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Cosmological Structure Formation from Soft Topological Defects

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Abstract

Some models have extremely low–mass pseudo–Goldstone bosons that can lead to vacuum phase transitions at late times, *after* the decoupling of the microwave background. This can generate structure formation at redshifts $z \gtrsim 10$ on mass scales as large as $M \sim 10^{18} M_\odot$. Such low energy transitions can lead to large but phenomenologically acceptable density inhomogeneities in "soft topological defects" (e.g., domain walls) with minimal variations in the microwave anisotropy, as small as $\delta T/T \lesssim 10^{-6}$. This mechanism is independent of the existence of hot, cold, or baryonic dark matter. It is a novel alternative to both cosmic string and to inflationary quantum fluctuations as the origin of structure in the Universe.

The problem of generating cosmological structure (galaxies, clusters, voids, peculiar velocities, etc.) in a universe that appears very homogeneous and isotropic on the largest scales is, perhaps, the major problem in physical cosmology today. Previously, attempts at solutions have involved generating density fluctuations (either quantum mechanically, the usual Gaussian inflationary fluctuations; or topologically, such as cosmic strings) at a very early cosmological epoch (e.g., Grand Unified [GUT] epoch when $kT \sim 10^{14}$ GeV) which survive to serve as seeds at the galaxy formation epoch at $kT \sim 10^{-2}$ eV. In some scenarios these seeds gravitationally accrete large quantities of non-baryonic dark matter, whereas in others they explode and push the baryons about. Rather than go into a detailed commentary on each of these models, let us merely note that the following combination of observations has been difficult (but maybe not impossible) for any existing model to satisfy:

1. Microwave background anisotropy[1] $\delta T/T \lesssim 2 \times 10^{-5}$.

2. Quasars and some galaxies exist[2] by redshifts $z \gtrsim 3$.

3. Large scale peculiar velocities[3] depart from the Hubble flow up to $\Delta v \sim 600$ km/sec on scales of $R \gtrsim 30$ Mpc.

4. Structure, clustering, foam, voids, etc. exist[4] on scales of $\gtrsim 20$ Mpc.

5. Clusters of galaxies appear more strongly correlated[5] than galaxies.

People favoring some theoretical models may dismiss some items so as not to have to discard their model prematurely. For example, proponents of cold dark matter and Gaussian fluctuations don't like #3 and #5, and most Gaussian fluctuation models with linear growth must argue that #2 is a statistical tail and not representative of the bulk of galaxy formation. Moreover, #1 forces a fine-tuning of the GUT phase transition.

The purpose of this note is to propose a completely different alternative, where the fluctuations are generated *after* decoupling, thus inducing minimal fluctuations in

$\delta T/T$. Furthermore, the fluctuations will be associated with "soft" topological structures, having fixed internal energy densities typically of order $m_{neutrino}^4$. At some redshift $z_{decoupling} \gg z_1 \gg 1$ we will have $\delta\rho/\rho \sim 1$, so density fluctuations immediately grow nonlinearly subsequent to z_1. This avoids the long linear growth period in previous models that makes it relatively difficult to have galaxies form from small initial amplitude by redshifts $z \gtrsim 1$. Large variations in density can also produce large peculiar velocities on scales up to the horizon at the time of the transition, $\sim 100\,\mathrm{Mpc}$ today (in present distances, the horizon at redshift $z \gg 1$ is $3000z^{-1/2}\,\mathrm{Mpc}$). Observed structures as large as $10^{18}M_\odot$ have been claimed by Tully[6] and cannot be easily accommodated by models where fluctuation scales must be limited by the horizon scale at decoupling. Another possible feature of our mechanism is that the fluctuations associated with the critical point phenomena of a phase transition have a scale–free or fractal character out to the horizon at the epoch they form. Such a fractal seems to provide a natural way of understanding the cluster–cluster correlations[7] as well as producing patterns that resemble the observed large scale structure. Much of what we say here is generic to any late–time phase transition (including, e.g., one in the hidden E_8 sector of $E_8 \times E_8$ superstring theories that could create the observed structure and be otherwise unobservable), though in the Appendix we present a specific model, intended primarily as an "existence proof" of the viability of such schemes.

Our initial thought as to why one might expect such a late–time phase transition came from considerations of the possible cosmological implications of low–mass pseudo–Goldstone bosons which can arise naturally in a variety of GUT settings. Pseudo–Goldstones, such as massless familons[8], arise when the pattern of masses of the observed fermions is associated with a spontaneously broken, continuous (but ungauged), exact symmetry. With further small explicit breakings of these symmetries, familons acquire minuscule masses, e.g., in the "schizon" models of Hill and Ross[9] these are typically of order $m_\phi \sim m_f^2/f_\phi$, where $f_\phi \sim 10^{15}\,\mathrm{GeV}$ to $10^{16}\,\mathrm{GeV}$ is a generic GUT scale, and m_f the mass of the associated family of fermions. The resulting masses of these pseudo–Goldstone bosons can have interesting astrophysical implications if the associated fermion family is taken to be the neutrinos. What

are the neutrino masses? An examination of MSW[10] mixing solution to the solar neutrino problem suggests neutrinos have a small mass, $m_\nu \sim 10^{-2}$ eV With these typical values for neutrino masses, one thus estimates the Compton wavelength of the neutrino–schizon to be $r = f_\phi/m_\nu^2 \sim 100$ pc to 1 Mpc (for 10^{-1} to 10^{-3} eV neutrino masses), with $f_\phi \sim 10^{15}$ GeV. This is a remarkable result for a cosmological distance scale, coming from the ratio of two high–energy particle physics mass scales and is suggestive of a possible mechanism for formation of large–scale structure.

What dynamics can lead to a late time phase transition? In the Appendix we write down the low–energy effective Lagrangian for a particular scheme and find that the potential for the neutrino–schizon, ϕ, is given by:

$$V(\phi) = -c(T)m_\nu^4 \cos(2\phi/f) \tag{1}$$

which implies $m_\phi^2 = 2c(0)m_\nu^4/f_\phi^2$. Here $c(T)$ is a temperature dependent coefficient of the form $\sim \log(T/\mu)/4\pi^2$, valid for $T \gtrsim m_\nu$ and $c(0) \sim -\log(\mu/m_\nu)/4\pi^2$, where μ is a renormalization scale as described in the Appendix. We shall assume that $\mu \gtrsim m_\nu$ and that the initial state of the ϕ–field is given by a quantum–mechanical vacuum (a gaussian wave–functional in ϕ) localized around a field configuration in a minimum of $V(\phi)$, e.g., $\langle\phi\rangle = 0$. We emphasize that this need not require that the pseudo–Goldstones are in thermal equilibrium (they generally are not); the other matter fields contribute to $c(T)$ through Feynman loops with thermal expectation values so long as they have a thermal density matrix (they themselves need not be in equilibrium; the thermal density matrix applies after decoupling).

In general as the universe cools, the pseudo–Goldstone bosons can undergo a phase transition if, for example, at some temperature T_c the coefficient $c(T)$ can become zero and, for subsequent lower temperatures, change sign. Considering the particular model described above this phase transition occurs at $T_c = \mu$. We define this to correspond to a redshift z_0, and we shall further assume that $c(T)$ evolves monotonically, as is the case above. The pseudo–Goldstone field now acquires a new VEV, with a discrete degeneracy that leads to topological defects, here typically domain walls. It

is not unnatural to choose $\mu \sim \lambda m_\nu$, with λ a dimensionless parameter that can range as small as $\mathcal{O}(1)$; we shall typically consider $\lambda \sim 10$. A prediction for μ depends upon the precise details of the model at the GUT energy scales. The fact that the late time phase transition can, in principle, naturally occur is sufficient for our present discussion.

This phase transition most likely proceeds quantum mechanically, in close analogy to inflationary phase transitions. The wave–function, $\Psi(\phi_k)$, to find a given Fourier component of the pseudo–Goldstone field, ϕ_k, satisfies the Schroedinger equation:

$$\frac{1}{2}\left[-\frac{\delta^2}{\delta\phi_k^2} + \{k^2 + V''(0)\}\phi_k^2\right]\Psi(\phi_k) = i\frac{\partial}{\partial t}\Psi(\phi_k) \tag{2}$$

(where we assume localization around $\phi_k = 0$ in writing the $V''(0)$ term; this term is time dependent through $c(T)$ and $T(t)$). As the potential $V(\phi)$ changes sign the wave–function uniformly spreads in ϕ_k provided the momentum k is not too large. Using the potential of eq.(1) we have spreading provided,

$$k^2 - 4|c(0)|m_\nu^4/f_\phi^2 \leq 0 \tag{3}$$

Of course, some components corresponding to small k will begin to spread as $T \sim T_c$, but ultimately as $T \to 0$ all $\Psi(\phi_k)$ will spread with k satisfying eq.(3); this relation establishes a critical $k_c^2 = 4c(0)m_\nu^4/f_\phi^2$ above which no spreading occurs. Spreading of the wave–function continues for a time $t \sim f_\phi/m_\nu^2$ until it becomes equally likely to find a new VEV, either $v_1 = \pi f_\phi$ or $v_2 = -\pi f_\phi$, Thus, some regions of space will have the wave–function collapse around v_1, while others around v_2. There will necessarily be domain walls between these regions. The domain walls have the transverse structure of the kink–soliton of the sine–Gordon equation:

$$\frac{\phi}{f_\phi} = 4\tan^{-1}\left[\exp\left(m_\phi(x - \beta t)/\sqrt{(1 - \beta^2)}\right)\right] - \pi \tag{4}$$

with $v/c = \beta$. The thickness, r, is thus $m_\phi^{-1} \sim f_\phi/c(0)m_\nu^2$ which is also of order the initial characteristic spacing at the time of the phase transition, k_c^{-1} and is the

characteristic time scale for the duration of the phase transition. The thickness of these soft domain walls can range from 100 pc to 1 Mpc for 1 to 10^{-3} eV neutrino masses, with $f \sim 10^{15}$ GeV. Since the domain wall thickness is of order k_c^{-1} at the initial redshift of the phase transition, z_0, the field configurations after this transition are very ill–defined and the domain walls are hard to distinguish and are distributed randomly, often lying on top of one another. The quantum state remains highly excited. It will relax, however, by Hubble red–shifting and the wall interspacing will typically become $k_c^{-1}(1 + z_0)/(1 + z)$ at subsequent redshift, $z \leq z_0$, while the walls themselves will relax to the well–defined kink–soliton configurations of eq.(4). In addition, there may be slow recombination of structures and evolutionary effects, analogous to the evolution of cosmic string networks.

At the epoch of the phase transition, a redshift z_0, the average cosmological energy density in the ϕ field configurations is of order $c(0)m_\nu^4$. This energy density will now redshift, the leading contribution behaving like a domain wall which we write as:

$$\rho_{walls} \sim c(0)m_\nu^4 \left(\frac{1 + z}{1 + z_0}\right)^{1+\kappa} \tag{5}$$

where the parameter $\kappa \geq 0$ is introduced to model the various evolutionary effects (parenting processes, domain wall annihilation and decay, etc.; there are, of course, subleading contributions to eq.(4), e.g., the ϕ field will oscillate about the true vacuum value between the domain walls, in analogy to the axion field, but this energy redshifts as $(1 + z)^3$ and may be neglected). On the other hand, matter redshifts as

$$\rho_{matter} = \rho_{matter}(z_1) \left(\frac{1 + z}{1 + z_1}\right)^3 \tag{6}$$

In the center of a domain wall we would have a fixed, non–redshifting energy density of order $c(0)m_\nu^4$. Let us assume that at the special redshift, z_1, the matter energy density becomes equal to $c(0)m_\nu^4$ (this is determined once the value of the neutrino mass is specified, thus for $m_\nu \sim 10^{-2}$ we have $z_1 \sim 10$). Indeed, z_1 can in principle equal z_0, but we expect in general that $z_1 \leq z_0$. At the redshift z_1 the domain

walls will appear as $\mathcal{O}(1)$ density contrasts, $\delta\rho/\rho \sim 1$ and large–scale structure will begin to form rapidly. Comparing eq.(5) and eq.(6) at $z = 0$ gives a relationship for the closure fraction of domain walls to matter:

$$\frac{\Omega_{walls}}{\Omega_{matter}} = \frac{(1 + z_1)^3}{(1 + z_0)^{1+\kappa}} \tag{7}$$

This implies that we have a closure constraint in the universe of the domain wall energy density, $\Omega_{walls} \lesssim 1$, with $\Omega_{matter} \approx 0.03$ to 0.10, implying that the redshifts z_0 and z_1 satisfy:

$$30 \text{ to } 10 \gtrsim z_1^3/z_0^{(1+\kappa)} \tag{8}$$

For example, in the extreme case that we close the universe with domain wall energy by saturating inequality (8), and further assume $\kappa \approx 0$ we might expect $z_0 \sim 10^2$, and then $z_1 \sim 10$ is the redshift at which $\delta\rho/\rho \sim 1$, which requires that $m_\nu \sim (\rho_{matter}(z_1))^{1/4} \sim 10^{-2}$ eV.

The important point here is that, because the fluctuations are immediately non-linear, at $z \sim z_1$, they will immediately grow into larger structures. The scenario is viable regardless of whether or not dark matter is present, or if the dark matter is of the hot or cold variety. Moreover, by having a phase transition occuring after microwave background decoupling at $kT \sim 1\,\text{eV}$, current $\delta T/T$ limits are not so restrictive.

However, $\delta T/T$ is roughly related to f_ϕ as follows. Consider the shift in energy of light falling into the center of a static domain wall with central energy density $\rho \sim c(0)m_\nu^4$ of characteristic thickness $r \sim f_\phi/c(0)m_\nu^2$. We thus have:

$$\frac{\delta T}{T} \sim G\rho r^2 \sim \frac{f_\phi^2}{M_{Planck}^2}. \tag{9}$$

This is actually a crude *upper bound* for the net $\delta T/T$ generated in this scenario, since there will generally be a cancellation of blue–shift against red–shift in the limit

of a completely adiabatic transition. For us, the value of f_ϕ is, therefore, probed by measurements of $\delta T/T$ while $\delta\rho/\rho$ is *independent of* f_ϕ. Thus, in the present scenario $\delta\rho/\rho$ can be large at z_1, while $\delta\rho/\rho$ in other scenarios, such as cosmic strings, is constrained by the upper limit on $\delta T/T$ to be small.

Indeed, conventional phase transitions occuring prior to decoupling are plagued by the requirement that all fluctuations be small.[11] Hence, associated defects like domain walls were required to be outside of the horizon or else to be "inflated" away. The inverse of this problem is that other topological structures like cosmic strings which also form at GUT scales would require fine tuning to avoid also being inflated away (only strings forming after inflation survive—but why should a GUT string form after inflation?). A late–time phase transition producing soft topological defects, however, has the advantage of producing structure without a conflict with inflation. The phase transition for any such low mass particles will be analogous to the axion phase transition that has been well studied theoretically. Wasserman[12], in a preliminary treatment of late time phase transitions without an underlying particle-physics motivation focused on the small perturbative fluctuations. We focus instead on the large topological defect effects; a real late time phase transition would have both, however the topological defects have the advantage that a fixed non–redshifting energy density is available in the cores of the defects which allow $\delta\rho/\rho \sim 1$. The energy scale of the topological defect is relatively low; hence the structures are "soft," e.g., their core sizes are very large and they have internal energy densities of order m_ν^4 in the schizon model (compared to GUT strings or monopoles having microscopic cores with energy scales of $\sim (10^{15})$ GeV). Nonetheless, the density contrasts can be high, after some redshift z_1, with domain walls or membranes having variations relative to the surrounding true vacuum of order unity or greater.

How the generic soft topological defects actually evolve, thus providing an estimate of κ in the above expressions, is a difficult question and no doubt varies with model specifics. As in the case of cosmic strings, the evolution of domain wall or "cosmic membrane" networks is non-trivial (in the string case, much debate still continues on this problem). The soft cosmic membranes can both accrete or repel

matter.[14] To nearby matter, because of large surface tension, a domain wall is gravitationally repulsive[14]; this will mimic *anti-biased* hot dark matter.[17] Closed bubbles have positive mass as seen from outside at distances greater than the radii of the bubbles, and will serve as accretion centers just as loops of cosmic string. If accreting, then the subsequent evolution of the gravitating baryons and dark matter is that of fragmenting sheets, but with collapse times that are much more rapid than standard linear growth. If repelling, then the matter between the walls will be compressed and collapse, somewhat in analogy to the scenario of Ostriker et al.[15] In fact, repelling late–time cosmic membranes may enable an explosive galaxy formation mechanism to work without having to invoke primordial magnetic fields and superconducting strings, or other contrivances. The initial pattern of membranes will be determined by the zeroes of a random field, and thus will resemble the pattern of caustics seen in hot dark matter models[16]. Finally, scale-free critical fluctuations within one domain may provide yet another possible source of large-scale structure.

As mentioned above, the microwave background anisotropies expected in this class of model are remarkably small. To first order they are zero, since no fluctuations need exist prior to electromagnetic decoupling at the time of recombination; in fact, the surface of last scattering for the microwave photons could be perfectly smooth. (This is even less than with cosmic strings, where string induced gravitational fluctuations exist at last scattering.) The only $\delta T/T$ effects that are induced are due to the second order differential blueshift/redshift resulting from the propagation of the microwave photon through evolving transparent density fluctuations. As shown in eq.(9) the fluctuations induced by the domain walls are limited by roughly f_ϕ^2/M_{Planck}^2. Present observational limits on $\delta T/T$ thus constrain $f_\phi \lesssim 5 \times 10^{16}$ GeV which clearly allows f_ϕ to be of GUT scale. In fact, note that for $f_\phi \lesssim 10^{16}$ GeV the maximum implied $\delta T/T$ due to the domain walls themselves is $\lesssim 10^{-6}$. Once we consider such small $\delta T/T$ we must be careful not to ignore the effects due to the eventual propagation of the microwave photons through the transparent potential wells of the created structures. While we need not consider the Sachs–Wolf effect on the surface of last scattering, Rees and Sciama[18] note that the present existence of large structures will produce

a second–order effect in $\delta T/T$ for $\Omega = 1$ universes. However, the magnitude of this effect we estimate to be only $\delta T/T \sim 6 \times 10^{-8} M_{18} h_0^3 R_{100}^2$ where M_{18} is the mass of the structure in units of $10^{18} M_\odot$, R_{100} is the size of the structure in units of 100 Mpc, and h_0 is the usual Hubble constant in units of 100 km/sec/Mpc. Thus, even the largest structures claimed by the most ambitious observers[6] would not yield observable effects. Note that even if $\Omega \neq 1$ or induced Sachs–Wolf effects enter, late–time transitions would still only yield effects of order $\delta T/T \sim \delta M/M_0$ where δM is the mass of the fluctuation and $M_0 \approx 10^{23} h_0^{-1} M_\odot$ is the horizon mass today. Thus, the largest scales presently observed where $\delta M \sim 10^{17} M_\odot$ only yield $\delta T/T \sim 10^{-6}$, and then only on angular scales $\Delta\theta \sim$ (a few degrees), or larger. All smaller scales would yield even smaller effects. It is clear that a late time phase transition produces the smallest possible $\delta T/T$ of any proposed scenario. If measurements of $\delta T/T$ are ever reported at levels of a few $\times 10^{-6}$, they would directly constrain and possibly measure f_ϕ via eq.(9).

Observationally there is also a limit on the fraction of critical density in the ϕ field today, Ω_ϕ, due to the induced large scale velocities. From the present data[3] on $R \sim 40$ Mpc we know that $\Omega_\phi(1+z_\phi)\delta\rho/\rho \lesssim 0.2$, where z_ϕ is the redshift of the phase transition and $\delta\rho/\rho$ is the density variation in the ϕ field. Thus, for $\delta\rho/\rho \sim 1$ we have $\Omega_\phi \leq 0.2/(1 + z_\phi)$. This constraint sets bounds on the evolution of the ϕ field structures, including κ from eq.(5).

The recent observations[19] of an excess at sub–millimeter wavelengths in the microwave background may, if real, also be explained with the help of a late–time phase transition. In particular, this non–linear growth model may be the only way to have significant star formation at $z \gtrsim 30$. Hogan, et.al.[20] argue that such star formation could create the necessary ionization. It should also be noted that energy released by the phase transition itself or by decay or annihilation of topological defects might provide an alternate source for ionization.

Laboratory tests for the model vary with the specific details. If the late–time transition is associated with neutrino–schizons then there must exist small neutrino

masses near the MSW range. The GALLEX and Baksan gallium experiments coupled with the ^{37}Cl experiment and the proposed D_2O experiment will eventually confirm or deny the MSW explanation for the solar neutrino puzzle. If MSW, then are there neutrino associated schizons? Our mechanism requires a generic pseudo–Goldstone boson which will be hard to detect directly, but its brethren associated with charged leptons or quarks produce potentially observable new phenomena, e.g., new composition dependent pseudo–Gravitational forces, as detailed in ref. (9). The observation of such effects and non–zero neutrino masses would be compelling circumstantial evidence for possible cosmological effects proposed here. As a generic mechanism, it could even occur in the hidden sector of $E_8 \times E_8$ superstring theories and be impossible to observe except by its role on galaxy formation.

Obviously much work remains to be done to examine the details of this class of models. In particular, the astrophysics of the detailed large scale structure that is generated by such late time fluctuations is only sketched here; and full hydrodynamic calculations will have to be carried out. Furthermore, detailed particle physics models will have to be developed to see if all the preferred properties really exist in a fully consistent model. Eventually we would hope to make detailed quantitative predictions about the model vis-à-vis large scale structure. However, the present large scale structure observations are still quite qualitative. Quantitative statistical measures have yet to definitively describe the apparent structure in a reproducible manner. Anecdotally, voids, filaments, sheets, bubbles or sponges appear, depending on the analyses used and on the rapporteur. Conceivably cosmic membranes could make any or all of these structures depending on how they evolve. Hopefully, specific quantitative predictions will be made before the observational data converge. Our purpose here is to alert readers to the fact that an alternative to the standard galaxy formation scenarios may exist. The physics it relies upon is not any more exotic than the GUT physics that the standard scenarios utilize. At low energy scales the model might even be testable in the laboratory. In any case, it may be the only model that can survive limits on $\delta T/T \lesssim 10^{-6}$.

Appendix

Here we present a simple neutrino–schizon model which can produce a phase transition at late times in a natural way. We do not discuss the fundamental Higgs structure at the GUT level which allows such a model to be natural; this can be done along the lines of the discussion in ref.(9).

We assume two species of neutrino, ν_1 and ν_2, and a pseudo–Goldstone boson ϕ with decay constant f_ϕ. ϕ appears as a phase factor in the low energy effective Lagrangian. The mass terms for the neutrinos are assumed to take the form:

$$\bar{\nu}_{1L}\nu_{1R}\left(m + \epsilon e^{i\phi/f_\phi}\right) + \bar{\nu}_{2L}\nu_{2R}\left(m - \epsilon e^{i\phi/f_\phi}\right) + h.c. \tag{A.1}$$

The special choice of eq.(A.1) actually corresponds to a discrete symmetry under which $\nu_1 \leftrightarrow \nu_2$ and $\phi/f_\phi \to \phi/f_\phi + \pi$. In the limit $m = 0$ the theory has a continuous chiral symmetry realized nonlinearly with ϕ in analogy with the Peccei–Quinn symmetry and the axion (actually, we may associate ϕ with the m terms by a redefinition of the neutrino fields, so the chiral symmetry may be regarded as present in the limit $\epsilon = 0$ as well). The presence of both nonzero m and ϵ implies the explicit breaking of the chiral symmetry; the ϕ field will acquire a mass. Note that we can rewrite eq.(A.1), after a ϕ–dependent phase redefinition of the ν_i, as:

$$\bar{\nu}_1\nu_1\left(m^2 + \epsilon^2 + 2m\epsilon\cos\phi/f_\phi\right)^{1/2} + \bar{\nu}_2\nu_2\left(m^2 + \epsilon^2 - 2m\epsilon\cos\phi/f_\phi\right)^{1/2} \tag{A.2}$$

The discrete symmetry protects the mass of the ϕ field from being quadratically divergent when loops are considered. However, we do obtain an induced logarithmically divergent mass term for ϕ of the form:

$$-m^2\epsilon^2\ln(\Lambda/m)\cos(2\phi/f_\phi)/4\pi^2 \tag{A.3}$$

Thus, we must introduce a renormalization counterterm for the induced ϕ mass. In principle this can be arbitrarily large, but it is not an unnatural fine–tuning to choose a counterterm of order the result of eq.(A.3); viewed another way, eq.(A.3) implies

a renormalization group equation for the ϕ mass such that, if we define the mass to be zero at some large energy scale Λ, it will never grow large at low energies and will be given more or less by eq.(A.3). We thus have for the renormalized ϕ mass the expression:

$$-m^2\epsilon^2 \ln(\mu/m)\cos(2\phi/f_\phi)/4\pi^2 \tag{A.4}$$

where μ summarizes the choice of renormalization condition.

We now wish to examine the system at finite temperature (Note: this does not imply that the neutrinos are in thermal equilibrium; it merely implies that they exist at finite density in a thermal density matrix, which can be a relic of an earlier epoch in which they were in equilibrium). The neutrino bilinears at high temperature have the behavior:

$$\langle\bar\nu\nu\rangle_T \to m_\nu T^2/4\pi^2 + m_\nu^3 \ln(T/m_\nu)/8\pi^2 \qquad T > m_\nu \tag{A.5}$$

Here m_ν is the full physical mass of the neutrino as read off in eq.(A.2):

$$m_{\nu 1} = \left(m^2 + \epsilon^2 + 2m\epsilon\cos\phi/f_\phi\right)^{1/2}; \quad m_{\nu 2} = \left(m^2 + \epsilon^2 - 2m\epsilon\cos\phi/f_\phi\right)^{1/2} \tag{A.6}$$

Thus, substituting the finite temperature expectations of eq.(A.5) into eq.(A.2) and adding the zero–temperature ϕ mass of eq.(A.4), gives the temperature dependent ϕ mass:

$$m^2\epsilon^2 \ln(T/\mu)\cos(2\phi/f_\phi)/4\pi^2 \tag{A.7}$$

where the T^2 terms have cancelled (these are analogues of the quadratic divergence at zero temperature and cancel owing to the discrete symmetry; note that the overall sign of eq.(A.7) is irrelevant as we can always shift $\phi/f_\phi \to \phi/f_\phi + \pi/2$).

We see from this result that the temperature of the phase transition is controlled by the renormalization mass, μ. For $T > \mu > m_\nu$ the potential has the positive sign,

and for $m_\nu < T < \mu$, the negative sign. If we choose $\epsilon \sim m_\nu \sim 10^{-2}$ eV, $f_\phi \sim 10^{15}$ GeV and $\mu \sim 10m_\nu$, we have the scenario outlined in the text of this paper.

The unsatisfying aspect of this is that we do not have a *prediction* for the quantity μ. This would have to come from a detailed understanding of the full GUT theory, which we do not know. There is nothing in principle wrong with a value of $\mu \sim m_\nu$, though one might say that we are making a special choice of the log–interval of μ; however, this is not too special a choice since μ between 10^{-2} to 1 eV is acceptable, and this is therefore only a particular selection of 1 in 13 log–intervals between 10^{-2} eV and 10^{24} eV, the GUT scale. Nevertheless, this model (taken together with a demonstration of the naturalness in the full GUT theory as in ref.(9)) illustrates that a late–time phase transition is not at all unreasonable and may even be dictated in some models. The above model is simply a toy; there are no doubt large classes of models admitting this phenomenon.

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