# Coupled-resonator optical waveguide: a proposal and analysis

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We propose a new type of optical waveguide that consists of a sequence of coupled high-$Q$ resonators. Unlike other types of optical waveguide, waveguiding in the coupled-resonator optical waveguide (CROW) is achieved through weak coupling between otherwise localized high-$Q$ optical cavities. Employing a formalism similar to the tight-binding method in solid-state physics, we obtain the relations for the dispersion and the group velocity of the photonic band of the CROW's and find that they are solely characterized by coupling factor $\kappa_1$. We also demonstrate the possibility of highly efficient nonlinear optical frequency conversion and perfect transmission through bends in CROW's. 

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Two mechanisms have been proposed and used in the past for optical waveguiding.[1] The most widely used is waveguiding by total internal reflection, as illustrated in Fig. 1(a). Another mechanism, Bragg waveguiding, in which waveguiding is achieved through Bragg reflection from a periodic structure, has also been demonstrated.[1,2] Figure 1(b) illustrates an example of Bragg reflection provided by a periodic Bragg stack.[2]

In this Letter we propose a new type of waveguide based on coupling of optical resonators, the coupled-resonator optical waveguide (CROW). Figure 1(c) shows a possible realization of such a waveguide based on evanescent-field coupling between the high-$Q$ whispering-gallery modes of individual microdisk cavities.[3] Another possible realization is shown in Fig. 1(d), in which the individual resonators consist of defect cavities[4,5] embedded in a two-dimensional (2D) periodic structure (a 2D photonic crystal).[6,7] These defect resonators are designed such that their resonant frequency falls within the forbidden gap of the surrounding 2D structure, which permits high-$Q$ optical modes. The coupling in this case is due to the evanescent Bloch waves. In both realizations of the CROW we assume sufficiently large separation between the individual resonators that the resonators are weakly coupled. Consequently, we expect that the eigenmode of the electromagnetic field in such a coupled-resonator waveguide will remain essentially the same as the high-$Q$ mode in a single resonator. At the same time one must take into account the coupling between the individual high-$Q$ modes to explain the transmission of the electromagnetic waves. This coupling is exactly the optical analog of the tight-binding limit in condensed-matter physics,[8] in which the overlap of atomic wave functions is large enough that corrections to the picture of isolated atoms are required yet at the same time is not large enough to render the atomic description completely irrelevant. The individual resonators in the CROW are the optical counterpart of the isolated atoms, and the high-$Q$ mode in the resonators corresponds to the atomic wave function.

In the spirit of the tight-binding approximation, we take the eigenmode $\mathbf{E}_K(\mathbf{r}, t)$ of a CROW as a linear combination of the high-$Q$ modes $\mathbf{E}_\Omega(\mathbf{r})$ of the individual resonators along a straight line parallel to the $\mathbf{e}_z$ axis [see Figs. 1(c) and 1(d)]. Denoting the coordinate of the center of the $n$th resonator as $z = nR$, we have

$$\mathbf{E}_K(\mathbf{r}, t) = E_0 \exp(i\omega_K t) \sum_n \exp(-inKR) \times \mathbf{E}_\Omega(\mathbf{r} - nR\mathbf{e}_z)\,. \tag{1}$$

It is straightforward to show that the waveguide mode $\mathbf{E}_K(\mathbf{r}, t)$ satisfies the Bloch theorem. Consequently we can limit the wave vector $K$ to the first Brillouin zone, i.e., $-\pi/R \le K \le \pi/R$. By writing $\mathbf{E}_K(\mathbf{r}, t)$ in this form, we have assumed $\mathbf{E}_\Omega(\mathbf{r})$ to be nondegenerate.

$\mathbf{E}_K(\mathbf{r}, t)$ satisfies the Maxwell equations, which leads to (in Gaussian units)

$$\nabla \times (\nabla \times \mathbf{E}_K) = \epsilon(\mathbf{r}) \frac{{\omega_K}^2}{c^2} \mathbf{E}_K\,, \tag{2}$$

where $\epsilon(\mathbf{r})$ is the dielectric constant of the system (of coupled resonators) and $\omega_K$ is the eigenfrequency of the waveguide mode. Similarly, $\mathbf{E}_\Omega(\mathbf{r})$ satisfies Eq. (2) but with $\epsilon(\mathbf{r})$ replaced with $\epsilon_0(\mathbf{r})$, the dielectric constant of the single resonator, and $\omega_K$ replaced with the single-resonator mode frequency $\Omega$. We can take $\mathbf{E}_\Omega(\mathbf{r})$ to be real and normalize it to unity according to $\int \mathrm{d}^3\mathbf{r}\epsilon_0(\mathbf{r})\mathbf{E}_\Omega \cdot \mathbf{E}_\Omega = 1$.

After substituting Eq. (1) into Eq. (2), multiplying both sides from the left-hand side by $\mathbf{E}_\Omega(\mathbf{r})$ and spatially integrating, we find the dispersion relation for the waveguide mode $\mathbf{E}_K(\mathbf{r}, t)$ to be

$${\omega_K}^2 = \Omega^2 \frac{\left[1 + \sum_{n\neq 0} \exp(-inKR)\beta_n\right]}{\left[1 + \Delta\alpha + \sum_{n\neq 0} \exp(-inKR)\alpha_n\right]}\,, \tag{3}$$

where $\alpha_n$, $\beta_n$, and $\Delta\alpha$ are defined as

 

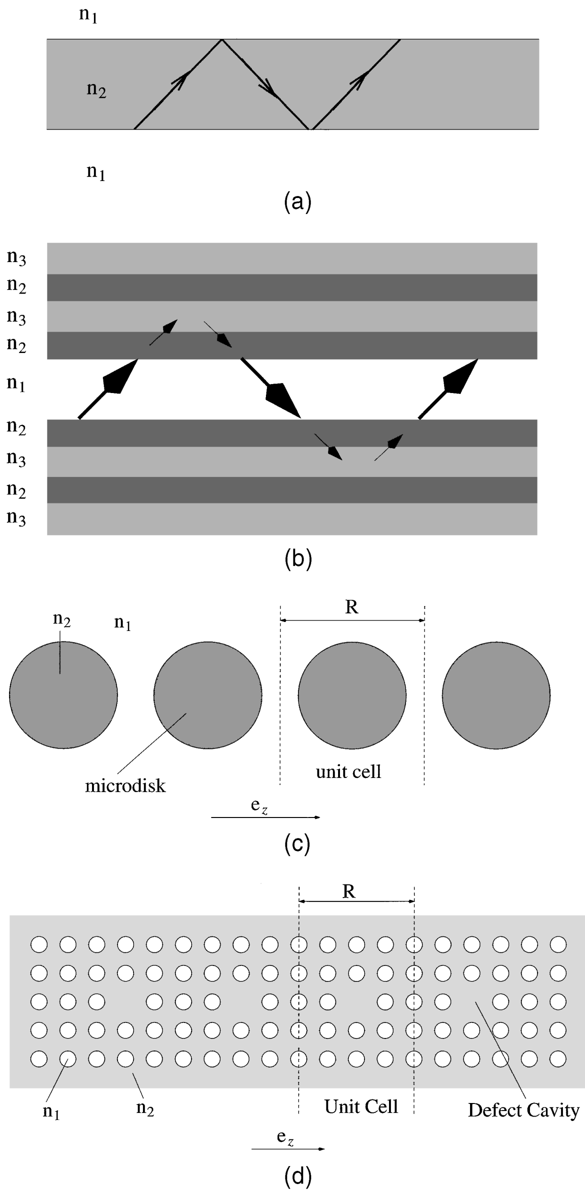


Fig. 1. Three types of waveguiding: (a) waveguiding achieved through total internal reflection at the interface between a dielectric medium with a high refractive index $n_2$ and a low refractive index $n_1$. (b) Bragg waveguiding achieved by reflection from periodic Bragg stacks. (c) CROW, with waveguiding that is due to coupling between individual microdisks. $R$ is the size of a unit cell, and $\mathbf{e}_z$ is the direction of the periodicity for the coupled resonators. (d) CROW realized by coupling of the individual defect cavities in a 2D photonic crystal. $R$ and $\mathbf{e}_z$ are defined the same as in (c).

$$\alpha_n = \int \mathrm{d}^3\mathbf{r}\epsilon(\mathbf{r})\mathbf{E}_\Omega(\mathbf{r}) \cdot \mathbf{E}_\Omega(\mathbf{r} - nR\mathbf{e}_z), \qquad n \neq 0, \tag{4a}$$

$$\beta_n = \int \mathrm{d}^3\mathbf{r}\epsilon_0(\mathbf{r} - nR\mathbf{e}_z)\mathbf{E}_\Omega(\mathbf{r}) \cdot \mathbf{E}_\Omega(\mathbf{r} - nR\mathbf{e}_z), \qquad n \neq 0, \tag{4b}$$

$$\Delta\alpha = \int \mathrm{d}^3\mathbf{r}[\epsilon(\mathbf{r}) - \epsilon_0(\mathbf{r})]\mathbf{E}_\Omega(\mathbf{r}) \cdot \mathbf{E}_\Omega(\mathbf{r}). \tag{4c}$$

If the coupling between the resonators is sufficiently weak, we can keep only the nearest neighbor coupling, i.e., $\alpha_n = 0$ and $\beta_n = 0$ if $n \neq 1, -1$. From symmetry considerations, we also require that $\alpha_1 = \alpha_{-1}$ and $\beta_1 = \beta_{-1}$. Finally, we assume $\alpha_1$, $\beta_1$, and $\Delta\alpha$ to be small. Putting all these observations together, we simplify Eq. (3) to

$$\omega_K = \Omega\left[1 - \frac{\Delta\alpha}{2} + \kappa_1 \cos(KR)\right], \tag{5}$$

where we define the coupling factor $\kappa_1$ as

$$\kappa_1 = \beta_1 - \alpha_1 = \int \mathrm{d}^3\mathbf{r}[\epsilon_0(\mathbf{r} - R\mathbf{e}_z) - \epsilon(\mathbf{r} - R\mathbf{e}_z)] \times \mathbf{E}_\Omega(\mathbf{r}) \cdot \mathbf{E}_\Omega(\mathbf{r} - R\mathbf{e}_z). \tag{6}$$

A dispersion diagram is shown in Fig. 2. This dispersion relation defines a photonic band formed by the coupling of the high-$Q$ modes in the individual resonators, which can be denoted the CROW band. From Eq. (5), the group velocity is found to be

$$v_g(K) = \frac{\mathrm{d}\omega_K}{\mathrm{d}K} = -\Omega R \kappa_1 \sin(KR), \tag{7}$$

which can be quite small for a weakly coupled CROW. Notice that both the dispersion and the group velocity are characterized by $\kappa_1$ only.

A particularly appealing feature of the CROW is the possibility of making lossless and reflectionless bends. It is obvious from symmetry considerations that if the individual resonator mode possesses an $n$-fold rotational symmetry one can make a perfect $2\pi/n$ bend, since the coupling of the corner resonator to its two immediate neighbors is identical. This bend is illustrated in Fig. 3. The transmission coefficient through the bend is 100% throughout the entire CROW band. This property is in contrast with the bent photonic crystal waveguide that has been proposed by Mekis *et al.*[9] in which complete transmission occurs only at certain frequencies.

Another important application envisaged for the CROW is nonlinear optical frequency conversion. Using second-harmonic generation as an example, we

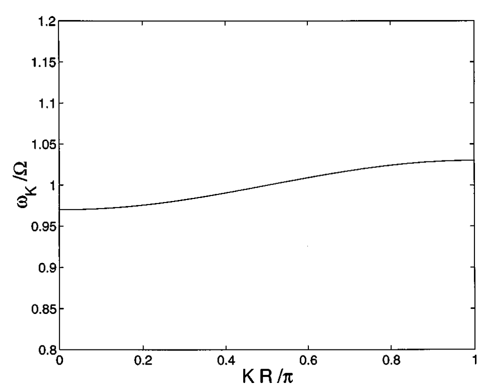


Fig. 2. Dispersion diagram of a CROW band. The dispersion relation is plotted according to Eq. (5), with $\Delta\alpha = 0$ and $\kappa_1 = -0.03$. $\Omega$ is the resonant frequency of the single high-$Q$ cavity. $K$ is the wave vector of the CROW band.

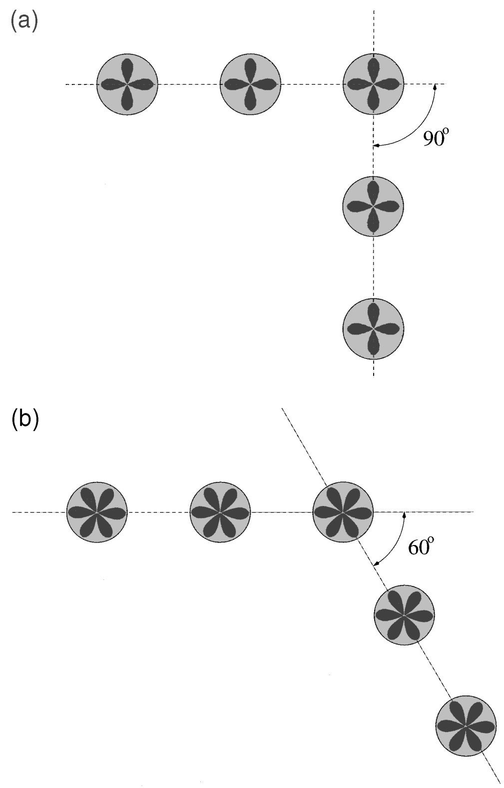


Fig. 3. Two realizations of the CROW bend with complete transmission. The gray regions represent the microcavities that are coupled together to form the CROW. The black regions inside the individual microcavities represent the high-$Q$ optical modes in each microcavity, which have $n$-fold rotational symmetry. (a) $n = 4$, (b) $n = 6$.

can use the unique $\omega(K)$ dispersion characteristics of the CROW to satisfy the phase-matching condition $K(2\omega) = 2K(\omega)$. In addition, the highly concentrated optical field in the CROW can also increase the conversion efficiency for a given input power $P$. In fact, the propagating power flux $P$ in the CROW is proportional to the group velocity of the CROW band[1] [see Figs. 1(c) and 1(d)]:

$$P = \frac{1}{8\pi R}\, v_{\omega,g} {E_0}^2. \tag{8}$$

Consequently the small group velocity of the CROW band can result in a large optical field with only a modest amount of power flux. From Eq. (7), it is obvious that at $K \approx \pi/R$, $-\pi/R$ (band edge) or $K \approx 0$ (band center), the group velocity $v_g$ of the CROW band will be close to 0. Since the efficiency of nonlinear optical processes (second-harmonic generation, for example) is proportional to some power of the electric-field strength,[10] it is possible to use the CROW band to greatly enhance the efficiency of these processes. Another interesting application is as a superresonator, i.e., a resonator of resonators, formed by folding a CROW back upon itself. Since the folding angles of the superresonator are determined by the symmetry of the single resonator modes, the superresonator itself will possess the same, or simply related, symmetry. A higher-rank resonator can also be formed whose basic elements are the superresonators. This resonator opens the way to a hierarchy of self-similar, or fractal, resonators.

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