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Cosmological constraints on $f(R)$ gravity theories within the Palatini approach

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ABSTRACT

We investigate $f(R)$ theories of gravity within the Palatini approach and show how one can determine the expansion history, $H(a)$, for an arbitrary choice of $f(R)$. As an example, we consider cosmological constraints on such theories arising from the supernova type Ia, large-scale structure formation, and cosmic microwave background observations. We find that the best fit to the data is a non-null leading order correction to the Einstein gravity. However, the current data exhibits no significant trend toward such corrections compared to the concordance ΛCDM model. Our results show that the oft-considered $1/R$ models are not compatible with the data. The results demonstrate that background expansion alone can act as a good discriminator between modified gravity models when multiple data sets are used.

Key words. gravitation – cosmological parameters – cosmology: observations – cosmology: theory – large-scale structure of Universe

1. Introduction

The combination of Einstein's General Relativity (GR) and ordinary matter, as described by the standard model of particle physics (Eidelman et al. 2004), cannot explain the current cosmological observations. Key observations confronting the matter-only universe are the luminosity-redshift relationship from observations of supernovae of type Ia (SNIa, Riess et al. 2004), the matter power spectrum of large-scale structures as inferred from galaxy redshift surveys such as the Sloan Digital Sky Survey (SDSS, Tegmark et al. 2004), and the 2dF Galaxy Redshift Survey (2dFGRS, Colless et al. 2001), and from the anisotropies in the cosmic microwave background radiation (CMBR, Spergel et al. 2003). Two exotic components are required in the matter-energy budget of the Universe to account for the results from all of these cosmological probes within GR. These two components are *dark matter*, a collisionless and pressureless fluid that contributes about 25% of the universe energy budget, and a negative pressure fluid called *dark energy*. Currently the dark energy component dominates the energy density of the universe, causing accelerating expansion.

Despite the very good agreement between the so-called concordance model and the astrophysical data, the nature of dark matter and dark energy is one of the greatest mysteries of modern cosmology. In fact, none of the dark matter candidates from high energy physics beyond the standard model (Bertone et al. 2005; Ellis 2000; Brookfield et al. 2005; Amendola 2000) have ever been observed. One should bear in mind that the existence of the dark sector is only inferred from the motion of ordinary matter in a gravitational field, so one can ask whether the necessity of including dark matter and dark energy in the energy budget might not be a sign of our lack of understanding of gravitational physics (Lue et al. 2004a,b; Lue 2003). A natural alternative to adding new exotic fluids is to modify gravitational physics. Several ways of modifying gravity have been proposed to dispense with dark matter (Sanders & McGaugh 2002; Milgrom 1994; Moffat 2004; Bekenstein 2005; Skordis et al. 2006; Sellwood & Kosowsky 2001) or dark energy (Dvali et al. 2000; Deffayet 2001; Dvali & Turner 2003; Arkani-Hamed et al. 2002; Nojiri & Odintsov 2003, 2004a,b, 2005; Abdalla et al. 2005; Freese & Lewis 2002; Meng & Wang 2003; Dolgov & Kawasaki 2003; Shao et al. 2005; Gong et al. 2004; Lima 2004; Ahmed et al. 2004; Bento et al. 2002, 2003). They can account for the observations with some degree of success (Lue et al. 2004a,b; Lue 2003; Bento et al. 2002, 2003; Deffayet et al. 2002; Carroll et al. 2005; Multamaki et al. 2003, 2004; Koivisto et al. 2005; Elgaroy & Multamaki 2005; Amarzguioui et al. 2005; Multamaki & Elgaroy 2004).

In this article we investigate a family of alternative models to the dark energy paradigm based on a generalization of the Einstein-Hilbert Lagrangian. These models are called nonlinear theories of gravity (Magnano & Sokolowski 1994; Allemandi et al. 2006) or $f(R)$ theories (Bronnikov & Chernakova 2005; Cognola et al. 2005; Nunez & Solganik 2004; Ezawa et al. 2003; Barraco et al. 2000; Schmidt 1998; Rippl et al. 1996; Barrow & Ottewill 1983), since the scalar curvature R in the Einstein-Hilbert Lagrangian is replaced by a general function $f(R)$. The main motivation for this generalization is that higher-order terms in curvature invariants (such as R^2, $R^{\mu\nu}R_{\mu\nu}$, $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$, etc.) have to be added to the effective Lagrangian of the gravitational field when quantum corrections are considered (Buchbinder et al. 1992; Birrell & Davies 1982; Vilkovisky 1992; Gasperini & Veneziano 1992). Furthermore, there is no a priori reason to restrict oneself to the simple Einstein-Hilbert action when a more general formulation is allowed. Higher-order terms in the gravitational action also have interesting consequences in cosmology, like natural inflationary behavior at early times (Starobinsky 1980; Barrow & Cotsakis 1988, 1991; La & Steinhardt 1989), and late time acceleration of the Universe (Meng & Wang 2003, 2004b; Carroll et al. 2005; Capozziello et al. 2005). Consequently, several authors have investigated whether such theories are indeed compatible with current cosmological observations, big bang nucleosynthesis, and solar system constraints in its weak field limit (Clifton & Barrow 2005; Barrow & Clifton 2006; Quandt & Schmidt 1991; Dominguez & Barraco 2004; Sotiriou 2005a,b; Olmo 2005a,b; Capozziello 2002; Carloni et al. 2005; Barraco et al. 2002; Hwang & Noh 2001; Mena et al. 2005; Allemandi et al. 2004a,b, 2005). Most of these investigations have been model dependent and the conclusions are somewhat contradictory (Flanagan 2004; Vollick 2004).

When dealing with $f(R)$ theories of gravity, the choice of the independent fields to vary in the action is a fundamental issue. The so-called Palatini approach considers the metric and the connection to be independent of each other, and the resulting field equations are in general different from those one gets from varying only the metric in the so-called metric approach. The two approaches lead to the same equations only if $f(R)$ is linear in R. The correct choice of approach to derive the field equations is still a hot topic of research. Initially, $f(R)$ theories were investigated in the metric approach. However, since this method leads to fourth-order equations and the Palatini approach leads to second-order equations, the latter is appealing because of its simplicity. Moreover, the equations resulting from the metric approach seem to have instability problems in many interesting cases (Dolgov & Kawasaki 2003; Chiba 2003) that the Palatini approach does not have. However, recent work has cast doubts on these instabilities (Cembranos 2005). In the present paper we concentrate on the Palatini approach.

The aim of this article is to use current cosmological data to consider possible deviations from GR by combining a number of different cosmological observations. The data used are the latest Supernovae Ia gold set (Riess et al. 2004), the CMBR shift parameter (Bond et al. 1997), the baryon oscillation length scale (Eisenstein et al. 2005), and the linear growth factor at the 2dFGRS effective redshift (Hawkins et al. 2003; Wang & Tegmark 2004). As far as we are aware, this is the first time one uses all of the main cosmological data sets in order to constrain these models.

Newtonian and Solar system constraints within the Palatini formulation have been analyzed by several authors, but there doesn't seem to be any consensus on this topic at present time. To illustrate this we list briefly the conclusion reached by a few authors: Sotiriou (2005a) claims that "any reasonable $f(R)$ model will give the correct Newtonian limit". Meng & Wang (2004a) claim that all models with inverse powers of R give a correct Newtonian limit; i.e. there are no Newtonian constraints on such models. Dominguez & Barraco (2004), however, claim that the Newtonian limit places very strong constraints on the form of $f(R)$. Using a scalar-tensor representation of the $f(R)$ model, Olmo (2005b) also finds that there are Newtonian constraints on the Lagrangian.

The structure of the paper is as follows. In Sect. 2 we deduce and summarize the basic equations and properties of general $f(R)$ gravities in the Palatini approach. In Sect. 3, we investigate observational constraints based on the evolution of the background of the Universe. In particular, we consider fits to the SNIa and CMB shift parameter from the WMAP data. In Sect. 4, we analyze the evolution of linear perturbations and probe large-scale structure formation in these models using the linear growth factor derived from the 2dFGRS data. Finally, Sect. 5 contains a summary of our work and our conclusions.

2. General $f(R)$ gravity theories

The action that defines $f(R)$ gravity theories in the Palatini formalism is

$$S[f;g,\hat{\Gamma},\psi_{\rm m}] = -\frac{1}{2\kappa}\int {\rm d}^4x\,\sqrt{-g}f(R) + S_{\rm m}[g_{\mu\nu},\psi_{\rm m}] \tag{1}$$

where $\kappa = 8\pi G$, $S_{\rm m}[g_{\mu\nu},\psi_{\rm m}]$ is the matter action that depends only on the metric $g_{\mu\nu}$ and on the matter fields $\psi_{\rm m}$; $R \equiv R(g,\hat{\Gamma}) = g^{\mu\nu}R_{\mu\nu}(\hat{\Gamma})$ is the generalized Ricci scalar; and $R_{\mu\nu}(\hat{\Gamma})$ is the Ricci tensor of the affine connection $\hat{\Gamma}$, which in the Palatini approach is independent of the metric. The generalized Riemann tensor is given by (Vollick 2003):

$$R^{\alpha}_{\mu\nu\beta} = \hat{\Gamma}^{\alpha}_{\mu\nu,\beta} - \hat{\Gamma}^{\alpha}_{\mu\beta,\nu} + \hat{\Gamma}^{\lambda}_{\mu\nu}\hat{\Gamma}^{\alpha}_{\beta\lambda} - \hat{\Gamma}^{\lambda}_{\mu\beta}\hat{\Gamma}^{\alpha}_{\nu\lambda}\,. \tag{2}$$

We define the Ricci tensor by contracting the first and the third indices of the Riemann tensor. The field equations will be obtained using the Palatini formalism i.e. we vary both with respect to the metric and to the connection.

Varying the above action with respect to the metric, we obtain the generalized Einstein equations

$$f'(R)R_{\mu\nu}(\hat{\Gamma}) - \frac{1}{2}f(R)g_{\mu\nu} = -\kappa T_{\mu\nu}\,, \tag{3}$$

where $f'(R) \equiv {\rm d}f/{\rm d}R$, and $T_{\mu\nu}$ is the energy momentum tensor

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\rm m}}{\delta g^{\mu\nu}}\cdot \tag{4}$$

Varying with respect to the connection $\hat{\Gamma}$ and contracting gives us the equation that determines the generalized connection (Vollick 2003):

$$\hat{\nabla}_{\alpha}\left[f'(R)\sqrt{-g}g^{\mu\nu}\right] = 0, \tag{5}$$

where $\hat{\nabla}$ is the covariant derivative with respect to the affine connection $\hat{\Gamma}$. This equation implies that we can write the affine connection as the Levi-Civita connection of a new metric $h_{\mu\nu} = f'(R)g_{\mu\nu}$. The Levi-Civita connections of the metrics $g_{\mu\nu}$ and $h_{\mu\nu}$ are then related by a conformal transformation. This allows us to write the affine connection as

$$\hat{\Gamma}^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\mu\nu} + \frac{1}{2f'}\left[2\delta^{\sigma}_{(\mu}\partial_{\nu)}f' - g^{\sigma\tau}g_{\mu\nu}\partial_{\tau}f'\right], \tag{6}$$

where $\Gamma^{\sigma}_{\mu\nu}$ is the Levi-Civita connection of the metric $g_{\mu\nu}$.

The generalized Ricci tensor can now be written as

$$R_{\mu\nu} = R_{\mu\nu}(g) - \frac{3}{2}\frac{\nabla_{\mu}f'\nabla_{\nu}f'}{f'^2} + \frac{\nabla_{\mu}\nabla_{\nu}f'}{f'} + \frac{1}{2}g_{\mu\nu}\frac{\nabla^{\mu}\nabla_{\mu}f'}{f'}\cdot \tag{7}$$

Here $R_{\mu\nu}(g)$ is the Ricci tensor associated with $g_{\mu\nu}$, and ∇_{μ} the covariant derivative associated with the Levi-Civita connection of this metric.

Since we are interested in cosmological solutions, we consider the spatially flat FRW metric,

$${\rm d}s^2 = -{\rm d}t^2 + a(t)^2\delta_{ij}{\rm d}x^i{\rm d}x^j, \tag{8}$$

and the perfect fluid energy-momentum tensor $T^{\nu}_{\mu} = \mathrm{diag}(-\rho, p, p, p)$. Taking the trace of Eq. (3) gives

$$Rf'(R) - 2f(R) = -\kappa T, \tag{9}$$

where $T = g^{\mu\nu}T_{\mu\nu} = -(\rho - 3p)$. Equation (9) can in certain special cases be solved explicitly for $R = R(T)$, but this is not possible in general.

The generalized Friedmann equation can be derived straightforwardly using the generalized Ricci tensor (7). This gives

$$\left(H + \frac{1}{2}\frac{\dot{f}'}{f'}\right)^2 = \frac{1}{6}\frac{\kappa(\rho + 3p)}{f'} - \frac{1}{6}\frac{f}{f'}, \tag{10}$$

which agrees with the result in Allemandi et al. (2004a,b) (note the sign difference in the definition of the Einstein equation)[1].

If the equation of state of the fluid, $p = p(\rho)$, is known, one can use the continuity equation of the fluid together with Eq. (9) to express $\dot{R}$ as

$$\dot{R} = -3H\frac{\left(1 - 3p'(\rho)\right)\left(\rho + p(\rho)\right)}{Rf''(R) - f'(R)}. \tag{11}$$

Using the Friedmann equation, it is easy to see that now $H = H(\rho, R)$, which together with Eq. (9) forms an algebraic set of equations from which one can in principle determine $H = H(R)$ for any fluid.

In the case of a constant equation of state, $p = w\rho$, we get $\kappa\rho = (Rf' - 2f)/(1 - 3w)$ and hence the generalized Friedmann equation can be written as

$$H^2 = \frac{1}{6(1 - 3w)f'}\frac{(1 + 3w)Rf' - 3(1 + w)f}{\left(1 - \frac{3}{2}(1 + w)\frac{f''(Rf' - 2f)}{f'(Rf'' - f')}\right)^2}. \tag{12}$$

Similarly, from Eq. (9), we can write $a = a(R)$:

$$a = \left(\frac{1}{\kappa\rho_0(1 - 3w)}(Rf' - 2f)\right)^{-\frac{1}{3(1+w)}}, \tag{13}$$

where $\rho_0 = \rho(a_0)$, and we have chosen $a_0 = 1$. Using these two equations, (12) and (13), it is now straightforward to determine the expansion history, $H(a)$, for any $f(R)$.

In the special case $w = 0$, applicable when the universe is matter dominated, these equations reduce to

$$H^2 = \frac{1}{6f'}\frac{Rf' - 3f}{\left(1 - \frac{3}{2}\frac{f''(Rf' - 2f)}{f'(Rf'' - f')}\right)^2}, \tag{14}$$

$$a = \left(\frac{1}{\kappa\rho_0}(Rf' - 2f)\right)^{-\frac{1}{3}}. \tag{15}$$

2.1. The leading correction to Einstein gravity

To investigate to what extent observations allow deviations from general relativity (where $f(R) = R$), we consider the following gravity Lagrangian:

$$f(R) = R\left(1 + \alpha\left(-\frac{R}{H_0^2}\right)^{\beta - 1}\right), \tag{16}$$

[1] This differs from the results of Wang & Meng (2004) by $3H\dot{f}'/f'$.

where α, β are dimensionless parameters (note that in our notation R is negative). Specializing to a matter-dominated universe ($w = 0$), we get from Eq. (9) the relation between the curvature scalar and matter density:

$$\kappa\rho_{\rm m} = -R\left(1 + \alpha(2 - \beta)\left(-\frac{R}{H_0^2}\right)^{\beta - 1}\right). \tag{17}$$

We wish to recover standard behavior at early times. This implies that the correction term in the Lagrangian must vanish for large $|R|$, and hence we must demand that $\beta < 1$. Furthermore, we must demand that the right-hand sides of Eqs. (14) and (17) always remain positive, which restricts the parameter space further.

Defining $\Omega_{\rm m} \equiv \kappa\rho^0_{\rm m}/(3H_0^2)$ and choosing units so that $H_0 = 1$, we can solve for R_0 from Eq. (17). Consistency then requires that substituting the obtained value of R_0 into Eq. (12) must give $H_0 = 1$. Hence, given α and β, $\Omega_{\rm m}$ is fixed. As an example, consider the case $\beta = 0$, which corresponds to the ΛCDM model. From Eq. (12), we have $H_0^2 = (3\alpha H_0^2 - 2R_0)/6$ and from Eq. (17), $3\Omega_{\rm m}H_0^2 = 2\alpha H_0^2 - R_0$, so $\Omega_{\rm m} = 1 + \alpha/6$ is fixed.

3. Observational constraints from background evolution

Armed with the modified Friedmann equation, we can now consider the constraints arising from cosmological observations. In this section we consider quantities related to the background expansion of the Universe: the SNIa luminosity distance-redshift relationship, the CMBR shift parameter, and the baryon oscillation length scale.

3.1. CMBR shift parameter

The CMBR shift parameter (Bond et al. 1997; Melchiorri et al. 2003; Odman et al. 2003) in a spatially flat universe is given by

$$\mathcal{R} = \sqrt{\Omega_{\rm m}H_0^2}\int_0^{z_{\rm dec}}\frac{{\rm d}\tilde{z}}{H(\tilde{z})}, \tag{18}$$

where $z_{\rm dec}$ is the redshift at decoupling. The WMAP team (Spergel et al. 2003) quotes $z_{\rm dec} = 1088^{+1}_{-2}$ and $\mathcal{R} = 1.716 \pm 0.062$. In writing the shift parameter in this form, we have implicitly assumed that photons follow geodesics determined by the Levi-Civita connection. In Koivisto (2005), this is shown to be the case if there is no torsion present. Furthermore, in order to use the shift parameter, the evolution of the universe needs to be standard up to very late times so that at decoupling we recover standard matter-dominated behavior. We therefore restrict our analysis to $\beta < 1$.

Since we do not have an explicit expression for the Hubble parameter in terms of the redshift, it is useful to rewrite the shift parameter in terms of the curvature scalar, R:

$$\begin{aligned}\mathcal{R} &= \sqrt{\Omega_{\rm m}H_0^2}\int_0^{z_{\rm dec}}\frac{{\rm d}z}{H(z)}\\ &= \sqrt{\Omega_{\rm m}H_0^2}\int_{R_{\rm dec}}^{R_0}\frac{a'(R)}{a(R)^2}\frac{{\rm d}R}{H(R)}\\ &= \frac{1}{3^{4/3}}\left(\Omega_{\rm m}H_0^2\right)^{1/6}\int_{R_0}^{R_{\rm dec}}\frac{Rf'' - f'}{(Rf' - 2f)^{2/3}}\frac{{\rm d}R}{H(R)}.\end{aligned} \tag{19}$$

The constraints arising from the CMBR shift parameter can be seen in Fig. 1.

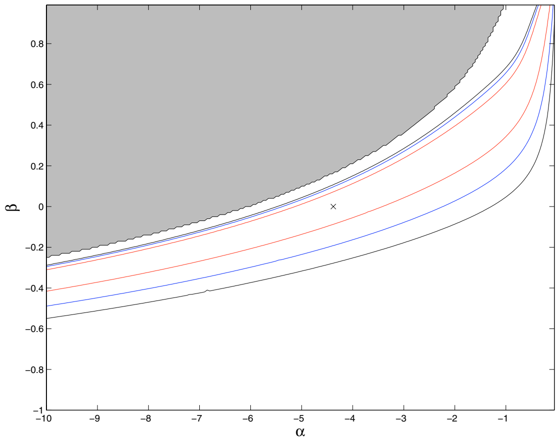

Fig. 1. The 68, 95, and 99% confidence contours arising from fitting the CMBR shift parameter. The parameter values corresponding to the concordance ΛCDM model ($\Omega_{\rm m} = 0.27$, $\Omega_\Lambda = 0.73$ or $\alpha = -4.38$, $\beta = 0$) are marked with a cross. The gray area represents a section of the parameter space that is not allowed.

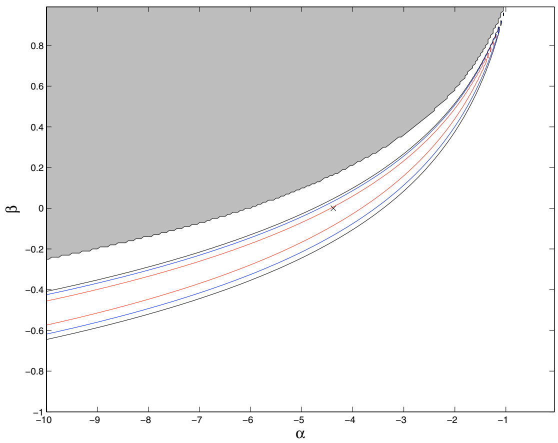

Fig. 2. The 68, 95, and 99% confidence contours arising from fitting the SN Ia data. The parameter values corresponding to the concordance ΛCDM model ($\Omega_{\rm m} = 0.27$, $\Omega_\Lambda = 0.73$ or $\alpha = -4.38$, $\beta = 0$) are marked with a cross. The gray area represents a section of the parameter space that is not allowed.

3.2. SNIa constraints

To incorporate measurements from SNIa, it is useful to rewrite the expression for the luminosity distance as

$$\begin{aligned} D_{\rm L}(z) &= (1+z)\int_0^z \frac{{\rm d}\tilde{z}}{H(\tilde{z})} \\ &= \sqrt{\Omega_{\rm m}H_0^2}\frac{1}{a(R)}\int_R^{R_0}\frac{a'(R)}{a(R)^2} \\ &= \frac{1}{3}\sqrt{\Omega_{\rm m}H_0^2}\,(Rf'-2f)^{1/3} \\ &\quad\times\int_{R_0}^{R}\frac{Rf''-f'}{(Rf'-2f)^{2/3}}\frac{{\rm d}R}{H(R)}. \end{aligned} \tag{20}$$

For the supernova data, we use the "Gold data set" from Riess et al. (2004). The contour plots showing the constraints on the parameters from the supernovae can be seen in Fig. 2. To get these plots we marginalized over the Hubble parameter h.

With the added information from the CMBR in the form of the shift parameter, the situation improves as one can see from Fig. 3. Still quite a large degeneracy persists on the 99% level,

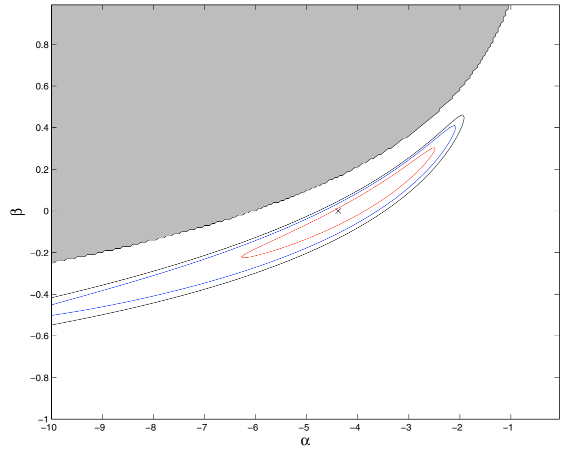

Fig. 3. The combined 68, 95, and 99% confidence contours arising from fitting the SN Ia and the CMBR shift parameter data. The parameter values corresponding to the concordance ΛCDM model ($\Omega_{\rm m} = 0.27$, $\Omega_\Lambda = 0.73$ or $\alpha = -4.38$, $\beta = 0$) are marked with a cross. The gray area represents a section of the parameter space that is not allowed.

but on the 68% level, the model is quite well-constrained and centered around the concordance ΛCDM model.

3.3. Baryon oscillations

The baryon oscillations in the galaxy power spectrum are imprints from acoustic oscillations prior to recombination, which are also responsible for the the acoustic peaks seen in the CMBR temperature power spectrum. The physical length scale associated with the oscillations is set by the sound horizon at recombination, which can be estimated from the CMBR data (Spergel et al. 2003). Measuring the apparent size of the oscillations in a galaxy survey allows one to measure the angular diameter distance at the survey redshift. Together with the angular size of the CMB sound horizon, the baryon oscillation size is a powerful probe of the properties and evolution of the universe.

The imprint of the primordial baryon-photon acoustic oscillations in the matter power spectrum therefore provides us with a "standard ruler" via the dimensionless quantity A (Linder 2003, 2005; Hu & Sugiyama 1996; Eisenstein & Hu 1998; Eisenstein & White 2004):

$$A = \sqrt{\Omega_{\rm m}}E(z_1)^{-1/3}\left[\frac{1}{z_1}\int_0^{z_1}\frac{{\rm d}z}{E(z)}\right]^{2/3}, \tag{21}$$

where $E(z) = H(z)/H_0$.

Recently the acoustic signature associated with the baryonic oscillations has been identified at low redshifts in the distributions of luminous red galaxies in the Sloan Digital Sky Survey (Eisenstein et al. 2005), with a value of

$$A = D_{\rm v}(z=0.35)\frac{\sqrt{\Omega_{\rm m}H_0^2}}{0.35c} = 0.469 \pm 0.017, \tag{22}$$

where

$$D_{\rm v}(z) = \left[D_{\rm M}(z)^2\frac{cz}{H(z)}\right]^{1/3}, \tag{23}$$

and $D_{\rm M}(z)$ is the comoving angular diameter distance. For instance, we have $D_{\rm v}(z=0.35) = 1334$ Mpc in the case of ΛCDM with $\Omega_{\rm m} = 0.3$, $\Omega_\Lambda = 0.7$ and $h = 0.7$.

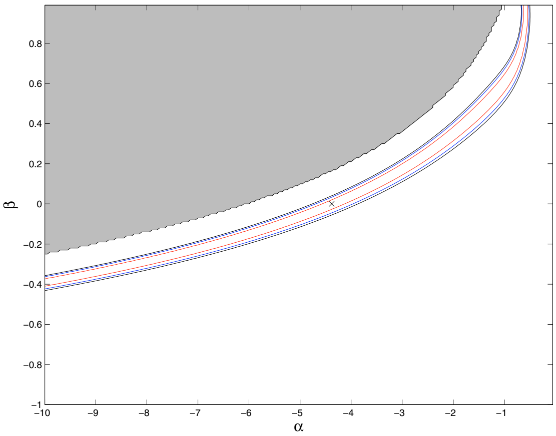

Fig. 4. The 68, 95, and 99% confidence contours arising from fitting the length scale associated with the baryon oscillations. The parameter values corresponding to the concordance ΛCDM model ($\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$ or $\alpha = -4.38$, $\beta = 0$) are marked with a cross. The gray area represents a section of the parameter space that is not allowed.

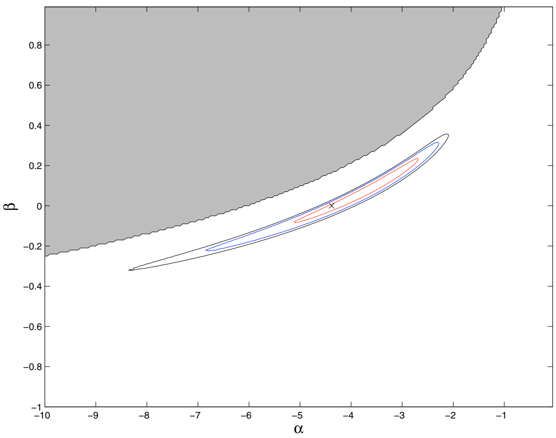

Fig. 5. The 68, 95, and 99% confidence contours arising from fitting the combined data from the SNIa, CMBR shift parameter and the length scale associated to the baryon oscillations. The parameter values corresponding to the concordance ΛCDM model ($\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$ or $\alpha = -4.38$, $\beta = 0$) are marked with a cross. The gray area represents a section of the parameter space that is not allowed.

Using the observed baryon oscillation-length scale, one can constrain the cosmological model. The confidence contours for the modified gravity model we are concerned with in this paper, are shown in Fig. 4. Once again there is a large degeneracy in the α vs. β plane, similar to the case when we fit SNIa (Fig. 2) and the CMBR shift parameter (Fig. 1). Such degeneracies are strongly restricted, however, when one combines all of the data sets in one single plot (see Fig. 5).

Using the baryon oscillation-length scale and combining it with the SNIa and the CMBR shift parameter data hence imposes strong constraints on possible deviations from Einstein's General Relativity. This also demonstrates how combining the current data, one can efficiently study and constrain cosmological models from the background expansion only. The best-fit model to the three data sets is $\alpha = -3.6$ and $\beta = 0.09$, which is slightly different from the ΛCDM model; but the ΛCDM model is well within the 68% confidence contour.

4. Large-scale structure: formalism and constraints

So far we have only considered observables related to the background evolution. In order to get further information, it is useful to go beyond these "zeroth order" tests and consider perturbations within $f(R)$ models. Knowledge of the evolution of perturbations allows one to confront models with large-scale structure observations from galaxy surveys. Several authors have investigated cosmological perturbations in generalized gravity theories in the metric approach (Hwang 1991a,b). However within the Palatini formalism, the first steps were made only very recently (Koivisto & Kurki-Suonio 2005). Here we follow a spherical collapse formalism (Lue et al. 2004a; Multamaki et al. 2003) where, by requiring that a general gravity theory respects the Jebsen-Birkhoff theorem[2], one derives the modified gravitational force law necessary to describe the evolution of the density perturbations at astrophysical scales.

4.1. Spherical collapse in f(R) theories

Although the spherical collapse model has been set and used for a long time (Peebles 1993; Padmanabhan 1993) and in many different contexts (Lue et al. 2004a; Mota & Barrow 2004a,b; Mota & van de Bruck 2004; Clifton et al. 2005), it has not been applied previously to nonlinear gravity theories. Lue et al. (2004a) start by assuming a generalization of the Jebsen-Birkhoff theorem: for any test particle outside a spherically symmetric matter source, the metric observed by that test particle is equivalent to that of a point source of the same mass located at the center of the sphere. With this one assumption, one can deduce the Schwarzschild-like metric of the new hypothetical gravity theory.

Armed with the Schwarzschild-like metric one can then investigate the evolution of spherical matter overdensities and compare it to the latest large-scale structure data. The idea is to consider a uniform sphere of dust. Imagine that the evolution inside the sphere is exactly cosmological, while outside the sphere is empty space, whose metric (given the Jebsen-Birkhoff theorem) is Schwarzschild-like (as defined by the metric Eq. (2.2) in Lue et al. 2004a). The mass of the matter source (as determined by the form of the metric at short distances) is unchanged throughout its time-evolution. The surface of the spherical mass therefore charts out the metric throughout space as the sphere expands with time, as long as we demand that the cosmological metric just inside the surface of the sphere smoothly matches the Schwarzschild solution just outside. In order to see how the metric depends on the mass of the central source, we just take a sphere of dust of a different initial size and watch its surface chart out a new metric. The procedure for determining the metric from the cosmological evolution is described in Lue et al. (2004a), so we refer the reader to this article for further information and details.

An open question is the validity of the Jebsen-Birkhoff theorem in $f(R)$ theories. Although one cannot explicitly show that the only possible solution to the field equations, when a spherically symmetric ansatz is inserted into them, is the Schwarzschild metric, there are strong indications that this is indeed the case. In fact, this was shown for the case of $f(R) = R + R^2$, where $R^2 = R^{\beta}{}_{\alpha}{}^{\mu\nu} R^{\alpha}{}_{\beta\mu\nu}$ (Ramaswamy & Yasskin 1979). And it was generalized later for any type of invariant of the form R^2, even in the case of a non-null torsion (Neville 1980).

[2] What is commonly known as Birkhoff's theorem was first probed by the Norwegian J. T. Jebsen in 1921, two years prior to Birkhoff's work. See physics/0508163 for more historical details.

The authors also claimed that similar results would most probably be valid even for the case of higher-order curvature invariants, such as R^3, R^4, etc. However, there is no mathematical proof of this as yet, even though there are several studies and proofs for other complex cases such as multidimensional gravities, Einstein-Yang-Mills systems, and conformally transformed metrics (Brodbeck & Straumann 1993; Bronnikov & Melnikov 1995).

4.1.1. Growth of perturbations

We want to follow the evolution of a lump of matter throughout the history of our Universe. We start by considering a Universe with a Schwarzschild-like metric,

$$ds^2 = g_{00}(t, r)dT^2 - g_{rr}(t, r)dr^2 - r^2 d\Omega, \tag{24}$$

where the metric components are uniquely determined from a given $a(t)$ (Lue et al. 2004a):

$$g_{00} = E^2\left[1 - \left(\frac{r}{a}\right)^2 \dot{a}^2\right] \tag{25}$$

$$g_{rr}^{-1} = 1 - \left(\frac{r}{a}\right)^2 \dot{a}^2,$$

where $E = \sqrt{g_{00}g_{rr}}$ is a constant.

In order to investigate the evolution of density perturbations in this scenario, we use the spherical collapse model (Peebles 1993). Consider a top-hat overdensity $\delta(t)$ of a spherical distribution of dust with mass M and radius r defined by

$$1 + \delta = \frac{M}{\frac{4\pi}{3}\rho r^3}, \tag{26}$$

where $\rho(t)$ is the background matter density. Using the geodesic equation as expressed by differentiating Eq. (25) with respect to t, we get

$$\ddot{r} = -\frac{1}{2}\frac{d}{dr}g_{rr}^{-1} = rH_0^2\left[g(x) - \frac{3}{2}xg'(x)\right], \tag{27}$$

where $g(x)$ is defined via a generalized Friedman equation

$$g(x) = \frac{H^2}{H_0^2}, \tag{28}$$

and where we have expressed ρ in terms of a dimensionless quantity x defined as

$$x \equiv \frac{8\pi G\rho}{3H_0^2} = \Omega_m a^{-3}. \tag{29}$$

Using the constraint from the Jebsen-Birkhoff theorem, one can calculate the evolution of an overdensity by following the geodesic of a spherical mass, without the need to consider what is happening outside the spherical mass itself. This is only possible if spherically symmetric configurations respect the metric Eqs. (25) (Lue et al. 2004a). Differentiating Eq. (26) twice with respect to time, and using Eq. (27), one obtains a new equation for $\delta(t)$

$$\ddot{\delta} + 2H\dot{\delta} - \frac{4}{3}\frac{1}{1+\delta}\dot{\delta}^2 = 3(1+\delta)H_0^2\left[\frac{3}{2}x(1+\delta)g'(x(1+\delta)) - g(x(1+\delta))\right] - 3(1+\delta)H_0^2\left[\frac{3}{2}xg'(x) - g(x)\right]. \tag{30}$$

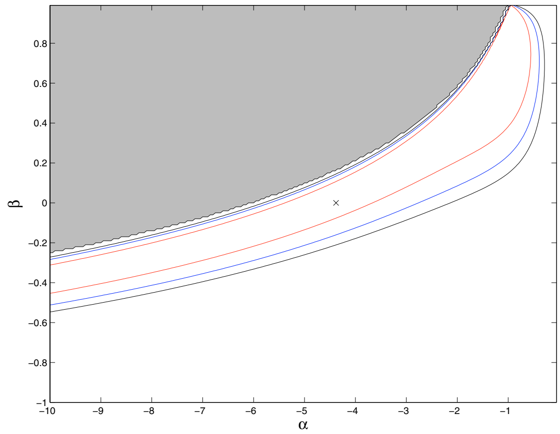

Fig. 6. The 68, 95, and 99% confidence contours arising from fitting the linear growth factor using the spherical collapse approach. The parameter values corresponding to the concordance ΛCDM model ($\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$ or $\alpha = -4.38$, $\beta = 0$) are marked with a cross. The gray area represents a section of the parameter space that is not allowed.

At linear order in perturbation theory, Eq. (30) gives:

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho\delta\left[g'(x) + 3xg''(x)\right]. \tag{31}$$

In our case, both H and ρ are expressed as functions of the scalar curvature R and not the time explicitly. We therefore need to rewrite Eq. (31) in terms of derivatives of R. The equation to be solved will then have the form

$$\frac{d^2\delta}{dR^2} + A(R)\frac{d\delta}{dR} = B(R)\delta, \tag{32}$$

where $A(R)$ and $B(R)$ are two rather unattractive functions of the scalar curvature. Using Eq. (32), one can solve for the evolution of the linear growth factor and compare this with observations.

4.1.2. Constraints

The large-scale structure information we choose to use here is the linear growth rate $F(z_{2dF}) = 0.51 \pm 0.11$ measured by the 2dFGRS (Verde et al. 2002; Knop et al. 2003; Hawkins et al. 2003), where $F \equiv d\ln D/d\ln a$. We compare the theoretical value we get for the linear growth rate of our model with the value measured by the 2dFGRS at its effective redshift, $z_{2dF} = 0.15$. The constraints arising from the linear growth rate are plotted in Fig. 6. Combining this with all the other constraints leads to the confidence contours shown in Fig. 7.

5. Conclusions

We have investigated observational constraints on $f(R)$ theories within the Palatini formalism. In order to relate these theories to observations, we show how one can determine the expansion history for a given $f(R)$. In a matter dominated universe in particular, determining $H(a)$ is straightforward as expressed by Eqs. (14) and (15).

We have investigated the possible form of the leading correction to standard GR, parameterizing the Lagrangian for gravity as $\mathcal{L}_G = R - \alpha(-R/H_0^2)^\beta$, and used a combination of data sets to determine the allowed ranges of α and β. This is by no means an exhaustive study. Other interesting forms of $f(R)$ that are definitely worth studying include $f(R) = \ln(R)$ and perhaps especially $f(R) = R - c_1/R + c_2R^3$ (Sotiriou 2005b), but our main

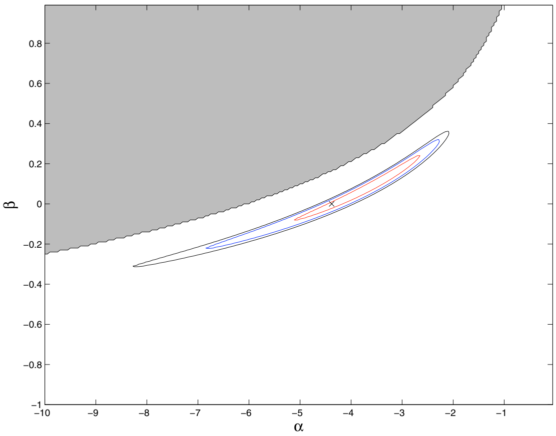

Fig. 7. The 68, 95, and 98% confidence contours arising from fitting to the Combined SNIa, shift parameter, baryon oscillations and the linear growth data sets. The parameter values corresponding to the concordance ΛCDM model ($\Omega_{\rm m} = 0.27$, $\Omega_\Lambda = 0.73$ or $\alpha = -4.38$, $\beta = 0$) are marked with a cross. The gray area represents a section of the parameter space that is not allowed.

purpose here has been to set up the formalism and demonstrate the effectiveness of combining the current data sets.

Using a combination of data sets that probe the background evolution, we find that the current data efficiently constrains the allowed parameter space of the leading correction to GR in the Palatini approach. The best-fit models to the individual data sets are $(\alpha, \beta) = (-10.0, -0.51)$ for the SNIa, $(\alpha, \beta) = (-8.4, -0.27)$ for the CMBR shift parameter, and $(\alpha, \beta) = (-1.1, 0.57)$ for the baryon oscillations. The best fit to the combination of these data sets is $(\alpha, \beta) = (-3.6, 0.09)$, but the ΛCDM concordance model is well within the 1σ contour. Note, however, that the commonly considered $1/R$ model is strongly disfavored by the data.

In order to bring in additional information from the current galaxy surveys, we also considered the growth of structures in these models of modified gravity. By assuming that the new gravitational physics obeys a limited version of the Jebsen-Birkhoff theorem, we can describe the evolution of overdensities in $f(R)$ gravity theories at sub-horizon scales. We find the best-fit model to the linear growth factor alone to be $(\alpha, \beta) = (-4.25, 0.05)$, but the allowed parameter range is degenerate and does not improve constraints derived from the background evolution. To fully utilize the information available from the galaxy survey in the form of the large-scale matter power spectrum, a more detailed analysis is needed along the lines presented in Koivisto & Kurki-Suonio (2005).

In summary, modified gravities provide us with an interesting alternative to the cosmological concordance model with a dominant dark energy component. Modern cosmological data can efficiently constrain such models. These data indicate that currently there is no compelling evidence for non-standard gravity.

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