TRANSLATION

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COSMOLOGICAL MODELS OF THE UNIVERSE

WITH ROTATION OF TIME'S ARROW

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КОСМОЛОГИЧЕСКИЕ МОДЕЛИ ВСЕЛЕННОЙ

С ПОВОРОТОМ СТРЕЛЫ ВРЕМЕНИ

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ABSTRACT

      The paper treats cosmological models of the universe with rotation of time's arrow. The hypothesis of cosmological CPT symmetry previously advanced by the author and the hypothesis of a many-sheeted open model with negative space curvature, with possible violation of CPT symmetry by an invariant combined charge, are presented. The statistical paradox of reversibility is discussed with respect to these models. The fundamental possibility of estimating (in absolute magnitude) the serial number of the expansion-contraction cycle corresponding to the present stage of evolution of the universe is pointed out for the many-sheeted model with rotation of time's arrow. The small dimensionless quantity δ^2/a^2 (where a is the hyperbolic radius and δ^{-3} the entropy density), which characterizes the average space curvature of the universe, is explained as a result of the evolution of the universe in the course of many successive expansion-contraction cycles.

COSMOLOGICAL MODELS OF THE UNIVERSE
WITH ROTATION OF TIME'S ARROW

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The equations of motion of classical and nonrelativistic quantum mechanics and of quantum field theory allow time reversal (in field theory, they do so simultaneously with the CP transformation). Statistical equations, however, are irreversible. This contradiction has been known since the end of the 19th century as the "reversibility paradox" of statistical physics (Zermelo and others). The traditional explanation relates irreversibility to initial conditions. If this were the case, however, the nonequivalence of the two directions of time would not be preserved in the worldview.

Contemporary cosmology is opening up the possibility of eliminating this paradox. The notion of an expanding universe now is generally accepted in cosmology. According to this concept, some time is characterized by the vanishing of the space metric tensor (below, for brevity this time of the "Friedmann singularity" is designated Φ). In 1966-1967 the present author suggested that cosmology

could consider not only times later than Φ, but also earlier
ones, though the statistical properties of the state of the
universe at time Φ are such that entropy increases both
forward and __backward__ in time:

$$\frac{dS}{dt} > 0 \qquad S(t) > S(0) \quad \text{when } t > 0$$
$$\frac{dS}{dt} < 0 \qquad S(t) > S(0) \quad \text{when } t < 0. \qquad (1)$$

Thus, it is assumed that when $t > 0$ the normal statis-
tical equations obtain, but when $t < 0$ time-reversed equations
obtain. This reversal applies to all nonequilibrium processes,
including information processes, i.e., it applies to vital
processes as well. The author called this situation a
"rotation of time's arrow." Rotation of time's arrow does
away with the reversibility paradox in the worldview [i.e.,]
the equivalence of the two directions of time that is inherent
in the equations of motion is restored.

Despite the lack of dynamic interaction between
regions of the universe with $t > 0$ and $t < 0$, the hypothesis
of rotation of time's arrow is physically meaningful--certain
assertions regarding the nature of the initial conditions at
point Φ should follow from it.

Let us consider the classical kinetic theory of gases as a
model example of the rotation of time's arrow. Let us
postulate at time $t = 0$ a spherically symmetric distribution
of atoms with respect to velocities at every point in space,

and an inhomogeneous distribution of density and temperature in space. Let us postulate (and this is especially significant) the absence at time $t = 0$ of a correlation between the relative positions and relative velocities of the atoms--in this case this is the "statistical condition" by using which the minimality of the entropy value at the point $t = 0$ is proven.

In Ref. 1 the author suggested cosmological CPT symmetry as an implementation of the rotation of time's arrow. According to this hypothesis, all events in the universe are completely symmetric with respect to the hypersurface corresponding to the instant of cosmological collapse Φ. Assuming $t = 0$ for this time, we require the presence of symmetry under the transformation $t \rightarrow -t$. The only exact symmetry that includes time reflection is the CPT symmetry. The singularity of the point Φ and neutrality with respect to all conserved charges follow from CPT symmetry. Let us define CPT-conjugate fields in an auxiliary half-space:

$$x_o = |t| \geq 0 \qquad -\infty < x_\alpha < +\infty$$

and let us designate these fields with indices a and b. Let us require:

For spinors $\psi^a = \gamma_5 \psi^b$

For the components of the tetrad (the index, referred to the frame, is given in parentheses)

$$e^{a}_{\;i(j)} = -\, e^{b}_{\;i(j)} \quad \text{(PT reflection)}$$

Let us map the field a onto the domain $t \geqslant 0$ and the field b onto the domain $t \leqslant 0$ (with appropriate change of the sign of $\ell^{b}_{c(j)}$). From the condition of continuity on the hypersurface we have $\quad \ell_{\alpha(j)}^{(b)} = 0$ (singularity of the point Φ) and $\psi(0) = \gamma_5 \psi(0)$, whence the current

$$j^{(0)} = \overline{\psi} \gamma_j \psi = \overline{\psi} \gamma_5 \gamma_j \gamma_5 \psi = -\overline{\psi} \gamma_j \psi = 0$$

(the condition for neutrality at the point Φ).

Neutrality of the universe requires that the observed baryon asymmetry arose in the course of nonequilibrium processes of expansion of the universe. Here we must assume destruction of baryon charge, but conservation of a combined charge of the type 3B + L is possible (see Refs. 1 and 2). The notations are B--baryon charge, L--lepton charge. Let us note, however, that in the schemes for unifying the strong, weak, and electromagnetic interactions that are most in favor now (e.g., in the SU_5 scheme) there is no such conservation law (the conservation of B - L also is approximate in most schemes).

CPT symmetry is not the only possible implementation of a rotation of time's arrow. It suffices to stipulate that the statistical conditions for the absence of correlations be fulfilled at time Φ. It is most natural to assume that the breakdown of CPT symmetry upon rotation of time's arrow is due to the presence of a finite invariant combined charge (if, of course, such exists and does not have a gauge field). Here the numerical value of the combined charge is not directly related to the residual baryon asymmetry that appears dynamically as the universe expands.

Rotation of time's arrow (with or without CPT symmetry) is possible both in the conventional open model of the universe and in models with infinite repetition of expansion-contraction cycles (in pulsating models or, in the author's terminology, "many-sheeted models"; see Ref. 2). Such models seem especially interesting to us by virtue of features inherent in them, and we will treat them in greater detail.

Let us note first that in these models cycles close to time Φ should differ significantly in properties from "later" cycles, for which the main statistical characteristics asymptotically approach their limiting values as $|n| \to \infty$ (n is the cycle number; $-\infty < n < \infty$). These limiting "self-reproducing" values correspond to a many-sheeted

model without rotation of time's arrow (see Ref. 2).
According to Ref. 2, in the many-sheeted model without
rotation of time's arrow, the space curvature and all con-
served charges (in the sense of the average values) should
be equal to zero. But in the model with rotation of time's
arrow these quantities should equal zero only asymptotically.
In this sense the many-sheeted model with rotation of
time's arrow is more general.

Thus, let us consider the many-sheeted model with
finite space curvature $1/a^2$ and, perhaps, a finite combined
charge. We will assume the curvature negative (a is the
hyperbolic radius), which apparently corresponds to obser-
vations. We also assume Einstein's cosmological constant
to be nonzero and of sign such that the corresponding
energy density of a vacuum is $\varepsilon < 0$. We make no assumptions
regarding the absolute value $|\varepsilon|$, but it is quite probable
that $|\varepsilon|$ is much less than the average density of matter at
the present time. The negative sign of ε corresponds to
violation of the symmetry of the vacuum state with $\varepsilon = 0$.

The dynamics of the universe is determined by
Einstein's equation $\frac{8\pi G}{3} T_0^0 = R_0^0 - \frac{1}{2} R$, which we will write in
the form (it is assumed that c [the speed of light] = 1)

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\left(\varrho + \varepsilon\right) + \frac{1}{a^2} \qquad (2)$$

where H is the Hubble constant, ρ the density of "ordinary" matter; $\rho \to 0$ and $1/a^2 \to 0$ as $a \to \infty$. Since $\varepsilon = \text{const} < 0$, at some value of a, H vanishes, and expansion is succeeded by contraction. The universe thus undergoes an infinite number of expansion-contraction cycles.

For the initial conditions in the vicinity of point Φ, the following assumptions are most natural (there are four versions, where σ is the entropy density and n_k the combined charge density, and $n_k \, a^3 = 0$ means that either there is no combined charge, or it is equal to 0).

$$1. \quad \sigma a^3 \approx 1 \qquad n_k a^3 \sim 1$$

$$2. \quad \sigma a^3 \sim 1 \qquad n_k a^3 = 0$$

$$3. \quad \sigma a^3 \to 0 \qquad n_k a^3 \sim 1$$

$$4. \quad \sigma a^3 \to 0 \qquad n_k a^3 = 0$$

Cosmological CPT symmetry corresponds to versions 2 and 4. In the case of versions 1 and 3 CPT symmetry is broken by the presence of combined charge, which should lead to significant differences in the details of the appearance of the universe in positive and negative cycles. Versions 1 and 2 correspond to a hot model of the universe, versions 3 and 4 to a cold model. The cold model seems a natural implementation of the rotation of time's arrow, but on the

whole there are neither theoretical nor experimental data for selecting a specific version.

The entropy σa^3 in a co-moving volume a^3 increases with each cycle. Let us assume that if $|n|$ increases by 1, the entropy increases by a factor ν; estimating this number, which would be possible in principle, would require allowance for basic nonequilibrium processes. At present (in "our" cycle n_i) the entropy is $\sigma a^3 \sim n_\gamma/h^3$. n_γ is the relic radiation photon density. It is assumed that the density ρ is subcritical. For versions 1 and 3 we have an estimate of the serial number n_i of our cycle (for illustration it is assumed that $\nu = 1.1$):

$$|n_i| \sim \frac{\ln|n_\gamma H^{-3}|}{\ln \nu} \sim \frac{\ln(10^{87})}{\ln 1.1} \sim 2 \cdot 10^3$$

In the cold versions, additional cycles are needed to create the initial entropy. In version 4 the initial particles arise as a result of a large number of nearly void cycles through the small curvature proportional to $|\varepsilon|$.

Designating the relic radiation photon density as δ^{-3} ($\delta \sim 0.1$ cm), we have the very small dimensionless number $\delta^2/a^2 \sim 10^{-58}$, which characterizes the curvature of the universe (if, of course, the curvature is not identically zero, which as yet cannot be excluded). An important advantage of the many-sheeted model with rotation of time's

arrow is the possibility of a natural explanation of the appearance of this dimensionless number in the course of successive expansion-contraction cycles.

The asymptotic regime of complete similarity of successive cycles is described by equation (2), ignoring the term $1/a^2$. The solution of (2) has the form

$$a = a_n \left(\sin \frac{3}{2} a_o^{-1} t \right)^{2/3} \quad ; \quad a_o = \left\{ \frac{8\pi G}{3} |\varepsilon| \right\}^{-1/2}$$

The maximum hyperbolic radius of the n-th cycle, a_n, is determined from the condition $\rho(a_n) = |\varepsilon|$ and is proportional to $\nu^{\frac{|n|}{3}} \to \infty$ as $|n| \to \infty$. The length of each cycle is

$T_4 = \frac{2\pi}{3} a_0$. The baryon, lepton, and entropy densities at the corresponding times of each cycle are independent of $|n|$. Cycles closer to Φ (the initial regime) are described by equation (2), ignoring ρ (with the exception of relatively short time intervals at the start and end of each cycle). Ignoring ρ, we have $a = a_0 \sin a^{-1} t$, and the length of the cycles is $T_H = \pi a_0$. The transition from the initial regime to the asymptotic regime is determined by the condition $\rho(a_0) = |\varepsilon|$, and will occur at a cycle number $n_2 > n_1$ (on the assumption that now $\rho < \rho_k$). However, the baryon asymmetry n_B/n_γ now has an asymptotic value, since it is determined by the initial stage of the process of expansion of the universe.

The stability of the picture of successive collapses described above has not been investigated.

The "reversibility" paradox, the hypothesis of cosmological CPT symmetry, and versions of the many-sheeted model are discussed in the present paper.

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REFERENCES

1. A. D. Sakharov. Pis'ma v ZhETF, $\underline{5}$, 32, 1967.

2. A. D. Sakharov. Zhurnal eksperimental'noi i teoreticheskoi fiziki, $\underline{76}$, 4, 1179, 1979.