Costly Enforcement of Property Rights and the Coase Theorem

Alexander R.W. Robson and Stergios Skaperdas*

May 2002

Abstract

We examine a setting in which property rights are initially ambiguously defined. Whether the parties go to court to remove the ambiguity or bargain and settle privately, they incur enforcement costs. When the parties bargain, a version of the Coase theorem holds. Despite the additional costs of going to court, other ex post inefficiencies, and the absence of incomplete information, however, going to court may be an equilibrium or ex ante Pareto-superior over settlement; this is especially true in dynamic settings whereby a court decision saves on future enforcement costs. When the parties do not negotiate and go to court the Coase theorem ceases to hold, and a simple rule for the initial assignment of rights maximizes net surplus. (JEL C70, K40)

*Robson: School of Economics, Faculty of Economics and Commerce, Australian National University, Canberra, ACT, 0200, Australia. Email: alex.robson@anu.edu.au Ph +61 2 6125 4909. Skaperdas: Department of Economics, University of California, Social Science Plaza Irvine, CA 92697-5100 USA. Email: sskaperd@uci.edu Ph (949) 824 4167.
A paper circulated under the same title and dated March, 6, 2000 contained the basic structure examined in this paper, but did not develop many of the results reported here. For comments, we would like to thank participants at the WZB-CEPR contests conference that took place in Berlin, November 2001.

Perhaps because the Coase theorem is not a Theorem in the mathematical sense of the term, its meaning and implications are far from being settled more than four decades after its initial formulation in Ronald H. Coase (1960). For instance, Dan Usher (1998) has recently provided an elementary scrutiny of the possible meanings of the theorem and found them wanting, as suggested by the provocative title of his article: "The Coase Theorem is Tautological, Incoherent, or Wrong." At this late date, one would expect to have such issues settled, but it is surprising how little systematic follow-up has been to Coase's own call for examining the case of positive transaction costs (see, for example, Coase, 1992, page 717).

One line of research has focused on the role of incomplete information as a source of transaction costs. It is straightforward to show under such conditions that the initial assignment of property rights matters for efficiency (see for example Roger B. Myerson and Mark A. Satterthwaite, 1983; Richard D. McKelvey and Talbot Page, 1998). What is much harder to determine, however, is whether a third party with limited information – a "bumbling bureaucrat" in Joseph P. Farrell's (1987) terminology – can make the correct decision and pick the more efficient property rights structure. In some cases a bumbling bureaucrat can rely on simple enough pieces of information to make the correct decision, in others not. Then, other than that it depends on the particular circumstances little more can be said from such a viewpoint, a position that could be interpreted as being in favor of a weak version of the Coase theorem.

A neglected aspect of the study of property rights is that they are often costly to enforce and thus can be considered a significant component of the rarely defined and operationalized term of transaction costs. Apart from resorting to violence or the threat of it – a not uncommon form of enforcement in much of the world even nowadays – there are significant costs in securing title to assets in all economies. It is costly to enforce rights to standardized assets like real estate in developed economies, and there are even higher costs in claiming property rights on less standardized cases like intellectual property and nuisance disputes. In this paper we study the effect of enforcement costs and, in the main interpretation that we adopt, we focus on the effect of litigation costs that are incurred to secure either a better settlement or a favorable court decision.

To focus solely on the effects of costly enforcement, we do not allow for any income effects, incomplete information, bargaining costs, or other asymmetries. Our main assumption is that the parties are unable to commit not to engage in enforcement activities; in particular, they are unable to commit not to engage in exploratory litigation effort and other preparatory measures towards bringing a case to court. If commitment were possible and the parties could contract on the level of enforcement, then they could avoid them altogether. The assumption is thus analogous to the non-contractibility of relationship-specific investments in the theory of the firm (Sanford J. Grossman and Oliver D. Hart, 1986). Property rights are ambiguously defined in the sense that the court's decision is uncertain, though other things being equal one party has higher probability of winning and that party is said to have the initial, ambiguously defined property right. Conditional on this initial right, enforcement efforts influence the probability of each party's winning in court.

We first examine a static model in which the two parties can engage in bargaining both before and after going to court. Bargaining and settlement involve both the sharing of a larger surplus than otherwise and the saving of some of the costs of going to court. Settling before going to court is shown then to be subgame perfect. Equilibrium enforcement efforts under such a settlement are independent of the initial assignment of property rights. When, however, the parties cannot bargain and expect to go to court, enforcement costs can be low enough that at least one party can be better off ex ante by committing not to bargain. Moreover, in such conditions, who has the initial property right matters for efficiency and a simple rule can be used to decide who should be assigned that right.

Going to court resolves all or part of the uncertainty about who has the property right. The implication, then, is that court decisions can reduce or eliminate enforcement costs in he future. With that observation in mind, we next examine dynamic versions of our model. For a wide set of conditions, we find that going to court is a subgame perfect equilibrium despite the absence of incomplete information or other complications that would be typically associated with conflictual outcomes. The parties may decide to go to court because the resolution of uncertainty about property rights saves future enforcement costs. Bargaining and settlement can still take place once a court decision has been made and, in such a case, a version of the Coase theorem holds. As in the static model, however, tying one's hands not to negotiate can be more efficient and also a bumbling bureaucrat could follow a simple rule for assigning the more efficient property rights structure.

The possibility of going to court under complete information is also of interest for the literature on the economics of trials.[1] To our knowledge, such a possibility has not been rigorously demonstrated in other research. The closest papers to ours are Chulho Jung et al. (1995) and Cooter (1982). Jung et al. analyze a game of incomplete, asymmetric information in which players use valuable resources in order to influence the distribution of property rights. They obtain the result that low influence costs are less likely to be associated with Coasean bargaining. However, they consider only fixed, exogenous influence costs in their analysis, and do not explore bargaining possibilities in any great detail. Cooter, on the other hand, develops a "Hobbes Theorem" which suggests that the role of law is to minimize the inefficiencies that result when bargaining breaks down, by restricting the threats which parties can make against each other. The spirit of this result is similar to the results that we derive in the latter part of the paper.

I. The Basic Setting: The Rancher Versus the Farmer

We consider two parties, a *farmer* (f) and a *rancher* (r). The rancher undertakes an activity (say, raising cattle) that produces output $x \in [0, \infty)$ which yields profits or private benefits $B(x)$. We assume $B : [0, \infty) \to [0, \infty)$

[1]Robert D. Cooter and Daniel L. Rubinfeld (1989) represents an earlier survey of the literature. Our approach is based on the theory of contests; related contributions include Avery W. Katz (1988), Jack Hirshleifer and Evan Osborne (2001), and Amy Farmer and Paul Pecorino (1999). Other contributions delve deeper into the microanalytics of evidence production (see Andrew F. Daughety and Jennifer F. Reinganum, 2000, and Jesse Bull and Joel Watson, 2001). An analogous result for the occurrence of conflict and war has been shown in Michelle R. Garfinkel and Stergios Skaperdas (2000). In that setting, conflict can occur despite its costly nature because in dynamic setting there are compounding rewards to the winner and savings of future resources.

is bounded, increasing and strictly concave on its domain of definition. The production of x generates a cost of $C(x)$ to the farmer by, for example, having the cattle trample some of the farmer's crops. We assume $C : [0, \infty) \to [0, \infty)$ is bounded, increasing and strictly convex. We further assume that $B(0) = C(0) = 0$, and that $C(x) < B(x)$ for at least one $x \in (0, \infty)$. The assumptions on $B(x)$ ensure that there exists a unique $x_r \in (0, \infty)$ which maximizes the rancher's benefit $B(x)$. The farmer's optimal level of x is clearly 0. Let x^* denote the socially optimal level of production, so that $x^* = \arg\max_x \{B(x) - C(x)\}$. The assumptions on $B(x)$ and $C(x)$ensure that such a socially optimal level of externality exists, and that it is unique. It is also straightforward to show that $0 < x^* < x_r$; that is, the profit-maximizing level of the rancher's activity is higher than the socially optimal level of the activity which, in turn, is higher than what the farmer would most prefer. Figure 1 illustrates one possible set of $B(x)$ and $C(x)$ functions that satisfy the properties we have just described.

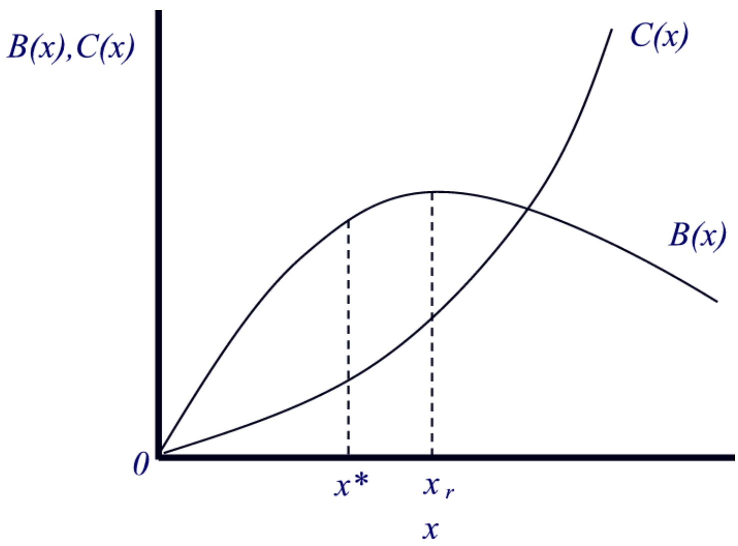

Figure 1

In the absence of third-party enforcement, laws, or any norms about who has the right to choose the level of activity x, private enforcement through the threat of violence would be the typical condition. Indeed one possible logical interpretation of the Coase Theorem is that "resources will be allocated efficiently regardless of whether or not there is assignment of property rights" (Usher, 1998, p.4) and therefore private enforcement through violence would be the setting one would want to examine in order to investigate the Coase Theorem in the presence of enforcement costs.[2] However, Coase's own writings and much of the subsequent literature presupposes the existence of laws, courts, enforcement, and assignment of property rights. Therefore, in the remainder we assume the presence of these institutions, although a limiting case of our model could be interpreted to apply to the case of violence as well.[3]

For our purposes here, we suppose that the parties can clarify their legal positions by going to court. To model this, we assume that the players engage in a *probabilistic contest* to enforce property rights, the outcome of which is ex ante uncertain. The winning player in the contest is awarded the property right to choose x unilaterally. The players can influence their winning probabilities in the contest by investing in "enforcement activities", a generic term which refers to the costs of hiring of counsel and expert witnesses, payments to other legal and scientific researchers and private investigators, and other disbursements associated with the civil litigation

[2] Usher does not claim that this is *the* appropriate statement of the Coase theorem: just that it is one of several possible interpretations.

[3] In particular, this is the case when the parameter φ (defined below) takes the value of 1/2 and the contest success function (also defined below) is interpreted as a technology of conflict. For related work, see the collections of articles in Garfinkel and Skaperdas (1996) and Hirshleifer (2001).

process. Specifically, we assume that the win probabilities depend directly on these enforcement activities and obey a *contest success function*.[4]

Let e_f and e_r be the amount that the farmer and the rancher invest in enforcement activities, and let $\varphi \in (0,1)$. Then the rancher's probability of winning can be described by the following function:

$$(1) \qquad p(e_r, e_f) \equiv \begin{cases} \dfrac{\varphi f(e_r)}{\varphi f(e_r) + (1-\varphi) f(e_f)} & \text{if } e_r + e_f > 0 \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

where f is a non-negative, continuous, increasing function, with $f(0) \geq 0$.[5] Note that since $p(e_r, e_f)$ is a probability for all values of (e_r, e_f), we must have that the probability of the farmer winning the contest is $1 - p(e_r, e_f)$.

The parameter $\varphi \in [0,1]$ represents the degree of "right" that the rancher and the farmer have over the choice of x. For example assuming $\varphi = 1$ or $\varphi = 0$ represents the case in which binding legal precedent, legislation, or the facts of the case completely favor either the rancher or the farmer in gaining the unilateral right to choose x. Any other value of $\varphi \in (0,1)$ represents a situation where the legal or factual situation is not completely biased in favor of either party. Thus, we can also think of φ as a measure of the degree

[4]The approach and functional form used here is examined in detail in the rent seeking literature, and was pioneered by Gordon Tullock (1980). It is utilized in many different contexts other than political economy, including, for example, the analysis of R&D contests in the theory of industrial organization (Avinash K. Dixit (1987)), and also in labor economics (Sherwin Rosen (1986)).

[5]We make more assumptions on f later to ensure the existence and uniquenessof pure strategy equilibria. This functional form is a special case of the n-player asymmetric rent seeking contest analyzed by Mark Gradstein (1995). For an axiomatization of the case $f(e) = e^m$, see Derek Clark and Christian Riis (1998). Amy Farmer and Paul Pecorino (1999), Antonio Bernardo et. al. (2000), and Hirshleifer and Osborne (2001) use this functional form. Fullerton and MacAfee (1999) provide an additional analytical justification for this functional form in the context of research tournaments.

of ambiguity of property rights, or even the general "effectiveness" of the legal system, where effectiveness refers to the law's ability to generate and sustain well defined, widely applicable rules.[6]

Although the nature of the true legal relationship between the parties is not completely clear at the outset, we assume that both parties know the value of φ with probability one. Thus, one way of thinking about φ is to regard it as the parties' common estimate of the true nature of the legal or factual relationship between them, given the particular characteristics of the legal environment.

We can further suppose that the ambiguity of property rights is given in the following sense: φ (and $1-\varphi$) can only take one of two values, φ' or $1-\varphi'$ (where $\varphi' > 1/2$). When $\varphi = \varphi'$, then the rancher can be said to posses the (ambiguous) property right to set x, and when $\varphi = 1-\varphi'$ it is the farmer who can be said to possess the (ambiguous) right to set x. The *level* of φ' should be considered to be beyond the control of government officials and, of course, beyond the control of the parties involved in the dispute. It is supposed to be part of the legal system that can be changed only through major changes in governance. However, the particular *assignment* of the ambiguous right could be made by administrative decision or regulation and its ambiguity is due to the fact that such a decision can be challenged in court.

Does assigning the property right in the sense just described make a difference? If not, we would then have a version of the Coase Theorem in the

[6]Another way to think about φ is to follow Hirshleifer and Osborne(2001) by assuming that φ represents a legal "fault factor" orthe "advantage of having truth on one's side." Alternatively, Katz (1988) and Farmer and Pecorino (1999) (who use a function somewhat similar to ours), call φ the "objective merits of the case."

presence of costly enforcement. If yes, the question emerges of whether a bumbling bureaucrat with minimal information at his disposal could make the right decision and assign the ambiguous property right to one party so that welfare is maximized.

We assume that both parties are risk neutral. If the parties were to go to court, then, their payoffs would be as follows:

$$V_r^c \equiv p(e_r, e_f)B_r - (1+\beta)e_r \tag{2}$$

and

$$V_f^c \equiv -p(e_r, e_f)C_r - (1+\beta)e_f \tag{3}$$

where $B_r = B(x_r)$, $C_r = C(x_r)$,and $\beta > 0$. These payoffs require some explanation. Consider equation (2). In the event of a rancher victory (which occurs with probability $p(e_r, e_f)$), he chooses $x = x_r$, gains B_r, and pays e_r. Should the rancher lose (with probability $1 - p(e_r, e_f)$), the farmer would choose $x = 0$ and the rancher's payoff is simply $0 - e_r = -e_r$. Weighting these payoffs by the appropriate win and loss probabilities gives the expression in (2). The expected payoff for the farmer is derived in a similar fashion to yield equation (3). Finally, we should mention that the parameter β represents the additional marginal cost of actually going to court over just gearing up to go to court.[7]

[7] Of course, there are different ways of modeling the costs of going to court: a fixed cost, a "melting" of part of the pie that is contested, and so on. None of out results depend on the particular way we model the costs of going to court.

II. Incentives to Bargain and Settle

Once a case goes to court and a decision is made about who has the right to choose x, the two parties would have the incentive to bargain over the actual choice of x. By definition, the total surplus is maximized at x^* and has a value of $S^* = B(x^*) - C(x^*)$. If the rancher were to win the right to choose x, he could be induced to choose that level x^* instead of his privately optimal level x_r in exchange for a large enough transfer from the farmer. Similarly, if the farmer were to be granted the right to choose x, a large enough transfer from the farmer could make him choose x^* instead of his optimal level of 0.

Whereas the surplus-maximizing choice of x provides incentives for ex post bargaining – for bargaining once a court decision has been made – there are also incentives for bargaining before going to court, as going to court entails additional costs. For this ex ante bargaining to take place and lead to a settlement that avoids the costs of going to court, the two parties would obviously have to agree on a choice of x, which we can assume to be the surplus-maximizing one, and on a transfer from one party to another that deters both parties from going to court. Clearly, with the two parties bargaining and settling, the going-to-court payoff functions in (2) and (3) would be inappropriate. To arrive at the appropriate definition of the payoff functions when bargaining and settlement are allowed and to further clarify the environment we are examining, there are four distinct stages in the game:

1. Both parties choose initial enforcement efforts e_r and e_f.

2. The parties negotiate in the shadow of the court and possibly settle.

3. If no settlement takes place, the case goes to court and the parties expend βe_r and βe_f resources on litigation.

4. Given the court's decision, the parties can negotiate and settle.

We should emphasize that the level of the initial enforcement efforts, e_r and e_f, is non-contractible. If that were not the case, the two parties could choose to set them equal to zero and avoid all the enforcement costs. Our approach is similar to the incomplete contracts approach to the theory of the firm (see, for example, Grossman and Hart, 1986, or Hart, 1995) in which the non-contractible quantities are relationship-specific investments.

In each bargaining situation we follow standard practice in supposing that the outcome of bargaining depends on (i) the surplus available for division; and (ii) on the disagreement (or threat) utilities that each party has in the event that bargaining breaks down. Moreover, we suppose that the parties, given their respective disagreement utilities, split the surplus. Because there are no income effects in our setting (intentionally so), we have transferable utility, and the Pareto frontier is a straight line, this supposition appears reasonable; it coincides not only with the Nash bargaining solution but also with any other symmetric bargaining solution.[8] It is also the only bargaining outcome that would not provide one side with more exogenous bargaining power than the other.

As is evident from our discussion on the incentives to bargain above, we sup-

[8]For noncooperative implementations of this solution, see Ken Binmore et al (1986). The appropriate noncooperative game for ourcase is the one in which there is an exogenous risk of breakdown of the bargaining process. When utility is not transferable, different symmetric bargaining solutions can have qualitatively different outcomes that can even be Pareto ranked in some instances – see Nejat Anbarci et al. (forthcoming).

pose that neither party will engage in a subgame-imperfect manner. Given that information is complete in our setting and going to court is costly, then we can expect the two parties to settle at the second stage and not go to court. The appropriate payoff functions in this case and one important property of theirs is described next.

Proposition 1 *(i) The ex ante bargaining payoff functions in stage 2 are:*

$$V_r^b(e_r, e_f) \equiv \frac{S^*}{2} + p(e_r, e_f)\left(\frac{B_r + C_r}{2}\right) + \frac{\beta}{2}e_f - \left(1 + \frac{\beta}{2}\right)e_r \tag{4}$$

and

$$V_f^b(e_r, e_f) \equiv \frac{S^*}{2} - p(e_r, e_f)\left(\frac{B_r + C_r}{2}\right) + \frac{\beta}{2}e_r - \left(1 + \frac{\beta}{2}\right)e_f \tag{5}$$

(ii) These same payoff functions would obtain if the stage of ex-post bargaining (stage 4) were not allowed to take place.

Unless otherwise noted, all proofs are to be found in the Appendix. The first term in each of the payoff functions represents the share of the total surplus S^* received by each party. The remaining terms largely reflect the relative disagreement payoffs of the two parties and the bargaining power that emanates from that source. The higher the probability of the rancher winning ($p(e_r, e_f) = \frac{\varphi f(e_r)}{[\varphi f(e_r)+(1-\varphi)f(e_f)]}$), the higher is the rancher's benefit B_r , and the higher is the cost to the farmer C_r, the higher is the rancher's payoff and the lower is the farmer's payoff. The costs of going to court (βe_r for the rancher and βe_f for the farmer) are actually shared by the two parties since bargaining takes place before the two parties incur them. As for the second part of Proposition 1, whether ex post bargaining is allowed or not does not make a difference, first, because bargaining costs are zero and, second, because transferable utility implies that the two parties can take full account in ex ante bargaining what can occur down the road.

III. When There is Settlement

We next examine the Nash equilibrium that emerges with the payoff functions in (4) and (5) under settlement. The following assumption is sufficient to ensure existence and uniqueness of equilibrium.[9]

Assumption 1: *The function f is twice continuously differentiable, with $f'' \leq 0$ everywhere on its domain of definition.*

In an interior equilibrium the choices of enforcement efforts (e_r^b, e_f^b) satisfy the following first-order conditions:

$$\frac{\partial V_r^b(e_r^b, e_f^b)}{\partial e_r} = \frac{\varphi(1-\varphi)f'(e_r^b)f(e_f^b)}{[\varphi f(e_r^b) + (1-\varphi)f(e_f^b)]^2}\frac{B_r + C_r}{2} - \left(1 + \frac{\beta}{2}\right) = 0 \tag{6}$$

$$\frac{\partial V_f^b(e_r^b, e_f^b)}{\partial e_f} = \frac{\varphi(1-\varphi)f(e_r^b)f'(e_f^b)}{[\varphi f(e_r^b) + (1-\varphi)f(e_f^b)]^2}\frac{B_r + C_r}{2} - \left(1 + \frac{\beta}{2}\right) = 0 \tag{7}$$

These two equations imply:

$$\frac{f'(e_r^b)}{f(e_r^b)} = \frac{f'(e_f^b)}{f(e_f^b)}$$

[9] Weaker conditions also suffice to ensure existence of equilibrium in our model – for example, Skaperdas (1992) and other authors show that as long as f is not "too convex" a nontrivial equilibrium will exist. The issue is also addressed by Farmer and Pecorino (1999) in the context of legal battles.

By Assumption 1, this is possible only if the enforcement efforts of the rancher and the farmer are identical ($e^b_r = e^b_f = e^b$). Thus, the probability of winning of the rancher if they were to go to court (which affects the share of the total surplus received by each party) would equal φ, while that of the farmer would be $1 - \varphi$.

Does it make difference for the size of the net surplus, whether the administrator or regulator initially assigns the (ambiguous) right to choose x to the rancher ($\varphi = \varphi'$) or the farmer ($\varphi = 1 - \varphi'$)? Given that the enforcement efforts are identical in equilibrium, either of the first-order conditions in (6) or (7) implies:

$$\frac{\varphi(1-\varphi)f'(e^b)}{f(e^b)}\frac{B_r + C_r}{2} = 1 + \frac{\beta}{2} \tag{8}$$

Note that the term $\varphi(1 - \varphi)$ in the left hand side of this equation equals $\varphi'(1-\varphi')$, regardless of whether $\varphi = \varphi'$ or $\varphi = 1-\varphi'$. Since this is the only place that the value of φ enters in the determination of the equilibrium effort e^b, that effort is independent of whether the rancher of farmer have been assigned the ambiguous right to choose x. Since the surplus S^* is fixed, any variations in efficiency can only occur through variations in the *level* of enforcement efforts. Therefore, as enforcement efforts do not vary with the *assignment* of rights, the net surplus does not depend on the assignment of rights either.

However, from (8) we can determine that the more ambiguous the property rights are, in the sense that the closer is φ' to $1/2$, the higher are the enforcement efforts and lower is the net surplus. We summarize our findings under settlement in the following:

Proposition 2 *Suppose the two parties bargain and settle before going to court. Then:*

(i) The rancher and the farmer choose identical enforcement efforts in equilibrium;

(ii) Given a level of ambiguity of property rights of $\varphi' > 1/2$*, these efforts and the net surplus available for division between the two parties are independent of the initial assignment of rights; and*

(iii) The more ambiguous are property rights (the closer is φ' *to* $1/2$*), the higher are the equilibrium enforcement efforts and the lower is the net surplus.*

For $f(e) = e$, we can analytically calculate the equilibrium. In particular, the equilibrium efforts and payoffs are:

$$e^b = \frac{\varphi(1-\varphi)(B_r + C_r)}{2+\beta} \tag{9}$$

$$V_r^b(e^b, e^b) = \frac{S^*}{2} + \varphi\frac{(\beta + 2\varphi)(B_r + C_r)}{2(2+\beta)} \tag{10}$$

$$V_f^b(e^b, e^b) = \frac{S^*}{2} - \varphi\frac{(4 + \beta - 2\varphi)(B_r + C_r)}{2(2+\beta)} \tag{11}$$

We will compare these values to others later.

IV. Settling Versus Going to Court

Although given some initial enforcement choices and that negotiations are allowed to take place the two parties have an incentive to settle, there are still reasons for examining the possibility of going to court. Would the enforcement efforts differ from those under settlement if the two parties expected to go to court and, if so, how? Does the version of the Coase theorem that appears to hold for the case of settlement continue to hold when the two parties expect to go to court? Are the ex ante equilibrium payoffs under settlement higher than those under going to court? If not, would there be a way for one or both parties to make an ex ante commitment not to go to court?

Therefore, we now consider the equilibrium under the payoff functions in (2) and (3); that is, we consider the game with stages 1 and 3 only. Again, Assumption 1 guarantees existence and uniqueness of equilibrium. The first order conditions for the rancher and farmer at the equilibrium (e_r^c, e_f^c) imply:

(12) $$\frac{\varphi f'(e_r^c)(1-\varphi)f(e_f^c)}{[\varphi f(e_r^c)+(1-\varphi)f(e_f^c)]^2}B_r = (1+\beta)$$

(13) $$\frac{\varphi f(e_r^c)(1-\varphi)f'(e_f^c)}{[(e_r^c)+(1-\varphi)f(e_f^c)]^2}C_r = (1+\beta)$$

Equating these two first order conditions yields the equilibrium condition:

(14) $$\frac{f'(e_r^c)}{f(e_f^c)} = c\frac{f'(e_f^c)}{f(e_f^c)}$$

where $c \equiv C_r/B_r (= C(x_r)/B(x_r))$, which is the ratio of social costs to benefits at the rancher optimal output x_r. Note that the assumptions on $B(x)$ and $C(x)$ do not restrict the value of c in anyway, although of course we always have $c > 0$. When $c > 1$,the costs to the farmer exceed the benefits to the rancher and therefore the social costs exceed the benefits of the activity when the rancher chooses the activity. When $c < 1$, the opposite holds.

To determine how the equilibrium efforts are related to this cost benefit ratio, define $g(e) \equiv f'(e)/f(e)$. Then, from equation (14) we have:

$$g(e_r^c) = cg(e_f^c) \tag{15}$$

By the definition of $g(e)$ we have:

$$g'(e) = \frac{f''(e)f(e) - [f'(e)]^2}{[f(e)]^2} < 0 \tag{16}$$

where the last inequality follows from Assumption 1 and the fact that $f(e)$ is an increasing function. Therefore g is monotonically decreasing on its domain of definition. From (16) we have

$$\begin{aligned} g(e_r^c) &> g(e_f^c) \quad \text{for } c > 1 \\ g(e_r^c) &= g(e_f^c) \quad \text{for } c = 1 \\ g(e_r^c) &< g(e_f^c) \quad \text{for } c < 1 \end{aligned} \tag{17}$$

which, together with the fact that g is a decreasing function demonstrates that e_r^c is greater or smaller than e_f^c as c is smaller or greater than 1. That is, the party with relatively more at stake puts more effort in equilibrium. If the cost that the farmer is trying to avoid is greater than the benefit that the rancher will receive, then the farmer will exert greater effort. If the benefit to the rancher were to be greater than the cost the farmer would incur, then it would be the farmer who would exert higher effort. This outcome does not occur in the case of the negotiated settlement because the two parties split the costs and benefits(because of the assumption of symmetry in bargaining), thus contest prizes of the same size, and exert the same amount of enforcement effort.

The enforcement efforts of course depend on φ. Given that the efforts of the adversaries differ when they expect to go to court and their payoff functions exhibit an asymmetry that the settlement payoffs do not have, the next issue to examine is whether assigning the initial ambiguous property right to the rancher ($\varphi = \varphi'$) or to the farmer ($\varphi = 1 - \varphi'$) makes a difference. For a given φ,consider the net equilibrium surplus when the parties expect to go to court:

$$
\begin{aligned}
&V_r^c(e_r^c, e_f^c) + V_f^c(e_r^c, e_f^c) \\
&= \frac{\varphi f(e_r^c)}{[\varphi f(e_r^c) + (1-\varphi) f(e_f^c)]}(B_r - C_r) - (1+\beta)(e_r^c + e_f^c) \\
&= \frac{\varphi f(e_r^c)}{[\varphi f(e_r^c) + (1-\varphi) f(e_f^c)]}B_r(1 - c) - (1+\beta)(e_r^c + e_f^c) \qquad (18)
\end{aligned}
$$

The second term in this surplus is the cost of enforcement efforts. The first term represents the expected net social benefit from the choice of x. [10] Note that this first term is positive or negative depending on whether the benefit B_r to the rancher is larger or smaller than the cost C_r to the farmer (or, whether c is smaller or greater than 1). Thus, for given enforcement efforts this term is maximized by assigning the ambiguous right to the rancher when $c < 1$, and assigning to the farmer when $c > 1$. It turns out that the whole net surplus is also maximized when this rule of property rights assignment is followed. We state this result as part of Proposition 3 below and prove it

[10] As well as in the rest of the paper, note that this term shows only the expected net benefit of the choice of x by the rancher because the the cost and benefits of the choice of x by the farmer have been normalized to 0.

in the Appendix. Part (i) has already been shown above.

Proposition 3 *Suppose the two parties expect to go to court and their payoff functions are as described in (2) and (3). Then:*
(i) The party with more at stake devotes more resources to enforcement activities:

$$e_r^c < e_f^c \quad for\ c > 1$$
$$e_r^c = e_f^c \quad for\ c = 1 \tag{19}$$
$$e_r^c > e_f^c \quad for\ c < 1$$

and
(ii) Given a level of ambiguity of property rights of $\varphi' > 1/2$, it is efficient to assign these rights to the party that has more at stake (to the rancher if $c < 1$, and to the farmer if $c > 1$).

The simplicity of the rule of assigning the ambiguous property right in this case makes the expectation that even a "bumbling bureaucrat" could possibly make in the right direction. [11] Although settlement does not involve the additional cost of going to court and production induces the maximal social surplus S^*, going to court could still be better for one or even both parties if the costs of enforcement were to be low enough compared to those under settlement. To make welfare comparisons we will calculate enforcement efforts and equilibrium payoffs under $f(e) = e$ and compare them to those in equations (9)-(11). In particular, under $f(e) = e$, the following relations

[11] Harold Demsetz (1972) states a similar rule in the context of assigning unambiguous property rights.

hold:

$$(20) \qquad e_r^c = \frac{\varphi(1-\varphi)cB_r}{(1+\beta)(\varphi+(1-\varphi)c)^2} \ , \ e_f^c = \frac{\varphi(1-\varphi)cC_r}{(1+\beta)(\varphi+(1-\varphi)c)^2}$$

$$(21) \quad V_r^c(e_r^c, e_f^c) = \frac{\varphi^2 B_r}{(\varphi+(1-\varphi)c)^2}$$

$$(22) \quad V_f^c(e_r^c, e_f^c) = -\frac{\varphi(\varphi+2(1-\varphi)c)C_r}{(\varphi+(1-\varphi)c)^2}$$

In comparing first the costs of enforcement under settlement with those under going-to-court, it should be noted that the figures in (20) should be multiplied by $(1+\beta)$ since going to court involves the additional cost of βe_i^c $(i = r \ , \ f)$. That is, whereas the total costs of enforcement under settlement are $2e^b$, those under going-to-court equal $(1+\beta)(e_r^c + e_f^c)$. Using (9) and (20), it is straight forward to show that settlement entails higher costs if and only if $\frac{2}{2+\beta} > \frac{c}{(\varphi+(1-\varphi)c)^2}$. This condition is satisfied when the value of c is sufficiently small or sufficiently large.[12] That occurs because when the effects on the two parties are sufficiently different, as they can be when going to court, both parties exert considerably lower efforts so that the additional cost of going to court can be overcome. Then, the payoffs under going-to-court in (21) and (22) could well be lower than their respective payoffs under settlement in (10) and (11). This is indeed the case, as shown by example in the Appendix, and stated in the following result:

Proposition 4 *At least one party may ex ante prefer going to court over bargaining and settlement.*

[12]In particular, it can be shown that the inequality holds when:
$c > \frac{[2+\beta-4\varphi(1-\varphi)]+\{(2+\beta)[(2+\beta)-8\varphi(1-\varphi)]\}^{1/2}}{4\varphi^2}$
and when:
$c < \frac{[2+\beta-4\varphi(1-\varphi)]-\{(2+\beta)[(2+\beta)-8\varphi(1-\varphi)]\}^{1/2}}{4\varphi^2}$.

If one or both parties were to ex ante prefer going to court over settlement, the game would have to be modified to allow the outcome of going to court as a subgame perfect equilibrium, for once at stage 2 both parties would prefer to settle regardless of the initial choice of enforcement effort. One party could, for example, commit not to bargain in advance by a burn-the-bridges act that cuts the lines of communication. We could also think of the same outcome obtaining when the bargaining costs are sufficiently high. What could an administrator or regulator do if the only information he had were the cost-benefit ratio c and had no knowledge of whether the parties would go to court or not? It would be reasonable to take the weakly preferred action of assigning the ambiguous right to choose x to the party with the higher stake.

V. When the Future Casts its Shadow

Thus far we have examined a setting with an one-time interaction between the two parties or, trivially, as a multi-period repetition of the same exact conditions and outcomes in every period. However, once the time dimension is brought in there are non-trivial dynamic considerations that enter the picture. On the one hand, if one side has the ambiguous property right and agrees to settle, could the property right become even more atrophied in the future (see James M. Buchanan, 1989)? On the other hand, when a court makes a decision it strengthens the property right of the winner and, presumably,reduces or eliminates the costs of future enforcement. Such considerations might drive one or both parties to go to court. To examine such a possibility we consider a non-trivial dynamic extension of the model we have analyzed thus far. For simplicity we allow for two periods.[13] The first period involves exactly the same characteristics and stages of the static model. If the parties have not gone to court in the first period, the second period also has the same characteristics and stage of the static model. If, however, the parties have gone to court in the first period,the court's decision stands in the second period as well and the party that has won has the complete right to choose x in that period too.[14]

Both parties discount the second period by the factor $\delta \in (0, 1]$. We do not explicitly model the possibility that negotiation and settlement could erode one's property right, but it will become clear that our findings would be, if anything, strengthened by allowing for such a possibility. Before going on, we should re-emphasize the basic assumption we have made: the initial enforcement costs in each period are non-contractible. That is, the two parties cannot write a binding contract in the first period about the level of enforcement costs they can incur in either period. Thus, enforcement costs can be eliminated in the second period only if they can be induced by a subgame perfect equilibrium, and that would be possible typically only when a court decision has unambiguously assigned property rights in the first period.

[13]The main ideas are easily generalizable to a finite horizon of arbitrary length and, with appropriate modifications, to an infinite horizon.

[14]We can allow for the right in the second period not to be perfectly defined, but strengthened relative to the first period, without changing the nature of the results. For example, after the court's decision in the first period, the winner's still ambiguous right in the second period could equal $\varphi'' > \varphi' > 1/2$, where φ' is the favored party's first period right. That approach could be further generalized by allowing a greater number of periods, with each court decision refining the property right of the winner. The highest court's decision could be thought of as providing the perfectly defined property right. Thus, our approach here is equivalent to the court's decision in the first period being final or not allowing any appeals.

V.A. Going to Court with Ex-post Bargaining

We will first show that going to court and then bargaining and settling is a subgame perfect equilibrium under some reasonable set of conditions. In the one-period model we have seen that whether ex post bargaining can take place or not does not make a difference for ex ante bargaining (Proposition 1). In the two-period model, though, that we just outlined,the resolution of uncertainty following a court decision has implications for the future that it did not have in the one-period model. Such a decision implies that one party has gained the unambiguous right to choose x now and in the future and thus the two parties do not have to incur any enforcement costs in the second period. By contrast, if a settlement were to be reached ex ante, enforcement costs will typically have to be incurred in the second period.

Consider any (e_r^1, e_f^1) pair of enforcements efforts that have be incurred in stage 1 of period 1. To derive the threat payoffs at stage2, we need to first examine what would occur in stage 4, once a court decision has been made. At that stage each part has paid e_i^1 (for $i = r, f$) in stage 1 and βe_i^1 at the court stage; these costs, because they are sunk, do not play any role in ex-post bargaining. There are two possible bargaining outcomes, depending on who has won in court. If the rancher has won, the rancher's threat payoff over the two periods would be $(1+\delta)B_r$ whereas the farmer's threat payoff would be $-(1+\delta)C_r$. Given that the surplus over the two periods is $(1+\delta)S^*$ and no enforcement costs are incurred in the second period, the split-the-surplus rule would imply the following payoffs if the rancher were to win:

$$W_{rr} = \frac{(1+\delta)(S^* + B_r + C_r)}{2} \quad \text{and} \quad V_{fr} = \frac{(1+\delta)(S^* - B_r - C_r)}{2}$$

If the farmer has won in court, then the threat payoffs for either party would be 0 (since the optimal choice of x for the farmer is 0 and $B(0) = C(0)$). The ex-post bargaining payoffs in that case would be:

$$W_{rf} = \frac{(1+\delta)S^*}{2} \quad \text{and} \quad W_{ff} = \frac{(1+\delta)S^*}{2}$$

Let $p^1 \equiv \frac{\varphi f(e_r^1)}{[\varphi f(e_r^1)+(1-\varphi)f(e_f^1)]}$. Then, expected two-period payoffs before going to court are:

$$W_r^e = p^1 W_{rr} + (1-p^1)W_{fr} - \beta e_r^1 \;\; \text{and} \;\; W_f^e = p^1 W_{fr} + (1-p^1)W_{ff} - \beta e_f^1$$

and substitution from the expressions above yields:

$$W_r^e = \frac{(1+\delta)S^*}{2} + p^1 \frac{(1+\delta)(1+c)B_r}{2} - \beta e_r^1 \qquad (23)$$

$$W_f^e = \frac{(1+\delta)S^*}{2} - p^1 \frac{(1+\delta)(1+c)B_r}{2} - \beta e_f^1 \qquad (24)$$

These are the expected payoffs of going to court and, if the parties were not to go to court, they represent the threat payoffs in stage 2. To have settlement at that stage, it is necessary and sufficient that the surplus under settlement be greater than $W_r^e + W_f^e = (1+\delta)S^* - \beta(e_r^1 + e_f^1)$, the sum of the parties' expected payoffs of going to court. The surplus from settlement equals $(1+\delta)S^*$ minus any additional enforcement costs. Because no court costs would be incurred, there would be no additional enforcement costs in the first period. In the second period, however, the parties would face exactly the same conditions as those in the one-period model and therefore they would incur the equilibrium cost of e^b each. Thus the net payoff from settlement would be $(1+\delta)S^* - 2\delta e^b$. Comparing this to the surplus of going to court and then bargaining, we determine that the parties will go to court if and only if:

$$2\delta e^b > \beta(e_r^1 + e_f^1) \tag{25}$$

The two parties will thus go to court if the enforcement efforts chosen in the first period are small enough. Low marginal cost of going to court (i.e., low β), low discounting of the future (high δ), and high one-period equilibrium efforts e^b. To determine whether equilibrium efforts will ever satisfy (25) first we need to define the appropriate payoff functions. For (e_r^1, e_f^1) combinations that satisfy (25), the parties will go to court and engage in ex-post bargaining; otherwise, the parties will settle ex-ante and split the surplus $(1+\delta)S^* - 2\delta e^b$ with the payoffs in (23) and (24) as threat payoffs. That is, the two-period payoff functions are:

(26)

$$W_r(e_r^1, e_f^1) = \begin{cases} \frac{(1+\delta)S^*}{2} + p^1\frac{(1+\delta)(1+c)B_r}{2} - (1+\beta)e_r^1 & \text{if } \; 2\delta e^b > \beta(e_r^1 + e_f^1) \\ \frac{(1+\delta)S^*}{2} - \delta e^b + p^1\frac{(1+\delta)(1+c)B_r}{2} + \frac{\beta}{2}e_f^1 - (1+\frac{\beta}{2})e_r^1 & \text{otherwise} \end{cases}$$

and

(27)

$$W_f(e_r^1, e_f^1) = \begin{cases} \frac{(1+\delta)S^*}{2} - p^1\frac{(1+\delta)(1+c)B_r}{2} - (1+\beta)e_f^1 & \text{if } \; 2\delta e^b > \beta(e_r^1 + e_f^1) \\ \frac{(1+\delta)S^*}{2} - \delta e^b - p^1\frac{(1+\delta)(1+c)B_r}{2} + \frac{\beta}{2}e_r^1 - (1+\frac{\beta}{2})e_f^1 & \text{otherwise} \end{cases}$$

For $f(e) = e$, it can be shown that going-to-court occurs if and only if $\frac{\beta}{2-\beta} < \delta$, or when the marginal cost of going to court is not too high and the second period is not discounted heavily.[15] This result can be shown more generally. Moreover, regardless of whether the two parties go to court,the initial assignment of property rights does not affect total enforcement efforts.

Proposition 5 *Consider the two-period model, whereby going to court in the first period determines who has the property rights in both periods. Then:*
(i) There are combinations of costs of going to court (β) and discount factors (δ) for which going to court and settling ex post is the subgame perfect equilibrium; and
(ii) Whether the two parties bargain ex post or ex ante in equilibrium, and given a level of ambiguity of property rights of $\varphi' > 1/2$, the equilibrium efforts and the net surplus available for division between the two parties are independent of the initial assignment of rights.

V.B. Going to Court Without Settlement

Even when the parties go to court, part *(ii)* of Proposition 5 shows that a version of the Coase theorem holds. The fact that there is settlement after the parties go to court is critical for this result, for settlement allows the two parties to split the prize that they are going after which in turn induces identical enforcement efforts in equilibrium.

There are, however, at least two potential problems with bargaining and settlement in a dynamic context. First, as mentioned earlier, any kind of

[15]From (9), we have $e^b = \frac{\varphi(1-\varphi)(1+c)B_r}{2+\beta}$. Assuming $2\delta e^b > \beta(e_r^1 + e_f^1)$ in (26) and 27), we find that $e_r^1 = e_f^1 = e^p = \frac{\varphi(1-\varphi)((1+\delta)1+c)B_r}{2(1+\beta)}$. It is straightforward to show that $2\delta e^b > \beta(e_r^1 + e_f^1) = 2\beta e^p$ if and only if $\frac{\beta(2+\beta)}{1+\beta} < \frac{2\delta}{1+\delta}$, which in turn is equivalent to $\frac{\beta}{2-\beta} < \delta$.

bargaining – whether ex ante or ex post – would be difficult to take place without inducing some erosion of a party's property right. If for example the farmer had acquired the right to choose x but acquiesced to choose x^* in exchange for some transfer from the rancher, the rancher could possibly use that choice of x as evidence against the rancher's right at some point in the future.

Second, evidence suggests that very little bargaining, if at all, takes place after court decisions are made. For example, Ward Farnsworth (1999, at page 373)[16] "examines twenty nuisance cases and finds no bargaining after judgment in any of them; nor did the parties' lawyers believe that bargaining would have occurred if judgment had been given to the loser. The lawyers said that the possibility of such bargaining was foreclosed not by the sorts of transaction costs that usually are the subject of economic models, but by animosity between the parties and their distaste for bargaining over the rights at issue." Animosity and the use of emotions for strategic purposes has been noted by some economists (Thomas C. Schelling, 1960; Hirshleifer, 2001, Chapter 10) as a commitment device. Is it possible, then, as it was in the static model that going to court could yield higher ex ante payoffs than those that allow for bargaining? To answer that question, we first define the two-period payoff functions when both parties expect to go to court:

$$W_r^c(e_r^1, e_f^1) \equiv p(e_r^1, e_f^1)(1+\delta)B_r - (1+\beta)e_r^1 \tag{28}$$

[16]Quoted in http://www.cooter-ulen.com, supplement to Cooter and Thomas Ulen (2000).

$$W_f^c(e_r^1, e_f^1) \equiv -p(e_r^1, e_f^1)(1+\delta)C_r - (1+\beta)e_f^1 \tag{29}$$

Note that these expected payoff functions differ from those of the one-period model in (2) and (3) only in that the first term of each of them is multiplied by $(1+\delta)$. A moment's reflection can show why this is a sensible property. Since the parties will go to court in the first period, the court's decision will determine who has the property right in both periods, and no bargaining will ever take place; what matters is the total "prize" over the two periods which is the sum of the first period prize and the discounted sum of the second period prize. All enforcement effort is undertaken in the first period.

Given this similarity of the payoff functions of going to court of the one-period and two-period models, it is trivial to show the same properties of equilibrium for the two-period model as those described in Proposition 3. Furthermore, although the welfare comparisons are not exactly the same in the two-period model as they were in the one-period model, a two-period version of Proposition 4 holds here as well: Going to court can be better for at least one party than allowing any bargaining. We summarize these findings in the following:

Proposition 6 *Consider the two-period model. Suppose the two parties expect to go to court and their payoff functions are described in (28) and (29). Then:*

(i) The party with more at stake devotes more resources to enforcement activities:

$$e_r^{1c} < e_f^{1c} \quad \textit{for } c > 1$$
$$e_r^{1c} = e_f^{1c} \quad \textit{for } c = 1$$
$$e_r^{1c} > e_f^{1c} \quad \textit{for } c < 1$$

(ii) Given a level of ambiguity of property rights of $\varphi' > 1/2$, it is efficient to assign these rights to the party that has more at stake (to the rancher if $c < 1$; to the farmer if $c > 1$); and
(iii) At least one party may ex ante prefer going to court over bargaining and settlement.

VI. Concluding Remarks

We have intentionally kept any asymmetries of information or power and concavities or income effects outside the model so that the conditions conform as closely as possible to the basic formulation of the Coase theorem with zero transaction costs. This way we have been able to focus on the effect of enforcement costs. What is somewhat surprising is the possibility that going court can be an equilibrium or ex ante Pareto superior when the costs of enforcement are taken into account. That is when not only the Coase theorem does not hold, but also a very simple rule – based on the cost and benefits of the activity that produces the externality – can be used to assign the more efficient property rights structure. This optimality of a targeted assignment of property rights comes about because the absence of a negotiated settlement introduces an asymmetry in the payoffs of the two parties, which translates in different enforcement efforts. The introduction of other asymmetries in the model would similarly induce different enforcement efforts. Two types of asymmetries that could be readily introduced are differential bargaining power or a liquidity constraint for one party that limits its ability to incur enforcement costs. Despite the different enforcement efforts that would be induced, it is unclear whether simple rules for the initial assignment of property rights can be found as we found for the case of going to court subsequent without negotiation.

Appendix

Proof of Proposition 1, Part (i): Consider any given (e_r, e_f) and the associated win probability of the rancher $p \equiv \frac{\varphi f(e_r)}{[\varphi f(e_r)+(1-\varphi)f(e_f)]}$. Our objective is to find the appropriate payoff functions taking into account that bargaining and settlement will take place. The disagreement or threat payoffs at the ex post bargaining stage (stage 2) are those that would be induced from going to court. In turn, these payoff would depend on what can be expected to occur at the stage of ex post bargaining. We therefore proceed by backward induction, beginning with the last stage of the game of ex post bargaining. Because the court has decided at this stage, there are two possible bargaining outcomes depending on whether the rancher or the farmer has won the right to choose x. If the rancher has won the threat payoffs would be B_r for the rancher and $-C_r$ for the farmer. Given that the surplus is S^*, the split-the-surplus rule would then imply the following payoffs for the two parties:

$$V_{rr} = \frac{S^* + B_r + C_r}{2} \quad \text{and} \quad V_{fr} = \frac{S^* - B_r - C_r}{2}$$

Note that no e_r or e_f appear in these expressions because enforcement expenditures have already been incurred at the initial and court stages of the game and thus represent sunk costs at the ex post bargaining stage. If the rancher were to win the right to choose x, the disagreement payoffs for the rancher and the farmer would both be 0, implying the following ex post bargaining payoffs:

$$V_{rf} = \frac{S^*}{2} \quad \text{and} \quad V_{ff} = \frac{S^*}{2}$$

The expected payoffs of the two parties just before going to court would be:

$$V_r = pV_{rr} + (1-p)V_{fr} - \beta e_r \text{ and } V_f = pV_{fr} + (1-p)V_{ff} - \beta e_f$$

Substitution from the expressions above then yields:

$$\begin{aligned} V_r &= \frac{S^*}{2} + p\frac{B_r + C_r}{2} - \beta e_r \\ V_f &= \frac{S^*}{2} - p\frac{B_r + C_r}{2} - \beta e_f \end{aligned}$$

Note that the costs of going to court for each party, βe_r for the rancher and βe_f for the farmer, are included here since the have yet to be incurred at the ex ante bargaining stage (stage 2). At that stage, the split-the-surplus rule then implies, the following payoffs:

$$\begin{aligned} V_r^{ab} &= \frac{S^* + V_r - V_f}{2} = \frac{S^*}{2} + p\frac{B_r + C_r}{2} + \frac{\beta e_f}{2} - \frac{\beta e_r}{2} \\ V_f^{ab} &= \frac{S^* + V_f - V_r}{2} = \frac{S^*}{2} - p\frac{B_r + C_r}{2} + \frac{\beta e_r}{2} - \frac{\beta e_f}{2} \end{aligned}$$

The payoff functions in Proposition 1 are obtained by subtracting the expenditures of each party at the first stage of the game (e_r for the rancher and e_f for the farmer).

Part (ii): To prove the second part of the Proposition, suppose the game would end without any negotiations once a court decision were to be made. Then, with the initial choices (e_r, e_f) given, the expected payoffs before going to court are:

$$V_r' = pB_r - \beta e_r \text{ and } V_f' = -pC_r - \beta e_f$$

Again, the payoffs at the ex ante bargaining stage would then be $V_r^{ab\prime} = \frac{S^*+V_r'-V_f'}{2}$ and $V_f^{ab\prime} = \frac{S^*+V_f'-V_r'}{2}$. It is a matter or simple algebra to show that these payoff are identical to those in V_r^{ab} and V_f^{ab} above. Hence, the payoff functions would be the same as those in the statement of Proposition 1.

Proof of Proposition 3, Part (ii): Suppose, at the rancher's optimum x_r, the cost to the farmer is higher than the benefit to the rancher, so that $B(x_r) < C(x_r)$. We need to compare the sum of the equilibrium efforts $e_f^c + e_r^c$ when $\varphi = 1 - \varphi'$ to the same sum when $\varphi = \varphi'$. Let us define the notation:

$$\underline{e_f} \equiv e_f^c\big|_{\varphi=1-\varphi'} \quad , \quad \underline{e_r} \equiv e_r^c\big|_{\varphi=1-\varphi'} \quad , \quad \overline{e_f} \equiv e_f^c\big|_{\varphi=\varphi'} \quad \text{and} \quad \overline{e_r} \equiv e_r^c\big|_{\varphi=\varphi'}$$

Recall from the first order conditions that, in any Nash equilibrium, we must have:

$$g(e_r) = cg(e_f)$$

where $g(\cdot)$ is a monotonically decreasing function and $c \equiv C(x_r)/B(x_r)$. Therefore we have:

$$g(\overline{e_r}) = cg(\overline{e_f})$$

and:

$$g(\underline{e_r}) = cg(\underline{e_f})$$

These conditions also means that $\underline{e_f}$, $\overline{e_f}$ and $\underline{e_r}$, ,$\overline{e_r}$ must "move" in the same direction. To see this, suppose that $\underline{e_r} < \overline{e_r}$. Then:

$$cg(\underline{e_f}) = g(\underline{e_r}) > g\left(\overline{e_r}\right) = cg(\overline{e_f})$$

and so:

$$g(\underline{e_f}) > g(\overline{e_f})$$

and therefore $\underline{e_f} < \overline{e_f}$. Conversely, if $\underline{e_r} > \overline{e_r}$, then we must also have $\underline{e_f} > \overline{e_f}$ by the same reasoning. Thus, to prove the result, it suffices to show that $c > 1$ implies that $\underline{e_r} < \overline{e_r}$. Suppose, to the contrary that $c > 1$ and $\underline{e_r} \geq \overline{e_r}$. Then, we also have $\underline{e_f} \geq \overline{e_f}$. The first order conditions for the rancher imply that when $\varphi = 1 - \varphi'$, we have:

$$1 = \frac{(1-\varphi')\,\varphi' f'(\underline{e_r}) f(\underline{e_f})}{[(1-\varphi')\,f(\underline{e_r}) + \varphi' f(\underline{e_f})]^2} B(x_r)$$

and, when $\varphi = \varphi'$, we have:

$$1 = \frac{(1-\varphi')\,\varphi' f'(\overline{e_r}) f(\overline{e_f})}{[\varphi' f(\overline{e_r}) + (1-\varphi')\,f(\overline{e_f})]^2} B(x_r)$$

These two conditions imply that:

$$\frac{f'(\underline{e_r}) f(\underline{e_f})}{[(1-\varphi')\,f(\underline{e_r}) + \varphi' f(\underline{e_f})]^2} = \frac{f'(\overline{e_r}) f(\overline{e_f})}{[\varphi' f(\overline{e_r}) + (1-\varphi')\,f(\overline{e_f})]^2}$$

Let us consider the function:

$$p_r(e_r, e_f) = \frac{\varphi(1-\varphi) f'(e_r) f(e_f)}{[\varphi f(e_r) + (1-\varphi) f(e_f)]^2}$$

It is straightforward to show that this function is decreasing in each of its arguments separately when $c > 1$ and $\varphi < 1/2$. To see this, note that:

$$\operatorname{sgn} \frac{\partial p_r}{\partial e_r} = \operatorname{sgn} \left\{ f''(e_r)[\varphi f(e_r) + (1-\varphi)\,f(e_f)] - 2\varphi \left[f'(e_r)\right]^2 \right\} < 0$$

where the inequality follows from the assumption that $f'' < 0$. Also, when

$c > 1$ and when $\varphi < 1/2$, we have:

$$\begin{aligned}\operatorname{sgn}\frac{\partial p_r}{\partial e_f} &= \operatorname{sgn}\left\{\left[\varphi f(e_r) + (1-\varphi)\, f(e_f)\right] - 2\,(1-\varphi)\, f(e_f)\right\} \\ &= \operatorname{sgn}[\varphi f(e_r) - (1-\varphi)\, f(e_f)] \\ &< \operatorname{sgn}\left[\varphi f(e_f) - (1-\varphi)\, f(e_f)\right] \\ &= \operatorname{sgn}\left[f(e_f)\,(2\varphi - 1)\right] < 0\end{aligned}$$

where the second last inequality follows from the fact that $e_r < e_f$ when $c > 1$ and the final inequality follows from the fact that $2\varphi - 1 < 0$ when $\varphi < 1/2$. Therefore, assuming that $c > 1$ and $\overline{e_r} \leq \underline{e_r}$ we have:

$$\begin{aligned}\frac{f'(\underline{e_r})f(\underline{e_f})}{[(1-\varphi')\, f(\underline{e_r}) + \varphi' f(\underline{e_f})]^2} &\leq \frac{f'(\overline{e_r})f(\underline{e_f})}{[(1-\varphi')\, f(\overline{e_r}) + \varphi' f(\underline{e_f})]^2} \\ &\leq \frac{f'(\overline{e_r})f(\overline{e_f})}{[(1-\varphi')\, f(\overline{e_r}) + \varphi' f(\overline{e_f})]^2} \\ &< \frac{f'(\overline{e_r})f(\overline{e_f})}{[\varphi' f(\overline{e_r}) + (1-\varphi')\, f(\overline{e_f})]^2} = \frac{f'(\underline{e_r})f(\underline{e_f})}{[(1-\varphi')\, f(\underline{e_r}) + \varphi' f(\underline{e_f})]^2}\end{aligned}$$

The first inequality follows from the fact that $p_r(e_r, .)$ is a decreasing function of e_r and the assumption that $\overline{e_r} \leq \underline{e_r}$. The second inequality follows from the fact that $p_r(., e_f)$ is a decreasing function of e_f when $\varphi = 1 - \varphi' < 1/2$ and the fact that $\overline{e_r} \leq \underline{e_r}$ also implies that $\overline{e_f} \leq \underline{e_f}$. The last inequality follows from the fact that, since $c > 1$ implies that $\overline{e_r} < \overline{e_f}$, we must also have $(1-\varphi')\, f(\overline{e_r}) + \varphi' f(\overline{e_f}) > \varphi' f(\overline{e_r}) + (1-\varphi')\, f(\overline{e_f})$. The last equality, which follows from the equality of the first order conditions when $\varphi = 1 - \varphi'$ and $\varphi = \varphi'$, gives a contradiction. Thus, it must be the case that $\underline{e_r} < \overline{e_r}$, from which it also follows that $\underline{e_f} < \overline{e_f}$, and so $\underline{e_f} + \underline{e_r} < \overline{e_f} + \overline{e_r}$, as required. The second part of the result, that $c < 1$ implies that $\overline{e_f} + \overline{e_r} < \underline{e_f} + \underline{e_r}$, can be proved in a similar fashion.

Proof of Proposition 4: Our objective is to find parameter values for which $V_r^c(e_r^c, e_f^c)$ in (21) attains a higher value than $V_r^b(e^b, e^b)$ in (10) or $V_f^c(e_r^c, e_f^c)$ in(22) has a lower value than $V_f^b(e^b, e^b)$ in (11). First, note that $V_r^c(e_r^c, e_f^c) > V_r^b(e^b, e^b)$ is equivalent to:

$$\varphi B_r \left(\frac{\varphi}{(\varphi + (1-\varphi)c)^2} - \frac{(\beta + 2\varphi)(1+c)}{2(2+\beta)} \right) > \frac{S^*}{2}$$

Note that the left-hand-side of this inequality is continuous in c and its limit as $c \to 0$ exists and equals the value of the left-hand-side at $c = 0$. That limit can be shown to equal $B_r(2-\varphi)/2 > B_r/2$. Note that $B_r = B(x_r) \geq B(x^*)$ since x_r maximizes $B(x)$. Therefore, $B_r(2-\varphi)/2 > B(x^*)/2 > (B(x^*) - C(x^*))/2 = S^*/2$ and the limit of the left–hand-side of the equation above as $c \to 0$ is strictly greater than its right-hand-side. Hence,for c sufficiently close to 0, we must have $V_r^c(e_r^c, e_f^c) > V_r^b(e^b, e^b)$. Next, to the farmer's equilibrium payoffs, $V_f^c(e_r^c, e_f^c) > V_f^b(e^b, e^b)$ can be shown to be equivalent to:

$$\varphi B_r \left(\frac{(4+\beta-2\varphi)(1+c)}{2(2+\beta)} - \frac{(\varphi + 2(1-\varphi)c)c}{(\varphi + (1-\varphi)c)^2} \right) > \frac{S^*}{2}$$

Again, we will follow the same method as in the case of the rancher above and consider the limit of the left-hand-side as $c \to 0$ of the inequality above which equals $\varphi B_r \frac{4+\beta-2\varphi}{2(2+b)}$. For φ sufficiently large, this limit can be shown to be greater than $S^*/2$, and therefore the for c close enough to 0, we must have $V_f^c(e_r^c, e_f^c) > V_f^b(e^b, e^b)$. [Note that the conditions for the farmer's payoff being higher under going-to-court are more stringent than the equivalent conditions for the rancher. However, the opposite can be shown to hold when c is sufficiently small. In that case the conditions for the farmer are much less stringent, whereas for large c it is impossible for the rancher's payoff under going-to-court to be higher than that under bargaining.]

Proof of Proposition 5, Part (i): Suppose initially that (25) is satisfied ($2\delta e^b > \beta(e_r^1, e_f^1)$) and derive the implied Nash equilibrium using the payoff functions in (26). Such an equilibrium is symmetric with $e_r^1 = e_f^1 = e^p$, which is implicitly defined by:

$$\frac{\varphi(1-\varphi)f'(e^p)}{f(e^p)}\frac{(1+\delta)(1+c)B_r}{2} - (1+\beta) = 0$$

Condition (25) then reduces to $\delta e^b > \beta e^p$, where e^b is implicitly defined in (8) (note that $(1+c)B_r = B_r + C_r$). (25) is automatically satisfied for combinations of $\beta = 0$ and any $\delta > 0$. Both e^b and e^p are differentiable, and therefore continuous,functions of β. Thus, (25) must be satisfied for other combinations of β and δ, with β close enough to zero.

Part (ii): From the implicit definition of e^p above, it is clear that e^p does not depend on whether $\varphi = \varphi'$ or $\varphi = 1 - \varphi'$. Therefore, when the two parties bargain ex ante, equilibrium efforts and net surplus are independent of the initial assignment of rights. When the two parties bargain ex ante, it is straightforward to show the same result.

Proof of Proposition 6: The proofs of parts (i) and (ii) of the Proposition are virtually identical to the proofs of parts (i) and $(ii)$of Proposition 3. (The only difference is that the payoff functions in the two period model are $(1+\delta)$ multiples of the one period payoff functions, but the comparative statics can easily be shown to be identical.) We therefore concentrate on proving part (iii), a major part of which is identical to the proof of Proposition 4. As in that proof, we consider the case of $f(e) = e$. Then, the equilibrium payoffs under (28) and (29) are:

$$\begin{aligned} W_r^c &= \frac{\varphi^2(1+\delta)B_r}{(\varphi+(1-\varphi)c)^2} \\ W_f^c &= -\frac{\varphi(\varphi+2(1-\varphi)c)(1+\delta)cB_r}{(\varphi+(1-\varphi)c)^2} \end{aligned}$$

Note that these payoffs are just $V_i^c(e_i^c, e_i^c)$ multiplied by $(1+\delta)$. We need to compare these payoffs to those that correspond to the equilibrium under (either ex ante or ex post) settlement with the payoff functions in (26) and (27). When the two sides settle ex ante, the comparison is identical to that in the proof of Proposition 4, except that all payoffs are to be multiplied by $(1+\delta)$ without affecting the comparisons. When the two sides settle ex post under (26) and (27), with $2\delta e^b > \beta(e_r^1 + e_f^1)$ in equilibrium, the equilibrium payoffs become:

$$\begin{aligned} W_r^{ep} &= \frac{(1+\delta)S^*}{2} + \frac{\varphi^2(1+\delta)(1+c)B_r}{2} \\ W_f^{ep} &= \frac{(1+\delta)S^*}{2} - \frac{\varphi(2-\varphi)(1+\delta)(1+c)B_r}{2} \end{aligned}$$

Note first that $W_r^c > W_r^{ep}$ if and only if:

$$\varphi^2 B_r \left(\frac{1}{(\varphi+(1-\varphi)c)^2} - \frac{1+c}{2} \right) > \frac{S^*}{2}$$

The left-hand-side of this inequality is continuous in c and its limit as $c \to 0$ exists and equals the value of the left-hand-side at $c = 0$. That limit can be shown to equal $B_r(2-\varphi^2)/2 > B_r/2$. Note that $B_r = B(x_r) \geq B(x^*)$ since x_r maximizes $B(x)$. Therefore, $B_r(2-\varphi)/2 > B(x^*)/2 > (B(x^*) - C(x^*))/2 = S^*/2$ and the limit of the left–hand-side of the equation above as $c \to 0$ is strictly greater than its right-hand-side. Hence, for c sufficiently close to 0, we must have $W_r^c > W_r^{ep}$. Next, we have $W_f^c > W_f^{ep}$ if and only if:

$$\varphi B_r \left(\frac{(2-\varphi)(1+c)}{2} - \frac{(\varphi + 2(1-\varphi)c)c}{(\varphi + (1-\varphi)c)^2} \right) > \frac{S^*}{2}$$

Again, as above, the left-hand-side of this inequality is continuous in c and its limit as $c \to 0$ exists and equals the value of the left-hand-side at $c = 0$. The limit at $c = 0$ equals $\varphi(2-\varphi)B_r/2$, which for sufficiently large φ is greater than $S^*/2$ and the above inequality holds. By continuity, then, $W_f^c > W_f^{ep}$ for c small enough and large enough φ. Thus, as required in part *(iii)* of the Proposition, we have found conditions under which going to court and never negotiating is preferable by at least one party.

References

[1] Anbarci, Nejat, Skaperdas, Stergios and Syropoulos, Constantinos (2001) "Comparing Bargaining Solutions in the Shadow of Conflict: How Norms Against Threats Can Have Real Effects," *Journal of Economic Theory,* forthcoming.

[2] Bernardo, Antonio E., Talley, Eric and Welch, Ivo (2000) "A Theory of Legal Presumptions," *Journal of Law, Economics, and Organization*,16(1), 1-49.

[3] Binmore, Ken., Rubinstein, Ariel. and Wolinsky, Asher (1986) "The Nash Bargaining Solution in Economic Modeling," *Rand Journal of Economics*, 17, 176-188.

[4] Buchanan, James M. (1989) "Notes on Irrelevant Externalities, Enforcement Costs, and the Atrophy of Property Rights," in Robert D.Tollison and Victor J. Vanberg (eds), *Explorations Into Constitutional Economics*, Texas A&M University Economics Series, no. 9, College Station, TX: Texas A&M University Press, pp. 359-70.

[5] Bull, Jesse and Watson, Joel (2001) "Evidence Disclosure and Verifiability," Department of Economics, University of California, San Diego.

[6] Clark, Derek. and Riis, Christian (1998) "Contest Success Functions: An Extension," *Economic Theory,* 11, 201-204.

[7] Coase, Ronald H. (1960) "The Problem of Social Cost " *Journal of Law and Economics*, 3, 1-44.

[8] ——————(1992) "The Institutional Structure of Production," *American Economic Review*, September, 82, 713-719.

[9] Cooter, Robert D. (1982) "The Cost of Coase," *Journal of Legal Studies*, 11(1), 1-34.

[10] ______________ and Ulen, Thomas (2000) *Law and Economics,* Reading, MA: Addison Wesley.

[11] ______________ and Rubinfeld, Daniel L. (1989), "The Economic Analysis of Legal Disputes and their Resolution," *Journal of Economic Literature*, September, 27, 1067-97.

[12] Daughety, Andrew F. and Reinganum, Jennifer F. (2000) "On the Economics of Trials: Adversarial Process, Evidence, and Equilibrium Bias," *Journal of Law, Economics, and Organization*, 16(2), 365-94.

[13] DeMeza, David. (1998) "Coase Theorem," in P. Newman (ed.), *The New Palgrave Dictionary of Economics and the Law*, pp. 270-282, New York: Stockton Press.

[14] Demsetz, Harold (1972) "When Does the Rule of Liability Matter?" *Journal of Legal Studies*, 1(1), 13-28.

[15] Dixit, Avinash K. (1987) "Strategic Behavior in Contests" *American Economic Review*, 77(5), 891-898.

[16] Farmer, Amy. and Pecorino, Paul (1999) "Legal Expenditure as a Rent Seeking Game", *Public Choice*, September, 100, 271-288.

[17] Farnsworth, Ward. (1999) "Do Parties in Nuisance Cases Bargain After Judgment? A Glimpse Inside the Cathedral," *University of Chicago Law Review*, 66, 373-436.

[18] Farrell, Joseph P. (1987) "Information and the Coase Theorem," *Journal of Economic Perspectives*, 1, 113-129.

[19] Fullerton, Richard L. and McAfee, R. Preston (1999) "Auctioning Entry into Tournaments," *Journal of Political Economy*, 107(3), 573-605.

[20] Garfinkel, Michelle R. and Skaperdas, Stergios (eds). (1996) *The Political Economy of Conflict and Appropriation*, New York: Cambridge University Press.

[21] ____________ and ____________ (2000) "Conflict Without Misperceptions or Incomplete Information," *Journal of Conflict Resolution*, December, 44(6), 793-807.

[22] Gradstein, Mark (1995) "Intensity of Competition, Entry and Entry Deterrence in Rent Seeking Contests ", *Economics and Politics* , 7(1): 79-91.

[23] Grossman, Sanford J. and Hart, Oliver D. (1986) "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy*, 84, 691-719.

[24] Hart, Oliver D. (1995) *Firms, Contracts, and Financial Structure*, Oxford: Clarendon Press.

[25] Hirshleifer, Jack (2001), *The Dark Side of the Force*, New York: Cambridge University Press.

[26] ____________ and Osborne, Evan (2001) "Truth, Effort and the Legal Battle", *Public Choice*, 108, 169-195.

[27] Jung, Chulho, Krutilla, Kerry, Viscusi, W. Kip and Boyd, Roy (1995) "The Coase Theorem in a Rent-Seeking Society" *International Review of Law and Economics, 15, 259-268.*

[28] Katz, Avery W. (1988) "Judicial Decisionmaking and Litigation Expenditure," *International Review of Law and Economics*, 8, 127-143.

[29] McKelvey, Richard D. and Page, Talbot (1998) "An Experimental Study of the Effect of Private Information in the Coase Theorem", Caltech Social Science Working Paper No. 1018, April.

[30] Myerson, Roger B. and Satterthwaite, Mark A. (1983) "Efficient Mechanisms for Bilateral Trading" *Journal of Economic Theory*, 28, 265-281.

[31] Rosen, Sherwin (1986) "Prizes and Incentives in Elimination Tournaments" *American Economic Review*, 76(4), 701-715.

[32] Schelling, Thomas C. (1960) *The Strategy of Conflict*, Cambridge, MA: Harvard University Press.

[33] Skaperdas, Stergios (1992) "Cooperation, Conflict and Power in the Absence of Property Rights " *American Economic Review*, 82, 720-39.

[34] Skaperdas, Stergios (1996) "Contest Success Functions " *Economic Theory*, 7, 283-290.

[35] Tullock, Gordon (1980) "Efficient Rent Seeking " in J. Buchanan, R. Tollison and G. Tullock (eds) *Toward a Theory of the Rent Seeking Society*, College Station: Texas A & M Press.

[36] Usher, Dan (1998) "The Coase Theorem is Tautological, Incoherent or Wrong," *Economics Letters*, 61, 3-11.