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Coupled flows and oscillations in asymmetric rotating plasmas

A. R. Karimov,[1,a)] L. Stenflo,[2] and M. Y. Yu[3]
[1]Institute for High Temperatures, Russian Academy of Sciences, Izhorskaya 13/19, Moscow 127412, Russia
[2]Department of Plasma Physics, Umeå University, SE-90187 Umeå, Sweden
and Department of Physics, Linköping University, SE-58183 Linköping, Sweden
[3]Department of Physics, Institute for Fusion Theory and Simulation, Zhejiang University, 310027
Hangzhou, China and Institut für Theoretische Physik I, Ruhr-Universität Bochum,
D-44780 Bochum, Germany

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Nonlinear coupling among the radial, axial, and azimuthal flows in an asymmetric cold rotating plasma is considered nonperturbatively. Exact solutions describing an expanding or contracting plasma with oscillations are then obtained. It is shown that despite the flow asymmetry the energy in the radial and axial flow components can be transferred to the azimuthal component but not the vice versa, and that flow oscillations need not be accompanied by density oscillations. © *2009 American Institute of Physics*. [doi:10.1063/1.3247875]

I. INTRODUCTION

Studies of nonlinear wave phenomena in plasmas usually consider finite but small perturbations of a given equilibrium or steady state.[1,2] The wave dynamics, such as dispersion, etc., are still mainly governed by the characteristics of the corresponding linear mode. Nonlinear behavior appears because the latter becomes unstable and grows, so that wave-particle and/or wave-wave interactions must be taken into account. That is, the resulting nonlinear wave retains many of the characteristics of the corresponding linear mode. Such problems can be investigated by performing small-amplitude expansion of the parameters involved and deriving weakly nonlinear evolution equations. As most of the common linear waves have similar dispersion and propagation behavior, their nonlinear properties are often governed by one of the paradigm evolution equations,[1,2] whose derivation, mathematical properties, as well as solutions are well understood.

Mathematically exact solutions describing plasma wave dynamics have been of interest for many years.[1,3–11] Such exact solutions for highly nonlinear dynamic equilibrium states are useful in verifying new analytical and numerical schemes for solving nonlinear partial differential equations and as starting points for numerical investigations of more realistic problems.[12–16] In a recent paper, an exact model for cold plasma motion is used to investigate energy transfer among the different degrees of freedom in an expanding (or contracting) rotating plasma. In the model, the governing cold-plasma equations are solved nonperturbatively by first constructing a basis solution for the inertial (force free) motion of the electron (e) and ion (i) fluids. It is found that the flow and oscillation energies tend to concentrate into the azimuthal flow component.[11] That is, the energy in the azimuthal degree of freedom is not convertible into the other degrees of freedom, but that in the latter can be converted into the azimuthal flow. However, the model assumes that the flow vorticity $\boldsymbol{\omega}$ is only in the axial direction, a restriction that might have been responsible for the mentioned result. Here we relax this condition and investigate how the violation of cylindrical flow symmetry affects the energy transfer among the different flow directions. In particular, we shall allow for nonvanishing azimuthal electron and ion vorticities or $\omega_{\varphi j}\equiv\partial_r v_{zj}-\partial_z v_{rj}\neq 0$ $(j=i,e)$, where r, φ, and z are the cylindrical coordinates, and v_{rj} and v_{zj} are the radial and axial flow velocities.

II. MODEL OF THE ASYMMETRIC PLASMA FLOW

The present investigation extends our earlier work[11] on the coupling of axial and radial oscillations in a rotating cold plasma. Instead of the basis flow structure there, here we consider the basis structure

$$\mathbf{v}_j = rV_{rj}(t)\mathbf{e}_r + rV_{\varphi j}(t)\mathbf{e}_\varphi + rV_{zj}(t)\mathbf{e}_z, \tag{1}$$

$$\mathbf{E} = r\varepsilon_r(t)\mathbf{e}_r + r\varepsilon_\varphi(t)\mathbf{e}_\varphi + r\varepsilon_z(t)\mathbf{e}_z, \tag{2}$$

where $\mathbf{e}_{r,\varphi,z}$ are unit vectors, $V_{rj}(t)$, $V_{\varphi j}(t)$, $V_{zj}(t)$, $\varepsilon_r(t)$, $\varepsilon_\varphi(t)$, and $\varepsilon_z(t)$ are time-dependent functions to be determined. We note that Eqs. (1) and (2) differ from that in Ref. 11 only in the last (z-component) terms. Substituting Eqs. (1) and (2) into the cold-plasma equations of motion, we obtain

$$d_t n_j + 2V_{rj}n_j = 0, \tag{3}$$

$$d_t V_{rj} + V_{rj}^2 - V_{\varphi j}^2 = \mu_j(\varepsilon_r + V_{\varphi j}B_z - V_{zj}B_\varphi), \tag{4}$$

$$d_t V_{zj} + V_{rj}V_{zj} = \mu_j(\varepsilon_z + V_{rj}B_\varphi), \tag{5}$$

$$d_t V_{\varphi j} + 2V_{rj}V_{\varphi j} = \mu_j(\varepsilon_\varphi - V_{rj}B_z), \tag{6}$$

where $\mu_e=-1$, $\mu_i=m_e/m_i$, and m_e and m_i are the electron and ion masses, respectively, and $\mathbf{B}$ is the magnetic field. We have assumed that the electron density is spatially homogeneous or $n_e=n_e(t)$. Note that n_j depends directly only on V_{rj}, which is however related to the other velocity components. From the Poisson's equation we then get

[a)]Electronic mail: akarimov@mtu-net.ru.

$$n_i = n_e(t) + 2\varepsilon_r(t), \tag{7}$$

and the ion and electron continuity equations (3) are also satisfied. From the Maxwell's equations we have

$$d_t\varepsilon_r = n_e(V_{re} - V_{ri}) - 2\varepsilon_r V_{ri}, \tag{8}$$

$$d_t\varepsilon_\varphi = n_e(V_{\varphi e} - V_{\varphi i}) - 2\varepsilon_r V_{\varphi i}, \tag{9}$$

$$d_t\varepsilon_z = n_e(V_{ze} - V_{zi}) - 2\varepsilon_r V_{zi}, \tag{10}$$

$$d_t B_z = -2\varepsilon_\varphi, \tag{11}$$

$$d_t B_\varphi = \varepsilon_z. \tag{12}$$

We note that the above equation system differs from that in Ref. 11 only by the Eq. (5) for the axial velocity component and the Eq. (12) for the azimuthal magnetic field component. However, as shown below, these differences lead to a number of new physical features.

III. ASYMMETRIC PLASMA FLOW

In the flow field (1) there appears an azimuthal component of the vorticity,

$$\boldsymbol{\omega}_j = -V_{zj}(t)\mathbf{e}_\varphi + 2V_{\varphi j}(t)\mathbf{e}_z, \tag{13}$$

which confirms that besides the azimuthal flow rotation in the plane z=const, there is also flow rotation in the plane φ=const. Since the velocity and electric fields [given by Eqs. (1) and (2), respectively] have the same form, there is now also an azimuthal magnetic field component, which leads to additional $\mathbf{v}\times\mathbf{B}$ force components that can affect the flow dynamics.

The difference in the z-components of the flow velocities between the flow fields of Ref. 11 and here is clearly shown in the basis, or purely inertial, velocity profiles. The latter can be obtained by setting $\mu_e=\mu_i=0$, so that Eq. (5) reduces to

$$d_t V_{zj} + V_{rj}V_{zj} = 0, \tag{14}$$

which has the solution $V_{zj}=\beta_{j0}\exp(-\int V_{rj}(t)dt)$, with β_{j0} being an integration constant. Here the radial and axial flow components are strongly coupled, especially for $V_{rj}(t)<0$, when there is flow compression in the radial direction. One can thus have axial acceleration in the asymmetric flow owing to energy transfer from the radial flow. That is, the axial component of the basis velocity field is very different from that of Ref. 11 for symmetric flow, where the axial flow is not affected by the radial flow at all. On the other hand, for $V_{rj}(t)>0$ the difference between the v_{zj} in the symmetric and asymmetric cases is insignificant, since v_{zj} decreases with time and there is no enhancement of axial flow.

Equations (3)–(12) can be integrated numerically. Depending on the initial conditions, there exists a wide variety of solutions, representing possible dynamic states of an asymmetric cold plasma in cylindrical geometry in the presence of external and/or self-consistent electric and magnetic fields. In order to limit our study to familiar phenomena and to compare the results with the symmetric case, we shall use initial conditions similar to that in Ref. 11. We recall that for the symmetric case energy in the azimuthal flow is not convertible into the other degrees of freedom, but that in the latter can be converted into the azimuthal one. These results were obtained for $V_{rj}(0)=V_{\varphi j}(0)=10^{-2}$, $\varepsilon_r(0)=\varepsilon_\varphi(0)=0$, $n_0=1$, and $\mu=10^{-5}$. Here we shall also use these conditions, together with $\varepsilon_z(0)=0$. The other initial conditions will be varied. For convenience, we shall normalize all velocities by $|V_{re}(0)|$.

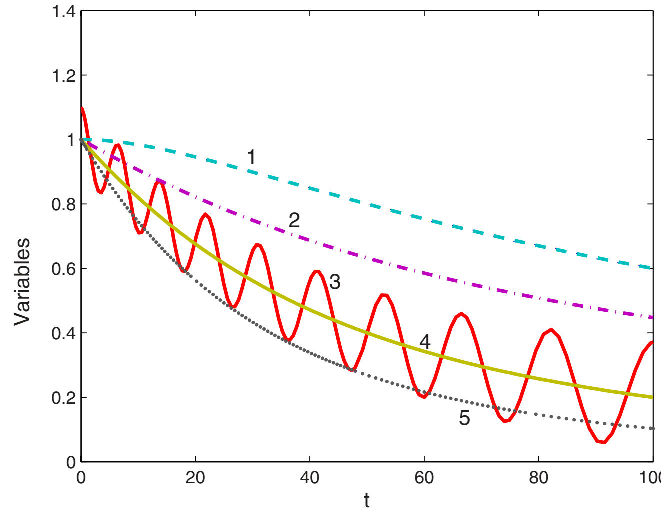

FIG. 1. (Color online) The time dependence of the radial, axial, and azimuthal flow variables $V_{re,ri}$ (curve 1), $V_{ze,zi}$ (curve 2), $V_{\varphi e}$ (curve 3), $V_{\varphi i}$ (curve 4), and the density n_e (curve 5) for an expanding flow [$V_{rj}(0)>0$]. The initial conditions are $V_{rj}(0)=V_{zj}(0)=10^{-2}$, $V_{\varphi e}(0)=1.1\times10^{-2}$, and $V_{\varphi i}(0)=10^{-2}$. All curves except n_e are normalized by $V_{re}(0)$.

First, we consider a state having oscillations in the azimuthal component of the electron velocity, with the ion velocity remaining unperturbed. The numerical results are shown in Fig. 1. The corresponding basis flow involves only decreasing velocity components, as expected for an expanding plasma. Figure 1 shows that for this case we have only oscillations in the azimuthal electron motion, and there is no density oscillation. This is because in the present mode structure the density is driven only by the radial motion [Eq. (3)]. Similar results were also obtained for the symmetric case.[11]

Next we consider a flow with oscillations in the axial direction. Figure 2 shows a typical result. Again we see that the nonlinear oscillations of the electron velocity remain mainly in the z-component, with only insignificant oscillations in the other components. Thus, the oscillation energy is preserved in the axial motion and not transferred to the other degrees of freedom. This result differs from that of the symmetric case, where relaxation of the oscillation energy into the other directions can occur. We note that in both Figs. 1 and 2 the overall structure of the velocity fields remains that of the basis solution for expanding flows, since in the examples here we are only interested in solutions that can be understood in terms of familiar phenomena associated with linear or weakly nonlinear plasma motion.

Finally we consider radially converging plasma flows,[11] in which the radial flow can reverse its direction and give rise to temporal plasma compression. In order to compare with the results of the expanding flow, we shall use the same

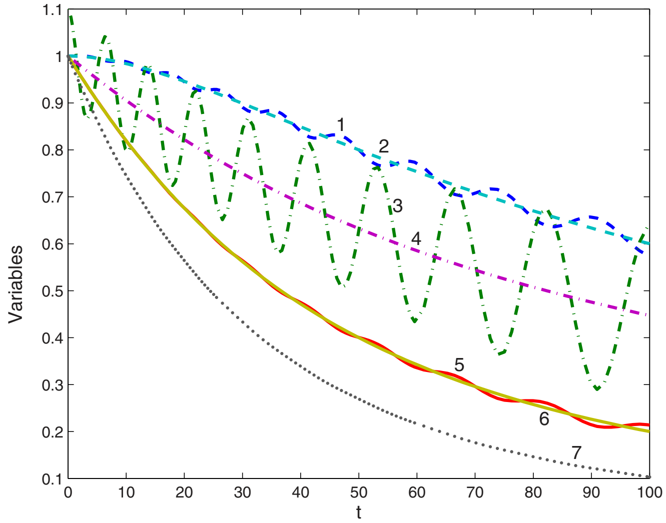

FIG. 2. (Color online) The time dependence of V_{re} (curve 1) and V_{ri} (curve 2), V_{ze} (curve 3) and V_{zi} (curve 4), $V_{\varphi e}$ (curve 5) and $V_{\varphi i}$ (curve 6), and n_e (curve 7) for an expanding flow. For $V_{rj}(0)=V_{\varphi j}(0)=10^{-2}$, $V_{ze}(0)=1.1\times10^{-2}$, and $V_{zi}(0)=10^{-2}$. All curves except n_e are normalized by $V_{re}(0)$.

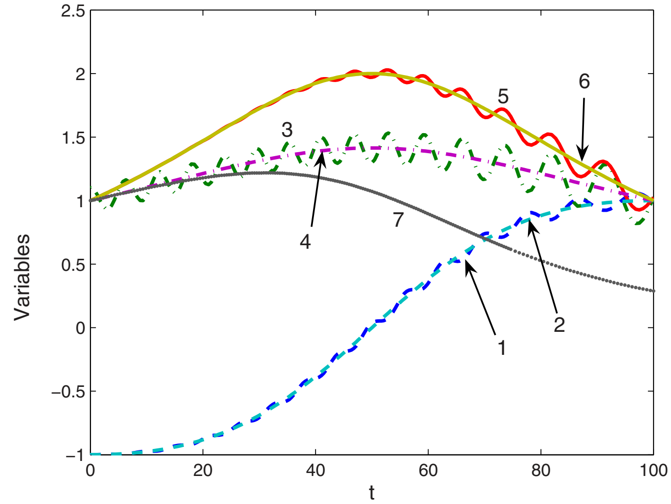

FIG. 4. (Color online) The time dependence of V_{re} (curve 1), V_{ri} (curve 2), V_{ze} (curve 3), V_{zi} (curve 4), $V_{\varphi e}$ (curve 5), $V_{\varphi i}$ (curve 6), and n_e (curve 7) for $V_{rj}(0)=-10^{-2}$, $V_{\varphi j}(0)=10^{-2}$, $V_{ze}(0)=1.1\times10^{-2}$, and $V_{zi}(0)=10^{-2}$. All curves except n_e are normalized by $V_{re}(0)$.

initial conditions as that in the above calculations, except that now we set $V_{re,ri}(0)=-10^{-2}$. The results are presented in Figs. 3 and 4. Comparing Figs. 1 and 2 with Figs. 3 and 4, we see that the overall flow structure of a converging plasma flow is very different from that of an expanding one. On the other hand, Fig. 3 shows that again the nonlinear oscillations are preserved in the azimuthal mode, with weak spreading into the other directions. However, Fig. 4 shows that oscillation energy in the axial direction spreads rather easily to the other directions. Nevertheless, Figs. 2 and 4 show that in both cases the axial oscillation energy is very weakly transferred to the other degrees of freedom. One may conclude that the axial oscillation energy practically remains in this degree of freedom. This is in contrast to that of an expanding flow, where there is spread of the oscillation energy out of the axial mode.

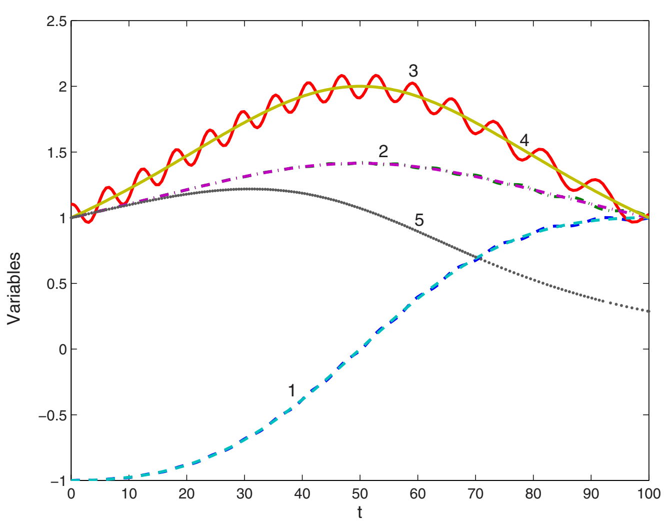

FIG. 3. (Color online) The time dependence of V_{re} and V_{ri} (curve 1), V_{ze} and V_{zi} (curve 2), $V_{\varphi e}$ (curve 3), $V_{\varphi i}$ (curve 4) and n_e (curve 5) for a contracting flow. For $V_{rj}(0)=-10^{-2}$, $V_{zj}(0)=10^{-2}$, and $V_{\varphi e}(0)=1.1\times10^{-2}$, $V_{\varphi i}(0)=10^{-2}$. All curves except n_e are normalized by $V_{re}(0)$.

IV. DISCUSSION

In this paper we have considered mathematically exact solutions of the cold fluid equations for an electron-ion plasma together with the Maxwell's equations for asymmetric, cylindrical plasma flows with axial as well as azimuthal vortical motion. Transfer of oscillation energy among the three degrees of freedom is investigated. The approach differs from most existing works[1,2,5] in that the spatial structure of the nonlinear eigenmode is predetermined.[4,9,11] It is found that the flow asymmetry tend to enhance the axial acceleration of the plasma fluid, and the oscillation energy tend to remain in the azimuthal and axial directions. In fact, the oscillation energy in the radial mode tends to be transferred to the other degrees of freedom. This behavior is valid for all the numerical solutions we have obtained. However, whether they reflect a general property of highly nonlinear plasma flows still remains to be proved, in particular, whether the flow and oscillation energies in nonequilibrium flows always tend to be concentrated in the rotational motion. In a sense, the problem is somewhat similar to investigating the time-dependent generalization of the Arnold–Beltrami–Childress flow, which possesses nontrivial current-line topology owing to the Beltrami condition $\boldsymbol{\omega}_j \propto \mathbf{v}_j$, and has been intensively studied because of its unique chaos properties and relevance to certain numerical schemes.[17,18] Furthermore, here we have considered cylindrical geometry, with the finite size of the system in axial direction ignored. It should thus be of interest to examine similar flows in the toroidal and spherical geometries, which have more natural rotational degrees of freedom.

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