

Cosmological fluctuations produced near a singularity

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Summary. The perturbations of a uniform Friedmannian universe, leading to galaxy formation, are explained by the strings, formed during the symmetry loss of vacuum of a complex Higgs field with mass characteristic of grand unification. Difficulties are pointed out inherent to phase transition, particle decay and black hole evaporation as sources of growing perturbations.

Observational cosmology presents at least three puzzles. Why is the Universe (1) nearly Friedmannian – with metric perturbations less than 1 per cent on all scales yet investigated; (2) why is it nearly flat: even a curvature of the order of the inverse horizon today corresponds to average curvature of the order of 10^{-30} of the inverse horizon near the singularity; (3) why is the Universe not exactly uniform, having initial perturbations of the order of 1–0.1 per cent on the scale of clusters of galaxies (and perhaps on other scales too).

We do not ask the fourth question – why does the Universe exist, but assuming that (1) and (2) are granted somehow, we concentrate on the third question on the origin of initial perturbations.

We assume the exact uniformity of the metric and composition at some moment, when the time is several times larger than Planckian, 10^{-43} s, so that a classical metric is meaningful. The metric involves Einstein's equations of motion and density of matter. At extremely high density full local equilibrium is established, therefore non-uniform initial composition is incompatible with the assumed uniformity of the total density (Ellis *et al.* 1979). The degree of density uniformity is even better than the uniformity of the metrics, $\delta\rho/\rho = (ct/\lambda)^2 \delta g/g$ for $\lambda \gg ct$ and is a quasi-isotropic (i.e. locally Friedmannian) solution.

The fluctuations are less than those in equilibrium with an external isothermal bath; there is not enough time to establish normal thermodynamical fluctuations characteristic of a stationary system. To be exact, if such a bath existed and a stationary non-expanding Universe could be brought in contact with the bath, the equilibrium would be unstable and no finite thermodynamical perturbations would establish on scales larger than the Jeans length. The onset of fluctuations by random dissipative forces would give an initial amplitude to the growing modes of gravitational instability. The overall picture does not contradict general relativity, if a large cosmological constant is allowed.

The build up of fluctuations due to viscosity during expansion is too slow in a uniform plasma to be the source of perturbations needed for galaxy formation from an initial state with

uniform metric in the evolutionary Friedmannian Universe. The discrepancy is most flagrant for large scales in space.

A new hope is connected with modern broken symmetry field theory. As pointed out by Kirzhnits & Linde (1974) the general feature of these theories is the symmetry restoration at ultra-high temperature. During cooling, phase transition must occur to the cold broken symmetry situation. The phase transition is likely to induce non-uniformity much larger than ideal gas fluctuations.

In this paper I will try to use topological excitations, particularly strings, as the immediate cause of perturbations.

Strings with mass per unit length of the order of $\sigma = m^2$ if $\hbar = c = 1$ or $\sigma = m^2 c / \hbar \text{ g cm}^{-1}$ are predicted to be formed after cooling in the simplest version of one scalar complex field interacting with one vector field (Nielsen & Olesen 1973). They are similar to Abrikosov magnetic strings in type II superconductors, being consequences of the Ginzburg–Landau theory of superconductivity.

Kibble (1976) points out that the linear mass density in theories of Weinberg–Salam type is too small to be of cosmological importance. With $m = 100 \text{ GeV}$, $\sigma = 10^{-6} \text{ g cm}^{-1}$, assuming string density of the order of $1/c^2 t^{2\star}$ one obtains mass density $\rho_s = 10^{-27} t^{-2} \text{ g cm}^{-3}$ which is negligible compared with the average density of the Universe $(5-8) \cdot 10^5 t^{-2} \text{ g cm}^{-3}$.

But the now fashionable GUT – grand unification theories – involve particle masses which are sizeable fractions of the Planckian mass $M = G^{-1/2} = 10^{-5} \text{ g}$. If the characteristic mass is written as $m = \epsilon M$, ϵ say of the order of 10^{-2} , then $\sigma = \epsilon^2 / G$ and $\rho_s = \epsilon^2 / G t^2$ which should be compared with the critical density $\rho_c = 1/6\pi G t^2$ or $\rho = 3/32\pi G t^2$ (depending on $p = 0$ or $p = \rho c^2/3$).

One obtains $\rho_s/\rho_c = 25\epsilon^2 \sim 2.5 \times 10^{-3}$ for $\epsilon = 10^{-2}$. This is approximately the amplitude of perturbations needed. Of course, the Universe with strings is rather strange.

Strings frozen in plasma would change their density like a^{-2} , i.e. like t^{-1} in radiation-dominated or $t^{-4/3}$ in a matter-dominated Universe (a – the radius of the Universe, $d \ln a / dt = H$ where H is Hubble constant).

In this approximation the strings would soon be dominant. The tension along a string is equal to its energy per unit length. Therefore the equation of state of a chaotic ensemble of strings is $p = -\rho c^2/3$. Expansion is connected with work done on pulling strings.

The order of magnitude estimate given above with ρ_s proportional to t^{-2} and to ρ implies more complicated behaviour of strings.

They are not simply pulled, but they actively diminish their length due to annihilation of closed loops, rectification of curved parts of strings and recombining of intersecting strings. Due to all these processes they are moving (which increases the average \bar{p} over $-\rho c^2/3$) and they are giving energy to the surrounding plasma. At $\rho_s/\rho \sim 10^{-3}$ the input of energy seems to be admissible from the point of view of spectral distortion of cosmic microwave radiation (Sunyaev & Zeldovich 1970).

The most difficult question concerns the local effects of these exotic strings on the microwave background. The mechanics of strings is not clear. Their gravitational potential is of the order of $\phi \cong c^2 \epsilon^2 \ln(t/t_{\text{pl}})^\dagger$, with $t_{\text{pl}} = (G\hbar/c^5)^{1/2} = 10^{-43} \text{ s}$. A closed loop at rest is characterized by the average potential $\bar{\phi} = c^2 l \epsilon^2 / r$; with $l = 2\pi r$ this is $\bar{\phi} = 2\pi c^2 \epsilon^2$.

* The order of magnitude estimate $c^{-2} t^{-2}$ for surface density comes from the belief that strings are moving with velocity of the order of c and that they intercommute and annihilate when they meet one another.

† The gravitational potential on the surface of a string of density σ , radius r and mean distance from other strings R is of order $|\phi| = G\sigma \ln(R/r)$. Inserting $\sigma = \epsilon^2/G$, $r = r_{\text{pl}}/\epsilon$, $R = ct$ and neglecting ϵ in the logarithmic term we obtain this estimate.

When the loop is shrinking slowly due to friction by surrounding plasma, ϕ remains the same. But if the loop is shrinking without friction, its effective mass is conserved, the decrease of length is compensated by the kinetic energy of transverse motion (the motion along the string is unobservable, the string is Lorentz-invariant to this motion; formally this is connected with the equality of tension and linear density). For shrinking loops $\sigma' = \sigma/\sqrt{1-\beta^2}$, $\sigma' l' = \sigma l_0$, $m = \text{const}$. If a circular loop shrinks remaining a circle, $l' = 2\pi r$ and finally the loop transforms in a black hole when $r = \epsilon^2 r_0 \ll r_0$. But it is a degenerate case. An irregular loop will oscillate and it is difficult to estimate the probability of a black hole formation in the general case, especially with account of plasma friction.

It must be stressed that the underlying physical theory i.e. the broken symmetry and grand unification concept are not yet confirmed. Moreover, possible variants of the theory give diverging predictions about the existence of strings (Polyakov 1975). Therefore all said above is no more than a tentative hypothesis. Short comments should be added on previous work aiming at explanation of perturbations.

In a very naïve cold Universe picture (Zeldovich 1962), solid hydrogen breakup was considered. The physical idea was fantastic, but an important methodical point remains: local events give rise to a quadratic spectrum on a large scale $\sqrt{(\delta\rho/\rho)_k^2} \sim k^2$, $\delta M/M \sim M^{-7/6}$.

This is a general law, connected with conservation of mass and momentum. It was discussed in a review article by Zeldovich (1965).

Kirzhnits & Linde (1974) first predicted a second-order transition, which goes smoothly and does not induce large scale perturbations as a consequence of broken vacuum symmetry. But in a later work they pointed out the possibility of a first-order transition with super-saturation and nucleation (Kirzhnits & Linde 1976; Linde 1979).

The discrete broken symmetry (Lee 1973) leads to formation of walls between domains with opposite signs of CP violation. These domains were shown to be too heavy to exist in Nature and this is an argument against the underlying CP-violation explanation (Zeldovich *et al.* 1974; Kibble 1976).

Press (1979) published an important work. The task of obtaining perturbation from first principles is formulated. But the immediate cause of perturbations is thought to be the energy density depending on the phase distribution after the establishment of broken symmetry. In the usual theories the phase gradient is totally compensated by the gauge field. No phase gradient terms occur unless one works without the gauge field, admitting the phase to be a long-range Goldstone scalar field. No such field is known in Nature. Instead of a distributed phase gradient, strings are formed when the field topology is appropriate.

Hogan (1979) assumes the existence of heavy particles with abnormally long decay time. Perhaps the small primordial black holes (PBH) can play the same role. Hogan stresses the statistical character of the radioactive decay. A distribution of PBH mass would give dispersion of their evaporation time.

But there is a very important point about theories of this type, which is shared by theories of random nucleation in first-order phase transitions.

Common to all these theories is the variation in the equation of state of the matter filling the Universe. Between initial and final ultra-relativistic gas $p = \rho c^2/3$, one has a period plagued with heavy particles, PBH or nucleation. It could be expressed as $p = \rho c^2/3 - \pi$, with π being the deviation $\pi = \pi(\rho, x)$ such that $\pi = 0$ at $\rho > \rho_i$ and $\rho < \rho_f$: it exists in a limited density region. In contrast to the main part $p = \rho c^2/3$, the contribution described by π is fluctuating, it is randomly distributed in space. It is thought that the π on x dependence will generate the perturbations. We are interested in perturbations on a scale much larger than the momentary value of the horizon ($\lambda \gg ct$) when the random equation of state perturbation is present, $\pi \neq 0$.

For perturbations on such a scale one can neglect the interaction of the domains under consideration. Every single domain follows the flat Friedmann evolution pattern with the given local average equation of state.

Of course $\rho(t)$ depends on the $\pi(t)$ and variations of $\pi(x, t)$ are leading to definite non-uniform, x -dependent, variations of the density, $\rho = \bar{\rho}(t) + \delta\rho(x, t)$.

Order of magnitude estimates give density perturbations at the moment when pressure fluctuations end

$$\left| \frac{\delta\rho}{\rho} \right|_{\rho < \rho_f, t = \text{const} > t_f} \cong \frac{\pi(\rho_i - \rho_f)}{c^2(\rho_i + \rho_f)^2}.$$

They are proportional to the fluctuations of pressure (due for example to random fluctuations of the moment of particle decay or new phase formation).

It is tempting to use these density perturbations as seeds of subsequent galaxy formation etc. The pressure fluctuations are statistically independent in regions causally not connected ($\lambda > ct$ at the fluctuation epoch). Therefore one would expect white noise i.e. flat Fourier spectrum of $\delta\rho/\rho$ in this mechanism, which is even more than that needed for the large-scale structure of the Universe.

But, in fact, all these expectations are erroneous! The point is, that in this case, the density perturbations are compensated by local Hubble constant perturbations. To demonstrate this behaviour, we integrate the Friedmann equations with the result that the solution after the end of the equation of state perturbation is $\rho = 3/32\pi G(t + \tau(x))^2$ and fluctuations of equations of state are changing the density due to their influence on the constant $\tau(x)$ i.e.

$$\frac{\tau(x)}{t_f} \cong \frac{\pi(\rho_i - \rho_f)}{2c^2(\rho_i + \rho_f)^2}.$$

But the shift of a solution by a time τ variable in space, $\tau = \tau(x)$, produces a decreasing perturbation mode^{*}. This point has already been clarified in an early review article of the author (Zeldovich 1965). This is seen immediately by decomposition of the formula for ρ , giving

$$\delta\rho/\rho = -2\tau(x)/(t + \tau)$$

decreasing like t^{-1} at $t \gg t_f > \tau$. This is not to say that the π -fluctuations are totally vanishing.

In the second approximation involving interaction of the adjacent regions, the growing modes of perturbations is excited. But its amplitude is smaller compared with the total amplitude in the ratio $(ct_f/\lambda_f)^2 \sim (\kappa_f ct_f)^2$. Instead of the flat spectrum, corresponding to white noise, the growing perturbations are characterized by the amplitude proportional to κ^2 . This actually kills the hope of explaining the structure of the Universe on a galactic scale by early random processes. The temperature ~ 100 GeV corresponds to $z \sim 3 \times 10^{14}$, $t \sim 10^{-10}$ s. A perturbation now on scale of 1 Mpc $= 3 \times 10^{24}$ cm had λ of the order of 10^{10} cm when ct was 3 cm. Therefore, the dimensionless $(ct/\lambda)^2$ factor is equal to 10^{-19} .

In a strongly exaggerated case assume $\delta\rho/\rho \sim 1$ on the causal scale ~ 1 cm. This would give in white noise approximation $\delta\rho/\rho \sim (ct/\lambda)^{3/2} \sim 10^{-28}$. The amplification of growing perturbations in radiation dominated regime is $t \sim z^2 \sim 10^{29}$. This seems to be enough to give $\delta\rho/\rho \sim 1$ or 10 now, at $z = 0$ with the white noise amplitude – but the extra 10^{-19} factor in the amplitude of the growing mode makes the perturbations hopelessly small.

^{*} This approach to perturbations was initiated by Barenblatt & Zeldovich (1959).

The fluctuations of the equation of state are generating entropy perturbations which actually have a white noise flat spectrum but they do not grow until decoupling at the modest $z \sim 10^3$.

This analysis confirms, that what one needs are just filament tensions in order to move matter. So we return to the first part of the paper.

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