_*)$ as a function of the radius for a given time t_* and the perturbations for different values of the frequency ω^2. Fig. 1b shows the gauge field $P(r, t_*)$ and its perturbation. In both cases the unperturbed static solution is represented by the solid line.

2 The string field $R(r, t)$ is plotted against t/t_0 with $t_0 = H_0^{-1}$ in fig.a and against r/r_0 in fig.b for a radiation dominated Universe. The oscillations are damped for large r and large t.

3 The string field $R(r, t)$ is plotted against t/t_0 in fig.a and against r/r_0 in fig.b for a matter dominated Universe. The oscillations are damped for large r and large t.

4 The string field $R(r, t)$ (dotted line) and the gauge field component $P(r, t)$ (solid line) are plotted here against r/r_0 for different time steps label by $t_1, t_2, \ldots$. The oscillations presented in the global case are also present in the local case.

5 The energy density ϵ as a function of r for different values of t

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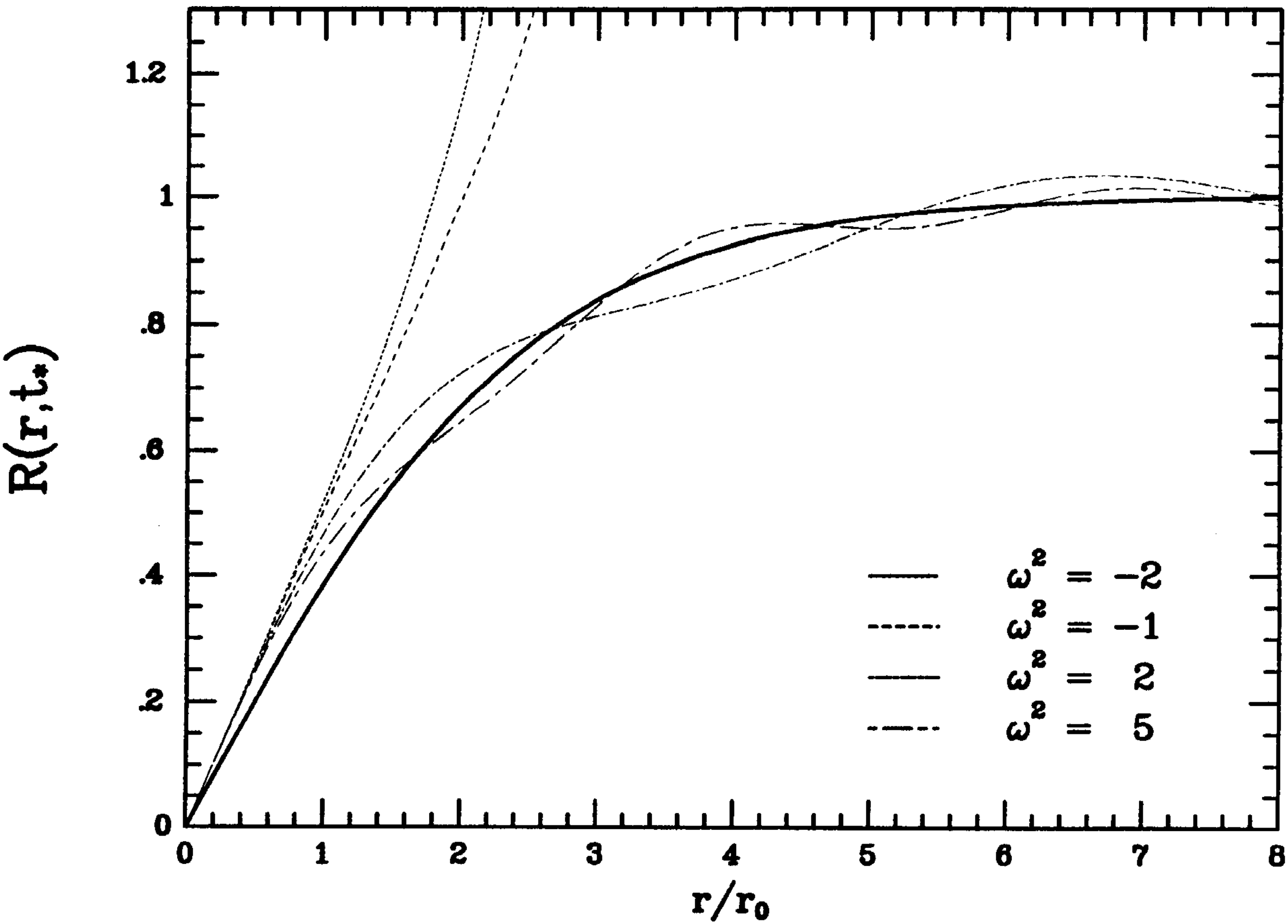

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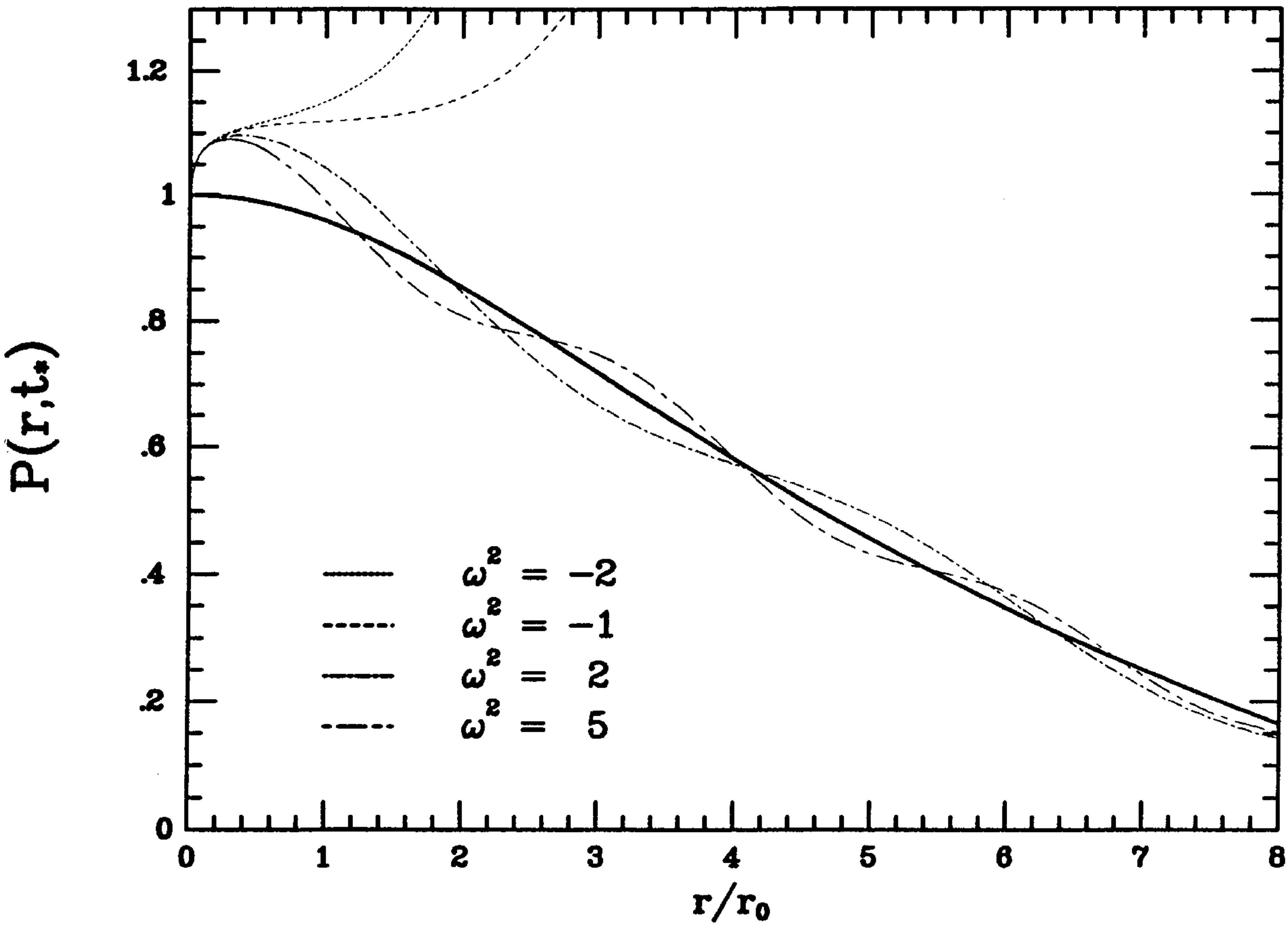

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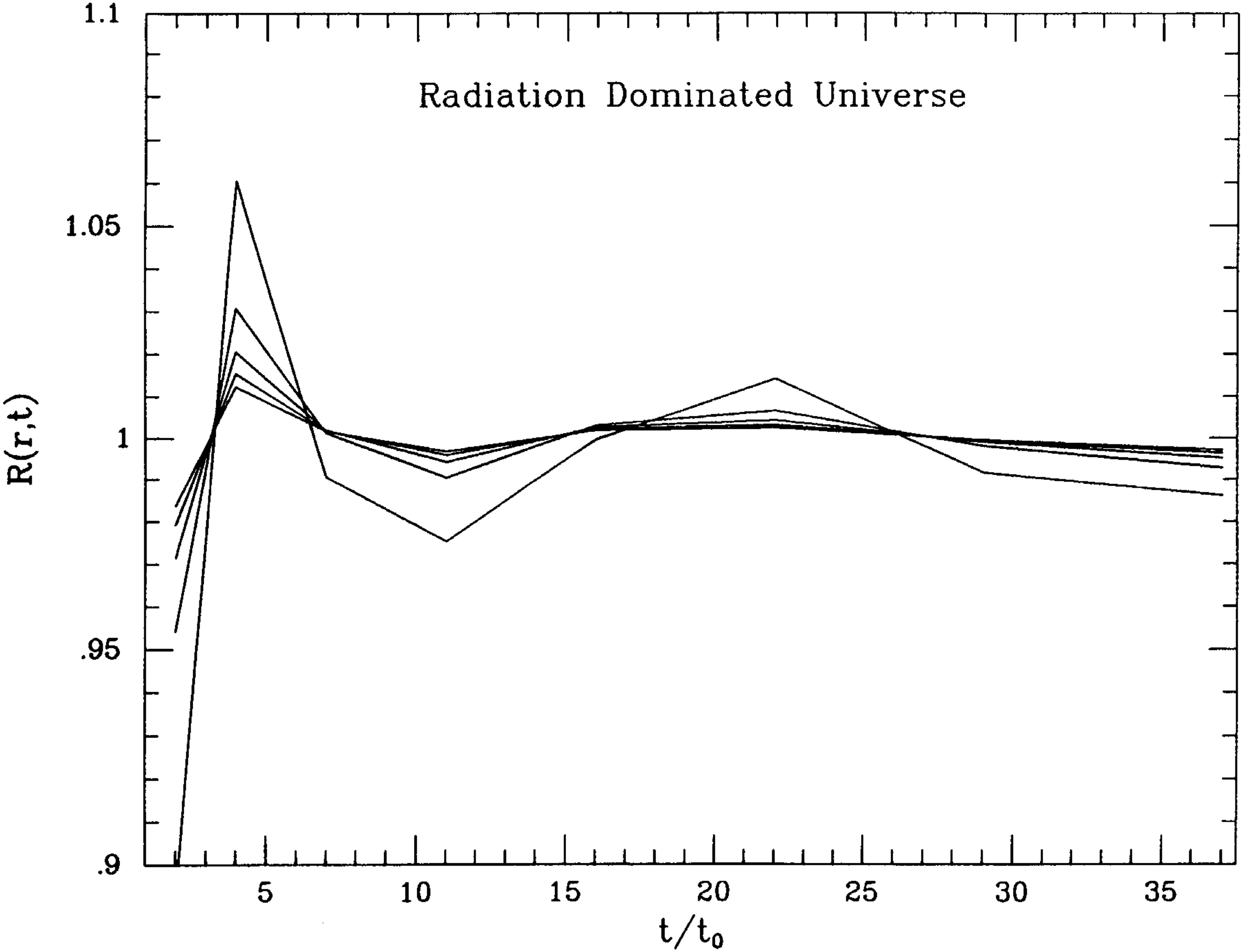
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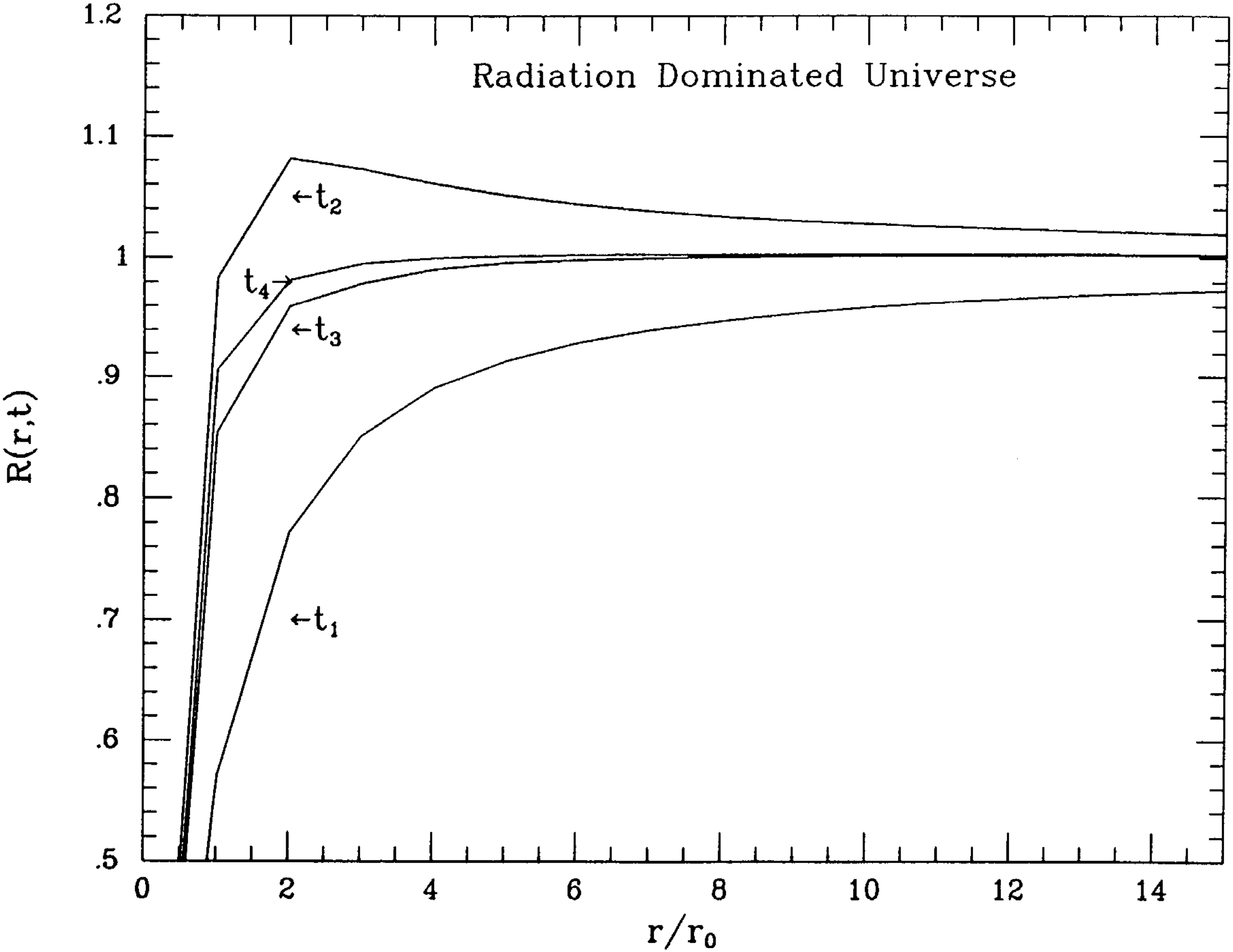
.95
.9
5
10
15
20
25
30
35
t/t0

2a


Radiation Dominated Universe
R(r,t)
r/r0
←t2
t4→
←t3
←t1
1.2
1.1
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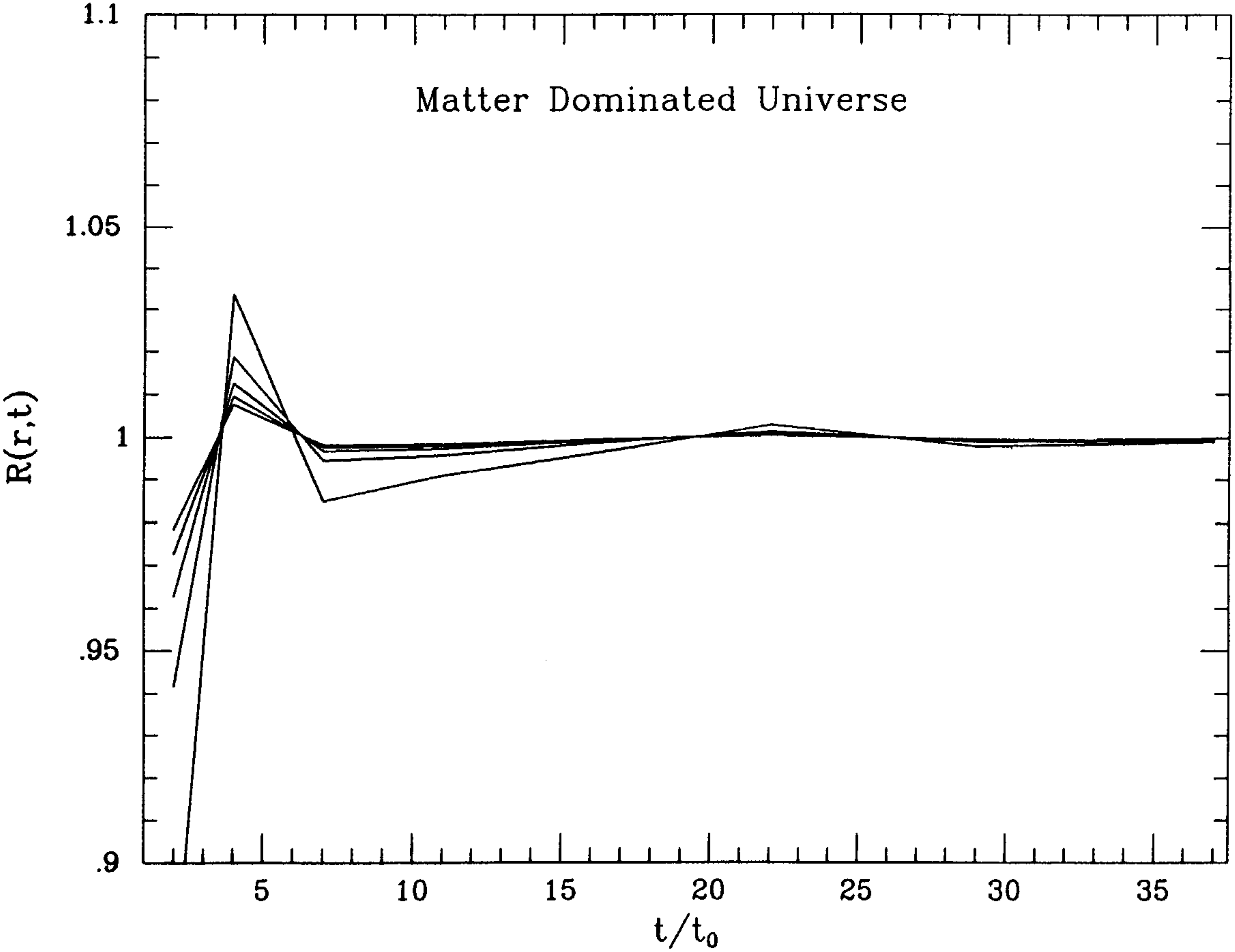
.9
.8
.7
.6
.5
0
2
4
6
8
10
12
14

3b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.95
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R(r,t)
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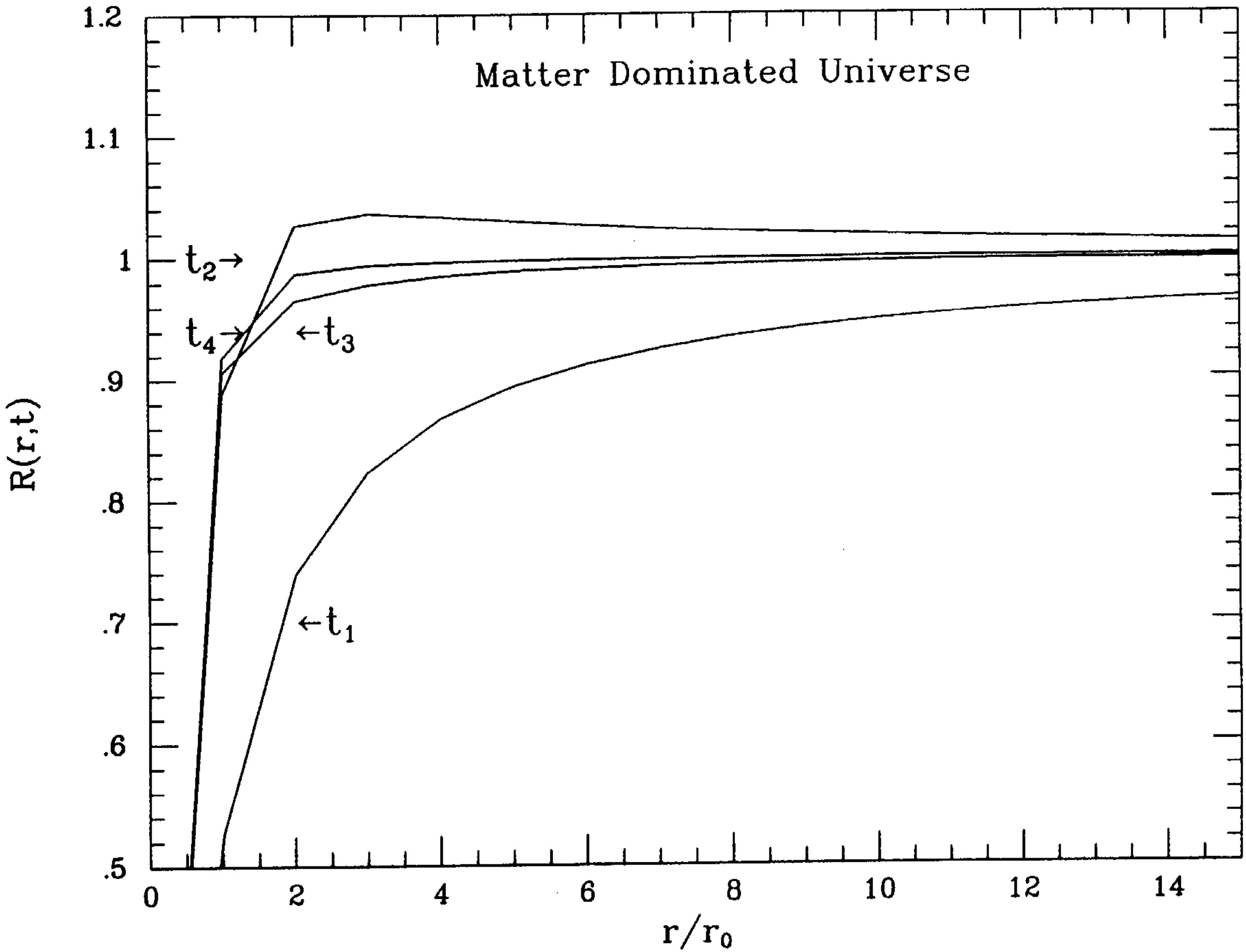
t/t0

3a


Matter Dominated Universe
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t2→
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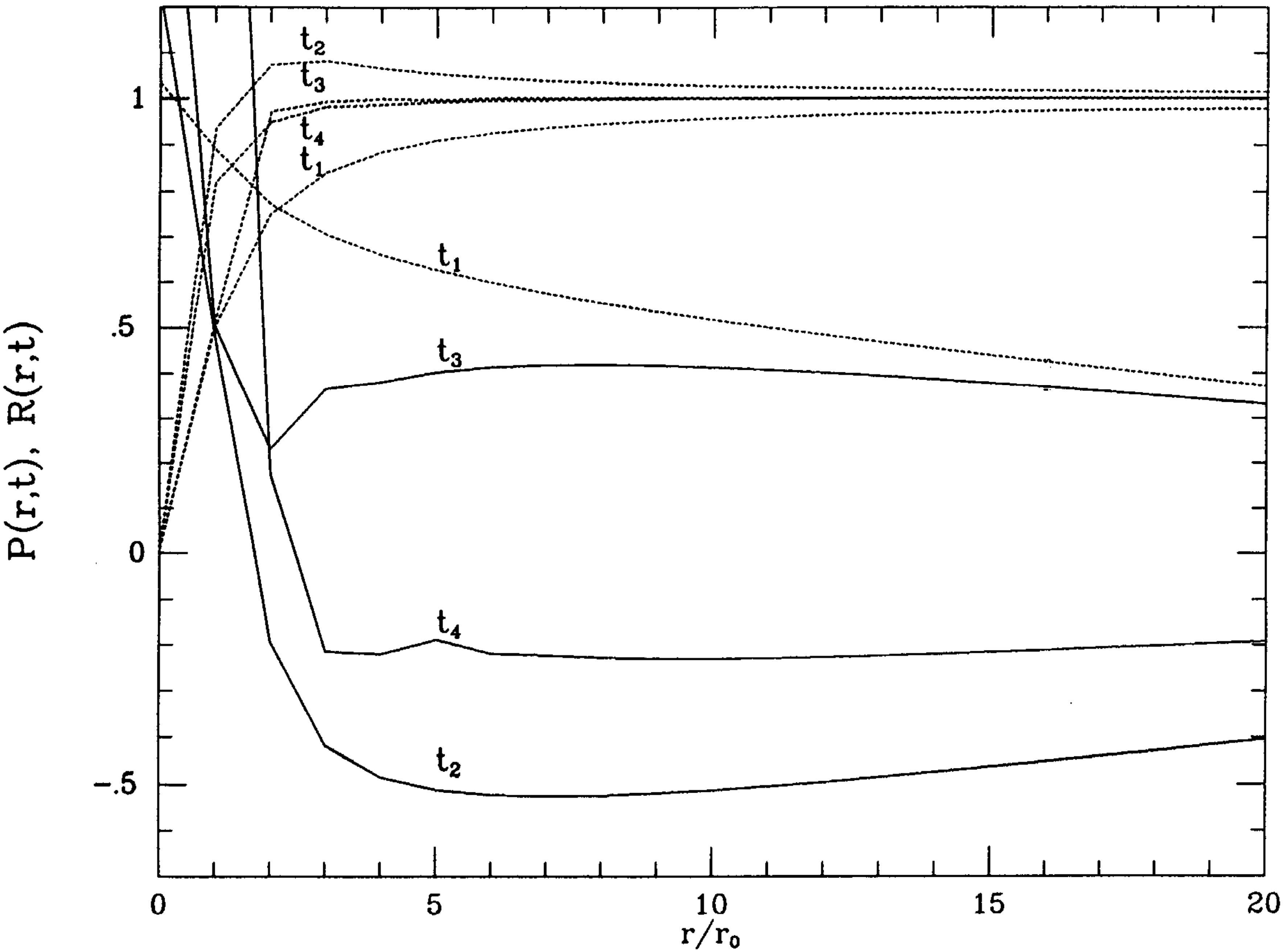

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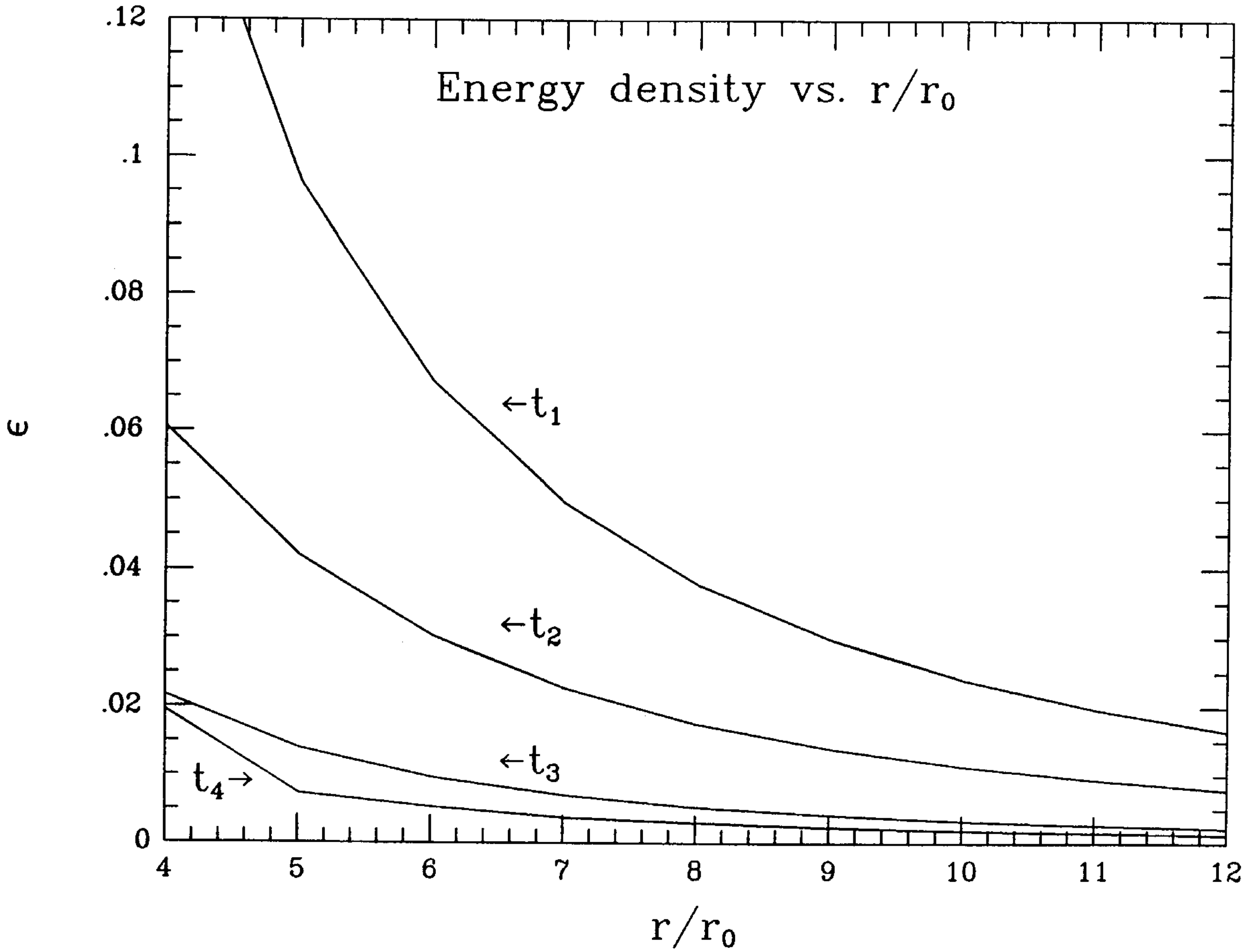

Energy density vs. r/r0
ε
.12
.1
.08
.06
.04
.02
0
4
5
6
7
8
9
10
11
12
r/r0
←t1
←t2
←t3
t4→