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COURNOT OLIGOPOLY WITH UNCERTAIN ENTRY

by

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### INTRODUCTION

The study of oligopoly has progressed along two lines. The first deals with behavior of a firm viewing existing rivals as responsive to its actions. The critical assumption in this analysis is the firm's conjecture about rivals' reactions to its price or quantity decisions. Cournot [4] provided the classic supposition that rivals will maintain their current level of output in response to a change in the given firm's output level. This implies price matching by rivals. Questions addressed include those of existence and stability of equilibrium in an oligopolistic market, its possible convergence to the competitive solution as the number of firms increases indefinitely, and whether the approach is monotone (quasi-competitiveness). A recent synthesis is provided by Ruffin [11].

The second strand of inquiry focuses on the firm's behavior regarding potential rivals, especially actions designed to preserve positive profits. The strategy of pricing to retard or preclude entry, "limit pricing", has received the most intense study. A summary of earlier theoretical developments is provided by Bhagwati [2]; more recent contributions include Gaskins [6], Pyatt [10], Baron [1], and Kamien and Schwartz [8,9].

The first of these two lines of inquiry, focused on interaction among existing rivals, may be viewed as a short-run theory of oligopoly. The second approach, with its attention to consequences of potential rivalry, forms the

complementary long-run theory. This paper constitutes an attempt to bridge these parallel developments, an objective shared with Fisher [5].

In the next section we present a model of a firm that views its existing rivals in accordance with the classic Cournot assumption and views potential rivals in the manner posited in our paper [8]. The timing of rival entry is regarded as a random variable whose probability distribution is dependent on current industry price. After obtaining necessary conditions for maximization of the firm's long-run expected profits, we extend the first four theorems of Ruffin's paper to the case in which potential rivalry is recognized. The classic Cournot oligopoly model is compared with the present one and our results are summarized.

#### THE MODEL

We posit an industry composed of  $n$  identical firms; the output of the  $i^{\text{th}}$  is denoted by  $x_i$ . Each firm chooses its output rate independently, following the Cournot assumption that the output rates of other firms are fixed. As Ruffin argues [11, p. 494-5], we need only consider a symmetric equilibrium in which all firms behave identically;  $x_i = x$ ,  $i = 1, \dots, n$ . Industry output  $Q = nx$  and we let  $Q_i = Q - x_i$ . We denote by  $f(Q) = p$  the inverse demand relation between industry price and quantity. We suppose that  $f(Q)$  is stationary through time and twice differentiable, with negative slope  $f'$ .

Since all firms are identical, we study a single representative firm. We assume the firm is cognizant of possible entry into the industry and believes current price affects its rapidity.

Speed of entry is of course only one dimension of potential rivalry, the other being entrant's size. We confine attention to the rapidity issue,

believing it requires a more novel extension of the traditional Cournot model. The size-of-entrant facet of the problem might be accommodated through the classic asymmetric model with potential entrants regarded as dormant existing rivals.

We assume that potential entrants are attracted by current price rather than current profit. The latter supposition would permit counter-intuitive conclusions, as Baron observed. Specifically, if profit signals potential entrants then the existing firm may price either below or above the monopoly price, for either will reduce profit. However, if price is the gauge for potential entrants, the threat of entry will unambiguously lead the firm to price below the monopoly price, in accordance with the conventional wisdom; see Kamien and Schwartz [8]. Similar arguments apply to oligopoly.

The posited direct relationship between price and speed of entry is based in part on the notion that the more rapidly a potential entrant sets up a productive facility the more costly it is. The higher current price, the larger the profits anticipated by the potential entrant and the greater his eagerness to expedite construction of a production facility, possibly incurring higher entry costs thereby. Further, profit opportunities need not be instantly apparent. Potential investors require time to search out these opportunities. The greater the profit potential, the more rapidly such an opportunity may be found by those prepared to act upon it.

The firm is assumed certain about the behavior of existing rivals but uncertain about the actions of prospective rivals. Specifically, the firm supposes that the probability of rival entry into the  $n$ -firm industry at any particular time, given that the  $(n+1)^{st}$  firm has not yet appeared, is an

increasing convex function  $h(p)$  of current price  $p$ . Let  $F(t)$  denote the probability that the  $(n+1)^{\text{st}}$  firm will enter by time  $t$ , so the conditional probability of entry at time  $t$  is  $F'(t)/(1-F(t))$  where prime denotes derivative. Thus

$$h(p(t)) = F'(t)/(1-F(t))$$

with  $h(0) = 0$ ,  $h'(p) \geq 0$ ,  $h''(p) \geq 0$  for  $p \geq 0$ .

We denote by  $V(n+1)$  the maximum present value of the firm's total expected profit stream after entry of the  $(n+1)^{\text{st}}$  firm. Upon entry, the new firm becomes indistinguishable from the  $n$  previously extant firms in its behavior towards both existing and future rivals. In brief, the arrival of a new firm causes the industry to advance to an  $n+1$  firm Cournot oligopoly. The function  $V(n+1)$  is assumed known to the firm through a process of maximization via backward induction as will be shown immediately in recursion equation (1).

The firm's cost function is denoted by  $C(x)$  and is assumed to be twice differentiable with  $C(0) = 0$ . Current profit  $xf(Q_i + x) - C(x)$  is supposed quasiconcave in output and to attain a finite maximum. The firm's objective can be posed as an optimal control problem:

$$(1) \quad V(n) = \max_x \int_0^{\infty} e^{-rt} \{ [xf(Q_i + x) - C(x)](1-F) + rV(n+1)F \} dt$$

subject to

$$(2) \quad F' = h(f(Q_i + x))(1-F)$$

$$F(0) = 0$$

The time dependence of the variables in (1) and (2) has been suppressed

for expository convenience. The discount rate is  $r$ . Current output  $x(t)$  is the control variable and the probability of rival entry by time  $t$ ,  $F(t)$ , is the state variable.

The term in square brackets in the integrand of (1) is firm profit prior to entry of the  $(n+1)^{st}$  firm. It is multiplied by the probability the entrant has not yet appeared. The second term,  $rV(n+1)$ , represents the uniform flow equivalent of expected profits after entry and is multiplied by the probability entry has occurred. Note that it is industry price  $f(Q_i + x)$  that affects entry. Viewing the output of others,  $Q_i$ , as fixed, the firm can affect this price through its choice of output. It may forego some current profits if that will lengthen the expected period during which the industry contains just  $n$  members.

Applying standard methods for solving the problem posed in (1) and (2), we form the Hamiltonian

$$(3) \quad H = e^{-rt} \{ [xf(Q_i + x) - C(x)](1-F) + rV(n+1)F \} + \lambda h(f(Q_i + x))(1-F)$$

According to the Maximum Principle, an optimal output path  $x$  will maximize  $H$ . Assuming this maximum achieved for positive, finite  $x$ , we set  $\partial H / \partial x = 0$  which implies

$$(4) \quad e^{-rt} [f(Q_i + x) + xf'(Q_i + x) - C'(x)] + \lambda h'(f(Q_i + x)) f'(Q_i + x) = 0$$

The multiplier  $\lambda(t)$  satisfies the differential equation

$$(5) \quad \lambda' = \frac{-\partial H}{\partial F} = e^{-rt} [xf(Q_i + x) - C(x) - rV(n+1)] + \lambda h(f(Q_i + x))$$

with transversality condition

$$(6) \quad \lim_{t \rightarrow \infty} \lambda(t)F(t) = 0$$

(We assume the transversality condition appropriate for a finite planning horizon continues to obtain as the horizon is extended indefinitely.)

SOLUTION AND DISCUSSION

As noted, we consider only the symmetric solution. If each firm assumes  $Q_i$  is exogenous and solves (1) - (2), the industry equilibrium result consistent with their actions will be  $Q_i = (n-1)x$ . Furthermore, it can be shown as in [8] that a constant level of output  $x(t) = x$  throughout the period in which the industry consists of just  $n$  firms will be an optimal solution to the problem posed in (1) and (2). With a symmetric output plan constant over time, (2) may be integrated to

$$(7) \quad F(t) = 1 - e^{-h(Q)t}$$

where  $Q = nx$ .

With  $x$  and  $Q_i$  constant functions of time and  $F$  given by (7), (1) can be integrated directly to give

$$(8) \quad V(n) = \max_x \frac{xf(Q_i + x) - C(x) - rV(n+1)}{r+h(f(Q_i + x))} + V(n+1)$$

According to (8) the present value of the firm in an industry with  $n$  firms is equal to its value when there are  $n+1$  firms plus a transient bonus, quasi-rent, representing the advantage to being in an  $n$ -firm industry rather than one with  $n+1$  firms. The bonus is the difference between the firm's profit in an industry of  $n$  firms and its (uniform flow equivalent of) expected profit after the industry has been enlarged, all capitalized at a rate  $r+h$  reflecting both the discount rate and the expected speed of entry, or duration of the quasi-rent. At the optimal output or price, the quasi-rent or capitalized transient return will be nonnegative; current price cannot optimally

be so low that entry would raise the incumbents' profits. See [8] for a more technical discussion of this last point in the context of monopoly.

The optimal constant output rate  $x$  satisfies, differentiating (8),

$$(9) \quad f(Q) + [x-g(x,Q,n)]f'(Q) - C'(x) = 0$$

where

$$(10) \quad g(x,Q,n) \equiv [xf(Q) - C(x) - rV(n+1)]h'(f(Q))/[r+h(f(Q))] \geq 0$$

(One can also obtain (9) by integrating (5) with (6) for the symmetric constant output plan to find  $x$  and then combining this result with (4)). The sufficient condition for a maximum of (8) at the point  $x$  obeying (9)

$$(11) \quad (x-g)f'' + (2-g_1-g_2)f' - C'' < 0$$

is assumed satisfied there. The  $g_i$ 's are first partial derivatives of  $g$ . This condition corresponds to Ruffin's expression (1), the assumption of local concavity of the firm's profit function in its own output, for any given  $Q_i$ .

In interpreting (9), we observe that a marginal increase in output will have the usual immediate effect on total profits, measured by the current marginal revenue and marginal cost terms, and also a longer run effect on future profits. Specifically the increase in output will lower price, thereby also marginally reducing the conditional probability of entry. This lengthens the expected duration of the transient quasi-rent and so increases the expected capitalized value. The longer run impact of marginal output on total profit is reflected in the term  $-gf' \geq 0$ . The nonnegativity of  $g$



follows from the discussion of (8) and the assumption on  $h'$ .

We take up first the situations in which  $g > 0$ . The case of  $h' > 0$  corresponds to Hicks' concept of oligopolistic expectations in which the firm believes that (the probability of) rival entry is responsive to its current actions, [7]. In this instance, it follows from (9) that

$$f(Q) + xf'(Q) - C'(x) < 0$$

But negative current marginal profits, together with the assumption that current profit is a quasiconcave function of output, implies that the firm's optimal output exceeds the current-profit-maximizing level. In other words, the threat of entry causes the firm to choose a larger output, and thereby lower industry price, than it would in the static or myopic Cournot oligopoly setting without consideration of potential entry.

There are two situations in which  $g = 0$  so the optimal output is unaffected by the threat of entry, as further study of (9) - (10) discloses. The first case is that of  $h' = 0$ , corresponding to Hicks' firm with polypolistic expectations; it recognizes the possibility of entry but believes it independent of its own actions. Obviously, the firm does not sacrifice current profits if it believes it futile.

The second instance occurs when the first bracketed term in (10) is zero, so entry does not change the incumbents' current profits. This can only happen if current profit  $xf(Q) - C(x) = 0$ , and if there is free exit so  $V(n+1)$  need not be negative. The situation of zero current profit with a finite number  $n$  of firms constitutes a Chamberlinian equilibrium [3], as will be seen. The price  $f(Q) = C(x)/x$  in this case. Substitution into (9) yields

$$C(x)/x + xf'(Q) - C'(x) = 0$$

so

$$C(x)/x - C'(x) = -xf'(Q) > 0$$

Thus zero current profit implies that average cost exceeds marginal cost at the optimal output rate, so the average cost curve is downward sloping there. This implies tangency between the demand curve and the average cost curve at the optimal output as in the Chamberlinian equilibrium.

It is worth noting that if  $F' = 0$ , then by definition  $h = 0$  and  $\lambda' = 0$  from (5). But  $F' = 0 = \lambda'$  constitute the stationary state of the differential equations system (2) and (5). In other words, the stationary state of this model corresponds to a Chamberlinian equilibrium with a finite number of firms in which entry has ceased.

If the first bracketed term in (10) does not become zero for a finite  $n$  it surely will as  $n \rightarrow \infty$ . As the number of firms increases indefinitely, individual firm output  $x \rightarrow 0$ . In this case, since  $g \rightarrow 0$  we conclude from (9) that

$$\lim_{x \rightarrow 0} [f(Q) + xf'(Q) - C'(x)] = f(Q) - C'(0) = 0$$

Therefore  $p = f(Q) = C'(0)$  just as in Ruffin's study of Cournot oligopoly. Ruffin argued that convergence of the Cournot oligopoly to perfect competition can be shown if and only if the limiting marginal cost equals the marginal cost under perfect competition, namely minimum average cost. But in the limit firm size tends to zero, so the industry converges to the competitive equilibrium if and only if  $C'(0) = \text{minimum average cost}$ . These arguments likewise

apply here in view of (13). Thus Ruffin's Theorems 1 and 2 apply to Cournot oligopoly with pricing to retard entry:

1. If average cost is non-decreasing, so its minimum is attained at  $x = 0$  (equalling marginal cost there), then the solution tends to the competitive solution as  $n \rightarrow \infty$ .
2. If average cost is U-shaped, so that  $C'(0)$  exceeds the minimum average cost, then the solution does not tend to the competitive solution as  $n \rightarrow \infty$  and the limiting price exceeds the competitive price.

We now turn to the quasi-competitive characteristics of our model. More specifically, we seek conditions under which industry output will rise and individual firm output will shrink with an increase in the number of firms in the industry (counterparts of Ruffin's Theorems 3 and 4.)

Append to (9) the equation

$$(12) \quad Q - xn = 0$$

The left sides of this pair of equations can be viewed as dependent on the three variables  $x$ ,  $Q$ , and  $n$ . Viewing  $n$  as the independent variable and differentiating totally yields in matrix notation

$$(13) \quad \begin{bmatrix} (x-g)f'' + (1-g_2)f' & (1-g_1)f' - C'' \\ 1 & -n \end{bmatrix} \begin{bmatrix} dQ/dn \\ dx/dn \end{bmatrix} = \begin{bmatrix} g_3 f' \\ x \end{bmatrix}$$

Equations (9) and (12) will, according to the implicit function theorem,

define functions  $x = x(n)$ ,  $Q = Q(n)$  in a neighborhood of the equilibrium if the Jacobian, namely the determinant of the coefficient matrix in (13), is nonvanishing there. This determinant is denoted  $D$  and equals

$$(14) \quad -(n-1)[(x-g)f'' + (1-g_2)f'] - [(x-g)f'' + (2-g_1-g_2)f' - C''] \equiv D$$

Since the second bracketed expression in  $D$  is identical to (11) and therefore negative by assumption,  $D$  will certainly be positive if

$$(15) \quad (x-g)f'' + (1-g_2)f' < 0$$

We assume (15) to hold; it is sufficient for positivity of (14) and is the counterpart to Ruffin's assumption A.7 that the firm's marginal revenue is steeper than the demand function.

Solving for  $dQ/dn$  and  $dx/dn$  using (13) and Cramer's rule yields

$$(16) \quad dQ/dn = - \left[ ((1-g_1)f' - C'')x + f'g_3n \right] / D$$

$$(17) \quad dx/dn = \left[ (x-g)f'' + (1-g_2-g_3/x)f' \right] x/D$$

which show how industry output and firm output respond to an increase in the number of firms in the industry. Since

$$(18) \quad g_1 = (f-C')h'/(r+h) > 0, \quad g_3 = -rV'(n+1)h'/(r+h) > 0$$

$$g_2 = (x-g)f'h'/(r+h) + gh''f'/h'$$

it appears that industry and firm output may either expand or contract after entry of another firm .

We can interpret certain conditions under which (16) and (17) are signed. In view of (18), a sufficient condition for industry output to rise with the number of firms is that the coefficient of  $x$  in (16) be negative:

$$(19) \quad (1-g_1)f' - C'' < 0$$

Since the left side of (9) is (current and future) marginal profit of output, the left side of (19) is its rate of change with incremental firm output, industry output held fixed. Thus, if marginal profitability falls with firm output, then industry output rises with  $n$ . Condition (15), which indicates that a ceteris paribus increment in industry output reduces the marginal profitability of the firm's output, is not sufficient for negativity of (17), in view of (18). It should be observed that (15) and (19) together are sufficient but not necessary for (11). The firm's output will fall with an increase in  $n$  if that causes the marginal profitability of the firm's output to decline. These interpretations of the signing of (16)-(17) correspond directly to those in [11] and are identical when  $g \equiv 0$ , as Ruffin has pointed out to us.

#### SUMMARY

The classic Cournot oligopoly model has been extended to include possible entry. Recognition of potential rivals leads the firm to select an output that exceeds the myopic, current profit maximizing level. This behavior persists as long as the incumbent firms' quasi-rent is positive and therefore worth protecting. A steady state occurs in our extended Cournot oligopoly when current profit is zero, an event that will obtain as the numerical size of the industry expands indefinitely. Thus the limiting behavior of our model

coincides with that of the traditional one. We have also shown that if profits vanish in an industry composed of a finite number of firms, that equilibrium displays Chamberlinian characteristics, namely tangency between the demand curve and the average cost curve. An interesting feature of our model is that though our firm expects existing rivals to match its price movements, it will, as we have demonstrated in [9], appear to price along a more elastic demand curve than it actually faces. Finally, we have displayed the formal similarities between the two models regarding their quasi-competitiveness.