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Cosmography and cosmic acceleration

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ABSTRACT

We investigate the prospects for determining the accelerating history of the Universe from upcoming measurements of the expansion rate $H(z)$. In our analyses, we use Monte Carlo simulations based on w cold dark matter models to generate samples with different characteristics and calculate the evolution of the deceleration parameter $q(z)$. We show that a cosmographic (and, therefore, model-independent) evidence for cosmic acceleration [$q(z < z_t) < 0$, where z_t is the transition redshift] will be possible only with an accuracy in $H(z)$ data expected in some planned surveys. A brief discussion about the prospects for reconstructing the dark energy equation of state from the parameters $H(z)$ and $q(z)$ is also included.

Key words: cosmological parameters – dark energy – distance scale.

1 INTRODUCTION

The determination of cosmographic parameters, such as H_0 and q_0, has a long and interesting history in cosmology (see e.g. Sandage 1970). In particular, the evolution of such parameters provides a unique and direct method to map the expansion history of the Universe in a model-independent way. Since all evidence we have so far for the current cosmic acceleration are indirect (Padmanabhan 2003; Frieman 2008; Caldwell & Kamionkowski 2009), extracting the evolution of these two parameters from future redshift surveys constitutes one of the major challenges in observational cosmology.[1]

In this paper, we investigate how well cosmography may provide a model-independent way to check the reality of cosmic acceleration. Specifically, we study the evolution of the deceleration parameter from the $H(z)$ data which are to become available by some planned projects. To this end, we performed a Monte Carlo (MC) simulation based on w cold dark matter (wCDM) models [of which the lambda cold dark matter (ΛCDM) model is a special case with $w \equiv p/\rho = -1$, where p stands for the dark energy pressure and ρ for its energy density]. We generated three samples of $H(z)$ with increasing accuracy and derived $q(z)$ from numerical differentiation. We show that a cosmographic evidence for cosmic acceleration will be possible only with a great (maybe out of the perspective of some current planned surveys) accuracy in $H(z)$ data. It is worth mentioning that the determination of $q(z)$ will also allow us to estimate the transition redshift, z_t, which corresponds to the epoch when the Hubble expansion switched from a decelerating to an accelerating phase. Since $q(z) \propto w(z)H(z)^2/f(z, w)$ (see equation 4), the same technique can also be used to map the recent past evolution of $w(z)$ in a given model of dark energy.

2 DIRECT OBSERVATION OF $H(z)$

Recently, it has been shown that luminous red galaxies (LRGs) can provide us with direct measurements of the expansion rate $H(z)$ [Jimenez & Loeb 2002; see Ma & Zhang 2011; see also Zhang & Ma 2010, for a recent review on $H(z)$ measurements from different techniques].[2] This can be done by calculating the derivative of cosmic time with respect to redshift,

$$H(z) = -\frac{1}{(1+z)}\frac{dz}{dt}\,, \quad (1)$$

from measurements of age difference between two passively evolving galaxies at different z. Simon, Verde & Jimenez (2005, hereafter SVJ) have demonstrated the feasibility of directly measuring $H(z)$ using this differential age method and have provided nine determinations of the expansion rate at $z \neq 0$. More recently, Stern et al. (2010) have updated SVJ sample to 11 estimates of $H(z)$ lying in the redshift interval $0.1 \leq z \leq 1.75$. New age-redshift data sets for different galaxy velocity dispersion groups (seven LRG samples) have also been made available by the Sloan Digital Sky Survey collaboration (Carson & Nichol 2010).

Simon et al. (2005) have also pointed out that in the near future, the Atacama Cosmology Telescope (ACT)[3] is expected to provide

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[1] Due to the multiple integrals that relate cosmological parameters to cosmological distances, direct determinations of $H(z)$ may also reduce the so-called smearing effect that makes constraining the dark energy equation of state (EoS) w extremely difficult (Maor, Brustein & Steinhardt 2001a,b).

[2] Direct measurements of $H(z)$ at different redshifts will also be possible through measurements of the line of sight or radial component of baryonic acoustic oscillations (BAO) from large redshift surveys with the redshift precision of the order of $0.003(1 + z)$ (see e.g. Benitez et al. 2009).

[3] http://www.physics.princeton.edu/act/

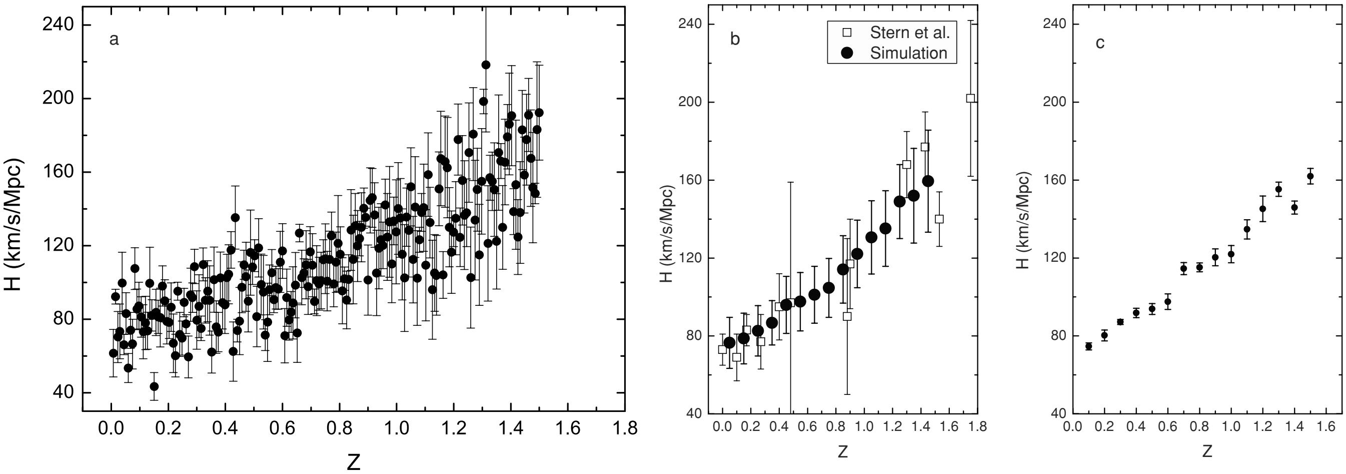

Figure 1. (a) An MC realization of 1000 simulated values of the Hubble parameter with 15 per cent accuracy based on Simon, Verde & Jimenez (2005). Only 200 points are shown for the sake of clarity. (b) Simulated values of $H(z)$ shown in the previous panel averaged over bins with width $\Delta z = 0.1$. Squares represent current $H(z)$ measurements of Stern et al. (2010). (c) Similar to panel (a) for the 3 per cent projection made by Crawford et al. (2010).

observations of over 500 galaxy clusters up to $z \simeq 1.5$. This, together with spectra to be acquired in other telescopes in Chile and Southern African Large Telescope (SALT) in South Africa, will provide a sample of more than 2000 passively evolving galaxies in the redshift range of $0 < z < 1.5$. From these observations, it will be possible to determine ∼1000 values of the Hubble parameter at a 15 per cent accuracy level if the ages of the galaxies are estimated with the 10 per cent error.

Following a similar approach, Crawford et al. (2010) examined the observational requirements to estimate $H(z)$ to a given precision. In their simulation, observations of LRGs are made at two redshifts, namely $z = 0.32$ and 0.51 (average 0.42), and they supposed that uncertainty on ages of individual galaxies lies in the range 0.05–2 Gyr. They estimated that the uncertainty on the mean ages will be 0.10, 0.05 and 0.03 Gyr and that the Hubble parameter can be measured with a precision of 10, 5 and 3 per cent. A study was also conducted on the constraints upon the exposure time per galaxy and on the number of galaxies observed. They concluded that with a total time of 17, 72 and 184 h, observing 80, 327 and 840 galaxies on the SALT will make it possible to recover $H(z)$ to 10, 5 and 3 per cent accuracy, respectively (Crawford et al. 2010; Crawford, private communication).

3 SIMULATED DATA SETS AND $q(z)$

In our analyses, we perform MC simulations which provide us with samples of $H(z)$ based on the flat ΛCDM model. For each 'measurement', we assume a Gaussian distribution of $H(z)$ centered at the value predicted by a flat wCDM with $w = -1$, with a standard deviation corresponding to the percentage accuracy predicted in future experiments. We shall examine two cases based on the projections made by Simon et al. (2005) and Crawford et al. (2010). For the former case, we simulated 1000 data points for $H(z)$, uniformly distributed between $z = 0$ and 1.5 with 15 per cent precision. In the latter case, we simulated 15 measurements of the Hubble parameter between redshifts 0 and 1.5 and equally spaced ($\Delta z = 0.1$). We take the errors estimated in Crawford et al. (2010) when recovering $H(z)$ from LRG's ages, namely 10, 5 and 3 per cent. Note that in the last case (the one explored throughout this paper), one needs 8400 LRGs in the whole interval $z = 0.1-1.5$. In all our simulations, we have adopted $H_0 = 74.2\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, which corresponds to the central value given by Riess et al. (2009) based on differential measurements of Cepheids variable observations and $\Omega_\mathrm{m} = 0.27$, as given by current CMB measurements (Komatsu et al. 2011).

Fig. 1(a) shows one realization of 1000 simulated values of $H(z)$ with 15 per cent accuracy according to Simon et al. (2005) for z in the range (0.0–1.5). The binned data points of this realization (with a bin width $\Delta z = 0.1$) are shown in Fig. 1(b). The error bars correspond to the standard deviation around the mean of individual values of $H(z)$ within each bin, which contains approximately 67 galaxies. For comparison, the observed values given by Stern et al. (2010) are also shown. In Fig. 1(c), we show the results of one MC realization taking the errors estimated by Crawford et al. (2010) with the resulting $H(z)$ data points at 3 per cent accuracy.

From the above $H(z)$ simulated data, one may derive evolution of the deceleration parameter $q(z)$, defined as

$$q(z) = \frac{1}{H(z)} \left[\frac{\mathrm{d}H(z)}{\mathrm{d}z} \right] (1+z) - 1. \qquad (2)$$

In order to do that one needs to compute the derivative of the $H(z)$ with respect to the redshift. However, while integration tends to smooth out data fluctuations, numerical differentiation tends to magnify errors and the scatter of the points. Thus, depending on the degree of scatter of $H(z)$ measurements, the gradient estimation may be useless. For equally spaced points, the derivative is calculated using finite difference approximation, that is, $\mathrm{d}H(z_i)/\mathrm{d}t \simeq [H(z_{i+1}) - H(z_{i-1})]/2\Delta z$, where $\Delta z = z_i - z_{i-1}$. We then substitute this into (2) and calculate $q(z_i)$ and the associated uncertainty by using the standard error propagation method.[4]

For our first simulation, shown in Figs 1(a) and (b), we use the binned points since the large scatter of the raw data makes them useless. The resulting values of the deceleration parameter from the MC realization of Fig. 1(b) are shown in Fig. 2(a). Although the points lie near the predicted curve of the ΛCDM model (black curve), we cannot expect a definitive evidence for a recent cosmic

[4] The uncertainty calculated from the standard error propagation method is considerably larger than the analytical results obtained directly from equation (2) due to finite spacing of the measurements in redshift (see e.g. Clarkson, Cortes & Bassett 2007 for a discussion). We have also verified the results shown in the figures above from the MC error analysis. Similar results are obtained.

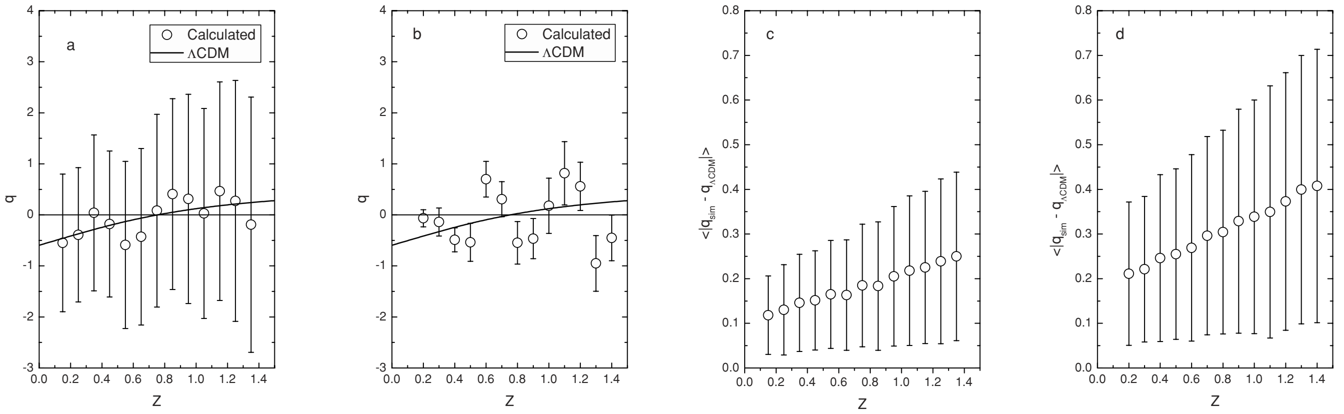

Figure 2. The evolution of $q(z)$ derived from the $H(z)$ points displayed in Fig. 1. Panel (a) corresponds to the 1000 binned values of Fig. 1(b), whereas panel (b) corresponds to the 15 $H(z)$ data points at 3 per cent accuracy shown in Fig. 1(b). (c) The difference between the simulated and the background model values of $q(z)$ for 1000 MC realization and 15 per cent accuracy in $H(z)$ observations. (d) The same as in panel (c) for the 15 $H(z)$ data points at 3 per cent accuracy shown in Fig. 1(b).

acceleration from these simulated $H(z)$ sample with 15 per cent accuracy. Note that the same is also true even when the evolution of $q(z)$ is derived from the 15 data points with 3 per cent accuracy following Crawford et al. (2010) (Fig. 2b). In this case, although the error of individual points is smaller, the determination of the deceleration parameter is less precise with considerable scattering of points and error bars allowing both a decelerating and accelerating universe today.

In order to quantify the above results, we also performed 1000 MC realizations to calculate the mean of the absolute values of the difference between the calculated and the background model values of $q_{\Lambda\mathrm{CDM}}(\mathrm{z_i})$, that is,

$$\langle \Delta q \rangle = \frac{1}{N} \sum_{i=1}^{N} |q_{\mathrm{sim}}(z_i) - q_{\Lambda\mathrm{CDM}}(z_i)|. \quad (3)$$

We plot this quantity for both cases discussed above in Figs 2(c) (1000 binned points) and (d) (15 points), with the error bars representing the standard deviation from the mean. As we see, $\langle \Delta q \rangle$ has a weak dependence on redshift for the first case, and in the interval $0.1 < z < 1.4$ it lies in the range ∼0.1–0.25, although the uncertainty is relatively large. From Fig. 2(d) we see that the departure of the calculated $q_{\mathrm{sim}}(z_i)$ from the expected value given by the model is larger than that in the former case, with $\langle \Delta q \rangle$ varying from 0.2 to 0.4. This clearly reflects the large scattering of points shown in Fig. 2(b).

In view of the above results, we also speculated about how precise should future observations be in order to provide a clear evidence for cosmic acceleration from cosmography. We did that for the method proposed by Crawford et al. (2010) and found that it will give very good results if the accuracy is improved to as much as 1 per cent. This is illustrated in Fig. 3. In this case, the calculated value of $q_{\mathrm{sim}}(z_i)$ is very close to the ΛCDM model predictions in almost the entire range of z, and the deviation from the expected value is as low as 0.07 and always <0.15. Naturally, the observational requirements for this case would be much more stringent than the previous ones, with the need of thousand galaxies and more observational time as well.

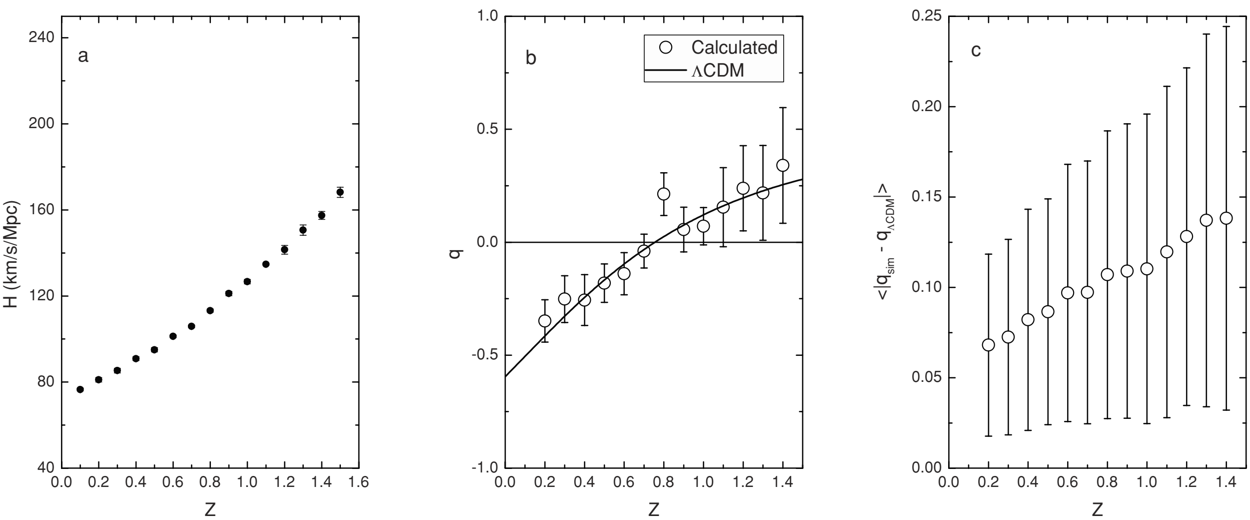

Figure 3. (a) The results of our simulations following Crawford et al. (2010) with a precision of 1 per cent in the $H(z)$ observations. (b) The evolution of $q(z)$ for the MC realization is shown in panel (a). With this precision in the $H(z)$ measurements, a cosmographic detection of the current cosmic acceleration can be obtained, as can also be seen from panel (c).

4 EQUATION-OF-STATE PARAMETER

By considering a dark-energy-dominated universe, a step further in the above discussion concerns the reconstruction of its EoS parameter w from the measurements of $H(z)$. In order to be consistent with the simulations presented earlier, we consider accelerating wCDM models with w lying in the interval [−1/3, −3/2]. In this context, equation (2) can be rewritten as

$$w(z) = \frac{2q(z) - 1}{3\Omega_w^0 (1+z)^{3(1+w)}} \left(\frac{H}{H_0}\right)^2 , \tag{4}$$

where Ω_w^0 is the current fractional contribution of dark energy to the critical density. Note that such procedure does not involve several integrations of $w(z)$, as in cosmological tests based on age or distance measurements and, therefore, may reduce the smearing effect on $w(z)$ determinations discussed earlier (see also Jimenez & Loeb 2002 for a different approach).

Fig. 4 shows the evolution of w with redshift calculated from the simulated measurements of $H(z)$ using the method proposed by Crawford et al. (2010) with 1 per cent accuracy. Although the input model ($w = -1$) is fairly recovered, we note that the estimated errors of the dark energy EoS (δw) increase considerably at the upper end of the redshift range. This can be more easily understood after solving equation (4) for w and calculating δw, which depends strongly on a competition between the different terms of equation (4) at the z interval considered. For the sake of completeness, we also performed the same analysis for the other two cases [with 15 and 3 per cent accuracy in $H(z)$] discussed earlier. In both, the scattering of the points is large enough to hamper any definitive conclusion on the expected evolution of the cosmic EoS [we refer the reader to Mörtsell & Clarkson (2009) and references therein for other model-independent reconstruction methods of $q(z)$ and $w(z)$ from the current data].

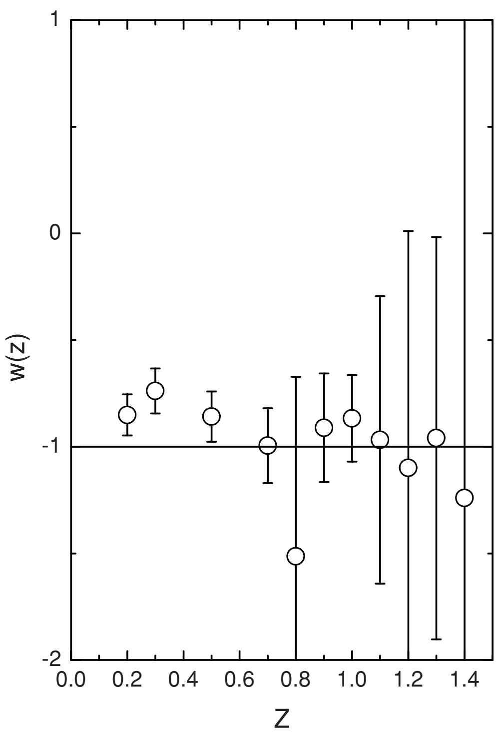

Figure 4. Evolution of w with redshift for 1 per cent accuracy in $H(z)$ observations following the method of Crawford et al. (2010). Although the background model is fairly recovered, the uncertainties increase considerably at the upper end of the redshift interval considered.

5 CONCLUSIONS

Cosmography explores the possibility of extracting the maximum amount of information from cosmological measurements, as well as the assumption that the universe can be modelled by the Friedmann–Robertson–Walker line element without assuming any dynamical theory to describe it. In this paper, differently from many analyses that study the current phase of cosmic evolution in the context of a given cosmological model, we have investigated the prospects for a cosmographic mapping of cosmic acceleration using two different projections of age determinations of passively evolving galaxies recently discussed in the literature (Simon et al. 2005; Crawford et al. 2010). We have found that a model-independent check of the reality of cosmic acceleration requires very accurate measurements of the expansion rate (∼1 per cent), which are in the perspectives of some current planned surveys. As a step further in this analysis, we have also used the direct relation between $q(z)$ and $w(z)$ to study possible constraints on the dark energy EoS from future observations (Fig. 4).

Finally, it is also worth emphasizing that the method discussed here can be applied regardless of the technique used to obtain $H(z)$ measurements. In this regard, we note that $H(z)$ measurements from radial BAO methods (despite the systematic errors introduced mainly by distortion effects) surpass the age method in precision and can extend the $H(z)$ measurements into deeper redshift ranges (Benitez et al. 2009; see also McDonald & Eisenstein 2007; Norman, Paschos & Harkness 2009) for expected measurements of the expansion rate at $z \leq 2.5$ by observing the Lyman-forest absorption spectra of high-z quasars). A detailed analysis involving expected high-z estimates of $H(z)$ from current planned BAO surveys is currently under investigation and will appear in a forthcoming communication.

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