Coupled-mode analysis of fiber-optic add–drop filters for dense wavelength-division multiplexing

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We present a coupled-mode theory of fiber-optic add–drop filters, which involve directional coupling between two fibers combined with fiber Bragg gratings defined inside the coupling region. The analysis self-consistently accounts for both the directional and the reflection coupling, and the propagation constants and structure of the supermodes of the combined structure are derived. We present a detailed analysis of a filter design based on identical fibers. The calculated device parameters satisfy the requirements for dense wavelength-division multiplexing applications. © 1997 Optical Society of America

Wavelength-division multiplexing is an attractive fiber-optic communications technique because it allows one to increase the transmission bandwidths of existing fiber-optic links by essentially the number of independent, closely spaced frequency channels. A challenging problem is the development of efficient add–drop devices that would enable one to add or drop optical signals at preselected wavelengths without substantial insertion loss and interference with other frequency channels in the link. A number of such filters based on fiber-grating technology[1] have been proposed and demonstrated.[2–5]

We present a coupled-mode analysis of a class of fiber-optic add–drop filters that consist of a directional coupler with a Bragg grating defined in it (Fig. 1). Regions I and III represent conventional directional couplers in which modes in (single-mode) fibers A and B exchange their energy.[6] Wavelength selectivity is provided by region II, which contains a reflection Bragg grating for a specified frequency channel. The optical waves in the selected wavelength band are reflected and coupled into the drop port of the device by use of the directional coupling provided by regions II and I. In the presence of both codirectional and contradirectional coupling (region II) the coupled-mode equations can be written as[6]

$$\frac{\mathrm{d}A_1}{\mathrm{d}z} = -i\kappa_{ab}B_1 \exp(i2\Delta\beta_{ab}z) - i\kappa A_2 \exp(i2\Delta\beta_a z)\,, \tag{1}$$

$$\frac{\mathrm{d}A_2}{\mathrm{d}z} = i\kappa_{ab}B_2 \exp(i2\Delta\beta_{ab}z) + i\kappa^* A_1 \exp(-i2\Delta\beta_a z)\,, \tag{2}$$

$$\frac{\mathrm{d}B_1}{\mathrm{d}z} = -i\kappa_{ab}^* A_1 \exp(-i2\Delta\beta_{ab}z) - i\kappa B_2 \exp(i2\Delta\beta_b z)\,, \tag{3}$$

$$\frac{\mathrm{d}B_2}{\mathrm{d}z} = i\kappa_{ab}^* A_2 \exp(i2\Delta\beta_{ab}z) + i\kappa^* B_1 \exp(-i2\Delta\beta_b z)\,, \tag{4}$$

where A_1, A_2, B_1, and B_2 are the corresponding slowly varying mode amplitudes; κ and κ_{ab} are the coupling strengths of the Bragg grating and the directional coupler, respectively.[6] The mismatch parameters $\Delta\beta_a$, $\Delta\beta_b$, and $\Delta\beta_{ab}$ are defined as

$$2\Delta\beta_a \equiv 2\beta_a - K_g = 2\beta_a - 2\pi/\Lambda_g\,, \tag{5}$$

$$2\Delta\beta_b \equiv 2\beta_b - K_g = 2\beta_b - 2\pi/\Lambda_g\,, \tag{6}$$

$$2\Delta\beta_{ab} \equiv \beta_a - \beta_b = \Delta\beta_a - \Delta\beta_b\,, \tag{7}$$

where β_a and β_b are propagation constants (unperturbed) in fibers A and B, respectively, and Λ_g is the period of the fiber Bragg grating. In Eqs. (1)–(4) we neglected the direct coupling between counterpropagating waves in *different* waveguides (i.e., coupling between A_1 and B_2 and between A_2 and B_1) because this kind of coupling is essentially proportional to the grating coupling strength κ times the mode overlap integral ($\sim\kappa_{ab}/\beta$) and thus is substantially weaker than codirectional and contradirectional coupling. Only in strongly coupled waveguides (with coupling distance $1/\kappa_{ab}$ of the order of several light wavelengths) can this mechanism make a substantial contribution.

Mode amplitudes A_1, A_2, B_1, and B_2 are expressed as

$$A_1(z) = \tilde{A}_1(z)\exp(i\Delta\beta_a z)\,, \tag{8}$$

$$A_2(z) = \tilde{A}_2(z)\exp(-i\Delta\beta_a z)\,, \tag{9}$$

$$B_1(z) = \tilde{B}_1(z)\exp(i\Delta\beta_b z)\,, \tag{10}$$

$$B_2(z) = \tilde{B}_2(z)\exp(-i\Delta\beta_b z)\,. \tag{11}$$

Then the system of coupled-mode equations (1)–(4) in

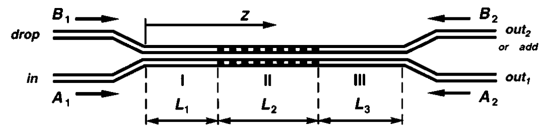

Fig. 1. Basic schematic of a fiber-optic add–drop filter.

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region II takes a form that can be presented in a matrix format:

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{pmatrix}\tilde{A}_1\\ \tilde{A}_2\\ \tilde{B}_1\\ \tilde{B}_2\end{pmatrix} = \begin{bmatrix} -i\Delta\beta_a & -i\kappa & -i\kappa_{ab} & 0\\ i\kappa^* & i\Delta\beta_a & 0 & i\kappa_{ab}\\ -i\kappa_{ab}^* & 0 & -i\Delta\beta_b & -i\kappa\\ 0 & i\kappa_{ab}^* & i\kappa^* & i\Delta\beta_b\end{bmatrix}\begin{pmatrix}\tilde{A}_1\\ \tilde{A}_2\\ \tilde{B}_1\\ \tilde{B}_2\end{pmatrix}. \tag{12}$$

The eigenvalues of the coupling matrix given by Eq. (12) are the propagation factors of the four eigenmodes in the Bragg-grating region:

$$s_{1,\ldots,4} = \pm\left\{|\kappa|^2 - \left[(|\kappa_{ab}|^2 + \Delta\beta_{ab}^2)^{1/2} \pm \frac{\Delta\beta_a + \Delta\beta_b}{2}\right]\right\}^{1/2}. \tag{13}$$

Depending on Bragg conditions and reflection coupling strength, these eigenmodes can be propagating or evanescent. The spatial structure of the eigenmodes (supermodes of region II) is determined by the eigenvectors of the coupling matrix [Eq. (12)]. The eigenvectors' components are the expansion coefficients for supermodes in terms of modes in individual fibers. Four supermodes are exponentially growing and exponentially decaying (with z) symmetric and antisymmetric eigenmodes of the directional coupler.

The response of an add–drop filter can be calculated as follows. First, the input waves [$A_1(0)$ and $B_1(0)$, here assumed to be zero, i.e., $B_1(0) \equiv 0$], are decomposed in terms of the two forward-propagating supermodes of the directional coupler (region I).[6] These two supermodes propagate (with their respective propagation constants) to the boundary with Bragg-grating region II and are decompose in terms of individual guide mode amplitudes, yielding $A_1(L_1)$ and $B_1(L_1)$. These values, combined with the boundary conditions $A_2(L_1 + L_2) = 0$ and $B_2(L_1 + L_2) = 0$ are used to find the amplitudes of the four supermodes in the Bragg-grating region. The total field of the supermodes is then decomposed at the boundary $z = L_1$ in terms of mode amplitudes in the individual waveguides, yielding the values of the backpropagating waves $A_2(L_1)$ and $B_2(L_1)$ at the interface with the directional coupler of region I. Then, similarly to the first step of the analysis, one uses these values to find the amplitudes of the two backpropagating supermodes in directional coupling region I and, finally, the amplitudes $B_2(0)$ and $A_2(0)$ at the output of the device. $B_2(0)$ represents the useful drop output, and $A_2(0)$ is the undesirable return loss. The purpose of region III is to direct the frequency channels outside the selected band into the output port (out_1 in Fig. 1) of the device [i.e., $|\kappa_{ab}|(L_1 + L_2 + L_3) = \pi n$].

The above approach allows us to analyze the device for the general case when the optical waves in waveguides A and B have arbitrarily different propagation constants. In what follows, we consider a substantially simpler (yet important, and realized experimentally[5]) case in which waveguides A and B are essentially identical; i.e., $\beta_a \equiv \beta_b$ and, therefore, $\Delta\beta_{ab} \equiv 0$ and $\Delta\beta_a \equiv \Delta\beta_b \equiv \Delta\beta$, where $\Delta\beta$ is the mismatch parameter (the same for both waveguides). The propagation factors of the four supermodes in the Bragg-grating region then become [Eq. (13)]

$$s_{1,\ldots,4} = \pm\,[|\kappa|^2 - (|\kappa_{ab}| \pm \Delta\beta)^2]^{1/2}. \tag{14}$$

Using the above procedure, we find the drop output:

$$B_2(0) = \frac{A_1(0)}{2}\,\frac{\kappa_{ab}^*}{|\kappa_{ab}|}\,\exp(-2i\Delta\beta L_1) \times \kappa^*\left[\frac{\exp(-2i|\kappa_{ab}|L_1)\tanh(s^+L_2)}{(-|\kappa_{ab}| - \Delta\beta)\tanh(s^+L_2) + is^+} - \frac{\exp(2i|\kappa_{ab}|L_1)\tanh(s^-L_2)}{(|\kappa_{ab}| - \Delta\beta)\tanh(s^-L_2) + is^-}\right], \tag{15}$$

and the return loss, or backreflection, of the filter:

$$A_2(0) = \frac{A_1(0)}{2}\,\exp(-2i\Delta\beta L_1) \times \kappa^*\left[\frac{\exp(-2i|\kappa_{ab}|L_1)\tanh(s^+L_2)}{(-|\kappa_{ab}| - \Delta\beta)\tanh(s^+L_2) + is^+} + \frac{\exp(2i|\kappa_{ab}|L_1)\tanh(s^-L_2)}{(|\kappa_{ab}| - \Delta\beta)\tanh(s^-L_2) + is^-}\right], \tag{16}$$

where $s^+ = (+)[|\kappa|^2 - (|\kappa_{ab}| + \Delta\beta)^2]^{1/2}$ and $s^- = (+)[|\kappa|^2 - (|\kappa_{ab}| - \Delta\beta)^2]^{1/2}$ [these are, in fact, the supermodes' propagation factors in region II, Eq. (14)].

The total power reflectivity R (drop + return) does not depend on L_1 and at central frequency ($\Delta\beta = 0$) is given by

$$R = \tanh^2(sL_2)\frac{|\kappa|^2}{|s|^2 + |\kappa_{ab}|^2\tanh^2(sL_2)}, \tag{17}$$

where s is the coupling strength at the central frequency of the Bragg grating: $s = (|\kappa|^2 - |\kappa_{ab}|^2)^{1/2}$, reduced because of the directional coupling. Equation (17) allows us to calculate the grating length L_2, given the required R and values of κ and κ_{ab}.

The length L_1 of the first directional coupling region, I, has to be optimized to minimize the backreflection at the central frequency of the filter; i.e., $|A_2(0)|^2$ has to be equal to 0 for $\Delta\beta = 0$. Using Eq. (16), we obtain the following expression for the optimal value of L_1:

$$\tan(2|\kappa_{ab}|L_1^{\mathrm{opt}}) = \frac{s}{|\kappa_{ab}|\tanh(sL_2)}. \tag{18}$$

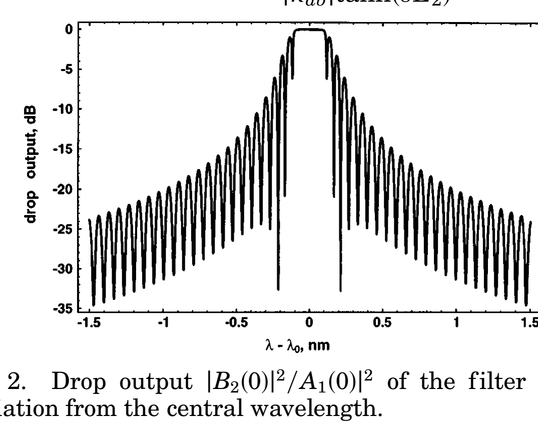

Fig. 2. Drop output $|B_2(0)|^2/A_1(0)|^2$ of the filter versus deviation from the central wavelength.

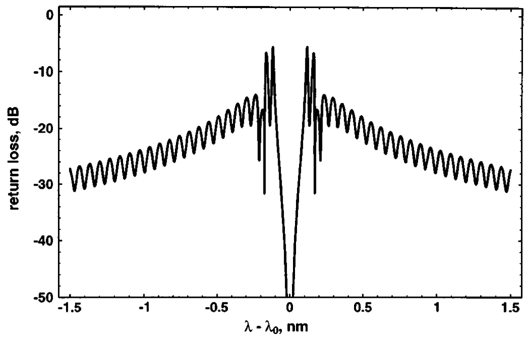

Fig. 3. Return loss $|A_2(0)|^2/|A_1(0)|^2$.

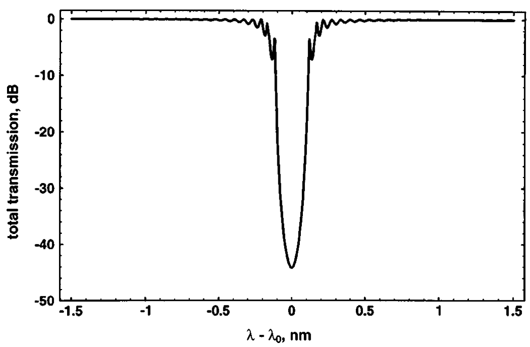

Fig. 4. Total transmission $1 - [A_2(0)|^2 + |B_2(0)|^2]/|A_1(0)|^2$.

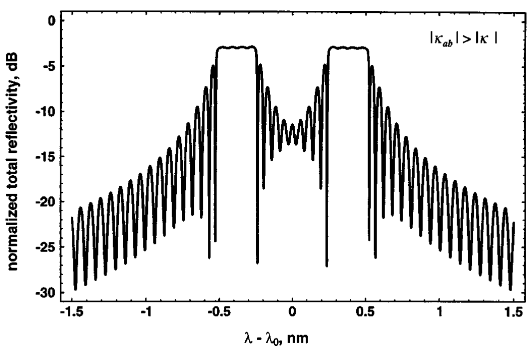

Fig. 5. Total reflectivity $[|A_2(0)|^2 + B_2(0)|^2]/|A_1(0)|^2$ of the filter for the case of strong directional coupling $|\kappa_{ab}| > |\kappa|(|\kappa_{ab}| = 15\ \mathrm{cm}^{-1}$, hence $\beta \gg |\kappa_{ab}|$; other parameters are the same as in Figs. 2–4). The two independent peaks correspond to almost 100% reflection of each of the two forward-propagating orthogonal supermodes.

It can be shown that for total reflectivity R close to unity and for small deviations of coupler length L_1 from its optimal value L_1^{opt}, the normalized return loss at the central frequency of the filter (i.e., for $\Delta\beta = 0$) is $\approx 10\ \mathrm{Log}(4|\kappa_{ab}|^2|L_1 - L_1^{\mathrm{opt}}|^2)$ (in decibels).

We calculate the normalized drop efficiency (Fig. 2), return loss (Fig. 3), and total transmission (Fig. 4) of the device for a typical set of parameters: $|\kappa| = 5\ \mathrm{cm}^{-1}$ (corresponding to an index change in the fiber core of 2.5×10^{-4}), $|\kappa_{ab}| = 1\ \mathrm{cm}^{-1}$, $L_2 = 1.175$ cm, and central wavelength $\lambda_0 = 1550$ nm. We calculated the value of $L_1 = 0.685$ cm by using Eq. (18) to achieve zero return loss at central wavelength λ_0. The device has ~0.2-nm total bandwidth in the drop output and exhibits relatively weak interchannel cross talk (< -20 dB at $\Delta\lambda = \pm 1$ nm; Fig. 2). The cross talk can be decreased substantially by appropriate grating apodization.[7] The signal extraction efficiency i.e., the residual transmission through the device at the resonant wavelength λ_0, can be as low as < -40 dB (for the chosen values of κ and L_2). The minimum L_3 is equal to 1.28 cm to satisfy the condition $|\kappa_{ab}|(L_1 + L_2 + L_3) = \pi$.

We also investigate the case when the directional coupling dominates the contradirectional one, i.e., $|\kappa_{ab}| > |\kappa|$ (Fig. 5). The reflectivity-versus-wavelength curve exhibits two maxima, which are off the central frequency. This can be explained as follows. The propagation constants of the copropagating symmetric and antisymmetric supermodes of the directional coupler differ by $2|\kappa_{ab}|$ (in fact, they are $\beta \pm |\kappa_{ab}|$ for forward-propagating supermodes[6]). Therefore, in the Bragg-grating region the counterpropagating supermodes (symmetric and asymmetric pair) become resonant with the grating periodicity at essentially different frequencies, i.e., when $\Delta\beta = +|\kappa_{ab}|$ (for slow, antisymmetric supermodes) and $\Delta\beta = -|\kappa_{ab}|$ (for fast, symmetric supermodes). If the bandwidth of the fiber grating is narrower than the difference in resonant frequencies, two separate maxima are observed, as shown in Fig. 5. The condition for normal operation (such as is shown in Figs. 2–4) of the add–drop device is $|\kappa| > |\kappa_{ab}|$; therefore strongly coupled waveguides are not suitable for this type of application.

In conclusion, we have developed a self-consistent coupled-mode analysis that accounts for simultaneous reflection and codirectional coupling in the Bragg-grating region. An all-fiber add–drop multiplexer based on identical fibers combined with a Bragg grating was analyzed. Using realistic parameters, we can envisage the following characteristics of the device: drop efficiency, >40 dB; return loss, >30 dB; and interchannel cross talk, <20 dB at $\Delta\lambda = \pm 1$ nm, which can be decreased substantially by grating apodization.

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