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Coupled coincidence point theorems for contractions without commutative condition in intuitionistic fuzzy normed spaces

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Abstract

Recently, Gordji et al. [Math. Comput. Model. 54, 1897-1906 (2011)] prove the coupled coincidence point theorems for nonlinear contraction mappings satisfying commutative condition in intuitionistic fuzzy normed spaces. The aim of this article is to extend and improve some coupled coincidence point theorems of Gordji et al. Also, we give an example of a nonlinear contraction mapping which is not applied by the results of Gordji et al., but can be applied to our results.

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1. Introduction

The classical Banach's contraction mapping principle first appear in [1]. Since this principle is a powerful tool in nonlinear analysis, many mathematicians have much contributed to the improvement and generalization of this principle in many ways (see [2-10] and others).

One of the most interesting is study to other spaces such as probabilistic metric spaces (see [11-15]). The fuzzy theory was introduced simultaneously by Zadeh [16]. The idea of intuitionistic fuzzy set was first published by Atanassov [17]. Since then, Saadati and Park [18] introduced the concept of intuitionistic fuzzy normed spaces (IFNSs). In [19], Saadati et al. have modified the notion of IFNSs of Saadati and Park [18].

Several researchers have applied fuzzy theory to the well-known results in many fields, for example, quantum physics [20], nonlinear dynamical systems [21], population dynamics [22], computer programming [23], fixed point theorem [24-27], fuzzy stability problems [28-30], statistical convergence [31-34], functional equation [35], approximation theory [36], nonlinear equation [37,38] and many others.

In the other hand, coupled fixed points and their applications for binary mappings in partially ordered metric spaces were introduced by Bhaskar and Lakshmikantham [39]. They applied coupled fixed point theorems to show the existence and uniqueness of a solution for a periodic boundary value problem. After that, Lakshmikantham and Ćirić

[40] proved some more generalizations of coupled fixed point theorems in partially ordered sets.

Recently, Gordji et al. [41] proved some coupled coincidence point theorems for contractive mappings satisfying commutative condition in partially complete IFNSs as follows:

Theorem 1.1 (Gordji et al. [41]). *Let* $(X, \preccurlyeq)$ *be a partially ordered set and* $(X, \mu, \nu, *, \Diamond)$ *a complete IFNS such that* (μ, ν) *has n-property and*

$$a \Diamond b \le ab \le a * b, \quad \forall a, b \in [0, 1]. \tag{1.1}$$

Let F: $X \times X \to X$ *and* $g : X \to X$ *be two mappings such that F has the mixed g-monotone property and*

$$\begin{aligned} &\mu(F(x, y) - F(u, v), kt) \ge \mu(gx - gu, t) * \mu(gy - gv, t), \forall x, y, u, v \in X, \\ &\nu(F(x, y) - F(u, v), kt) \le \nu(gx - gu, t) \Diamond \nu(gy - gv, t), \quad \forall x, y, u, v \in X, \end{aligned} \tag{1.2}$$

for which $g(x) \preccurlyeq g(u)$ *and* $g(y) \succcurlyeq z\, g(v)$, *where* $0 < k < 1$, $F(X \times X) \subseteq g(X)$, *g is continuous and g commuting with F. Suppose that either*

(1) *F is continuous or*
(2) *X has the following properties:*
  (a) *if* $\{x_n\}$ *is a non-decreasing sequence with* $\{x_n\} \to x$, *then* $gx_n \preccurlyeq gx$ *for all* $n \in \mathbb{N}$,
  (b) *if* $\{y_n\}$ *is a non-increasing sequence with* $\{y_n\} \to y$, *then* $gy \preccurlyeq gy_n$ *for all* $n \in \mathbb{N}$.

If there exist x_0, $y_0 \in X$ *such that*

$$g(x_0) \preccurlyeq F(x_0, y_0), \quad g(y_0) \succcurlyeq F(y_0, x_0),$$

then F and g have a coupled coincidence point in $X \times X$.

In this article, we improve the result given by Gordji et al. [41] without using the commutative condition and also give an example to validate the main results in this article. Our results improve and extend some couple fixed point theorems due to Gordji et al. [41] and other couple fixed point theorems.

2. Preliminaries

Now, we give some definitions, examples and lemmas for our main results in this article.

Definition 2.1 ([42]). A binary operation $*$: $[0,1]^2 \to [0,1]$ is called a *continuous t-norm* if ([0,1], $*$) is an abelian topological monoid, i.e.,

(1) $*$ is associative and commutative;
(2) $*$ is continuous;
(3) $a * 1 = a$ for all $a \in [0,1]$;
(4) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0,1]$.

Definition 2.2 ([42]). A binary operation $\Diamond$: $[0,1]^2 \to [0,1]$ is called a *continuous t-conorm* if ([0,1],$\Diamond$) is an abelian topological monoid, i.e.,

(1) $\lozenge$ is associative and commutative;
(2) $\lozenge$ is continuous;
(3) $a \lozenge 0 = a$ for all $a \in [0,1]$;
(4) $a \lozenge b \leq c \lozenge d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Using the continuous t-norm and t-conorm, Saadati and Park [18] introduced the concept of IFNSs.

Definition 2.3 ([18]). The 5-tuple $(X, \mu, \nu, *, \lozenge)$ is called an IFNS if X is a vector space, $*$ is a continuous t-norm, $\lozenge$ is a continuous t-conorm and μ, ν are fuzzy sets on $X \times (0, \infty)$ satisfying the following conditions: for all $x, y \in X$ and $s, t > 0$,

(IF_1) $\mu(x, t) + \nu(x, t) \leq 1$;
(IF_2) $\mu(x, t) > 0$;
(IF_3) $\mu(x, t) = 1$ if and only if $x = 0$;
(IF_4) $\mu(\alpha x, t) = \mu\left(x, \frac{t}{|\alpha|}\right)$ for all $\alpha \neq 0$;
(IF_5) $\mu(x, t) * \mu(y, s) \leq \mu(x + y, t + s)$;
(IF_6) $\mu(x,.): (0, \infty) \to [0,1]$ is continuous;
(IF_7) μ is a non-decreasing function on $\mathbb{R}^+$,

$$\lim_{t\to\infty} \mu(x,t) = 1, \quad \lim_{t\to 0} \mu(x,t) = 0;$$

(IF_8) $\nu(x, t) < 1$;
(IF_9) $\nu(x, t) = 0$ if and only if $x = 0$;
(IF_{10}) $\nu(\alpha x, t) = \nu\left(x, \frac{t}{|\alpha|}\right)$ for all $\alpha \neq 0$;
(IF_{11}) $\nu(x, t) \lozenge \nu(y, s) \geq \nu(x + y, t + s)$;
(IF_{12}) $\nu(x,\cdot): (0, \infty) \to [0,1]$ is continuous;
(IF_{13}) ν is a non-increasing function on $\mathbb{R}^+$,

$$\lim_{t\to\infty} \nu(x,t) = 0, \quad \lim_{t\to 0} \nu(x,t) = 1.$$

In this case, (μ, ν) is called an *intuitionistic fuzzy norm.*

Definition 2.4 ([18]). Let $(X, \mu, \nu, *, \lozenge)$ be an IFNS.

(1) A sequence $\{x_n\}$ in X is said to be *convergent* to a point $x \in X$ with respect to the intuitionistic fuzzy norm (μ, ν) if, for any $\varepsilon > 0$ and $t > 0$, there exists $k \in \mathbb{N}$ such that

$$\mu(x_n - x, t) > 1 - \varepsilon, \quad \nu(x_n - x, t) < \varepsilon, \quad \forall n \geq k.$$

In this case, we write $\lim_{n\to\infty} x_n = x$. In fact that $\lim_{n\to\infty} x_n = x$ if $\mu(x_n - x, t) \to 1$ and $\nu(x_n - x, t) \to 0$ as $n \to \infty$ for every $t > 0$.

(2) A sequence $\{x_n\}$ in X is called a *Cauchy sequence* with respect to the intuitionistic fuzzy norm (μ, ν) if, for any $\varepsilon > 0$ and $t > 0$, there exists $k \in \mathbb{N}$ such that

$$\mu(x_n - x_m, t) > 1 - \varepsilon, \quad \nu(x_n - x_m, t) < \varepsilon, \quad \forall n, m \geq k.$$

This implies $\{x_n\}$ is Cauchy if $\mu(x_n - x_m, t) \to 1$ and $\nu(x_n - x_m, t) \to 0$ as $n, m \to \infty$ for every $t > 0$.

(3) An IFNS $(X, \mu, \nu, *, \Diamond)$ is said to be *complete* if every Cauchy sequence in $(X, \mu, \nu, *, \Diamond)$ is convergent.

Definition 2.5 ([43,44]). Let X and Y be two IFNS. A function $g : X \to Y$ is said to be *continuous* at a point $x_0 \in X$ if, for any sequence $\{x_n\}$ in X converging to a point $x_0 \in X$, the sequence $\{g(x_n)\}$ in Y converges to a point $g(x_0) \in Y$. If $g : X \to Y$ is continuous at each $x \in X$, then $g : X \to Y$ is said to be *continuous* on X.

Example 2.6 ([41]). Let $(X, ||\cdot||)$ be an ordinary normed space and θ an increasing and continuous function from $\mathbb{R}^+$ into (0,1) such that $\lim_{t\to\infty} \theta(t) = 1$. Four typical examples of these functions are as follows:

$$\theta(t) = \frac{t}{t+1}, \quad \theta(t) = \sin\left(\frac{\pi t}{2t+1}\right), \quad \theta(t) = 1 - e^{-t}, \quad \theta(t) = e^{\frac{-1}{t}}.$$

Let $a * b = ab$ and $a \Diamond b \geq ab$ for all $a, b \in [0,1]$. If, for any $t \in (0, \infty)$, we define

$$\mu(x,t) = [\theta(t)]^{||x||}, \quad \nu(x,t) = 1 - [\theta(t)]^{||x||}, \quad \forall x \in X,$$

then $(X, \mu, \nu, *, \Diamond)$ is an IFNS.

The other basic properties and examples of IFNSs are given in [18].

Definition 2.7 ([41]). Let $(X, \mu, \nu, *, \Diamond)$ be an IFNS. (μ, ν) is said to satisfy the *n-property* on $X \times (0, \infty)$ if

$$\lim_{n\to\infty} [\mu(x, k^n t)]^{n^p} = 1, \quad \lim_{n\to\infty} [\nu(x, k^n t)]^{n^p} = 0$$

whenever $x \in X$, $k > 1$ and $p > 0$.

For examples for n-property see in [41]. Next, we give some notion in coupled fixed point theory.

Definition 2.8 ([39]). Let X be a non-empty set. An element $(x, y) \in X \times X$ is call a *coupled fixed point* of the mapping $F : X \times X \to X$ if

$$x = F(x,y), \quad y = F(y,x).$$

Definition 2.9 ([40]). Let X be a non-empty set. An element $(x, y) \in X \times X$ is call a *coupled coincidence point* of the mappings $F : X \times X \to X$ and $g : X \to X$ if

$$g(x) = F(x,y), \quad g(y) = F(y,x).$$

Definition 2.10 ([39]). Let $(X, \preccurlyeq)$ be a partially ordered set and $F : X \times X \to X$ be a mapping. The mapping F is said to has the *mixed monotone property* if F is monotone non-decreasing in its first argument and is monotone non-increasing in its second argument, that is, for any $x, y \in X$

$$x_1, x_2 \in X, x_1 \preccurlyeq x_2 \quad \Rightarrow \quad F(x_1, y) \preccurlyeq F(x_2, y) \tag{2.1}$$

and

$$y_1, y_2 \in X, y_1 \preccurlyeq y_2 \quad \Rightarrow \quad F(x, y_1) \succcurlyeq F(x, y_2). \tag{2.2}$$

Definition 2.11 ([40]). Let $(X, \preccurlyeq)$ be a partially ordered set and $F : X \times X \to X$, $g : X \to X$ be mappings. The mapping F is said to has the *mixed g-monotone property* if F is monotone g-non-decreasing in its first argument and is monotone g-non-increasing in its second argument, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, g(x_1) \preccurlyeq g(x_2) \quad \Rightarrow \quad F(x_1, y) \preccurlyeq F(x_2, y) \tag{2.3}$$

and

$$y_1, y_2 \in X, g(y_1) \preccurlyeq g(y_2) \quad \Rightarrow \quad F(x, y_1) \succcurlyeq F(x, y_2). \tag{2.4}$$

Definition 2.12 ([40]). Let X be a non-empty set and $F : X \times X \to X$, $g : X \to X$ be mappings. The mappings F and g are said to be *commutative* if

$$g(F(x, y)) = F(g(x), g(y)), \quad \forall x, y \in X.$$

The following lemma proved by Haghi et al. [45] is useful for our main results:

Lemma 2.13 ([45]). *Let X be a nonempty set and $g : X \to X$ be a mapping. Then, there exists a subset $E \subseteq X$ such that $g(E) = g(X)$ and $g : E \to X$* is *one-to-one.*

3. Main Results

First, we prove a coupled fixed point theorem for a mapping $F : X \times X \to X$ which is an essential tool in the partial order IFNSs to show the existence of coupled fixed point. Although the proof in Theorem 3.1 is not difficult to modify, it is an important theorem which is helpful in proving some coupled coincidence point theorems without commutative condition.

Theorem 3.1. *Let $(X, \preccurlyeq)$ be a partially ordered set and $(X, \mu, \nu, *, \diamond)$ a complete IFNS such that (μ, ν) has n-property and*

$$a \diamond b \leq ab \leq a * b, \quad \forall a, b \in [0, 1]. \tag{3.1}$$

Let $F : X \times X \to X$ be mapping such that F has the mixed monotone property and

$$\begin{aligned} \mu(F(x, y) - F(u, v), kt) &\geq \mu(x - u, t) * \mu(y - v, t), \ \forall x, y, u, v \in X, \\ \nu(F(x, y) - F(u, v), kt) &\leq \nu(x - u, t) \diamond \nu(y - v, t), \quad \forall x, y, u, v \in X, \end{aligned} \tag{3.2}$$

for which $x \preccurlyeq u$ and $y \succcurlyeq v$, where $0 < k < 1$. Suppose that either

(1) *F is continuous or*
(2) *X has the following properties:*
  (a) *if $\{x_n\}$ is a non-decreasing sequence with $\{x_n\} \to x$, then $x_n \preccurlyeq x$ for all $n \in \mathbb{N}$,*
  (b) *if $\{y_n\}$ is a non-increasing sequence with $\{y_n\} \to y$, then $y \preccurlyeq y_n$ for all $n \in \mathbb{N}$.*

If there exist $x_0, y_0 \in X$ such that

$$x_0 \preccurlyeq F(x_0, y_0), \quad y_0 \succcurlyeq F(y_0, x_0),$$

then F has a coupled fixed point in $X \times X$.

Proof. Let $x_0, y_0 \in X$ be such that

$$x_0 \preccurlyeq F(x_0, y_0), \quad y_0 \succcurlyeq F(y_0, x_0).$$

Since $F(X \times X) \subseteq X$, we can construct the sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$x_{n+1} = F(x_n, y_n), \quad y_{n+1} = F(y_n, x_n), \quad \forall n \geq 0. \tag{3.3}$$

Now, we show that

$$x_n \preccurlyeq x_{n+1}, \quad y_n \succcurlyeq y_{n+1}, \quad \forall n \geq 0. \tag{3.4}$$

In fact, by induction, we prove this. For $n = 0$, since $x_0 \preccurlyeq F(x_0, y_0) = x_1$ and $y_0 = F(y_0, x_0) \succcurlyeq y_1$, we show that (3.4) holds for $n = 0$. Suppose that (3.4) holds for any $n \geq 0$. Then, we have

$$x_n \preccurlyeq x_{n+1}, \quad y_n \succcurlyeq y_{n+1}. \tag{3.5}$$

Since F has the mixed monotone property, it follows from (3.5) and (2.1) that

$$F(x_n, y) \preccurlyeq F(x_{n+1}, y), \quad F(y_{n+1}, x) \preccurlyeq F(y_n, x), \quad \forall x, y \in X, \tag{3.6}$$

and also it follows from (3.5) and (2.2) that

$$F(y, x_n) \succcurlyeq F(y, x_{n+1}), \quad F(x, y_{n+1}) \succcurlyeq F(x, y_n), \quad \forall x, y \in X. \tag{3.7}$$

If we take $y = y_n$ and $x = x_n$ in (3.6), then we get

$$x_{n+1} = F(x_n, y_n) \preccurlyeq F(x_{n+1}, y_n), \quad F(y_{n+1}, x_n) \preccurlyeq F(y_n, x_n) = y_{n+1}. \tag{3.8}$$

If we take $y = y_{n+1}$ and $x = x_{n+1}$ in (3.7), then we get

$$F(y_{n+1}, x_n) \succcurlyeq F(y_{n+1}, x_{n+1}) = y_{n+2}, \quad x_{n+2} = F(x_{n+1}, y_{n+1}) \succcurlyeq F(x_{n+1}, y_n). \tag{3.9}$$

Hence, it follows from (3.8) and (3.9) that

$$x_{n+1} \preccurlyeq x_{n+2}, \quad y_{n+1} \succcurlyeq y_{n+2}. \tag{3.10}$$

Therefore, by induction, we conclude that (3.4) holds for all $n \geq 0$, that is,

$$x_0 \preccurlyeq x_1 \preccurlyeq x_2 \preccurlyeq \cdots \preccurlyeq x_n \preccurlyeq x_{n+1} \preccurlyeq \cdots \tag{3.11}$$

and

$$y_0 \succcurlyeq y_1 \succcurlyeq y_2 \succcurlyeq \cdots \succcurlyeq y_n \succcurlyeq y_{n+1} \succcurlyeq \cdots . \tag{3.12}$$

Define $\alpha_n(t) := \mu(x_n - x_{n+1}, t) * \mu(y_n - y_{n+1}, t)$. Then, using (3.2) and (3.3), we have

$$\begin{aligned} \mu(x_n - x_{n+1}, kt) &= \mu(F(x_{n-1}, y_{n-1}) - F(x_n, y_n), kt) \\ &\geq \mu(x_{n-1} - x_n, t) * \mu(y_{n-1} - y_n, t) \\ &= \alpha_{n-1}(t) \end{aligned} \tag{3.13}$$

and

$$\begin{aligned} \mu(y_n - y_{n+1}, kt) &= \mu(y_{n+1} - y_n, kt) \\ &= \mu(F(y_n, x_n) - F(y_{n-1}, x_{n-1}), kt) \\ &\geq \mu(y_n - y_{n-1}, t) * \mu(x_n - x_{n-1}, t) \\ &= \mu(y_{n-1} - y_n, t) * \mu(x_{n-1} - x_n, t) \\ &= \alpha_{n-1}(t). \end{aligned} \tag{3.14}$$

From the t-norm property, (3.13) and (3.14), it follows that

$$\alpha_n(kt) \geq \alpha_{n-1}(t) * \alpha_{n-1}(t). \tag{3.15}$$

From (3.1), we have

$$\alpha_{n-1}(t) * \alpha_{n-1}(t) \geq [\alpha_{n-1}(t)]^2. \tag{3.16}$$

By (3.15) and (3.16), we get $\alpha_n(kt) \geq [\alpha_{n-1}(t)]^2$ for all $n \geq 1$. Repeating this process, we have

$$\alpha_n(t) \geq \left[\alpha_{n-1}\left(\frac{t}{k}\right)\right]^2 \geq \cdots \geq \left[\alpha_0\left(\frac{t}{k^n}\right)\right]^{2n}, \tag{3.17}$$

which implies that

$$\mu(x_n - x_{n+1}, kt) * \mu(y_n - y_{n+1}, kt) \geq \left[\mu\left(x_0 - x_1, \frac{t}{k^n}\right)\right]^{2n} * \left[\mu\left(y_0 - y_1, \frac{t}{k^n}\right)\right]^{2n}. \tag{3.18}$$

On the other hand, we have

$$t(1-k)(1+k+\cdots+k^{m-n-1}) < t, \quad \forall m > n,\ 0 < k < t.$$

By property of t-norm, we get

$$\begin{aligned}
&\mu(x_n - x_m, t) * \mu(y_n - y_m, t)\\
&\geq \mu(x_n - x_m, t(1-k)(1+k+\cdots+k^{m-n-1}))\\
&\quad *\mu(y_n - y_m, t(1-k)(1+k+\cdots+k^{m-n-1}))\\
&\geq \mu(x_n - x_{n+1}, t(1-k)) * \mu(y_n - y_{n+1}, t(1-k))\\
&\quad *\mu(x_{n+1} - x_{n+2}, t(t-k)k) * \mu(y_{n+1} - y_{n+2}, t(1-k)k)\\
&\quad *\cdots\\
&\quad *\mu(x_{m-1} - x_m, t(1-k)k^{m-n-1}) * \mu(y_{m-1} - y_m, t(t-k)k^{m-n-1})\\
&\geq \mu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right) * \mu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\\
&\quad *\cdots\\
&\quad *\mu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right) * \mu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\\
&\geq \left[\mu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right]^{m-n} * \left[\mu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\right]^{m-n}\\
&\geq \left[\mu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right]^{m} * \left[\mu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\right]^{m}\\
&\geq \left[\mu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right]^{np} * \left[\mu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\right]^{np},
\end{aligned} \tag{3.19}$$

where $p > 0$ such that $m < n^p$. Sine (μ, ν) has the n-property, we have

$$\lim_{n\to\infty} \left[\mu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right]^{np} = 1$$

and so

$$\lim_{n\to\infty} \mu(x_n - x_m) * \mu(y_n - y_m) = 1. \tag{3.20}$$

Next, we claim that

$$\lim_{n\to\infty} \nu(x_n - x_m) \diamond \nu(y_n - y_m) = 0.$$

Define $\beta_n(t) := \nu(x_n - x_{n+1}, t) \diamond \nu(y_n - y_{n+1}, t)$. It follows from (3.2) and (3.3) that

$$\begin{aligned}
\nu(x_n - x_{n+1}, kt) &= \nu(F(x_{n-1}, y_{n-1}) - F(x_n, y_n), kt)\\
&\leq \nu(x_{n-1} - x_n, t) \diamond \nu(y_{n-1} - y_n, t)\\
&= \beta_{n-1}(t)
\end{aligned} \tag{3.21}$$

and

$$\begin{aligned}\nu(y_n - y_{n+1}, kt) &= \nu(y_{n+1} - y_n, kt)\\ &= \nu(F(y_n, x_n) - F(y_{n-1}, x_{n-1}), kt)\\ &\leq \nu(y_n - y_{n-1}, t)\diamond\nu(x_n - x_{n-1}, t)\\ &= \nu(y_{n-1} - y_n, t)\diamond\nu(x_{n-1} - x_n, t)\\ &= \beta_{n-1}(t).\end{aligned} \tag{3.22}$$

Thus, it follows from the notion of t-conorm, (3.21) and (3.22) that

$$\beta_n(kt) \leq \beta_{n-1}(t)\diamond\beta_{n-1}(t). \tag{3.23}$$

From (3.1), we have

$$\beta_{n-1}(t)\diamond\beta_{n-1}(t) \leq [\beta_{n-1}(t)]^2. \tag{3.24}$$

Thus, by (3.23) and (3.24), we get $\beta_n(kt) \leq [\beta_{n-1}(t)]^2$ for all $n \geq 1$. Repeating this process again, we have

$$\beta_n(t) \leq \left[\beta_{n-1}\left(\frac{t}{k}\right)\right]^2 \leq \cdots \leq \left[\beta_0\left(\frac{t}{k^n}\right)\right]^{2n}, \tag{3.25}$$

that is,

$$\nu(x_n - x_{n+1}, kt)\diamond\nu(y_n - y_{n+1}, kt) \leq \left[\nu\left(x_0 - x_1, \frac{t}{k^n}\right)\right]\diamond\left[\nu\left(y_0 - y_1, \frac{t}{k^n}\right)\right]^{2n}. \tag{3.26}$$

Since we have

$$t(1-k)(1+k+\cdots+k^{m-n-1}) < t, \quad \forall m > n,\ 0 < k < 1,$$

by the t-conorm property, we get

$$\begin{aligned}&\nu(x_n - x_m, t)\diamond\nu(y_n - y_m, t)\\ &\leq \nu(x_n - x_m, t(1-k)(1+k+\cdots+k^{m-n-1}))\\ &\quad\diamond\nu(y_n - y_m, t(1-k)(1+k+\cdots+k^{m-n-1}))\\ &\leq \nu(x_n - x_{n+1}, t(1-k))\diamond\nu(y_n - y_{n+1}, t(1-k)\\ &\quad\diamond\nu(x_{n+1} - x_{n+2}, t(1-k)k)\diamond\nu(y_{n+1} - y_{n+2}, t(1-k)k)\\ &\quad\diamond\cdots\\ &\quad\diamond\nu(x_{m-1} - x_{m,} t(1-k)k^{m-n-1})\diamond\nu(y_{m-1} - y_{m,} t(1-k)k^{m-n-1})\\ &\leq \nu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\diamond\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\\ &\quad\diamond\cdots\\ &\quad\diamond\nu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\diamond\nu\left(y_0 - y_1(1-k)\frac{t}{k^n}\right)\\ &\leq \left[\nu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right]^{m-n}\diamond\left[\nu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\right]^{m-n}\\ &\leq \left[\nu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right]^{m}\diamond\left[\nu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\right]^{m}\\ &\leq \left[\nu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p}\diamond\left[\nu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p},\end{aligned} \tag{3.27}$$

where $p > 0$ such that $m < n^p$. Sine (μ, ν) has the n-property, we have

$$\lim_{n\to\infty}\left[\nu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p} = 0$$

and so

$$\lim_{n\to\infty} \nu(x_n - x_m)\diamond\nu(y_n - y_m) = 0. \tag{3.28}$$

From (3.20) and (3.28), we know that the sequences $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X. Since X complete, there exist $x, y \in X$ such that

$$\lim_{n\to\infty} x_n = x, \quad \lim_{n\to\infty} y_n = y. \tag{3.29}$$

Next, we show that $x = F(x, y)$ and $y = F(y, x)$. If the assumption (1) holds, then we have

$$x = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} F(x_n, y_n) = F(\lim_{n\to\infty} x_n, \lim_{n\to\infty} y_n) = F(x, y) \tag{3.30}$$

and

$$y = \lim_{n\to\infty} y_{n+1} = \lim_{n\to\infty} F(y_n, x_n) = F(\lim_{n\to\infty} y_n, \lim_{n\to\infty} x_n) = F(y, x). \tag{3.31}$$

Therefore, $x = F(x, y)$ and $y = F(y, x)$, that is, F has a coupled fixed point.

Suppose that the assumption (2) holds. Since $\{x_n\}$ is non-decreasing and $x_n \to x$, it follows from (a) that $x_n \preccurlyeq x$ for all $n \in \mathbb{N}$. Similarly, we can conclude that $y_n \succcurlyeq y$ for all $n \in \mathbb{N}$. Then, by (3.2), we get

$$\begin{aligned}\mu(x_{n+1} - F(x, y), kt) &= \mu(F(x_n, y_n) - F(x, y), kt)\\ &\geq \mu(x_n - x, t) * \mu(y_n - y, t).\end{aligned} \tag{3.32}$$

Taking the limit as $n \to \infty$, we have $\mu(x - F(x, y), kt) = 1$ and so $x = F(x, y)$. Using (3.2) again, we have

$$\begin{aligned}\nu(y_{n+1} - F(y, x), kt) &= \nu(F(y, x) - y_{n+1}, kt)\\ &= \nu(F(y, x) - F(y_n, x_n), kt)\\ &\leq \nu(y - y_n, t)\diamond\nu(x - x_n, t)\\ &= \nu(y_n - y, t)\diamond\nu(x_n - x, t).\end{aligned} \tag{3.33}$$

Taking the limit as $n \to \infty$ in both sides of (3.33), we have $\nu(y - F(y, x), kt) = 0$ and then $y = F(y, x)$. Therefore, F has a coupled fixed point at (x, y). This completes the proof. □

Next, we prove the existence of coupled coincidence point theorem, where we do not require that F and g are commuting.

Theorem 3.2. *Let* $(X, \preccurlyeq)$ *be a partially ordered set and* $(X, \mu, \nu, *, \diamond)$ *a IFNS such that* (μ, ν) *has n-property and*

$$a\diamond b \leq ab \leq a * b, \quad \forall a, b \in [0, 1]. \tag{3.34}$$

Let $F : X \times X \to X$ *and* $g : X \to X$ be two *mappings such that F has the mixed g-monotone property and*

$$\begin{aligned}&\mu(F(x, y) - F(u, v), kt) \geq \mu(gx - gu, t) * \mu(gy - gv, t), \forall x, y, u, v \in X,\\ &\nu(F(x, y) - F(u, v), kt) \leq \nu(gx - gu, t)\diamond\nu(gy - gv, t), \quad \forall x, y, u, v \in X,\end{aligned} \tag{3.35}$$

for which $gx \preccurlyeq gu$ *and* $gy \succcurlyeq gv$, *where* $0 < k < 1$, $F(X \times X) \subseteq g(X)$, $g(X)$ *is complete and* g *is continuous. Suppose that either*

(1) *F is continuous or*
(2) *X has the following property:*
 (a) *if* $\{x_n\}$ *is a non-decreasing sequence with* $\{x_n\} \to x$, *then* $x_n \preccurlyeq x$ *for all* $n \in \mathbb{N}$,
 (b) *if* $\{y_n\}$ *is a non-increasing sequence with* $\{y_n\} \to y$, *then* $y \preccurlyeq y_n$ *for all* $n \in \mathbb{N}$.

If there exist $x_0, y_0 \in X$ *such that*

$$g(x_0) \preccurlyeq F(x_0, y_0), \quad g(y_0) \succcurlyeq F(y_0, x_0),$$

then F and g have a coupled coincidence point in $X \times X$.

Proof. Using Lemma 2.13, there exists $E \subseteq X$ such that $g(E) = g(X)$ and $g : E \to X$ is one-to-one. We define a mapping $\mathcal{A} : g(E) \times g(E) \to X$ by

$$\mathcal{A}(gx, gy) = F(x, y), \quad \forall gx, gy \in g(E). \tag{3.36}$$

As g is one to one on $g(E)$, so A is well-defined. Thus, it follows from (3.35) and (3.36) that

$$\mu(\mathcal{A}(gx, gy) - \mathcal{A}(gu, gv), kt) \geq \mu(gx - gu, t) * (gy - gv, t) \tag{3.37}$$

and

$$\nu(\mathcal{A}(gx, gy) - \mathcal{A}(gx, gy), kt) \leq \nu(gx - gu, t) \diamond \nu(gy - gv, t) \tag{3.38}$$

for all $gx, gy, gu, gv \in g(E)$ with $gx \preccurlyeq gy$ and $gy \succcurlyeq gv$. Since F has the mixed g-monotone property, for all $x, y \in X$, we have

$$x_1, x_2 \in X, gx_1 \preccurlyeq gx_2 \Rightarrow F(x_1, y) \preccurlyeq F(x_2, y) \tag{3.39}$$

and

$$y_1, y_2 \in X, gy_1 \succcurlyeq gy_2 \Rightarrow F(x, y_1) \preccurlyeq F(x, y_2). \tag{3.40}$$

Thus, it follows from (3.36), (3.39) and (3.40) that, for all $gx, gy \in g(E)$,

$$gx_1, gx_2 \in g(E), gx_1 \preccurlyeq gx_2 \Rightarrow \mathcal{A}(gx_1, gy) \preccurlyeq \mathcal{A}(gx_2, gy) \tag{3.41}$$

and

$$gy_1, gy_2 \in g(E), gy_1 \succcurlyeq gy_2 \Rightarrow \mathcal{A}(gx, gy_1) \preccurlyeq \mathcal{A}(gx, gy_2), \tag{3.42}$$

which implies that $\mathcal{A}$ has the mixed monotone property.

Suppose that the assumption (1) holds. Since F is continuous, $\mathcal{A}$ is also continuous. Using Theorem 3.1 with the mapping $\mathcal{A}$, it follows that $\mathcal{A}$ has a coupled fixed point $(u, v) \in g(X) \times g(X)$.

Suppose that the assumption (2) holds. We can conclude similarly in the proof of Theorem 3.1 that the mapping $\mathcal{A}$ has a coupled fixed point $(u, v) \in g(X) \times g(X)$.

Finally, we prove that F and g have a coupled coincidence point in X. Since (u, v) is a coupled fixed point of $\mathcal{A}$, we get

$$u = \mathcal{A}(u, v), \quad v = \mathcal{A}(v, u). \tag{3.43}$$

Since $(u, v) \in g(X) \times g(X)$, there exists a point $(\widehat{u}, \widehat{v}) \in X \times X$ such that

$$u = g\widehat{u}, \quad v = g\widehat{v}. \tag{3.44}$$

Thus, it follows from (3.43) and (3.44) that

$$g\widehat{u} = \mathcal{A}(g\widehat{u}, g\widehat{v}), \quad g\widehat{v} = \mathcal{A}(g\widehat{v}, g\widehat{u}). \tag{3.45}$$

Also, from (3.36) and (3.45), we get

$$g\widehat{u} = F(\widehat{u}, \widehat{v}), \quad g\widehat{v} = F(\widehat{v}, \widehat{u}). \tag{3.46}$$

Therefore, $(\widehat{u}, \widehat{v})$ is a coupled coincidence point of F and g. This completes the proof. □

Next, we give example to validate Theorem 3.2.

Example 3.3. Let $X = \mathbb{R}$, $a * b = ab \geq a \lozenge b$ for all $a, b \in [0,1]$ and $\theta(t) = e^{-\frac{1}{t}}$. Then, $(X, \mu, \nu, *, \lozenge)$ is a complete fuzzy normed space, where

$$\mu(x, t) = [\theta(t)]^{|x|}, \quad \nu(x, t) = 1 - [\theta(t)]^{|x|}, \quad \forall x \in X,$$

that (μ, ν) satisfies the n-property on $X \times (0, \infty)$. If X is endowed with the usual order as $x \preccurlyeq y \Leftrightarrow y - x \in [0, \infty)$, then $(X, \preccurlyeq)$ is a partially ordered set. Define mappings $F : X \times X \to X$ and $g : X \to X$ by

$$F(x, y) = 1, \quad \forall (x, y) \in X \times X$$

and

$$g(x) = x - 1, \quad \forall x \in X.$$

Since

$$g(F(x, y)) = g(1) = 0 \neq 1 = F(gx, gy)$$

for all $x, y \in X$, the mappings F and g do not satisfy the commutative condition. Hence, Theorem 2.5 of Gordji et al. [41] cannot be applied to this example. But, by simple calculation, we see that $F(X \times X) \subseteq g(X)$, g and F are continuous and F has the mixed g-monotone property. Moreover, there exist $x_0 = 1$ and $y_0 = 3$ with

$$g(1) = 0 \preccurlyeq 1 = F(1, 3)$$

and

$$g(3) = 2 \succcurlyeq 1 = F(3, 1).$$

Now, for any $x, y, u, v \in X$ with $gx \preccurlyeq gu$ and $gy \succcurlyeq gv$, we get

$$\begin{aligned} \mu(F(x, y) - F(u, v), kt) &= \mu(0, kt) \\ &= 1 \\ &\geq \mu(gx - gu, t) * \mu(gy - gv, t) \end{aligned} \tag{3.47}$$

and

$$\begin{aligned} \nu(F(x, y) - F(u, v), kt) &= \nu(0, kt) \\ &= 0 \\ &\leq \nu(gx - gu, t) \lozenge \nu(gy - gv, t), \end{aligned} \tag{3.48}$$

where $0 < k < 1$. Therefore, all the conditions of Theorem 3.2 hold and so F and g have a coupled coincidence point in $X \times X$. In fact, a point (2,2) is a coupled coincidence point of F and g.

Remark 3.4. Although Theorem 2.5 of Gordji et al. [41] is essential tool in the partially ordered fuzzy normed spaces to claim the existence of coupled coincidence points of two mappings. However, some mappings do not have the commutative property as in the above example. Therefore, it is very interesting to use Theorem 3.2 as another auxiliary tool to claim the existence of a coupled coincidence point.

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