Mon. Not. R. astr. Soc. (1975) **172**, 513–533.

COSMOLOGY FROM ANGULAR SIZE COUNTS OF EXTRAGALACTIC RADIO SOURCES

Vijay K. Kapahi

Radio Astronomy Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400 005, India

(Received 1975 April 8; in original form 1975 February 18)

SUMMARY

In this paper the cosmological implications of the observed angular sizes of extragalactic radio sources are investigated using (i) the log N–log θ relation, where N is the number of sources with an angular size greater than a value θ, for the complete sample of 3CR sources, and (ii) the θ_{median} *vs* flux density (S) relation derived from the 3CR, the All-sky, and the Ooty occultation surveys, spanning a flux density range of about 300 : 1. The method of estimating the expected $N(\theta)$ and $\theta_{\mathrm{m}}(S)$ relations for a uniform distribution of sources in space is outlined. Since values of $\theta \gtrsim 100''$ arc in the 3C sample arise from sources of small z, the slope of the $N(\theta)$ relation in this range is practically independent of the world model and the distribution of source sizes, but depends strongly on the radio luminosity function (RLF). From the observed slope, we derive the RLF in the luminosity range of about $10^{23} < P_{178} < 10^{26}$ W Hz^{-1} sr^{-1} to be of the form $\rho(P)\,dP \propto P^{-2\cdot 1}\,dP$.

It is shown that the angular size data provide independent evidence of evolution in source properties with epoch. It is difficult to explain the data with the simple steady-state theory even if identified QSOs are excluded from the source samples and a local deficiency of strong sources is postulated.

The simplest evolutionary scheme that fits the data in the Einstein–de Sitter cosmology indicates that (a) the local RLF steepens considerably at high luminosities, (b) the comoving density of high luminosity sources increases with z in a manner similar to that implies by the log N–log S data and by the V/V_{m} test for QSOs, and (c) the mean physical sizes of radio sources evolve with z approximately as $(1+z)^{-1}$. Similar evolutionary effects appear to be present for QSOs as well as radio galaxies.

1. INTRODUCTION

It is widely believed that the observed log N–log S relation for extragalactic radio sources does not agree with the predictions of reasonable world models, and provides evidence of strong evolutionary effects in the mean properties of radio sources with epoch (reviews by Longair 1971; Rees 1972). This interpretation of source counts, however, has often been questioned (e.g. Hoyle 1968; Brecher, Burbidge & Strittmatter 1971; Bolton 1971; Kellermann 1972), mainly because the distances of the vast majority of radio sources are not known. In particular, the cosmological origin of the redshifts of QSOs is not beyond doubt, while not enough redshifts have been measured for identified galaxies even among the stronger radio sources. Furthermore, only a small fraction of the fainter sources can be optically identified. It is thus desirable to use other cosmological tests that are free from these uncertainties and from radio and optical selection effects. In this paper we show how the statistics of the observed angular sizes of extragalactic radio sources over a range in flux density can be used for such a test. Our results provide independent support to the evolutionary interpretation of source counts.

1.1 *The $\theta(z)$ and $\theta(S)$ tests*

It is well known that the apparent angular size of an object of constant physical size at different redshifts depends on the cosmological model and provides, in principle, a powerful test of world models (Hoyle 1959; Sandage 1961). This test has had only limited application in the case of optical galaxies, mainly because (a) the test refers to *metric* diameter whereas one measures *isophotal* diameters of galaxies from their optical images (Sandage 1961, 1972) and (b) because it is difficult to record galaxy images at large redshifts where differences in the predictions of world models become important (see Baum 1972). Both these problems are largely overcome in the case of radio sources. As most radio sources from low-frequency surveys have a double structure, the angular separation between the components, which is used as a measure of the angular size, is a metric diameter. The test is, however, made difficult by the spread in angular rizes at any redshift; the inferred linear sizes in fact vary from <1 kpc to hundreds of kpc.

Plots of the largest angular size, θ, against z for QSOs indicate (Legg 1970; Miley 1971; Wardle & Miley 1974) a decrease in angular sizes with redshift. But the decrease in θ is found to be faster than expected in either the steady-state theory or in Friedman cosmologies, and suggests that quasars had smaller physical sizes at earlier epochs. Such a study of the $\theta(z)$ relation is, of course, limited to sources with measured redshifts and can therefore be affected by complex selection effects. Also, the method cannot at present be applied to radio galaxies because large redshifts have not been measured for galaxies.

Since the variation of both angular size and flux density with z, depends on the world model, the $\theta(S)$ relation can also be used as a statistical test of cosmology. Although such a relation introduces additional spread in the data due to the radio luminosity function, it has the advantage that decently large samples free of selection effects can be used. We shall apply this test in this paper to the relation between the median angular size θ_m, and flux density, derived by Swarup (1975; Paper I) from the All-sky (Robertson 1973) the 3CR (Bennett 1962) and the Ooty lunar-occultation surveys spanning a flux range of $\sim 300:1$.

A test combining the $\theta(z)$ relation and source counts has been used in a limited sense by Longair & Pooley (1969), who compared the number of sources with $\theta > 70''$ arc observed in the 5C surveys down to flux limits of 0·03 Jy (1 Jy = 10^{-26} W m^{-2} Hz^{-1}) at 408 MHz, with the number expected in different cosmologies, as predicted from the properties of the stronger sources in the 3CR catalogue. The observed number was found to be larger than predicted in all models that were considered, and could be explained by evolutionary effects in the numbers of intermediate luminosity sources with epoch. A similar test covering a smaller range in flux density has been performed by Fanaroff & Longair (1972).

1.2 *Angular size counts*

The smaller spread in the angular sizes of most radio sources compared to the spread in luminosities, and the apparently rare occurrence of sizes larger than ~ 1 Mpc, suggest that it should be fruitful to study the angular-size counts of radio sources, i.e. the log N–log θ relation, where N is the number of sources in a given region of the sky which have an angular size greater than a value θ. The flux density information is implicit in such a study as we are considering source surveys complete above a certain limiting flux density. It is clear that the log N–log θ relation contains information on the world model as well as on any evolutionary effects in the properties of the source population.

Consider first a survey in which the sensitivity limit is so low that every source is detected. If the sources are uniformly distributed in space and have a constant physical size then, for Euclidean geometry, N varies with distance as r^3 and θ as r^{-1}, so that $N(>\theta) \propto \theta^{-3}$. This would be true for any distribution of source sizes, since each range of sizes would give a number proportional to θ^{-3}. In an actual survey, only sources luminous enough to have a flux density greater than the sensitivity limit will contribute to the count at any θ. The slope of the log N–log θ relation would therefore be flatter than -3, the actual slope depending on the radio luminosity function. The method of estimating the expected $N(\theta)$ relations is outlined in Section 2.

In Section 3 we use the available angular-size information to construct the log N–log θ relation for a complete sample of 3CR sources down to $\theta = 10''$ arc, and compare the observed counts and the $\theta_m(S)$ relation derived in Paper I with the predictions of Section 2. We show that the counts for large values of θ ($\gtrsim 100''$ arc) provide a good determination of the local luminosity function for radio galaxies. The observed $N(\theta)$ and $\theta_m(S)$ relations cannot be explained by a uniform distribution of radio sources in space for reasonable world models, and in particular are incompatible with the predictions of the steady-state theory. In Section 4 we investigate the simplest type of evolutionary effects that are necessary to fit the observations, and conclude that evolution is required both in the space density and in the intrinsic sizes of radio sources.

We shall assume that QSOs are at cosmological distances implied by their redshifts, and treat them as indistinguishable from radio galaxies for the purposes of angular-size counts. As there seems to be little controversy about the cosmological origin of the redshifts of radio galaxies, we shall also consider a reduced sample of sources obtained by excluding the known QSOs from the complete samples. There are several reasons to believe (Bolton 1969) that most of the unidentified sources at moderate flux levels should be radio galaxies beyond the plate limits of the Palomar Sky Survey. Recent deep optical surveys in the positions of 3C sources appear to provide direct observational support for this conclusion (Kristian, Sandage & Katem 1974; Longair & Gunn 1975). The distribution of optical magnitudes of identified QSOs among the weaker Ooty sources also suggests QSO identifications to be essentially complete (M. N. Joshi, to be published). The smaller samples are therefore likely to consist mainly of radio galaxies. We shall come to very similar conclusions for both the samples considered.

2. PREDICTION OF ANGULAR SIZE COUNTS

In order to estimate the form of the expected $N(S, \theta)$ relation for a uniform distribution of radio sources in space we need to know the cosmological world model and the *luminosity–size function*, $\Phi(P, l)$, which can be defined such that the number of radio sources in a unit volume of space, that have radio luminosities (at a fixed emitted frequency) in the range P to $P+dP$, and projected linear sizes in the range l to $l+dl$ is given by

$$\Phi(P, l)\, dP\, dl.$$

2.1 *The luminosity–size function*

We shall express the luminosity–size function as a product of the *radio luminosity function* and the *radio size function*,

$$\Phi(P, l) = \rho(P)\, \psi(l), \tag{1}$$

where $\psi(l)$ is normalized such that

$$\int_0^\infty \psi(l)\, dl = 1.$$

The factorization of equation (1) appears to be justified as there is little evidence of a correlation between size and luminosity of radio sources. An anticorrelation between P and l might be expected if at large redshifts (therefore high luminosities) sources are seen to be individually younger (therefore of smaller size). But since the life times of radio sources have generally been inferred ($\sim 10^6$–10^9 yr) to be much smaller than the characteristic Hubble time, their age distribution should be practically independent of z. The plot of projected linear size against P_{178} (Longair & Macdonald 1969) for the complete sample of 3CR sources, is more or less a scatter diagram with only a slight tendency for sources with small physical sizes to have high luminosities. This tendency refers only to a few QSOs and can arise partly from evolutionary effects in source sizes at large redshifts as indicated by the θ–z plots. Very small source sizes are generally associated with QSOs of flat or inverted radio spectra. The fraction of such sources in low frequency catalogues is, however, very small (<5 per cent) even at the weak flux levels of the Ooty occultation survey (Kapahi, Joshi & Kandaswamy 1973). The possible evolutionary effects in source sizes can be minimized by considering only sources with small z. Mackay (1973) has made an l–P plot for all sources in the 3CR sample for which the redshift has been measured or estimated from optical magnitudes of identified galaxies to be <0.3. The plot is clearly a scatter diagram with a similar spread in the l values at different luminosities that range up to about 5×10^{26} W Hz^{-1} sr^{-1} (for a Hubble constant of $H = 50$ km s^{-1} Mpc^{-1}) at 178 MHz. At higher luminosities one has to consider QSOs of large z, for which it has been noted (Strom 1973; Wardle & Miley 1974) that the effects of any possible anticorrelation between l and P do not appear to exist in the θ–z data. It therefore appears quite reasonable to assume the radio-size function to be independent of the luminosity function.

2.1.1 *Radio luminosity function.* There is considerable uncertainty in the observationally determined form and scale of the radio luminosity function (RLF), especially at the high luminosity end. Apart from the difficulty of knowing the distance of all sources in a volume of space, the determination of the RLF requires knowledge of the world model, and if evolutionary effects are present, these must be known accurately in order to obtain the local form (at $z = 0$) of the RLF. The existing determinations of the RLF show that it is perhaps not unreasonable to express the function as a power law, $\rho(P) \propto P^{-\gamma}$, with the exponent $\gamma = 2.5 \pm 0.5$ over the entire range of luminosities relevant to radio galaxies and QSOs (von Hoerner 1973). For our purpose we shall, for simplicity, assume the RLF to be a truncated power law

$$\rho(P)\,dP = kP^{-\gamma}\,dP \quad \text{for} \quad P_L \le P_{178} \le P_u, \tag{2}$$
$$= 0 \quad \text{otherwise.}$$

We shall take $P_L = 10^{23}$ W Hz^{-1} sr^{-1} and $P_u = 2\times 10^{28}$ W Hz^{-1} sr^{-1} as indicated by the known luminosities of radio galaxies and QSOs. Our conclusions do not depend critically on these limits. In Section 3.2 we shall show that the observed $N(\theta)$ relation for the 3CR sample can be used to obtain the value of γ for $P_{178} \lesssim 10^{26}$ W Hz^{-1} sr^{-1}. Necessary modifications to the single power law form of equation (2) at higher luminosities will be considered in Section 4.

2.1.2 *Radio size function.* The spread in the inferred linear sizes of radio sources is likely to be due mainly to the age distribution of sources and the initial conditions in the explosions that give rise to double radio sources, such as the energy released in the components and the mechanisms of confinement of source components as they move outwards. Since these factors are not well understood, the expected form of the size function is not known. We shall therefore use an empirical method to derive a simple form of the size function that is consistent with observations.

In relativistic cosmologies the size function can in general depend on z. Since the relation between l and θ depends on the cosmological model and the exact form of the possible variation of l with z is not known, the local size function $\psi(l, z = 0)$ has to be derived from a complete sample of sources of small z. Mackay (1973) has considered such a sample of 64 sources from the 3CR catalogue which is likely to be reasonably complete for $z < 0.3$. A histogram of the projected linear sizes (in the Einstein–de Sitter model) for these sources (based on Fig. 3 of Mackay) is shown in Fig. 1(b).

Legg (1970) considered a distribution of true linear sizes in which all sizes up to a maximum value, l_0, are equally probable, i.e. if l_a is the actual linear size

$$\psi_a(l_a)\,dl_a = dl_a/l_0. \tag{3}$$

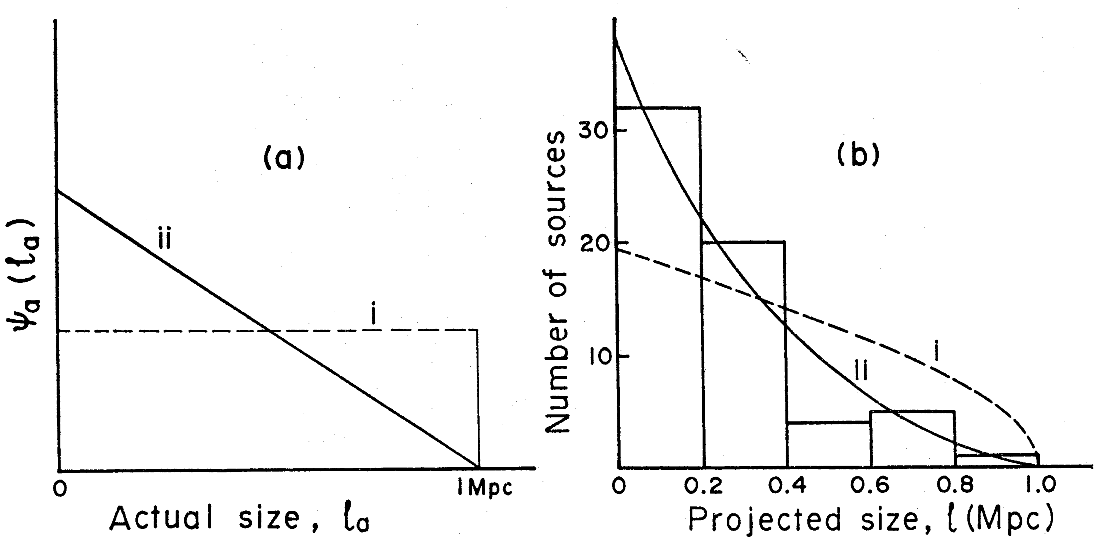

FIG. 1. *Probability distributions of (a) actual source sizes and (b) projected sizes, compared with the observed histogram based on Mackay (1973). One source (3C 236) with $l = 5.7$ Mpc (Willis* et al. *1974) lies outside the range covered by the histogram.*

If the sources are seen randomly projected on the plane of the sky, the corresponding size function for the projected linear sizes is given by

$$\psi(l)\,dl = (1/l_0)\cos^{-1}(l/l_0)\,dl. \tag{4}$$

This distribution, for a value of $l_0 = 1$ Mpc (Fig. 1(a)), is compared with the observed histogram in Fig. 1(b). It is clear that the size function of equation (4) predicts too many sources of large sizes as compared to the observations. We therefore try another distribution in which the probability of an actual size decreases linearly with l_a (Fig. 1(a)), i.e.

$$\psi_a(l_a)\,dl_a = (2/l_0)\{1-(l_a/l_0)\}\,dl_a. \tag{5}$$

For projected sizes this gives

$$\psi(l)\,dl = (2/l_0)[\cos^{-1}(l/l_0)-(l/l_0)\ln\{1+\sqrt{(l_0^2/l^2)-1}\}]\,dl \tag{6}$$

The size function of equation (6), with $l_0 = 1$ Mpc, gives a fairly good fit to the observations as shown in Fig. 1(b) and will be used for calculating the angular-size counts. The recent discovery of two radio galaxies of small z, with projected linear sizes of 2·0 and 5·7 Mpc (Willis, Strom & Wilson 1974) suggests that the local size function may have a weak tail extending to larger sizes. The exclusion of a few such large sources from the function of equation (6) does not, however, change our conclusions appreciably.

2.2 *The* $N(\theta)$ *relation*

As evolutionary effects in source properties may be more important than the differences in reasonable world models, we shall calculate the expected $N(\theta)$ relations only in the Einstein–de Sitter model ($\Lambda = 0$, $q_0 = \frac{1}{2}$) and in the steady-state model. For simplicity we first work out the relations in a static Euclidean universe and later indicate the changes necessary in the two world models.

The basic relations to be used are

$$P = (c/H)^2 \quad z^2 S \quad \text{and} \quad \theta = l/(c/H)\,z,$$

where c is the velocity of light and H is the Hubble constant, which is assumed to be $H = 50\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$.

For a given sensitivity limit S_0, the lower limit P_L to the RLF implies that all sources nearer than a redshift z_*, given by

$$z_* = (H/c)(P_L/S_0)^{1/2}$$

have a flux density $> S_0$. At the distance z_*, a source of maximum linear size l_0 subtends an angle

$$\theta_* = (H/c)\,l_0/z_*.$$

An observed angular size $\theta > \theta_*$ can therefore arise only from sources with $z < z_*$. For the 3CR sample we have $S_0 = 9$ Jy, so that $z_* = 0{\cdot}0057$ and $\theta_* = 6100''$ arc. As the largest angular size of a source in the 3CR sample is $\sim 3800''$ arc, the range $\theta > \theta_*$ is of no practical importance. For $\theta < \theta_*$, most of the contribution to angular-size counts comes from the region $z > z_*$. At any z above z_*, only sources luminous enough to be seen above the survey limit can contribute to the angular-size count and, since the fraction of such sources is controlled by the RLF, its form plays an important role in determining the counts. In order to see the importance of the

RLF, let us assume for simplicity that the power law RLF of equation (2) has no lower or upper cut-offs. The number of sources per steradian in the shell z to $z+dz$, with luminosities P to $P+dP$ and sizes l to $l+dl$ is

$$n(z, P, l) = (c/H)^3 z^2 k P^{-\gamma} \psi(l)\, dz\, dP\, dl$$

and the number that have a flux $S > S_0$ is

$$n(>S_0, z, l) = \int_{P = S_0 (c/H)^2 z^2}^{\infty} n(z, P, l)\, dP$$

$$= \frac{k(c/H)^{5-2\gamma}}{(\gamma-1)} S_0^{1-\gamma} z^{4-2\gamma} \psi(l)\, dz\, dl.$$

Changing the variable z to θ,

$$n(>S_0, \theta, l) = \frac{k S_0^{1-\gamma}}{(\gamma-1)} \theta^{2\gamma-6}\, d\theta\, l^{5-2\gamma} \psi(l)\, dl.$$

The angular-size count is given by

$$N(>S_0, >\theta) = \int_\theta^\infty d\theta \int_0^{l_0} n(>S_0, \theta, l)\, dl$$

$$= \frac{k S_0^{1-\gamma}}{(\gamma-1)(5-2\gamma)} \theta^{2\gamma-5} \int_0^{l_0} l^{5-2\gamma} \psi(l)\, dl. \tag{7}$$

Equation (7) shows that the slope of the log N–log θ relation is independent of the size function, but is very sensitive to the exponent, γ, of the RLF.

For calculating the angular-size counts for the RLF of equation (2), we shall express the $N(S, \theta)$ relation as an integral with respect to z. In the range of interest, $\theta < \theta_*$, contributions to $N(S, \theta)$ come from both the regions (i) $z < z_*$, and (ii) $z > z_*$ although, for the 3CR sample, the contribution from the first region is quite small. Let us consider the two regions separately:

(i) $z < z_*$

We have

$$n(>S_0, z, l) = \rho_0 (c/H)^3 z^2\, dz\, \psi(l)\, dl,$$

where

$$\rho_0 = \int_{P_L}^{P_u} \rho(P)\, dP.$$

Only sources that have projected sizes greater than $l = (c/H) z\theta$ at redshift z can subtend an angle $>\theta$. Let $Q(z, \theta)$ be the fraction of sources in shell z to $z+dz$ that have a projected size $>(c/H) z\theta$. We have

$$Q(z, \theta) = \int_{l=(c/H) z \theta}^{l_0} \psi(l)\, dl$$

which, on integration of equation (6), gives

$$Q(z, \theta) = (1-\xi^2)^{1/2} + \xi^2 \ln\left\{\frac{1+(1-\xi^2)^{1/2}}{\xi}\right\} - 2\xi \cos^{-1}(\xi) \tag{8}$$

where

$$\xi = (c/H)\, z\theta/l_0.$$

Therefore we have

$$n(>S_0, >\theta, z) = \rho_0(c/H)^3\, Q(z, \theta)\, z^2\, dz$$

and

$$N(>S_0, >\theta) = \rho_0(c/H)^3 \int_0^{z_*} z^2 Q(z, \theta)\, dz. \tag{9}$$

(ii) $z > z_*$

In this case

$$n(>S_0, >\theta, z) = (c/H)^3\, z^2 Q(z, \theta)\, dz \int_{P=(c/H)^2 z^2 S_0}^{P_u} kP^{-\gamma}\, dP$$

and

$$N(>S_0, >\theta) = \frac{(c/H)^3\, k}{(\gamma-1)} \int_{z_*}^{z_u} z^2\{(c/H)^{2-2\gamma} S_0^{1-\gamma} z^{2-2\gamma} - P_u^{1-\gamma}\}\, Q(z, \theta)\, dz. \tag{10}$$

The upper limit of integration, z_u, is the smaller of $z(\theta) = l_0/(c/H)\,\theta$ and $z(P) = (H/c)(P_u/S_0)^{1/2}$. The total count for the case $\theta < \theta_*$ is the sum of equations (9) and (10).

2.3 *The $\theta_m(S)$ relation*

In order to estimate the median value, θ_m, of the angular size as a function of flux density, we need to calculate angular size counts for differential ranges of S, i.e. the $N(S, >\theta)\, dS$ relation. We have

$$n(z, P, >\theta) = (c/H)^3\, z^2\, dzkP^{-\gamma}\, dPQ(z, \theta).$$

Changing the variable P to S,

$$n(z, S, >\theta)\, dz\, dS = k(c/H)^{5-2\gamma}\, z^{4-2\gamma} S^{-\gamma}\, dSQ(z, \theta)\, dz$$

and

$$n(S, >\theta)\, dS = k(c/H)^{5-2\gamma}\, S^{-\gamma}\, dS \int_{z_L}^{z_u} z^{4-2\gamma}\, Q(z, \theta)\, dz \tag{11}$$

where the lower limit of integration $z_L = (H/c)(P_L/S)^{1/2}$, and the upper limit z_u is decided as in equation (10) either by the maximum linear size, or by the maximum luminosity. The value of θ_m at flux density S can now be estimated from the $n(S, >\theta)\, dS$ relation.

2.4 *The calculations in Einstein–de Sitter and steady-state cosmologies*

The element of proper volume dV in the shell z to $z+dz$ at the present epoch, and the basic relations between P and S, and between θ and l, get modified in these two cosmologies as follows:

	Einstein–de Sitter	*Steady-state*
$\frac{dV}{4\pi}$	$= 4(c/H)^3\, X^2 Y^{-3/2}\, dz$	$(c/H)^3\, z^2 Y^{-3}\, dz$
$\frac{P_\nu}{S_\nu}$	$= 4(c/H)^2\, X^2 Y^{1+\alpha}$	$(c/H)^2\, z^2 Y^{1+\alpha}$
θ	$= lY/2(c/H)\, X$	$lY/(c/H)\, z$

where $X = 1 - Y^{-1/2}$, $Y = 1+z$, and a radio source is assumed to have a power-law spectrum of the form $S(\nu) \propto \nu^{-\alpha}$. Since the spectral index distribution of radio sources at low frequencies is known to peak around $\alpha = 0{\cdot}75$, with a relatively small dispersion, we shall for simplicity assume all sources to have $\alpha = 0{\cdot}75$.

With the above modifications the $N(>S_0, >\theta)$ and $n(S, >\theta)\,dS$ relations can be worked out as outlined in Sections 2.2 and 2.3. An important feature of the Einstein–de Sitter model is that, for a given size, θ does not decrease monotonically with z but has a minimum at $z = 1{\cdot}25$ and increases thereafter. This implies that the integrands of equations (10) and (11) must be integrated over two ranges on either side of $z = 1{\cdot}25$. In the steady-state model, θ approaches a minimum value $l/(c/H)$ asymptotically as $z \to \infty$.

3. COMPARISON WITH OBSERVATIONS

3.1 *Angular-size counts for the 3CR sample*

The Cambridge observations (Macdonald, Kenderdine & Neville 1968; Mackay 1969; Elsmore & Mackay 1970) of sources from the 3CR catalogue provide the best available data on the structures of a complete set of sources. The sample consists of 199 extragalactic sources believed to be complete to a limiting flux density of 9 Jy at 178 MHz, in a region of 4·25 sr of the sky defined by $|b| > 10°$ and $\delta > 10°$. Though the survey is not complete for sources of angular extent larger than $\sim 5'$ arc, it appears unlikely that an appreciable number of such sources have been missed (Mackay 1971). The sample has been used in Paper I for constructing the angular size-flux density relation. We use the same values of θ as in Paper I. Remarks on some individual sources are also given there. For constructing the log N–log θ relation, we need to go down to as low values of θ as possible. The Cambridge observations have been made at the principal frequency of 1407 MHz with a resolution of $23'' \times 23''$ cosec δ. In order to minimize the effects of insufficient angular resolution at low declinations, and in order to avoid using angular size data from other instruments at different frequencies, we have also considered a subset of the 3CR sample defined by $\delta > 40°$, covering 1·72 sr. For larger angular sizes ($\theta > 60''$) we have counted the sources from the entire sample of 199 sources, but at small angular sizes ($10'' \leq \theta < 60''$) we restrict the counts to the smaller sample and normalize the number to the area of the complete sample. Since the upper limits to the angular sizes of the unresolved sources in the complete sample and the smaller sample are about 40″ and 10″ arc respectively, the counts so derived are unlikely to have been seriously affected by insufficient resolution. Higher angular resolution is needed to extend the counts to $\theta < 10''$ arc. Due to the small fraction of QSOs (~ 22 per cent) in the 3CR sample, it is not practical to construct angular-size counts separately for quasars.

The observed counts are shown plotted in integral and differential forms in Figs 2 and 3, respectively, for the total sample. Counts for the sample without QSOs (Fig. 7(b)) differ from those of Fig. 2 only for values of $\theta < 65''$ arc, which is the largest θ for a QSO (3C 323.1) in the 3CR sample. The errors shown on the counts are statistical errors. Possible observational or instrumental selection effects in the angular size data are considered in Paper I and found unlikely to be serious. The differential counts (Fig. 3) do not show any evidence of an appreciable number of large θ sources having been missed in the 3CR catalogue. In the range of about

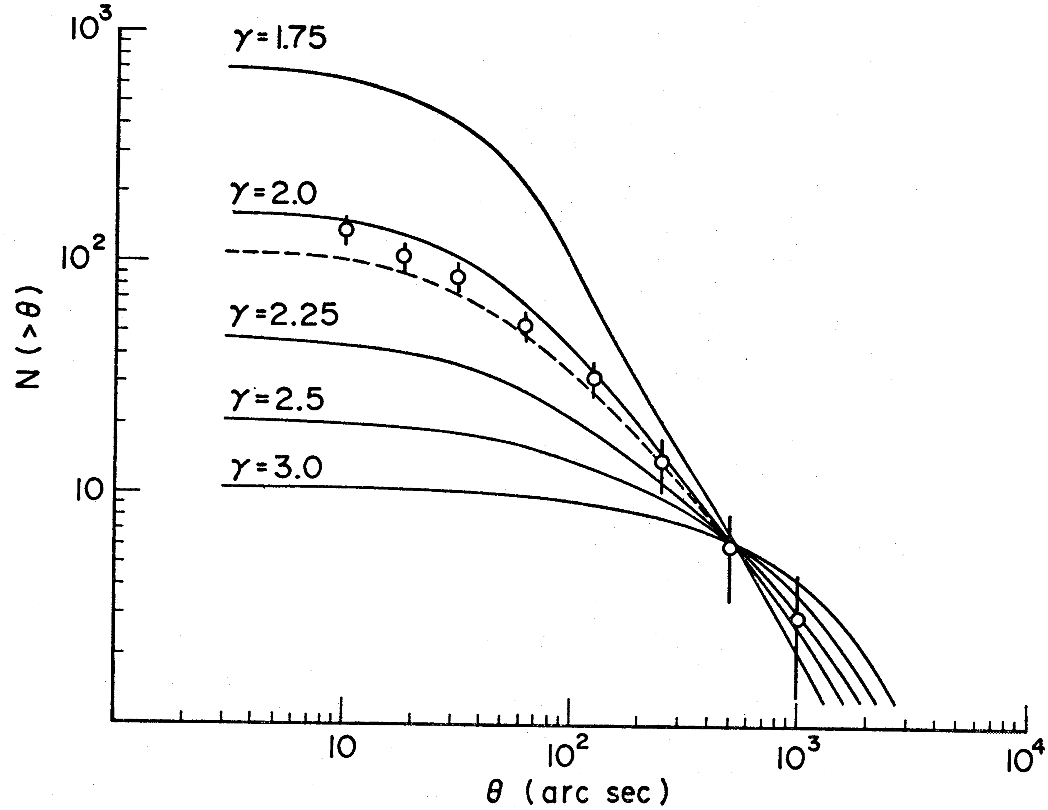

FIG. 2. *The observed log N–log θ relation for the 3CR complete sample, and the expected relations for different exponents of γ of the power-law luminosity function, in the Einstein–de Sitter cosmology. The dotted line is for γ = 2·0 in the steady-state theory.*

100–1000″ arc the log N–log θ relation appears to be linear with a slope of $-1{\cdot}1 \pm 0{\cdot}25$ and flattens gradually for smaller values of θ.

In Figs 2 and 3 we also show the predicted counts for different exponents of the RLF, calculated numerically by computer from the relations in Section 2. The value of the constant k of the RLF for each γ has been chosen so as to fit the observed counts at the large θ end. The strong dependence of the counts on γ is quite apparent. In interpreting the angular size counts, let us first consider the large angular-size part ($\theta \geq 125''$ arc) of the observed counts.

It is clear from the radio size function (equation (6)) that a vast majority of the sources that have $\theta > 125''$ arc must have small z. For the Einstein–de Sitter model about 90 per cent of the sources with $\theta > 125''$ arc should have $z < 0{\cdot}25$. There is a total of 32 sources with $\theta > 125''$ arc in the 3CR sample and 28 of these are optically identified; all of them with galaxies. Of the remaining four, one (3C 33.1) lies in an obscured region of the sky and two (3C 437.1 and 3C 435.1) have the largest values of θ in the sample (3800″ and 3348″ arc respectively), suggesting why there are no unambiguous identifications for them. Most of the 26 galaxies for which redshifts are known have $z \lesssim 0{\cdot}1$; the largest value being $z = 0{\cdot}239$ for 3C 284 (Burbidge & Strittmatter 1972). The sources with large angular sizes are thus radio galaxies most of which have luminosities in the range of $10^{24} < P_{178} < 10^{26}$ W Hz^{-1} sr^{-1}. Due to the small values of z involved the slope of the log N–log θ relation for these sources depends mainly on the exponent of the RLF and is practically independent of the cosmological model and the radio size function. Any

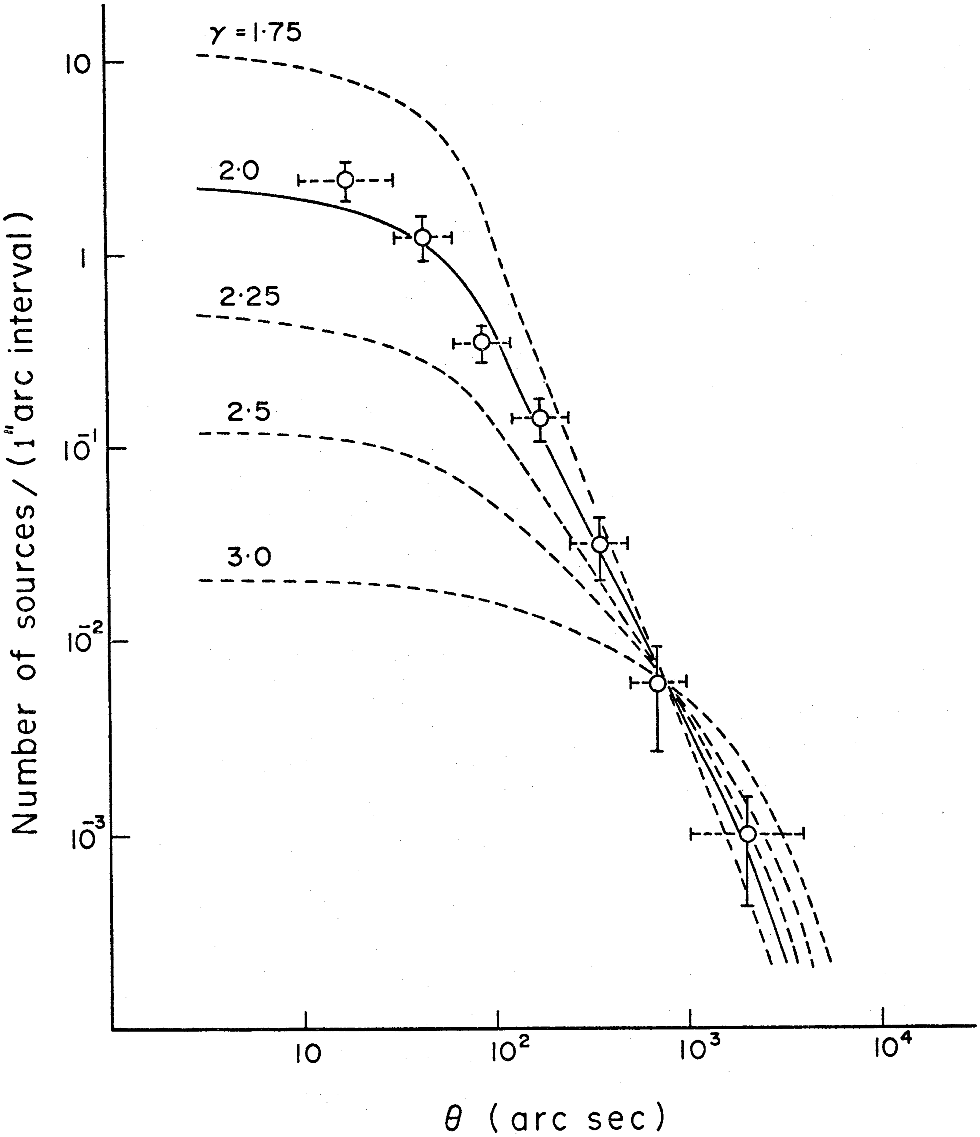

FIG. 3. *Differential angular-size counts for the 3CR sample.*

evolutionary effects with z in source density or source sizes are also relatively unimportant because of small z.

3.2 *The luminosity function for radio galaxies*

We find that the best fit to the data for $\theta > 125''$ arc is provided by a value of $\gamma = 2{\cdot}05$ in the Einstein–de Sitter model. Considering that the inclusion of a few radio sources of large angular size ($\theta > 500''$ arc) that may have been missed in the 3CR catalogue would tend to flatten the observed log N–log θ relation slightly, we shall take the RLF for radio galaxies in the range of about $10^{23} < P_{178} < 10^{26}$ W Hz^{-1} sr^{-1} to be given by

$$\rho(P)\, dP \propto P^{-2{\cdot}1 \pm 0{\cdot}15}\, dP. \qquad (12)$$

Merkelijn (1971) has determined the RLF at 400 MHz, based on the optical magnitudes of a complete sample of radio galaxy identifications from the Parkes catalogue, and finds that in the range $4\times10^{23} \lesssim P_{400} \lesssim 4\times10^{26}$ W Hz^{-1} sr^{-1} it can be expressed as a power law with $\gamma = 2{\cdot}06$ (from her tabulated values we estimate the error in the exponent to be comparable to that obtained from angular-size counts). From the observed increase with z in the number of sources stronger than a limiting flux density, Sholomitskii (1968) determined the RLF to be of power-law form with $\gamma = 2{\cdot}18$ in the entire range of about $10^{21} < P_{178} < 6 \times 10^{26}$ W Hz^{-1} sr^{-1}, covering both normal galaxies and radio galaxies (no error in γ was estimated). More recently, Mackay (1973) has determined the RLF from all sources in the 3CR sample which are estimated to have $z < 0{\cdot}3$. He finds $\gamma = 2{\cdot}16 \pm 0{\cdot}15$ in the range $10^{22} \lesssim P_{178} \lesssim 5 \times 10^{26}$ W Hz^{-1} sr^{-1}. The value of γ determined independently from the log N–log θ relation is thus in good agreement with other determinations based on measured redshifts or optical magnitudes of radio galaxies.

Hoyle & Burbidge (1970) have suggested that the available redshift data for radio galaxies in the 3CR catalogue support a value of $\gamma = 2{\cdot}5$ for the RLF. It is clear from Fig. 2, however, that such a steep form of the RLF is inconsistent with the observed log N–log θ relation. In fact values of $\gamma \gtrsim 2{\cdot}5$ are incompatible with observations even if possible evolutionary effects are taken into consideration. Although evolutionary schemes can be worked out for the RLF given by $\gamma = 2{\cdot}5$ to fit the log N–log S observations (von Hoerner 1973), it is difficult to fit the log N–log θ data at the high θ end where sources have small redshifts. For instance, if one considers simple density evolution of the form $(1+z)^{\beta}$, values of $\beta > 10$ are required for sources of $P_{178} \lesssim 10^{26}$ W Hz^{-1} sr^{-1} to fit the data at the large θ end. Such an evolutionary scheme predicts a large fraction of the fainter sources to be of low luminosity and small z, and cannot explain the observed $\theta_{\rm m}(S)$ relation at small S (Section 3.3). We conclude, therefore, that values of $\gamma \gtrsim 2{\cdot}5$ in the range of about 10^{23} to 10^{26} W Hz^{-1} sr^{-1} for radio galaxies can be ruled out on the basis of the observed angular size data.

In order to determine the form of the RLF for $P_{178} \gtrsim 10^{26}$ W Hz^{-1} sr^{-1} we must consider the observed log N–log θ relation at smaller values of θ, which can arise from sources at large z. As the slope then depends significantly on the cosmological model and on any evolutionary effects, the log N–log θ data alone cannot be used to determine the exponent of the RLF at high luminosities unambiguously. Before considering the possible form of the RLF at high luminosities we first examine if the angular size data are compatible with world models without evolutionary effects.

3.3 *No evolution*

We see from Fig. 2 that a value of $\gamma = 2$ in the entire range of luminosities gives a reasonable fit to the log N–log θ data down to the lowest value of θ. That the fit is only fortuitous is clear from Fig. 4 where we have compared the $\theta_{\rm m}(S)$ curves calculated for different value of γ, as outlined in Section 2.3, with the observed relation derived in Paper I. Fig. 4 shows that no power-law luminosity-function can fit the $\theta_{\rm m}$–S data over the entire range of flux density. The $\gamma = 2$ function predicts very little change in the median value of θ with flux density because for such an RLF there is a substantial contribution from sources at large redshifts even at high flux densities. While the steeper RLFs with $\gamma \gtrsim 2{\cdot}5$ predict a significant reduction in $\theta_{\rm m}$ with decreasing flux density, they predict much higher values of $\theta_{\rm m}$ at any S than are observed. Moreover, the discrepancy with observations cannot be explained by making the RLF steepen with increasing luminosity, because this predicts higher values of $\theta_{\rm m}$ at the lowest flux densities than those given by the $\gamma = 2$ function.

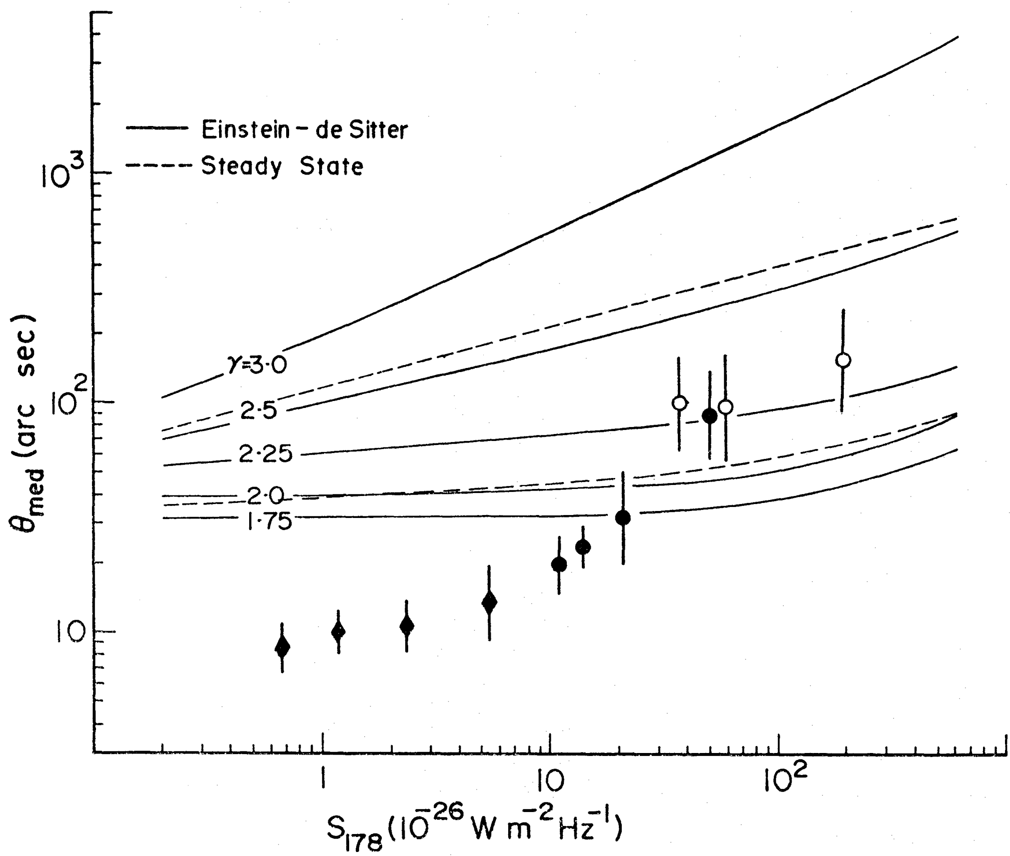

FIG. 4. *The observed $\theta_m(S)$ relation compared with the relations expected for different exponents of the luminosity function. In this and other figures the data from the All-sky catalogue and from the Ooty occultation survey at* 408 *MHz* (*Swarup* 1975) *have been translated to* 178 *MHz using a spectral index α = 0·75. Flux densities at* 178 *MHz have been used for the* 3*CR data.*

Fig. 4 illustrates the cosmological importance of the observed θ_m–S relation, which can be considered as a form of Hubble relation analogous to the S–$\bar{z}$ relation discussed by von Hoerner (1973). The use of θ_m in place of $\bar{z}$ (mean value of z) has the advantage that complete or unbiased samples of sources can be considered even at low flux densities whereas redshifts are not known for all sources even at high flux densities.

We note also from Figs 2 and 4 that the differences in the predictions of the two world models considered are much smaller than the discrepancy with observations. It appears, therefore, that the angular size data are incompatible with the predictions of simple cosmological models in which the comoving density or the physical sizes of sources do not change with epoch, and in particular with the predictions of the simple steady-state cosmology.

3.4 *Steady-state cosmology and the deficit of strong sources*

The differential flux density counts of radio sources indicate (Kellermann 1972; Mills, Davies & Robertson 1973) that in the range of about S_{408} = 1 to 10 Jy, the slope of the N–S relation does not differ significantly from that expected in a static Euclidean universe, and that the steep slope of the log N–log S is confined to sources with $S_{408} \gtrsim 10$ Jy. In a uniformly-filled expanding Universe the Euclidean slope can result only if the redshifts of the sources in the corresponding flux range are extremely small. Since the measured redshifts of many radio galaxies in the above flux range are <0.1, it is often pointed out that the evolutionary interpretation of source counts rests only on the acceptance of QSO redshifts being cosmological. In the Euclidean region of the 408 MHz counts, the value of θ_m decreases continuously with decreasing S and the statistics can be used to make a rough estimate of the median distance of sources other than QSOs.

In the 3CR sample (sensitivity limit of ~ 5.5 Jy at 408 MHz) the 51 galaxies with measured redshifts have median values of $z_m = 0.075$ and $\theta_m = 125''$. For the other 103 sources that are either unidentified or are galaxies of unknown z, we find $\theta_m \simeq 25''$ arc. On the simplest interpretation of a distance effect the 103 sources therefore have $z_m \sim 0.375$. In the range 1–5.5 Jy, z_m is likely to be even larger since θ_m decreases further to $\sim 10''$ arc. With such redshifts the Euclidean slope of the source counts can only be explained by evolutionary effects (see e.g. Longair & Rees 1972).

It has also been suggested that the log N–log S relation at high flux densities can be interpreted as a local deficiency of strong sources rather than as an excess of weak sources. The difficulties with such an explanation with regard to the number of 'missing sources' and the size of the local hole are well known (e.g. Ryle 1968; Longair & Rees 1972). If the local deficiency arises from a statistical fluctuation the observed $\theta_m(S)$ relation should not be affected by the addition of the 'missing sources' as these would be expected to have a similar distribution of angular sizes as the catalogued sources. A second possibility is to consider the missing sources to have large angular sizes. In fact it is conceivable that some of the deficiency comes from the difficulty of recognizing sources of large angular extent ($\theta \gtrsim 500''$ arc) in radio surveys. The effect of adding such missing sources would be to flatten the log N–log θ relation (Fig. 2) and consequently to steepen the local RLF ($\gamma > 2$). It is clear from Fig. 4, however, that a steeper RLF would only increase the discrepancy between the observed values of θ_m at the lowest flux densities, where presumably there are no missing sources, and those predicted by the steady-state theory.

The most favourable case for the steady-state theory would appear to require the missing sources to have small angular sizes for the RLF of $\gamma \sim 2$. To fit the observations at the smallest S (Fig. 4), the missing sources (of large S) must alter the distribution of linear sizes in local space so as to reduce the observed median value of l of ~ 200 kpc (Fig. 1(b)) by a factor of ~ 4. Such a modified size function would imply θ_m of only $\sim 15''$ arc at high flux densities. To reduce the observed θ_m, the missing sources must then have angular sizes $< 15''$ arc. Now, if the size of the local hole is to be less than say 100 Mpc (Hoyle 1968), the missing sources should have linear sizes less than ~ 8 kpc. It is hard to understand how such sources could have been missed.

4. EVOLUTIONARY EFFECTS

4.1 *Size evolution*

As is clear from Fig. 4, the observed θ_m–S relation implies that at small flux densities there should be a larger contribution to angular-size counts from distant sources, or the intrinsic sizes of radio sources must decrease with redshift. It is easily seen that evolution in source sizes with epoch alone cannot explain the observations. If the mean linear size of sources is assumed to vary as $(1+z)^{-n}$, such evolution reduces the number of sources ($N(>\theta)$, since a smaller range of z now contributes to the counts at any θ. But, at large and very small values of θ, size evolution has little effect on $N(>\theta)$ because large θ values arise from small z where evolution is unimportant, and in the limit $\theta \to 0$ all sources still contribute to $N(>\theta)$ so that the total count is not affected.

In order to include the effects of size evolution we assume the distribution of actual sizes at any z to be of the same mathematical form as equation (5), but with the maximum linear size l_0 decreasing with z; that is,

$$\psi_a(l_a, z)\, dl_a = \frac{2}{l_0(1+z)^{-n}}\left\{1 - \frac{l_a}{l_0(1+z)^{-n}}\right\} dl_a.$$

For the corresponding distribution of projected sizes we now calculate the $N(>\theta, >S_0)$ and $\theta_m(S)$ relations in the Einstein–de Sitter cosmology. It should be noted that size evolution can remove the occurrence of a minimum in the z–θ relation. For $n > 1$, the value of θ corresponding to size l_0 at $z = 0$ becomes a continuously-decreasing function of z.

The predicted $N(>\theta)$ relation for the 3CR sample, and the θ_m–S relation for $\gamma = 2{\cdot}0$ and $2{\cdot}5$, are shown in Figs 5 and 6 respectively, for $n = 1{\cdot}0$ and $3{\cdot}0$. While the $N(>\theta)$ relation for $\gamma = 2$ is still in fair agreement with observations, size evolution only reduces the value of θ_m at all flux densities (Fig. 6).

4.2 *Density evolution*

We require an increase in either the comoving density or the luminosity of radio sources with z. Although there has been much discussion in the literature on

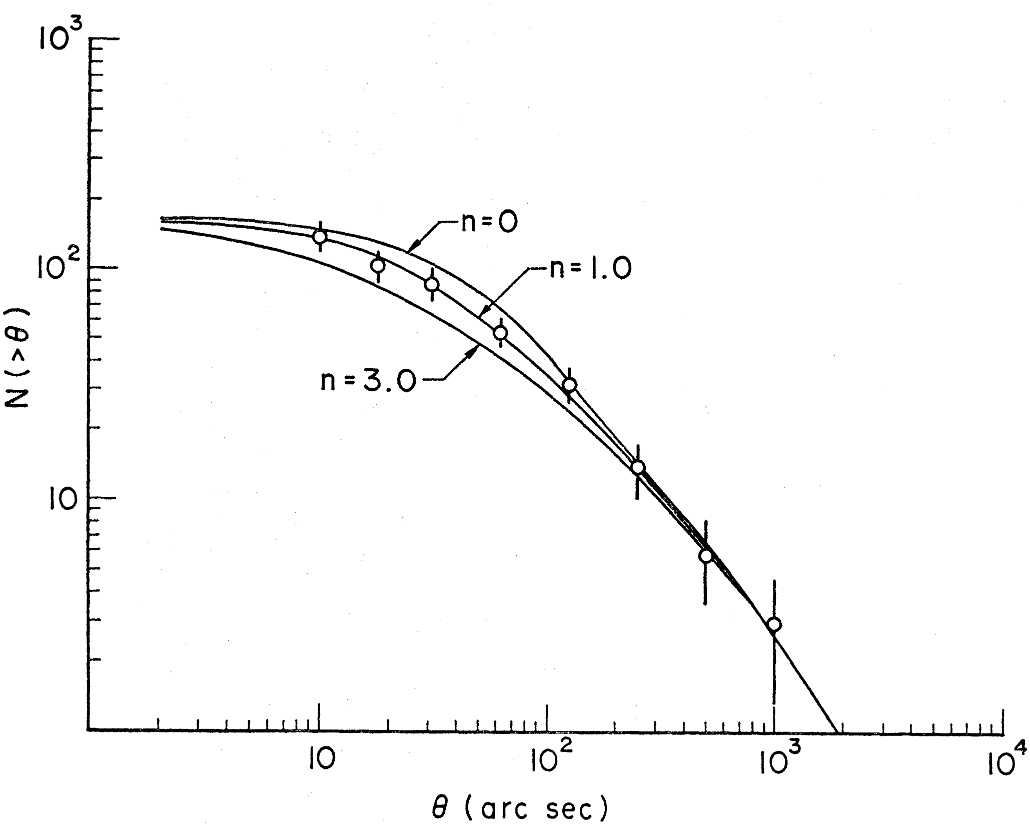

FIG. 5. *The observed log N–log θ relation for the 3CR sample compared with models for size evolution with $\gamma = 2{\cdot}0$.*

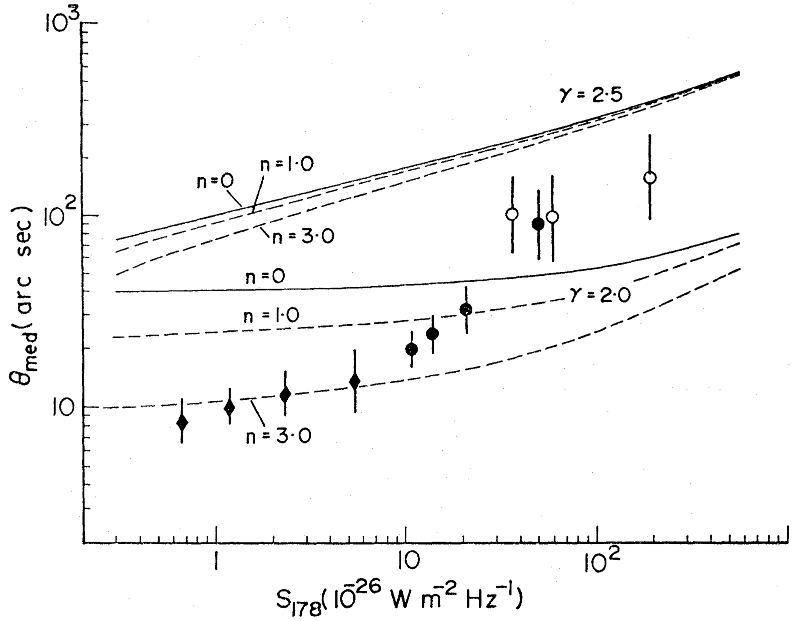

FIG. 6. *The observed $\theta_m(S)$ relation compared with models for size evolution.*

whether ' density evolution ' or ' luminosity evolution ' gives a better fit to the observed log N–log S relation, it seems clear (Longair & Scheuer 1970) that it is not possible from source counts alone to distinguish between the two possibilities. In the case of angular-size counts, since there is no clear evidence of a correlation between sizes and luminosities of radio sources, we do not expect it to be possible to distinguish between the two types of evolution. We shall therefore consider only the case of ' density evolution ' and estimate the approximate extent of evolution necessary to explain the angular-size data.

We consider only the simplest mathematical form of power-law evolution of the type $(1+z)^\beta$. It has been argued in Section 3.1 that evolution for sources with $P_{178} \lesssim 10^{26}$ W Hz^{-1} sr^{-1} is relatively unimportant. In order to fit the log N–log θ relation for the 3CR sample with density evolution for the high luminosity sources, the local luminosity function (at $z = 0$) must steepen at high luminosities compared to the value of $\gamma = 2{\cdot}1$ that was derived for lower luminosities; otherwise the calculated $N(>\theta)$ will exceed the observed value at small θ. For ease of computation we assume the local luminosity function to be given by a truncated power-law with two slopes, of the form

$$\rho(P, z = 0)\, dP = kP^{-2{\cdot}1}\, dP \quad \text{for} \quad P_L \leq P \leq P_m$$

and

$$= k'P^{-\gamma_2}\, dP \quad \text{for} \quad P_m < P \leq P_u.$$

For continuity at $P = P_m$, $k' = kP_m^{\gamma_2 - 2{\cdot}1}$. In the case of density evolution we assume $\rho(P)$ for $P \leq P_m$ to be independent of z, and

$$\rho(P, z)\, dP = k'P^{-\gamma_2}(1+z)^\beta\, dP \quad \text{for} \quad P_m < P \leq P_u.$$

We take $P_m = 10^{26}$ W Hz^{-1} sr^{-1} and determine the unknown parameters as follows. The value of θ_m at the highest flux densities (therefore small z) should be practically independent of evolutionary effects, depending principally on the value of γ_2. We therefore determine the value of γ_2 by calculating the $\theta_m(S)$ relation ignoring evolutionary effects, and requiring a fit with the observed θ_m at the highest flux densities. With this value of γ_2 we estimate the $N(>\theta)$ relation for the 3CR sample and determine the value of β that best fits the observed relation at the smallest values of θ, where size evolution is relatively unimportant. A comparison of the calculated $\theta_m(S)$ relation, including density evolution, with the observed data now shows that in order to fit the observations at lower flux densities, size evolution is also necessary in addition to density evolution. We include the effect of size evolution for sources of all luminosities, as outlined in Section 4.1, and estimate the value of n that fits the observed $\theta_m(S)$ relation.

Reasonable fits to the observed $N(\theta)$ and $\theta_m(S)$ relations can be obtained for values of P_m in the range of $\sim 10^{26}$ to 5×10^{26} W Hz^{-1} sr^{-1}; γ_2 in the range 2·7 to 2·9; β in the range 5 to 6; and n in the range 1·0 to 1·5. One set of parameters that gives a good fit is listed in Table I and the fit shown in Figs 7 and 8. In computing the $N(\theta)$ relation for the 3CR sample, the upper limit to $z(=2{\cdot}3)$ is determined by

TABLE I

Parameters of a satisfactory model

Luminosity function:	$P_m = 10^{26}$ W Hz^{-1} sr^{-1}
(178 MHz)	$P_L = 10^{23}$ W Hz^{-1} sr^{-1}
	$P_u = 2 \times 10^{28}$ W Hz^{-1} sr^{-1}
$\rho(P)\,dP \propto P^{-\gamma}\,dP$	$\gamma_1 = 2{\cdot}1,\ P_L \leq P \leq P_m$
	$\gamma_2 = 2{\cdot}8,\ P_m < P \leq P_u$
Density evolution ($P > P_m$):	
$(1+z)^{\beta}$	$\beta = 5{\cdot}5$
Size evolution:	
$(1+z)^{-n}$	$n = 1{\cdot}0,\ 1{\cdot}5$
Cut-off redshift:	$z_c = 3{\cdot}0$

the high-luminosity cut-off in the RLF. In the $\theta_m(S)$ relation, we have assumed a cut-off in the source density beyond $z_c = 3$. The cut-off is, however, not very critical and values up to $z_c = 5$ or 6 can still provide a reasonable fit to the observed θ_m values at the lowest flux values of the data.

The angular size data for samples obtained by excluding the known QSOs can be explained reasonably well by the same set of parameters as in Table I, but with the upper cut-off to the RLF reduced to $P_u = 5 \times 10^{27}$ W Hz^{-1} sr^{-1}, as is required to prevent radio galaxies of high flux densities from having high redshifts. The fit is shown in Figs 7 and 9.

5. CONCLUSIONS

The principal conclusions from our study of the angular-size counts of radio sources can be summarized as follows:

(1) The log N–log θ relation for the 3CR sample at large values of θ enables the local luminosity function for radio galaxies with $P_{178} \lesssim 10^{26}$ W Hz^{-1} sr^{-1} to be determined, using the redshift information only to obtain the range of intrinsic luminosities and the form of the radio size function which, however, is not very important because of the small redshifts involved. The derived form of the function, $\rho(P)\,dP \propto P^{-2{\cdot}1}\,dP$, is in good agreement with the conventional determinations.

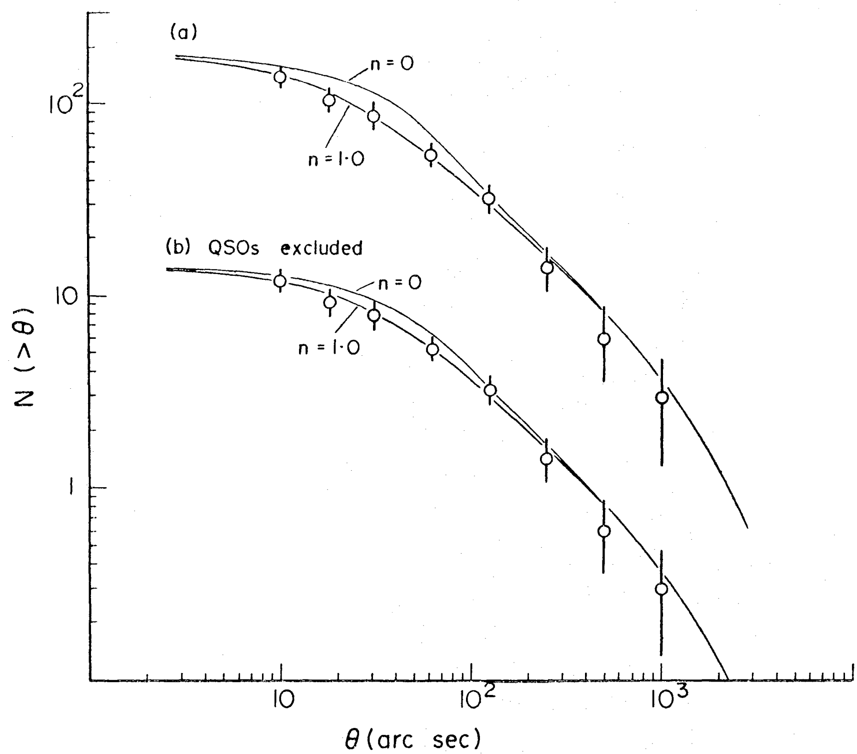

FIG. 7. *Angular-size counts predicted by the model given in Table I. The ordinate for the sample without QSOs should be lowered by a factor of* 10.

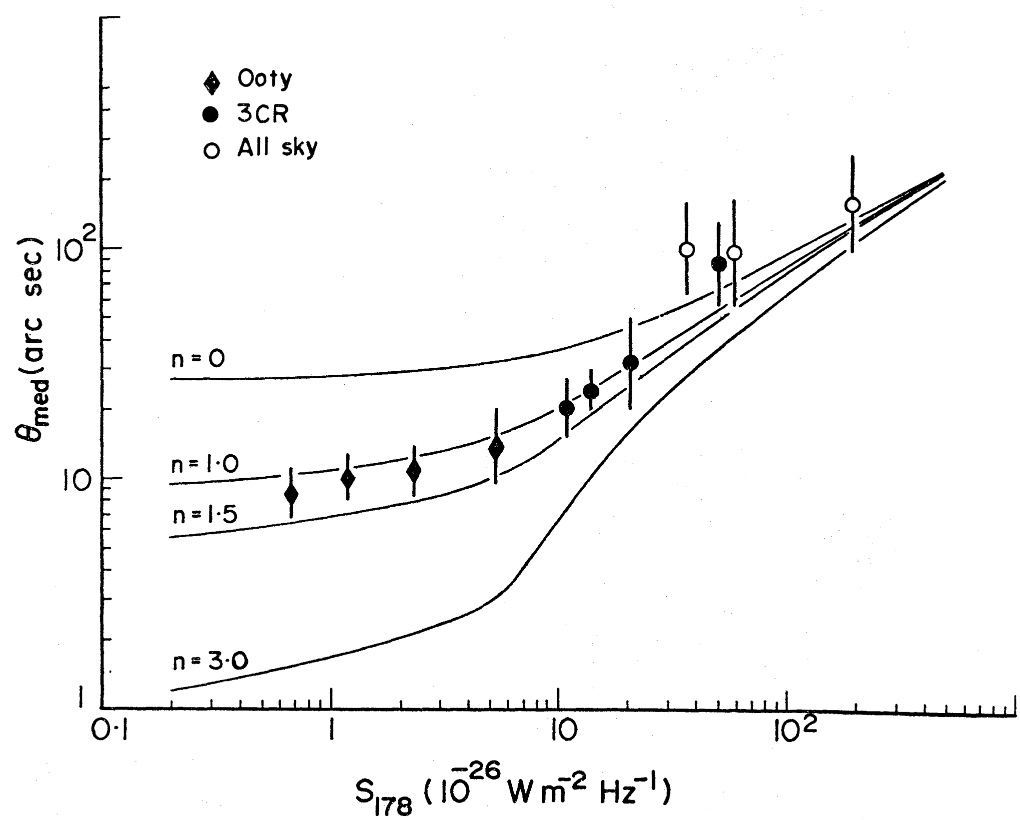

FIG. 8. *The observed $\theta_m(S)$ relation compared with predicted $\theta_m(S)$ curves for the model of Table I with different amounts of size evolution; $P_u = 2 \times 10^{28}$ W Hz^{-1} sr^{-1}.*

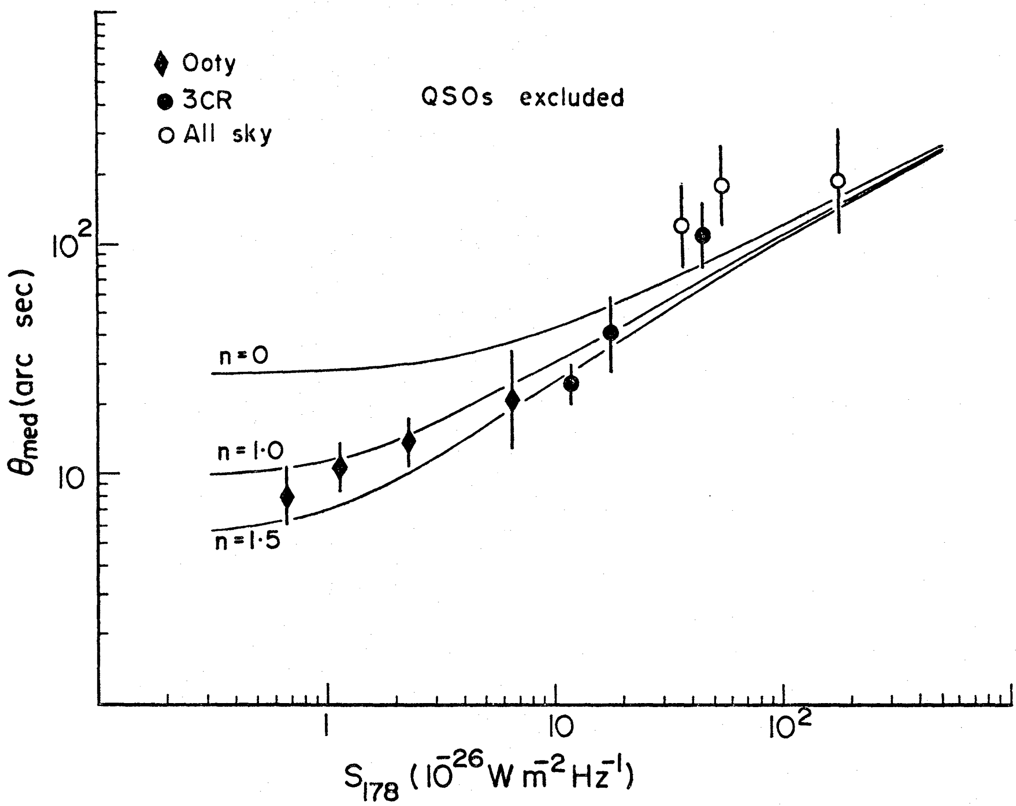

FIG. 9. *The observed $\theta_m(S)$ relation compared with predicted $\theta_m(S)$ curves for the model of Table I; $P_u = 5 \times 10^{27}$ W Hz^{-1} sr^{-1}.*

(2) On the reasonable assumption that the size function for radio sources is independent of radio luminosity, the $\theta_m(S)$ relation provides strong evidence for evolutionary effects in source properties, and these are more important than the geometrical differences of various world models. The angular-size data are incompatible with the simple steady-state theory.

The continuous decrease in angular sizes with flux density in the range of S_{408} = 10 to 1 Jy indicates the presence of evolutionary effects with epoch over a range of flux density in which the slope of the differential flux density counts does not differ significantly from the static Euclidean value.

(3) Evolution is required both in the space density (or luminosity) and in the physical sizes of radio sources. The simplest evolutionary scheme which fits the data in the Einstein–de Sitter cosmology shows that the local RLF is considerably steeper for $P_{178} \gtrsim 10^{26}$ W Hz^{-1} sr^{-1}, and that the co-moving density of high luminosity sources increases with epoch as $\sim(1+z)^{5.5}$. The amount of density evolution is similar to that inferred from the log N–log S data (e.g. Longair 1966; Doroshkevich, Longair & Zeldovich 1970) and from the volume–luminosity test for QSOs (e.g. Schmidt 1968). The local RLF, derived by Longair (1971) from the known optical identifications and a model-fitting technique to explain the log N–log S data, steepens continuously with increasing luminosity; the simplified power-law RLF with two slopes derived in this paper to fit the angular size data is in fact a reasonable approximation to Longair's model RLF. The form of the local RLF and the amount of density evolution implied by the angular size data are largely independent of flux density counts, since no use has been made of the detailed log N–log S relation except to require the total number of sources with

$S_{178} > 9$ Jy in the 3CR sample (i.e. $N(>\theta)$ at very small θ) to agree with the observed number.

The observed θ_m–S relation indicates that the mean physical sizes of radio sources evolve approximately as $(1+z)^{-1}$. An evolution of this nature is also required to explain the observed θ–z relation for QSOs. In Friedman cosmologies a decrease in source extent is readily explained by models that require an intergalactic medium to decelerate and confine the source components (De Young & Axford 1967), because the intergalactic density increases as $(1+z)^3$. From detailed numerical calculations, De Young (1971) has estimated the extent of double radio sources to decrease approximately as $(1+z)^{-0\cdot8}$. This is in fair agreement with size evolution required to explain the θ_m–S relation.

In single-burst models of double radio sources the maximum life times of electrons due to inverse Compton losses against the universal background radiation (Rees & Setti 1968; Christiansen 1969; van der Laan & Perola 1969) imply that the maximum source extents should vary even faster than $(1+z)^{-3\cdot5}$ (Wardle & Miley 1974). Such rapid evolution in source sizes is not supported by the angular size data (see Fig. 8) even allowing for the possible uncertainty in the local size function. Other difficulties with such models have been considered by van der Laan & Perola and by Wardle & Miley.

(4) Due to the small number of radio sources identified with QSOs (~22 per cent of the 3CR sources and ~13 per cent of the Ooty occultation sources) the exclusion of known QSOs from the source samples has little effect on the degree of evolution required to explain the angular size data. As there are several arguments to suggest that the unidentified sources in low frequency catalogues are likely to be mainly radio galaxies, the angular size data show that similar evolutionary effects in the RLF and in source sizes are present for QSOs as well as radio galaxies.

(5) The limited angular-size information in a recent deep survey with the Westerbork Synthesis telescope at 1415 MHz (Katgert & Spinrad 1974) suggests that the median value of θ may be approaching a constant value of ~6″ to 8″ arc at very low flux densities (Swarup 1975), in broad agreement with the predictions of the evolutionary scheme proposed in this paper. High-resolution observations of very weak radio sources should provide valuable information concerning the evolutionary behaviour of sources of intermediate and low luminosities.

ACKNOWLEDGMENT

It is a pleasure to thank Professor G. Swarup for his advice and encouragement. I thank a referee for suggesting improvements in the presentation of this paper.

REFERENCES

Baum, W. A., 1972. *External galaxies and quasi-stellar objects*, p. 393, ed. D. S. Evans, D. Reidel, Dordrecht.

Bennett, A. S., 1962. *Mem. R. astr. Soc.*, **68**, 163.

Bolton, J. G., 1969. *Astr. J.*, **74**, 131.

Bolton, J. G., 1971. *12th Intern. Conf. on Cosmic Rays, Invited and Rapporteur Papers*, p. 111, University of Tasmania.

Brecher, K., Burbidge, G. R. & Strittmatter, P. A., 1971. *Comm. Astrophys. Space Phys.*, **3**, 99.

Burbidge, E. M. & Strittmatter, P. A., 1972. *Astrophys. J.*, **172**, L37.

Christiansen, W., 1969. *Mon. Not. R. astr. Soc.*, **145**, 327.

De Young, D. S., 1971. *Astrophys. J.*, **167**, 541.
De Young, D. S. & Axford, W. I., 1967. *Nature*, **216**, 129.
Doroshkevich, A. G., Longair, M. S. & Zeldovich, Ya. B., 1970. *Mon. Not. R. astr. Soc.*, **147**, 139.
Elsmore, B. & Mackay, C. D., 1970. *Mon. Not. R. astr. Soc.*, **146**, 361.
Fanaroff, B. L. & Longair, M. S., 1972. *Mon. Not. R. astr. Soc.*, **159**, 119.
Hoyle, F., 1959. *Paris Symposium on Radio Astronomy*, p. 529, ed. R. N. Bracewell, Stanford University Press.
Hoyle, F., 1968. *Proc. R. Soc. London*, **308A**, 1.
Hoyle, F. & Burbidge, G. R., 1970. *Nature*, **227**, 359.
Kapahi, V. K., Joshi, M. N. & Kandaswamy, J., 1973. *Astrophys. Lett.*, **14**, 31.
Katgert, J. K. & Spinrad, H., 1974. *Astr. Astrophys.*, **35**, 393.
Kellermann, K. I., 1972. *Astr. J.*, **77**, 531.
Kristian, J., Sandage, A. & Katem, B., 1974. *Astrophys. J.*, **191**, 43.
Legg, T. H., 1970. *Nature*, **226**, 65.
Longair, M. S., 1966. *Mon. Not. R. astr. Soc.*, **133**, 421.
Longair, M. S., 1971. *Rep. prog. Phys.*, **34**, 1125.
Longair, M. S. & Gunn, J. E., 1975. *Mon. Not. R. astr. Soc.*, **170**, 121.
Longair, M. S. & Macdonald, G. H., 1969. *Mon. Not. R. astr. Soc.*, **145**, 309.
Longair, M. S. & Pooley, G. G., 1969. *Mon. Not. R. astr. Soc.*, **145**, 121.
Longair, M. S. & Rees, M. J., 1972. *Comm. Astrophys. Space Phys.*, **4**, 79.
Longair, M. S. & Scheuer, P. A. G., 1970. *Mon. Not. R. astr. Soc.*, **151**, 45.
Macdonald, G. H., Kenderdine, S. & Neville, A. C., 1968. *Mon. Not. R. astr. Soc.*, **138**, 259.
Mackay, C. D., 1969. *Mon. Not. R. astr. Soc.*, **145**, 31.
Mackay, C. D., 1971. *Mon. Not. R. astr. Soc.*, **154**, 209.
Mackay, C. D., 1973. *Mon. Not. R. astr. Soc.*, **162**, 1.
Merkelijn, J. K., 1971. *Astr. Astrophys.*, **15**, 11.
Miley, G. K., 1971. *Mon. Not. R. astr. Soc.*, **152**, 477.
Mills, B. Y., Davies, I. M. & Robertson, J. G., 1973. *Aust. J. Phys.*, **26**, 417.
Rees, M. J. & Setti, G., 1968. *Nature*, **219**, 127.
Rees, M. J., 1972. *External galaxies and quasi-stellar objects*, p. 407, ed. D. S. Evans, D. Reidel, Dordrecht.
Robertson, J. G., 1973. *Aust. J. Phys.*, **26**, 403.
Ryle, M., 1968. *A. Rev. Astr. Astrophys.*, **6**, 249.
Sandage, A., 1961. *Astrophys. J.*, **133**, 355.
Sandage, A., 1972. *Astrophys. J.*, **173**, 485.
Schmidt, M., 1968. *Astrophys. J.*, **151**, 393.
Sholomitskii, G. B., 1968. *Sov. Astr. AJ*, **11**, 756.
Strom, R. G., 1973. *Nature, Phys. Sci.*, **244**, 2.
Swarup, G., 1975. *Mon. Not. R. astr. Soc.*, **172**, 501.
van der Laan, H. & Perola, G. C., 1969. *Astr. Astrophys.*, **3**, 468.
von Hoerner, S., 1973. *Astrophys. J.*, **186**, 741.
Wardle, J. F. C. & Miley, G. K., 1974. *Astr. Astrophys.*, **30**, 305.
Willis, A. G., Strom, R. G. & Wilson, A. S., 1974. *Nature*, **250**, 625.