

# Covariant generalized holographic dark energy and accelerating universe

Shin'ichi Nojiri<sup>1,2,a</sup>, S. D. Odintsov<sup>3,4,5,6,b</sup>

<sup>1</sup> Department of Physics, Nagoya University, Nagoya 464-8602, Japan

<sup>2</sup> Kobayashi–Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, Nagoya 464-8602, Japan

<sup>3</sup> ICREA, Passeig Luis Companys, 23, 08010 Barcelona, Spain

<sup>4</sup> Institute of Space Sciences (IEEC-CSIC) C. Can Magrans s/n, 08193 Barcelona, Spain

<sup>5</sup> National Research Tomsk State University, Tomsk 634050, Russia

<sup>6</sup> Tomsk State Pedagogical University, Tomsk 634061, Russia

Received: 9 May 2017 / Accepted: 21 July 2017 / Published online: 8 August 2017

© The Author(s) 2017. This article is an open access publication

**Abstract** We propose the generalized holographic dark energy model where the infrared cutoff is identified with the combination of the FRW universe parameters: the Hubble rate, particle and future horizons, cosmological constant, the universe lifetime (if finite) and their derivatives. It is demonstrated that with the corresponding choice of the cutoff one can map such holographic dark energy to modified gravity or gravity with a general fluid. Explicitly,  $F(R)$  gravity and the general perfect fluid are worked out in detail and the corresponding infrared cutoff is found. Using this correspondence, we get realistic inflation or viable dark energy or a unified inflationary-dark energy universe in terms of covariant holographic dark energy.

## 1 Introduction

Quantum field considerations may play a fundamental role in the study of the early- and late-universe evolution. Indeed, if we consider a particle with mass  $m$ , the quantum correction to the (flat-space) vacuum energy density  $\rho_{\text{vacuum}}$  is given by

$$\rho_{\text{vacuum}} = \pm \frac{1}{(2\pi)^3} \int d^3k \left( \frac{1}{2} \sqrt{k^2 + m^2} \right). \quad (1)$$

Here the  $+$  ( $-$ ) sign corresponds to the bosonic (fermionic) particle. For large  $k$ , the integration diverges but because the observed value of the vacuum energy is very small, that is,  $(10^{-3} \text{ eV})^4$ , the divergence could be absorbed or canceled by

some as yet unknown mechanism.<sup>1</sup> On the other hand, if we consider small  $k$ , which corresponds to a large distance, there should be a minimum for  $k$ , nor there is, at least, a problem in the causality. We call the minimum an infrared cutoff and denote it by  $\Lambda_{\text{inf}}$ . Then we find  $k \gtrsim \Lambda_{\text{inf}}$ . If the minimum is really related with the causality, the infrared cutoff  $\Lambda_{\text{inf}}$  is related with the horizon radius  $L_{\text{H}}$  as  $\Lambda_{\text{inf}} \sim 1/L_{\text{H}}$ . Then the vacuum energy (1) could be estimated as  $\rho_{\text{vacuum}} \sim \mp m \Lambda_{\text{inf}}^3$  if the particle has non-vanishing mass,  $m \neq 0$  or  $\rho_{\text{vacuum}} \sim \mp \Lambda_{\text{inf}}^4$  if the particle is massless,  $m = 0$ . Hence, the small but non-vanishing vacuum energy might be regarded as the universe' dark energy and, in principle, might generate the accelerating expansion of the current universe.

Of course, the real situation could be much more complicated. For example, the infrared cutoff might be given by the temperature of the current universe  $T \sim 10^{-3} \text{ eV}$ ; then the energy scale of the vacuum energy in the present universe can be naturally given if we consider the massless particle,  $\rho_{\text{vacuum}} \sim \mp \Lambda_{\text{inf}}^4 \sim T^4 \sim (10^{-3} \text{ eV})^4$ . We should also note that the Stefan–Boltzmann law tells us that  $\rho \propto T^4$  although only the thermal energy cannot generate the accelerating expansion. In the curved space-time, the cutoff could depend on the space-time curvature or we may use the Hubble horizon as the horizon radius  $L \sim 1/H$ , with the Hubble rate  $H$ .

If we consider the holographic principle [4] (see also [5–24]), the vacuum energy could be proportional to the universe radius square  $L$ ,

$$\rho_{\text{vacuum}} = \frac{3c^2}{\kappa^2 L^2}. \quad (2)$$

<sup>a</sup> e-mail: nojiri@gravity.phys.nagoya-u.ac.jp

<sup>b</sup> e-mail: odintsov@ieec.uab.es

<sup>1</sup> In order to solve this problem, a topological model has been proposed [1] and the cosmology in this model has been investigated [2,3].

Here  $\kappa$  is the gravitational coupling and  $c$  is a constant. The problem of the original holographic dark energy model [4] where the infrared cutoff was chosen as the size of the event horizon is the fact that the corresponding FRW equations often do not correspond to any covariant gravity theory and even may not predict the acceleration of the universe. Subsequently, the generalized holographic dark energy has been proposed in Ref. [25] where the infrared cutoff is identified with a combination of the FRW universe parameters: the Hubble constant, particle and future horizons, cosmological constant and universe lifetime (if finite). Implicitly, the dependence from the derivatives of the corresponding FRW parameters was also assumed. Different versions of the cutoff corresponding to generalized holographic dark energy [25] have been considered in Refs. [26–41].

In this paper, we consider a generalized holographic dark energy with arbitrary cutoff which depends on the FRW universe parameters for the vacuum energy density  $\rho_{\text{vacuum}}$ . In fact, the choice of the infrared cutoff could be included in the definition of the quantum theory in curved space-time. Therefore, unitarity and causality could give important hints to find the correct definition of the infrared cutoff. The AdS/CFT correspondence might also give some indication because the infrared cutoff could be related with the ultraviolet cutoff in the corresponding gauge theory. However, there is no definite prediction at all for the choice of the infrared cutoff at least at present. Then we consider several possibilities for the cutoff choice. In the next section, we briefly review the holographic dark energy and introduce the covariant generalized holographic dark energy model. Section 3 is devoted to generalized holographic dark energy which is equivalent to  $F(R)$  gravity. In this way, consistent inflation naturally emerges from the covariant generalized holographic dark energy. In the last section, the reconstruction of the arbitrary fluid as generalized holographic dark energy is developed. The occurrence of a dark energy universe or even a unified inflation–dark energy universe in terms of such a theory is demonstrated.

## 2 Generalized holographic dark energy

Let us consider the spatially flat FRW universe

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3} (dx^i)^2. \quad (3)$$

Defining the Hubble rate  $H = \dot{a}(t)/a(t)$ , the first FRW equation has the following form:

$$\frac{3}{\kappa^2} H^2 = \rho_\Lambda, \quad (4)$$

which gives

$$H = \frac{c}{L}. \quad (5)$$

Here we assume that  $c$  is positive constant because the expanding universe is considered. It is well known that if one chooses the infrared cutoff  $\Lambda_{\text{inf}}$  to be the Hubble rate  $H$ , the accelerating universe cannot be realized. There are more possibilities for the choice of the infrared radius  $L$ . For example, one may choose  $L$  to be the particle horizon  $L_p$  or the future horizon  $L_f$ , which is defined by

$$L_p(t) \equiv a(t) \int_0^t \frac{dt'}{a(t')}, \quad L_f(t) \equiv a(t) \int_t^\infty \frac{dt'}{a(t')}. \quad (6)$$

For the FRW metric with the flat spatial part in (3), by choosing  $L$  as  $L_p$  or  $L_f$ , we find the following equation:

$$\frac{d}{dt} \left( \frac{c}{a(t)H(t)} \right) = \pm \frac{1}{a(t)}. \quad (7)$$

Here, the  $+$  ( $-$ ) sign corresponds to the particle (future) horizon. We can easily solve Eq. (7) and find

$$a(t) = a_0 t^{h_0}, \quad (8)$$

with

$$h_0 = \frac{1}{1 \pm \frac{1}{c}}. \quad (9)$$

Then, in the case  $L = L_f$ , the universe is accelerating because  $h_0 > 1$ . When  $c > 1$ , in the case  $L = L_p$ ,  $h_0$  becomes negative and the universe is shrinking. If one can change the direction of time as  $t \rightarrow t_s - t$ , instead of (8), we find

$$a(t) = a_0 (t_s - t)^{h_0}. \quad (10)$$

Then there will be a Big Rip singularity at  $t = t_s$ . Because we change the direction of time, the particle horizon becomes a future-like one,

$$L_p(t) \rightarrow \tilde{L}_f(t) \equiv a(t) \int_t^{t_s} \frac{dt'}{a(t')} = a(t) \int_{a(t)}^\infty \frac{da}{Ha^2}. \quad (11)$$

Note that if we choose  $L$  as a future horizon  $L = L_f$ , there is a solution describing the de Sitter space-time,

$$a(t) = a_0 e^{\frac{t}{l}} \quad \left( H = \frac{1}{l} \right). \quad (12)$$

If we choose, however, the particle horizon as  $L$ , the solution describing the de Sitter space-time does not exist. Additionally to the fact that not all choices of cutoff may lead to the accelerating universe, it is easy to see that the corresponding FRW equations cannot be obtained from some covariant action.

In general,  $L_\Lambda$  could be a combination (a function) of both  $L_p$  and  $L_f$  [42]. Furthermore, if there is a Big Rip singularity at  $t = t_s$  and therefore the lifetime of the universe is finite,  $L$  can also be a function of  $t_s$ . More general, it could be the case that  $L$  depends on the Hubble rate  $H$  and also the curvature as we have mentioned (see also Ref. [25]),

$$L = L(L_p, \dot{L}_p, \ddot{L}_p, \dots, L_f, \dot{L}_f, \ddot{L}_f, \dots, t_s, H, \dot{H}, \ddot{H}, \dots). \tag{13}$$

Some of the cutoffs given by (13) cannot be obtained from the covariant gravity theory. Still such a possibility should not be excluded because the FRW background breaks the covariance or at least the Lorentz symmetry, in some sense, spontaneously. A similar example might be the Casimir force, which could also appear because we break Lorentz invariance by the boundary conditions. We call the theory with the above cutoff covariant generalized holographic dark energy in the case where it may be equivalently described by some covariant theory. Later on, we shall give some examples of such a theory where the equivalence with modified gravity or fluid theory is established.

Let us demonstrate that the above model may unify the early-time inflation and the current accelerating expansion of the universe. We consider the case that  $c = 1$  and  $L$  is given by

$$\frac{1}{L} = \left(1 - \frac{1}{t_0 h_0}\right) \frac{1}{L_f} + \frac{2h_0}{t_0 h_1} + \frac{h_1}{t_0} \left(1 - \frac{h_0^2}{h_1^2}\right) L_f. \tag{14}$$

Here  $L_f$  is a future horizon defined by (6) and  $t_0, h_0$ , and  $h_1$  are positive constants and it is assumed that  $h_0 > h_1$ . Then the solution of the first FRW equation is given by

$$H = h_0 - h_1 \tanh \frac{t}{t_0} + \frac{h_1}{t_0 \left(h_0 - h_1 \tanh \frac{t}{t_0}\right) \cosh^2 \frac{t}{t_0}}. \tag{15}$$

In fact, one can check that

$$\frac{1}{L_f} = h_0 - h_1 \tanh \frac{t}{t_0}. \tag{16}$$

Then at the early universe  $t \rightarrow -\infty$ , the Hubble rate  $H$  goes to a constant  $H \rightarrow h_0 + h_1$ , which may be identified with the inflation and at the late universe  $t \rightarrow +\infty$ ,  $H$  goes to a constant again,  $H \rightarrow h_0 - h_1$ , which may be identified with the late-time accelerating expansion. Instead of (14), by using the scalar curvature  $R = 6(2H^2 + \dot{H})$ , we may discuss the model

$$\begin{aligned} \frac{1}{L^2} &= \frac{R}{12} - \frac{1}{2} \left\{ 1 - \frac{1}{h_1^2} \left(h_0 - \frac{1}{L}\right)^2 \right\} \\ &\times \left\{ \frac{h_1^2}{t_0^2} \left(1 - \frac{h_0^2}{h_1^2}\right) L_f^2 - \frac{h_1}{t_0} + \frac{1}{t_0^2} \right\}. \end{aligned} \tag{17}$$

Even in the model (17), the Hubble rate  $H$  (15) is again the solution. Hence, generalized holographic dark energy may unify the inflation with dark energy.

### 3 Holographic description of $F(R)$ gravity

Let us now demonstrate the correspondence between the generalized holographic dark energy of the previous section and  $F(R)$  gravity with the action

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} F(R) = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} (R + f(R)). \tag{18}$$

Because

$$\dot{L}_p = H L_p + 1, \quad \dot{L}_f = H L_f - 1, \tag{19}$$

we find

$$\begin{aligned} H &= \frac{\dot{L}_p}{L_p} - \frac{1}{L_p} = \frac{\dot{L}_f}{L_f} + \frac{1}{L_f}, \\ \dot{H} &= \frac{\ddot{L}_p}{L_p} - \frac{\dot{L}_p^2}{L_p^2} + \frac{\dot{L}_p}{L_p^2} = \frac{\ddot{L}_f}{L_f} - \frac{\dot{L}_f^2}{L_f^2} - \frac{\dot{L}_f}{L_f^2}, \end{aligned} \tag{20}$$

and therefore

$$\begin{aligned} R_{tt} &= -3(\dot{H} + H^2) = -3 \left( \frac{\ddot{L}_p}{L_p} - \frac{\dot{L}_p}{L_p^2} + \frac{1}{L_p^2} \right) \\ &= -3 \left( \frac{\ddot{L}_f}{L_f} + \frac{\dot{L}_f}{L_f^2} + \frac{1}{L_f^2} \right), \\ R_{ij} &= a^2(\dot{H} + 3H^2) \tilde{g}_{ij} = a^2 \left( \frac{\ddot{L}_p}{L_p} + \frac{2\dot{L}_p^2}{L_p^2} - \frac{5\dot{L}_p}{L_p^2} + \frac{3}{L_p^2} \right) \tilde{g}_{ij} \\ &= a^2 \left( \frac{\ddot{L}_f}{L_f} + \frac{2\dot{L}_f^2}{L_f^2} + \frac{5\dot{L}_f}{L_f^2} + \frac{3}{L_f^2} \right) \tilde{g}_{ij}, \\ R &= 6\dot{H} + 12H^2 = R_L \equiv 6 \left( \frac{\ddot{L}_p}{L_p} + \frac{\dot{L}_p^2}{L_p^2} - \frac{3\dot{L}_p}{L_p^2} + \frac{2}{L_p^2} \right) \\ &= 6 \left( \frac{\ddot{L}_f}{L_f} + \frac{\dot{L}_f^2}{L_f^2} + \frac{3\dot{L}_f}{L_f^2} + \frac{2}{L_f^2} \right). \end{aligned} \tag{21}$$

In the case of  $F(R)$  gravity (for a general review, see [43,44]) we have the following FRW equations:

$$\begin{aligned} 3H^2 &= -\frac{f(R)}{2} + 3(H^2 + \dot{H}) f'(R) \\ &- 3H \frac{df'(R)}{dt} + \kappa^2 \rho_{\text{matter}}, \end{aligned}$$

$$\begin{aligned}
 -3H^2 - 2\dot{H} &= \frac{f(R)}{2} - (\dot{H} + 3H^2) f'(R) \\
 &+ 6H \frac{df'(R)}{dt} + \frac{d^2 f'(R)}{dt^2} + \kappa^2 p_{\text{matter}}.
 \end{aligned}
 \tag{22}$$

Using the holographic language, we may rewrite (63) as follows:

$$\begin{aligned}
 3H^2 &= -\frac{f(R_L)}{2} + 3 \left( \frac{\ddot{L}_p}{L_p} - \frac{\dot{L}_p}{L_p^2} + \frac{1}{L_p^2} \right) f'(R_L) \\
 &- 3 \left( \frac{\dot{L}_p}{L_p} - \frac{1}{L_p} \right) \frac{df'(R_L)}{dt} + \kappa^2 \rho_{\text{matter}}, \\
 &= -\frac{f(R_L)}{2} + 3 \left( \frac{\ddot{L}_f}{L_f} + \frac{\dot{L}_f}{L_f^2} + \frac{1}{L_f^2} \right) f'(R_L) \\
 &- 3 \left( \frac{\dot{L}_f}{L_f} + \frac{1}{L_f} \right) \frac{df'(R_L)}{dt} + \kappa^2 \rho_{\text{matter}}, \\
 -3H^2 - 2\dot{H} &= \frac{f(R_L)}{2} - \left( \frac{\ddot{L}_p}{L_p} + \frac{2\dot{L}_p^2}{L_p^2} - \frac{5\dot{L}_p}{L_p^2} + \frac{3}{L_p^2} \right) f'(R_L) \\
 &+ 6 \left( \frac{\dot{L}_p}{L_p} - \frac{1}{L_p} \right) \frac{df'(R_L)}{dt} + \frac{d^2 f'(R_L)}{dt^2} \\
 &+ \kappa^2 p_{\text{matter}} \\
 &= \frac{f(R_L)}{2} - \left( \frac{\ddot{L}_f}{L_f} + \frac{2\dot{L}_f^2}{L_f^2} + \frac{5\dot{L}_f}{L_f^2} + \frac{3}{L_f^2} \right) f'(R_L) \\
 &+ 6 \left( \frac{\dot{L}_f}{L_f} + \frac{1}{L_f} \right) \frac{df'(R_L)}{dt} + \frac{d^2 f'(R_L)}{dt^2} \\
 &+ \kappa^2 p_{\text{matter}}.
 \end{aligned}
 \tag{23}$$

The above correspondence clearly shows how the arbitrary  $F(R)$  gravity may be mapped into the covariant generalized holographic dark energy. Similarly, the equivalence with other modified gravities like modified Gauss–Bonnet gravity, non-local gravity, string-inspired theory may be established.

As an example, we consider the case that

$$f(R) = \alpha R^2, \tag{24}$$

with a constant  $\alpha$ . This Starobinsky inflation model gives the following spectral index  $n_s$  and the scalar–tensor ratio  $r$  [45]:

$$n_s = 0.967, \quad r = 3.33 \times 10^{-3}, \tag{25}$$

which is consistent with results in Planck 2015 [46],

$$n_s = 0.968 \pm 0.006 \text{ (68\% CL)}, \quad r < 0.11 \text{ (95\% CL)}. \tag{26}$$

For the model (24), one gets

$$\begin{aligned}
 \frac{c^2}{\kappa^2 L^2} &= 18\alpha \left( -\frac{2\ddot{L}_p \dot{L}_p}{L_p^2} + \frac{2\ddot{L}_p}{L_p^2} - \frac{2\dot{L}_p \dot{L}_p^2}{L_p^3} + \frac{\dot{L}_p^2}{L_p^2} + \frac{4\dot{L}_p \dot{L}_p}{L_p^3} \right. \\
 &\left. - \frac{8\dot{L}_p}{L_p^3} + \frac{3\dot{L}_p^4}{L_p^4} - \frac{14\dot{L}_p^3}{L_p^4} + \frac{21\dot{L}_p^2}{L_p^4} - \frac{10\dot{L}_p}{L_p^4} \right).
 \end{aligned}
 \tag{27}$$

Hence, the correspondence between the  $F(R)$  gravity and the covariant generalized holographic dark energy models is established. This equivalence shows how to describe the accelerating universe in terms of generalized holographic dark energy.

#### 4 Generalized holographic dark energy description of perfect fluid

One may further develop the correspondence between the holographic dark energy and the perfect fluid. The equation of state (EoS) of the general perfect fluid is given by

$$p = -\rho + h(\rho). \tag{28}$$

Here  $p$  is the pressure and  $\rho$  is the energy density and  $h(\rho)$  is a function of the energy density  $\rho$ . By using (20), the conservation law,

$$0 = \dot{\rho} + 3H(\rho + p) = \dot{\rho} + 3Hh(\rho), \tag{29}$$

can be rewritten as follows:

$$0 = \dot{\rho} + 3 \left( \frac{\dot{L}_p}{L_p} - \frac{1}{L_p} \right) h(\rho) = \dot{\rho} + 3 \left( \frac{\dot{L}_f}{L_f} + \frac{1}{L_f} \right) h(\rho), \tag{30}$$

which gives

$$\ln \frac{L_p}{L_0} - \int^t \frac{dt}{L_p} = \ln \frac{L_f}{L_0} + \int^t \frac{dt}{L_f} = \frac{1}{3} \int \frac{d\rho}{h(\rho)}. \tag{31}$$

Here  $L_0$  is a constant which is introduced due to a dimensional considerations. Equation (31) can be algebraically solved with respect to  $\rho$ ,

$$\rho = \rho \left( \ln \frac{L_p}{L_0} - \int^t \frac{dt}{L_p} \right) = \rho \left( \ln \frac{L_f}{L_0} + \int^t \frac{dt}{L_f} \right). \tag{32}$$

Then we find that the perfect fluid is also described by the holographic language.

As a first simple example, we may consider the fluid with constant EoS parameter  $w$ ,  $p = w\rho$ , that is,

$$h(\rho) = (1 + w) \rho, \tag{33}$$

which gives  $a(t) \propto t^{\frac{2}{3(w+1)}}$ . Then Eq. (31) gives

$$\ln \frac{L_p}{L_0} - \int^t \frac{dt}{L_p} = \ln \frac{L_f}{L_0} + \int^t \frac{dt}{L_f} = \frac{1}{3(1+w)} \ln \frac{\rho}{\rho_0}, \tag{34}$$

that is,

$$\begin{aligned} \rho &= \rho_0 \left(\frac{L_p}{L_0}\right)^{3(1+w)} e^{-3(1+w) \int^t \frac{dt}{L_p}} \\ &= \rho_0 \left(\frac{L_f}{L_0}\right)^{3(1+w)} e^{3(1+w) \int^t \frac{dt}{L_f}}. \end{aligned} \tag{35}$$

For general perfect fluid, by using the FRW equations

$$\rho = \frac{3}{\kappa^2} H^2, \quad p = -\frac{1}{\kappa^2} (2\dot{H} + 3H^2) \tag{36}$$

one can reconstruct the holographic cutoff. If the Hubble rate  $H$  is given as a function of the cosmological time  $t$ ,  $H = J(t)$ , we have

$$p = -\rho - \frac{2}{\kappa^2} J' \left( J^{-1} \left( \kappa \sqrt{\frac{\rho}{3}} \right) \right). \tag{37}$$

Equation (37) can be regarded as a generalized equation of state, where

$$h(\rho) = -\frac{2}{\kappa^2} J' \left( J^{-1} \left( \kappa \sqrt{\frac{\rho}{3}} \right) \right). \tag{38}$$

In general, it is difficult to write down the explicit form of  $h(\rho)$  for general  $J(t)$  and also to solve (31) explicitly. We now just give an example, where

$$J(t) = J_1 t + J_2 t^{-1}. \tag{39}$$

By using (37), we may define the effective EoS parameter  $w_{\text{eff}}$  by

$$w_{\text{eff}} = \frac{p}{\rho} = -1 - \frac{2\dot{H}}{3H^2}. \tag{40}$$

Then  $H = J(t)$  in (39) gives

$$w_{\text{eff}} = -1 - \frac{2(J_1 - J_2 t^{-2})}{3(J_1 t + J_2 t^{-1})^2}. \tag{41}$$

One finds, for  $t \rightarrow +\infty$ , that  $w_{\text{eff}} \rightarrow -1$ , which corresponds to the accelerating expansion. On the other hand, when  $t \rightarrow 0$ ,  $w_{\text{eff}}$  behaves as  $w_{\text{eff}} \rightarrow -1 + \frac{2}{3J_2}$ . Especially if  $J_2 = \frac{2}{3}$ , the effective EoS parameter  $w_{\text{eff}}$  coincides with the dust EoS

parameter. Because Eq. (39) can be solved with respect to  $t$  as follows:

$$t = \frac{J \pm \sqrt{J^2 - 4J_1 J_2}}{2J_1}, \tag{42}$$

Eq. (38) gives

$$h(\rho) = \frac{1}{\kappa^2 J_2} \left( \frac{\kappa^2 \rho}{3} \mp \sqrt{\frac{\kappa^2 \rho}{3} \left( \frac{\kappa^2 \rho}{3} - 4J_1 J_2 \right)} \right). \tag{43}$$

We should note that the sign  $\mp$  corresponds to the sign  $\pm$  in Eq. (42). Then Eq. (31) gives

$$\begin{aligned} \ln \frac{L_p}{L_0} - \int^t \frac{dt}{L_p} &= \ln \frac{L_f}{L_0} + \int^t \frac{dt}{L_f} \\ &= \frac{1}{4J_1} \int \frac{d\rho}{\rho} \left( \frac{\kappa^2 \rho}{3} \pm \sqrt{\frac{\kappa^2 \rho}{3} \left( \frac{\kappa^2 \rho}{3} - 4J_1 J_2 \right)} \right). \end{aligned} \tag{44}$$

It is a little bit tedious to execute the integration and rather difficult to solve Eq. (44) with respect to  $\rho$  analytically.

In Ref. [47], the following EoS was investigated:

$$p = -\left(1 + \frac{\beta}{3}\right) \rho + \frac{G_3 \beta}{\kappa^2}, \tag{45}$$

with constants  $\beta$  and  $G_3$ . The EoS fluid (45) gives the following Hubble rate:

$$H^2 = G_2 a^\beta + G_3. \tag{46}$$

With the EoS (45),  $h(\rho)$  in (28) is given by

$$h(\rho) = -\frac{\beta}{3} \rho + \frac{G_3 \beta}{\kappa^2}. \tag{47}$$

Then Eq. (31) gives

$$\begin{aligned} \ln \frac{L_p}{L_0} - \int^t \frac{dt}{L_p} &= \ln \frac{L_f}{L_0} + \int^t \frac{dt}{L_f} = -\frac{1}{\beta} \int \frac{d\rho}{\rho - \frac{3G_3 \beta}{\beta \kappa^2}} \\ &= -\frac{1}{\beta} \ln \left( \frac{\rho - \frac{3G_3 \beta}{\beta \kappa^2}}{\rho_0} \right), \end{aligned} \tag{48}$$

and we obtain

$$\begin{aligned} \rho &= \frac{3G_3 \beta}{\beta \kappa^2} + \rho_0 \left(\frac{L_p}{L_0}\right)^{-\beta} e^{\beta \int^t \frac{dt}{L_p}} \\ &= \frac{3G_3 \beta}{\beta \kappa^2} + \rho_0 \left(\frac{L_f}{L_0}\right)^{-\beta} e^{-\beta \int^t \frac{dt}{L_f}}. \end{aligned} \tag{49}$$

This shows the equivalence of the specific perfect fluid universe with covariant generalized holographic dark energy for the specific cutoff.

Furthermore, in Ref. [48], the following EoS was proposed:

$$p = -\rho - 2 \cdot 3^{-\frac{\alpha-1}{2\alpha}} \alpha \kappa^{-\frac{\alpha+1}{\alpha}} f_0^{2-\alpha} \rho^{\frac{\alpha-1}{2\alpha}}. \tag{50}$$

Here  $\alpha$  and  $f_0$  are constants. The universe inspired by the EoS fluid (50) evolves with the following Hubble rate:

$$H(t) = f_0 |t_s - t|^\alpha \quad \text{or} \quad a(t) \propto e^{\frac{f_0}{\alpha+1} |t_s - t|^{\alpha+1}}. \tag{51}$$

The form of the Hubble rate (51) shows (see [49]):

- If  $-1 < \alpha < -\frac{1}{2}$ , a Type III singularity occurs.
- If  $\alpha < -1$ , a Type I singularity appears.
- If  $\alpha > 1$ , a Type IV singularity occurs.
- If  $-\frac{1}{2} < \alpha < 1$ , a Type II singularity occurs.

As the EoS fluid (50) gives

$$h(\rho) = -2 \cdot 3^{-\frac{\alpha-1}{2\alpha}} \alpha \kappa^{-\frac{\alpha+1}{\alpha}} f_0^{2-\alpha} \rho^{\frac{\alpha-1}{2\alpha}}, \tag{52}$$

Eq. (31) leads to

$$\begin{aligned} \ln \frac{L_p}{L_0} - \int^t \frac{dt}{L_p} &= \ln \frac{L_f}{L_0} + \int^t \frac{dt}{L_f} \\ &= -\frac{1}{2 \cdot 3^{\frac{\alpha+1}{2\alpha}} \alpha \kappa^{-\frac{\alpha+1}{\alpha}} f_0^{2-\alpha}} \int \frac{d\rho}{\rho^{\frac{\alpha-1}{2\alpha}}} \\ &= -\frac{1}{3^{\frac{\alpha+1}{2\alpha}} (\alpha + 1) \kappa^{-\frac{\alpha+1}{\alpha}} f_0^{2-\alpha}} \rho^{\frac{\alpha+1}{2\alpha}}. \end{aligned} \tag{53}$$

Then we find

$$\begin{aligned} \rho &= \left( -3^{\frac{\alpha+1}{2\alpha}} (\alpha + 1) \kappa^{-\frac{\alpha+1}{\alpha}} f_0^{2-\alpha} \left( \ln \frac{L_p}{L_0} - \int^t \frac{dt}{L_p} \right) \right)^{\frac{2\alpha}{\alpha+1}} \\ &= \left( -3^{\frac{\alpha+1}{2\alpha}} (\alpha + 1) \kappa^{-\frac{\alpha+1}{\alpha}} f_0^{2-\alpha} \left( \ln \frac{L_f}{L_0} + \int^t \frac{dt}{L_f} \right) \right)^{\frac{2\alpha}{\alpha+1}}. \end{aligned} \tag{54}$$

Again, the holographic description of the above perfect fluid is established.

As an example which unifies the inflation and the late-time accelerating expansion, one may consider the following model:

$$\rho(t) = \frac{\Lambda_e + \Lambda_l a(t)}{1 + a(t)}. \tag{55}$$

At the early universe, where  $a(t) \rightarrow 0$ , we find that  $\rho$  goes to a constant,  $\rho \rightarrow \Lambda_e$  and even at the late universe, where

$a \rightarrow \infty$ ,  $\rho$  goes to a constant  $\rho \rightarrow \Lambda_l$ . Hence, at the early universe and at the late universe, the asymptotically de Sitter universe is realized. The scale factor  $a$  behaves as  $a(t) \propto e^{t\sqrt{\frac{\kappa^2 \Lambda_e}{3}}}$  at the early universe and  $a(t) \propto e^{t\sqrt{\frac{\kappa^2 \Lambda_l}{3}}}$  at the late universe. The conservation law (29) gives

$$h(\rho(t)) = -\frac{\Lambda_e - \Lambda_l}{3(1 + a(t))^2}. \tag{56}$$

As Eq. (55) can be solved with respect to  $a(t)$  as

$$a(t) = \frac{\Lambda_e - \rho(t)}{\rho(t) - \Lambda_l}, \tag{57}$$

one gets

$$h(\rho) = \frac{(\rho - \Lambda_l)^2}{3(\Lambda_e - \Lambda_l)}. \tag{58}$$

Equation (31) gives

$$\begin{aligned} \ln \frac{L_p}{L_0} - \int^t \frac{dt}{L_p} &= \ln \frac{L_f}{L_0} + \int^t \frac{dt}{L_f} = (\Lambda_e - \Lambda_l) \\ &\times \int \frac{d\rho}{(\rho - \Lambda_l)^2} = -\frac{(\Lambda_e - \Lambda_l)}{\rho - \Lambda_l}, \end{aligned} \tag{59}$$

which can be solved with respect to  $\rho$  as follows:

$$\rho = \Lambda_l - \frac{(\Lambda_e - \Lambda_l)}{\ln \frac{L_p}{L_0} - \int^t \frac{dt}{L_p}} = \Lambda_l - \frac{(\Lambda_e - \Lambda_l)}{\ln \frac{L_f}{L_0} + \int^t \frac{dt}{L_f}}. \tag{60}$$

Thus, we succeed to establish the correspondence between the general perfect fluid with Einstein gravity and the covariant generalized holographic dark energy model. Due to the well-known equivalence between the fluid description and the scalar–tensor description (for a review, see [50]) one can extend the above correspondence to scalar–tensor theory.

### 5 Discussion

In summary, we proposed the generalized holographic dark energy model where the infrared cutoff is identified with a combination of the FRW parameters: the Hubble rate, particle and future horizons, cosmological constant and universe lifetime and the derivatives of the corresponding parameters. It is pointed out that, for simple and natural choices of the infrared cutoff, motivated by the considerations of the unitarity and causality or AdS/CFT related hints, the emerging universe may be not accelerating. Furthermore, it often happens that such a holographic model does not admit the covariant description. However, as we demonstrate in this

letter, with a more complicated choice of the infrared cutoff one can map the (covariant) generalized holographic dark energy to the modified gravity or to general relativity with a quite general fluid. Specifically, the examples of the arbitrary  $F(R)$  gravity and perfect fluids are worked out in detail. It is explicitly shown how to get the realistic inflationary universe, or a viable dark energy universe or even the unification of the inflation with dark energy epoch in the frameworks of specific covariant generalized holographic dark energy.

The results of this letter give a clear recipe on how to rewrite the arbitrary modified gravity including the scalar-tensor theory or gravity theory with fluid matter as the covariant holographic model. It may indicate that different descriptions of the universe evolution may have a common origin related with a yet not fully understood symmetry somehow related with holography.

We have established that  $F(R)$  gravity can be rewritten in the holographic language at the level of background equivalence. Then it might be interesting if we consider the equivalence at the perturbation level. In the standard formulation of the holographic dark energy, the infrared cutoff only depends on the time coordinate but the scalar curvature also depends on the space coordinates. Hence, in Eq. (21),  $L_p$  or  $L_f$  should also depend on the space coordinates when we consider the perturbation, although  $L_p$  and  $L_f$  should be usually spatially constant in the standard holographic dark energy. This indicates that there is no equivalence at the perturbation level between the holographic dark energy and the  $F(R)$  gravity. We expect that the spectral index  $n_s$  and the scalar-tensor ratio  $r$  are the same in both models but cosmological perturbations theory may lead to different results. Furthermore, the infrared cutoff can depend on the point where we are considering the theory. For example, near a black hole, the infrared cutoff should depend on the spatial coordinates in addition to the time coordinate. Such an infrared cutoff could also depend on the non-local quantities as the position of the black hole or the local fluctuation of the expansion and therefore it could be rather difficult to formulate the perturbation in such a complicated case.

As another example of the dark energy models, we may consider the viscous dark energy model (see [51] and the references therein) where a bulk viscosity in the cosmic fluid generates the accelerating expansion of the universe. In the viscous dark energy model, the conservation law of the fluid is modified by the bulk viscosity  $\zeta$  as follows:

$$\dot{\rho} + 3H(\rho + p) = 9\zeta H^2. \tag{61}$$

Then the FRW equations are modified as follows:

$$3H^2 = \kappa^2 \rho, \quad -3\dot{H}^2 - 2\dot{H} = \kappa^2 (p - 3\zeta H). \tag{62}$$

By comparing Eq. (62) with Eq. (63), we may identify

$$\begin{aligned} \rho &= \frac{1}{\kappa^2} \left\{ -\frac{f(R)}{2} + 3(H^2 + \dot{H})f'(R) - 3H\frac{df'(R)}{dt} \right\}, \\ p &= \frac{1}{\kappa^2} \left\{ \frac{f(R)}{2} - (\dot{H} + 3H^2)f'(R) + 6H\frac{df'(R)}{dt} \right. \\ &\quad \left. + \frac{d^2f'(R)}{dt^2} \right\} + 9\zeta H^2. \end{aligned} \tag{63}$$

In the holographic view point,  $H$  can be given by the infrared cutoff as shown in (20). Therefore the conservation law of the fluid in (61) can be further rewritten as

$$\begin{aligned} \dot{\rho} + 3H(\rho + p) &= \dot{\rho} + 3\left(\frac{\dot{L}_p}{L_p} - \frac{1}{L_p}\right)(\rho + p) \\ &= 9\zeta\left(\frac{\dot{L}_p}{L_p} - \frac{1}{L_p}\right)^2 \\ &= \dot{\rho} + 3\left(\frac{\dot{L}_f}{L_f} - \frac{1}{L_f}\right)(\rho + p) \\ &= 9\zeta\left(\frac{\dot{L}_f}{L_f} - \frac{1}{L_f}\right)^2. \end{aligned} \tag{64}$$

Further by using a more general infrared cutoff in (13), we may write (61) as follows:

$$\dot{\rho} + 3H(\rho + p) = 9\zeta H(L, \dot{L}, \ddot{L}, \dots, t_s, \dots)^2. \tag{65}$$

This indicates that viscous dark energy may be related with the holographic dark energy or vice versa.

It might also be interesting to consider the holographic dark energy in the brane cosmology [39, 52, 53]. In  $D$  dimensional space-time, the total energy in the region with a radius  $r$  does not exceed the mass of the maximum black hole in the region. In terms of the Schwarzschild radius  $r_s$ , the mass  $M$  is proportional to  $r_s^{D-3}$ ,  $M \propto r_s^{D-3}$ ; then the energy density  $\rho$  is restricted to

$$\rho \leq \frac{M|_{r_s=r}}{V} \propto r^{-2}. \tag{66}$$

Here  $V$  is the volume of the region, which is proportional to  $r^{D-1}$ . Then by choosing  $r$  to be the infrared cutoff  $L$ , we find  $\rho \propto L^{-2}$ . Hence, if we consider the  $D - 1$  dimensional brane in the  $D$  dimensional bulk space-time, an energy density which is proportional to  $L^{-2}$  is induced, and its behavior coincides with the behavior of the four dimensional holographic dark energy in (2). Therefore  $F(R)$  or any other modified gravity in the bulk and/or the brane can be rewritten as the holographic dark energy. This will be considered elsewhere.

**Acknowledgements** This work is supported (in part) by MEXT KAK-ENHI Grant-in-Aid for Scientific Research on Innovative Areas “Cosmic Acceleration” (No. 15H05890) (SN) and by MINECO (Spain), Project FIS2013-44881, FIS2016-76363-P(SDO) and by CSIC I-LINK1019 Project (SDO and SN).

**Open Access** This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP<sup>3</sup>.

## References

- S. Nojiri, Mod. Phys. Lett. A **31**(37), 1650213 (2016). doi:10.1142/S0217732316502138. arXiv:1601.02203 [hep-th]
- R. Saitou, Y. Gong, arXiv:1702.02806 [hep-th]
- T. Mori, D. Nitta, S. Nojiri, arXiv:1702.07063 [hep-th]
- M. Li, Phys. Lett. B **603**, 1 (2004). doi:10.1016/j.physletb.2004.10.014. arXiv:hep-th/0403127
- Q.G. Huang, Y.G. Gong, JCAP **0408**, 006 (2004). doi:10.1088/1475-7516/2004/08/006. arXiv:astro-ph/0403590
- B. Wang, E. Abdalla, R.K. Su, Phys. Lett. B **611**, 21 (2005). doi:10.1016/j.physletb.2005.02.026. arXiv:hep-th/0404057
- Y.S. Myung, Mod. Phys. Lett. A **20**, 2035 (2005). doi:10.1142/S0217732305018190. arXiv:hep-th/0501023
- Y.S. Myung, Phys. Lett. B **626**, 1 (2005). doi:10.1016/j.physletb.2005.09.003. arXiv:hep-th/0502128
- K. Enqvist, M.S. Sloth, Phys. Rev. Lett. **93**, 221302 (2004). doi:10.1103/PhysRevLett.93.221302. arXiv:hep-th/0406019
- S.D.H. Hsu, A. Zee, Mod. Phys. Lett. A **20**, 2699 (2005). doi:10.1142/S0217732305018839. arXiv:hep-th/0406142
- M. Ito, Europhys. Lett. **71**, 712 (2005). doi:10.1209/epl/i2005-10151-x. arXiv:hep-th/0405281
- P.F. Gonzalez-Diaz, arXiv:hep-th/0411070
- Q.G. Huang, M. Li, JCAP **0503**, 001 (2005). doi:10.1088/1475-7516/2005/03/001. arXiv:hep-th/0410095
- S. Nobbenhuis, Found. Phys. **36**, 613 (2006). doi:10.1007/s10701-005-9042-8. arXiv:gr-qc/0411093
- Y.G. Gong, B. Wang, Y.Z. Zhang, Phys. Rev. D **72**, 043510 (2005). doi:10.1103/PhysRevD.72.043510. arXiv:hep-th/0412218
- A.J.M. Medved, Mod. Phys. Lett. A **21**, 319 (2006). doi:10.1142/S0217732306019633. arXiv:hep-th/0501100
- X. Zhang, Int. J. Mod. Phys. D **14**, 1597 (2005). doi:10.1142/S0218271805007243. arXiv:astro-ph/0504586
- Y.G. Gong, Y.Z. Zhang, Class. Quantum Gravity **22**, 4895 (2005). doi:10.1088/0264-9381/22/22/014. arXiv:hep-th/0505175
- T. Padmanabhan, doi:10.1142/9789812700988\_0007. arXiv:gr-qc/0503107
- B. Guberina, R. Horvat, H. Stefancic, JCAP **0505**, 001 (2005). doi:10.1088/1475-7516/2005/05/001. arXiv:astro-ph/0503495
- F. Arevalo, P. Cifuentes, S. Lepe, F. Peña, Astrophys. Sp. Sci. **352**, 899 (2014). doi:10.1007/s10509-014-1946-3. arXiv:1308.5007 [gr-qc]
- L.P. Chimento, M.G. Richarte, Phys. Rev. D **85**, 127301 (2012). doi:10.1103/PhysRevD.85.127301. arXiv:1207.1492 [astro-ph.CO]
- Z. Zhang, M. Li, X.D. Li, S. Wang, W.S. Zhang, Mod. Phys. Lett. A **27**, 1250115 (2012). doi:10.1142/S0217732312501155. arXiv:1202.5163 [astro-ph.CO]
- X. Zhang, F.Q. Wu, Phys. Rev. D **72**, 043524 (2005). doi:10.1103/PhysRevD.72.043524. arXiv:astro-ph/0506310
- S. Nojiri, S.D. Odintsov, Gen. Relat. Gravit. **38**, 1285 (2006). doi:10.1007/s10714-006-0301-6. arXiv:hep-th/0506212
- L.N. Granda, A. Oliveros, Phys. Lett. B **669**, 275 (2008). doi:10.1016/j.physletb.2008.10.017. arXiv:0810.3149 [gr-qc]
- M. Khurshudyan, Astrophys. Sp. Sci. **361**(12), 392 (2016). doi:10.1007/s10509-016-2981-z
- S. Wang, Y. Wang, M. Li, arXiv:1612.00345 [astro-ph.CO]
- M. Khurshudyan, Astrophys. Sp. Sci. **361**(7), 232 (2016). doi:10.1007/s10509-016-2821-1. arXiv:1606.05264 [gr-qc]
- A. Khodam-Mohammadi, M. Malekjani, Mod. Phys. Lett. A **27**, 1250100 (2012). doi:10.1142/S0217732312501003. arXiv:1201.3200 [hep-th]
- M.H. Belkacemi, M. Bouhmadi-Lopez, A. Errahmani, T. Ouali, Phys. Rev. D **85**, 083503 (2012). doi:10.1103/PhysRevD.85.083503. arXiv:1112.5836 [gr-qc]
- L. Zhang, P. Wu, H. Yu, Eur. Phys. J. C **71**, 1588 (2011). doi:10.1140/epjc/s10052-011-1588-3
- M.R. Setare, M. Jamil, Gen. Relat. Gravit. **43**, 293 (2011). doi:10.1007/s10714-010-1087-0. arXiv:1008.4763 [gr-qc]
- K. Nozari, N. Rashidi, Int. J. Mod. Phys. D **19**, 219 (2010). doi:10.1142/S021827181001635X. arXiv:0912.4802 [gr-qc]
- A. Sheykhi, Phys. Lett. B **681**, 205 (2009). doi:10.1016/j.physletb.2009.10.011. arXiv:0907.5458 [hep-th]
- L. Xu, J. Lu, W. Li, Eur. Phys. J. C **64**, 89 (2009). doi:10.1140/epjc/s10052-009-1122-z. arXiv:0906.0210 [astro-ph.CO]
- H. Wei, Nucl. Phys. B **819**, 210 (2009). doi:10.1016/j.nuclphysb.2009.04.020. arXiv:0902.2030 [gr-qc]
- M.R. Setare, E.N. Saridakis, Phys. Lett. B **670**, 1 (2008). doi:10.1016/j.physletb.2008.10.029. arXiv:0810.3296 [hep-th]
- E.N. Saridakis, Phys. Lett. B **661**, 335 (2008). doi:10.1016/j.physletb.2008.02.032. arXiv:0712.3806 [gr-qc]
- M.R. Setare, Phys. Lett. B **644**, 99 (2007). doi:10.1016/j.physletb.2006.11.033. arXiv:hep-th/0610190
- F. Felegary, F. Darabi, M.R. Setare, arXiv:1612.03406 [gr-qc]
- E. Elizalde, S. Nojiri, S.D. Odintsov, P. Wang, Phys. Rev. D **71**, 103504 (2005). doi:10.1103/PhysRevD.71.103504. arXiv:hep-th/0502082
- S. Nojiri, S.D. Odintsov, Phys. Rep. **505**, 59 (2011). doi:10.1016/j.physrep.2011.04.001. arXiv:1011.0544 [gr-qc]
- M.R. Capozziello, M. De Laurentis, Phys. Rep. **509**, 167 (2011). doi:10.1016/j.physrep.2011.09.003. arXiv:1108.6266 [gr-qc]
- G. Hinshaw et al. [WMAP Collaboration], Astrophys. J. Suppl. **208**, 19 (2013). doi:10.1088/0067-0049/208/2/19. arXiv:1212.5226 [astro-ph.CO]
- P.A.R. Ade et al. [BICEP2 and Keck Array Collaborations], Phys. Rev. Lett. **116**, 031302 (2016). doi:10.1103/PhysRevLett.116.031302. arXiv:1510.09217 [astro-ph.CO]
- K. Bamba, S. Nojiri, S.D. Odintsov, D. Sáez-Gómez, Phys. Rev. D **90**, 124061 (2014). doi:10.1103/PhysRevD.90.124061. arXiv:1410.3993 [hep-th]
- S. Nojiri, S.D. Odintsov, V.K. Oikonomou, Phys. Lett. B **747**, 310 (2015). doi:10.1016/j.physletb.2015.06.016. arXiv:1506.03307 [gr-qc]
- S. Nojiri, S.D. Odintsov, S. Tsujikawa, Phys. Rev. D **71**, 063004 (2005). doi:10.1103/PhysRevD.71.063004. arXiv:hep-th/0501025
- K. Bamba, S. Capozziello, S. Nojiri, S.D. Odintsov, Astrophys. Sp. Sci. **342**, 155 (2012). doi:10.1007/s10509-012-1181-8. arXiv:1205.3421 [gr-qc]
- I. Brevik, Ø. Grøn, J. de Haro, S.D. Odintsov, E.N. Saridakis, arXiv:1706.02543 [gr-qc]
- E.N. Saridakis, JCAP **0804**, 020 (2008). doi:10.1088/1475-7516/2008/04/020. arXiv:0712.2672 [astro-ph]
- E.N. Saridakis, Phys. Lett. B **660**, 138 (2008). doi:10.1016/j.physletb.2008.01.004. arXiv:0712.2228 [hep-th]