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Cosmic Vacuum Strings and Domain Walls in Brans-Dicke Theory of Gravity

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September 6, 1994

Abstract

We investigate the gravitational field of cosmic vacuum strings and domain walls in the context of Brans-Dicke theory of gravity. Using the weak field approximation we find the solutions which describe the spacetime and the scalar field generated by these topological structures, comparing the results with the ones obtained in General Relativity.

*Partially supported by CNPq (Brazil)

1 Introduction

Cosmic vacuum strings and domain walls are exotic topological structures which have been introduced in Cosmology in the last two decades[1]-[3], and, despite the fact that so far no direct observational evidence for their existence has been found, the richness of the new ideas they brought along to this branch of Physics seems to justify their popularity at least among theoretical cosmologists. Cosmic strings, for example, are thought to play an essential role in some proposed theories of galaxies formation since they could act as 'seeds' for the nucleation of galaxies and cluster of galaxies[4]. Also, vacuum strings and domain walls are predicted by GUT models of the Universe. Although the predicted lifetime of these structures as well as their gravitational effects are still a matter of controversy, some authors strongly hold that they must have appeared when the Universe was cooling down after the early stages of hot big bang[2].

Historically, it was Vilenkin[1], who first found the solutions corresponding to the metrics generated by strings and domain walls in the context of General Relativity. Solving Einstein equations in the weak-field approximation, Vilenkin was able to show that the gravitational field of a vacuum string manifests itself as a global change in the topology of Minkowski spacetime. The Riemmann curvature vanishes everywhere except on the string, where it is singular. The spacetime commonly called 'conical' has a line element given by $ds^2 = dt^2 - dr^2 - \alpha^2 r^2 d\theta^2 - dz^2$, where $\alpha^2 = 1 - 8\mu G$ and μ is the linear energy density of the string.[1]. As is well known, this kind of conical geometry can produce several effects like gravitational lensing[5], pair production[6], bremsstrählung radiation[7], electrostatic self-interaction[8] and the so-called gravitational Aharonov-Bohm effect[9].

In this paper we consider vacuum strings and domain walls in Brans-

[1]This value of α was calculated by Vilenkin in the weak field approximation. Later, Hiscock[4], who found the string exact solution obtained $\alpha = 1 - 4\mu G$

Dicke theory of gravity. Our approach consists of working out the field equations in the weak field approximation in much the same way as Vilenkin did in solving the same problem in General Relativity. At this point we should quote an article published by Gundlach and Ortiz[10], who also considered strings in Brans-Dicke theory however through quite a diferent approach.

2 Brans-Dicke equations in the weak field approximation

Paralleling General Relativity one can linearize Brans-Dicke field equations by assuming that the metric $g_{\mu\nu}$ and the saclar field ϕ can be written as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad , \tag{1}$$

$$\phi = \phi_0 + \epsilon = G^{-1}(1 + G\epsilon) \quad , \tag{2}$$

where $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ is the Minkowski metric tensor, $h_{\mu\nu}$ is the linear part of $g_{\mu\nu}$, ϕ_0 is a constant which may be identified to G^{-1} (G being the Newtonian gravitational constant) and ϵ is a first-order term in the energy density μ. Clearly, it is assumed that both $|h_{\mu\nu}|$ and $|G\epsilon|$ are $\ll 1$. Thus, from these assumptions one is readily able to derive the following equations[11]:

$$R^{(1)}_{\mu\nu} = 8\pi G \left[T_{\mu\nu} - \frac{\omega + 1}{2\omega + 3} \eta_{\mu\nu} T \right] + G\epsilon_{,\mu,\nu} \quad , \tag{3}$$

and

$$\Box^{(1)}\epsilon = \frac{8\pi T}{2\omega + 3} \quad , \tag{4}$$

where $R^{(1)}_{\mu\nu}$ denotes the linearized Ricci tensor $R_{\mu\nu}$, i.e.,

$$R^{(1)}_{\mu\nu} = \frac{1}{2} \left(-\eta^{\beta\gamma} h_{\mu\nu,\beta,\gamma} + h^{\beta}_{\mu,\nu,\beta} + h^{\beta}_{\nu,\mu,\beta} - h_{,\mu,\nu} \right)$$

$$= \frac{1}{2}\left(h^{\beta}_{\mu,\nu,\beta} + h^{\beta}_{\nu,\mu,\beta} - h_{,\mu,\nu} - \Box^{(1)}h_{\mu\nu}\right) \quad , \tag{5}$$

with $h \equiv h^{\alpha}_{\alpha}$ and $\Box^{(1)}$ is the Minkowskian D'Alembertian.

A great simplification of the equations (3) is achieved if instead of the usual harmonic gauge of General Relativity $\left(\left(h^{\mu}_{\nu} - \frac{1}{2}\delta^{\mu}_{\nu}h\right)_{,\mu} = 0\right)$ one choses the Brans-Dicke gauge[11]

$$\left(h^{\mu}_{\nu} - \frac{1}{2}\delta^{\mu}_{\nu}h\right)_{,\mu} = G\epsilon_{,\nu} \quad . \tag{6}$$

(Later, this gauge will be put in a more general form when we examine the domain wall case.) From (6) one can easily show that

$$h^{\beta}_{\mu,\nu,\beta} + h^{\beta}_{\nu,\mu,\beta} - h_{,\mu\nu} = 2G\epsilon_{,\mu,\nu} \quad . \tag{7}$$

Then, the equation (3) reduces to

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) h_{\mu\nu} = -16\pi G\left[T_{\mu\nu} - \frac{\omega+1}{2\omega+3}\eta_{\mu\nu}T\right] \quad . \tag{8}$$

3 The Gravitational Field of Vacuum Strings

Now let us consider an infinite, static, straight vacuum string oriented along the z-axis and located at the origin of our coordinate system. As was shown by Vilenkin[1], its energy-momentum T^{μ}_{ν} may be written as

$$T^{\mu}_{\nu}(x,y) = \mu\delta(x)\delta(y)\cdot \mathrm{diag}\,(1,0,0,1), \quad , \tag{9}$$

where μ is the linear energy density of the vacuum string. Putting (9) into the field equations (4) and (8) we get the following:

$$\nabla^2 h_{00} = -\nabla^2 h_{33} = -G\nabla^2\epsilon = \frac{16\pi\mu G}{2\omega+3}\delta(x)\delta(y) \quad . \tag{10}$$

and

$$\nabla^2 h_{11} = \nabla^2 h_{22} = \frac{32\pi\mu G(\omega+1)}{2\omega+3}\delta(x)\delta(y) \quad . \tag{11}$$

Now, taking into account the cylindrical symmetry of the problem the solutions of the equations above are:

$$h_{00} = \quad -h_{33} = -G\epsilon = \tfrac{8\mu G}{2\omega+3}\ell n\left(\tfrac{r}{r_0}\right) \quad , \tag{12}$$

$$h_{11} = \qquad h_{22} = \tfrac{16\mu G(\omega+1)}{2\omega+3}\ell n\left(\tfrac{r}{r_0}\right) \quad , \tag{13}$$

where $r = [x^2 + y^2]^{1/2}$ and r_0 is an integration constant. In this way, we arrive at the metric line element which written in cylindrical coordinates is given by

$$ds^2 = (1+\frac{8\mu G}{2\omega+3}\ell n\left(\frac{r}{r_0}\right))(dt^2 - dz^2 - (1-8\mu G\ell n(\frac{r}{r_0}))(dr^2 + r^2 d\theta^2)) \cdot \tag{14}$$

Introducing a new coordinate r' by the transformation $r = r_0\left(\frac{r'}{a}\right)^b$, where $a = r_0(1-8\mu G)^{-1/2}$ and $b = (1-4\mu G)^{-1}$, and neglecting second-order terms in μG, this line element may be put in the form

$$ds^2 = \left(1+\frac{8\mu G}{2\omega+3}\ell n\left(\frac{r'}{r_0}\right)\right)\left(dt^2 - dz^2 - dr'^2 - (1-8\mu G)\, r'^2 d\theta^2\right) \quad . \tag{15}$$

Finally, defining a new angular variable by $(1-8\mu G)d\theta = d\theta'$, we end up with

$$ds^2 = \left(1+\frac{8\mu G}{2\omega+3}\ell n\left(\frac{r'}{r_0}\right)\right)\left(dt^2 - dz^2 - dr'^2 - r'^2 d\theta'^2\right) \quad . \tag{16}$$

Let us make some comments. First of all, it must be noted that r cannot be large, otherwise the weak-field approximation ceases to be valid. Second, looking at the equations (15) and (16) one realizes that they represent nothing but a conformal transformation of Vilenkin conical metric. (It is worthwhile mentioning that a very similar result is obtained by Gundlach and Ortiz[10] following a different method.).

We see that the effect caused by the presence of the scalar field is to bring curvature to the conical spacetime. Thus, in addition to the change of the global topology there is also a curved geometry. Indeed, from (12) and (13) one can evaluate the components of the Riemmann tensor which in the weak-field approximation are given by[12]

$$R_{\mu\nu\lambda\rho} = \frac{1}{2}\left(h_{\mu\rho,\nu\lambda} + h_{\nu\lambda,\mu\rho} - h_{\nu\rho,\mu\lambda} - h_{\mu\lambda,\nu\rho}\right) \quad . \tag{17}$$

A straightforward calculation shows that the non-vanishing components of $R_{\mu\nu\lambda\rho}$ are

$$R_{0101} = -R_{1313} = -\frac{4\mu G}{2\omega+3}\left(\frac{1-2\cos^2\theta}{r^2}\right) \quad ; \tag{18}$$

$$R_{0202} = -R_{2323} = -\frac{4\mu G}{2\omega+3}\left(\frac{1-2\sin^2\theta}{r^2}\right) \quad ; \tag{19}$$

$$R_{0102} = -R_{1323} = \frac{8\mu G}{2\omega+3}\frac{\sin\theta\cos\theta}{r^2} \tag{20}$$

and $$R_{1212} = \frac{-16\pi\mu G(\omega+1)}{2\omega+3}\delta(x)\delta(y) \quad . \tag{21}$$

However, due to the conformal character of the metric (16), the motion of photons is not affected by the local curvature. It is worthy of mention that if one takes the limit $\omega \rightarrow \infty$, only the component R_{1212} survives, which means that the curvature disappears everywhere except on the string and Vilenkin's result is reproduced. Analogously, if this limit is taken directly in either equation (15) or (16), then the rescaled conical metric tends to the corresponding General Relativity solution whereas from (12) the scalar field goes over G^{-1}.

4 The Gravitational field of vacuum domain walls

Let us turn to the case of domain walls, also investigated by Vilenkin, who solved this problem, first in the weak field approximation of General Relativity[1], and later by finding an exact solution[13].

We consider an infinite static plane wall parallel to the (y, z)-plane. If σ is a homogeneous vacuum energy surface distribution, then the energy-momentum tensor T^μ_ν must be given by[1]

$$T^\mu_\nu(x) = \sigma\delta(x).\mathrm{diag}(1,0,1,1) \quad \cdot \tag{22}$$

As we have mentioned earlier, for the sake of mathematical convenience we shall modify the gauge condition (6) to a more general form given by

$$\left(h^\mu_\nu - \frac{1}{2}\delta^\mu_\nu h\right)_{,\mu} = \left(1 - \frac{\alpha}{2}\right) G\epsilon_{,\nu} \quad , \tag{23}$$

where α is a free parameter. (This allows one to generate an infinite class of α-dependent solutions). With this choice we can show that the equation (3) takes the form

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) h_{\mu\nu} = -16\pi G\left(T_{\mu\nu} - \frac{\omega+1}{2\omega+3}\eta_{\mu\nu}T\right) - \alpha G\epsilon_{,\mu,\nu} \quad \cdot \tag{24}$$

Putting (22) into (24) and taking into account the fact that $h_{\mu\nu}$ and ϵ depend only on x, we get for the metric functions

$$\frac{d^2}{dx^2}h_{00} = -\frac{d^2}{dx^2}h_{22} = -\frac{d^2}{dx^2}h_{33} = -\frac{16\pi\sigma G\omega\delta(x)}{2\omega+3} \quad , \tag{25}$$

$$\frac{d^2}{dx^2}h_{11} = \frac{48\pi\sigma G(\omega+1-\frac{\alpha}{2})\delta(x)}{2\omega+3} \quad , \tag{26}$$

From (4) the equation for the scalar field will be given by

$$\frac{d^2\epsilon}{dx^2} = -\frac{24\pi\sigma\delta(x)}{2\omega+3} \quad . \tag{27}$$

The solutions of these equations are easily verified to be

$$h_{00} = -h_{22} = -h_{33} = -\frac{8\pi\sigma G\omega|x|}{2\omega+3} \quad , \tag{28}$$

$$h_{11} = \frac{24\pi\sigma G}{2\omega+3}(\omega + 1 - \frac{\alpha}{2})|x| \quad , \tag{29}$$

$$\text{and} \quad \epsilon = -\frac{12\pi\sigma|x|}{2\omega+3} \quad . \tag{30}$$

From these results a simple expression for the metric is obtained if one choses $\alpha = \frac{2}{3}(2\omega + 3)$:

$$ds^2 = \left[1 - \frac{8\pi\sigma G\omega|x|}{2\omega+3}\right]\left(dt^2 - dx^2 - dy^2 - dz^2\right) \quad , \tag{31}$$

which exactly reduces to Vilenkin's solution for large values of ω. We should point out that if α is taken independent of ω then when $\omega \to \infty$ the metric functions $h_{\mu\nu}$ also tend to Vilenkin's solution[1].

Thus we see that for domain walls the picture here is almost identical to that coming from General Relativity, since the only effect brought about by the scalar field consists of a simple redefinition of the vacuum energy density σ to $\sigma' = \frac{2\omega\sigma}{2\omega+3}$. If only first-order terms in σG are retained no curvature is generated in this case.

5 Final Remarks

Basically our motivation to investigate vacuum strings and domain walls outside the General Relativity scheme, specifically in Brans-Dicke theory, comes from the important role, we believe, scalar-tensor theories can play in our understanding of the early Universe (see, for example, [14],[15]), when topological structures like vacuum strings and domain walls may have existed. Physical processes like gravitational lensing[5], galaxies formation[16] pair production[6], bremsstrählung radiation[7], and electrostatic self-interaction[8], which may be viewed as caused by these structures should now be investigated assuming the curved spacetime background of eq.(16). We have left these subjects for future work.

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