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COVARIANT QUANTIZATION OF SUPERGRAVITY *)

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ABSTRACT

Unitarity requires a modification of the Faddeev-Popov quantization rules for Poincaré supergravity in second-order formalism. In particular a four-ghost coupling is needed. By introducing a scalar, a pseudoscalar and an axial vector auxiliary field, the gauge algebra closes and ordinary Faddeev-Popov quantization is applicable. Elimination of the auxiliary fields <u>after</u> quantization reproduces the four-ghost coupling.

The status of renormalizability of Poincaré supergravity is reviewed. The crucial question of three-loop finiteness is still unresolved. An explicit calculation of photon-photon scattering in the O(8) model is presented.

The gauge action of conformal supergravity is discussed. It arises naturally from gauging the graded conformal group, provided three constraints are imposed. These constraints turn out to be model-independent and to have a geometrical meaning. The gauge algebra closes and a connection between Poincaré and conformal supergravity arises.

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1. INTRODUCTION

Supergravity is by now two years old, old enough to allow an assessment of its successes and shortcomings. From the beginning it was hoped that supergravity would bring significant progress to two important problems of theoretical physics:

i) the unification of all interactions, gravity included;

ii) the construction of a satisfactory quantum theory of gravity.

Models of supergravity can, in principle, unify the interactions of nature because they unify all particles of a given model into one (or sometimes two or more) superparticles because all interactions arise by exchange of particles. To draw an analogy, consider nuclear physics where a proton and neutron form one nucleon. Exchange of the pion unifies the p-p, p-n and n-n forces into an isoscalar interaction. In supergravity, one superparticle can be a graviton, a photon, a gluon, an intermediate vector, an electron or a quark, etc., depending on the orientation of its "arrow in Fermi-Bose space", and exchange of this superparticle leads then at once to all interactions.

Supergravity is the gauge theory of supersymmetry (the latter is also sometimes called Fermi-Bose symmetry). Global supersymmetry models can be constructed in flat space-time and have nothing to do with gravity, but local supersymmetry requires curved space-time (see below) and is thus a theory of gravity. The symmetry between bosons and fermions gives all particles which constitute a superparticle the same mass. Thus the symmetry breaking is needed, and recently progress was made here[1]. [Specifically: in the spin $(2, 3/2) + (\frac{1}{2}, 0^+, 0^-)$ system one can choose a potential such that there is no cosmological term and the spin 3/2 acquires a mass through the Higgs mechanism.] However, much remains to be done before phenomenology is possible. In particular, even the largest of all models with only one superparticle [the so-called $0(8)$ model] has not enough internal symmetry to accommodate an $SU_3 \times SU_2 \times U_1$ group, and does not contain W^+, W^-, μ^- particles[2]. On the other hand, it is the only theory which predicts quark changes of $2/3$ and $-1/3$.

The problem of quantum gravity is its dimensional coupling constant. In the relativistic extension of Newton's theory (general relativity), gravitons couple to energy instead of mass, so that the effective coupling constant in gravity, and in gravity alone, depends itself on energy. Thus in a one-loop Feynman diagram one has as always the divergences due to a summation over all virtual energies, but in quantum gravity the integrand itself blows up with energy. Thus one has worse divergences than in, say, Yang-Mills theory, and true renormalizability can never occur, but only the two extremes: finiteness of the S matrix or non-renormalizability. Finiteness is only possible due to "miraculous" cancellations which follow from some symmetry, and there is a possibility that local supersymmetry is that symmetry, as we shall see.

2. BASICS OF SUPERSYMMETRY

Under a Fermi-Bose rotation a boson B rotates into a fermion F as follows

$$\delta B = F\epsilon$$

It follows that ε must be anticommuting and a half-integer spinor, in order to keep both sides of the equation the same (fermions must anticommute since otherwise their actions vanish identically). The simplest choice, ε a spin $\frac{1}{2}$ object, leads to supersymmetry. It follows that in supersymmetry the elementary building blocks are doublets of one fermion and one boson with adjacent spins $(J, J+\frac{1}{2})$ or $(J, J-\frac{1}{2})$.

Since the dimension of a boson is 1 and of a fermion 3/2, it follows that ε has dimension $-\frac{1}{2}$, so that in the inverse rotation $\delta F \sim B\varepsilon$, there is a gap of one unit of dimension. This gap can be filled by either a derivative, or the gravitational constant (or a mass, but since we are heading for a gauge theory we exclude masses). Thus

$$\delta F = \partial_\mu B\epsilon \quad \text{or} \quad \frac{1}{\kappa}(\partial_\mu + B)\epsilon \tag{1}$$

omitting gamma matrices, etc. The commutator yields*)

$$[\delta(\epsilon_1), \delta(\epsilon_2)]B = 2(\bar{\epsilon}_2\gamma^\mu\epsilon_1)\partial_\mu B + \partial_\mu\left(\frac{\bar{\epsilon}_1\epsilon_2}{\kappa}\right) \tag{2}$$

so that space-time symmetries ($\partial_\mu = P_\mu$ is part of the Poincaré group) and internal symmetries (the last term is an Abelian gauge transformation with parameter $\bar{\varepsilon}_1\varepsilon_2/\kappa$) become unified thanks to the introduction of the notion of fermionic symmetry. In operator notation, with $\delta(\varepsilon)B = [\bar{\varepsilon}Q, B]$ one thus has

$$[\bar{\epsilon}_1 Q, \bar{\epsilon}_2 Q] = 2(\bar{\epsilon}_2\gamma^\mu\epsilon_1)P_\mu + \text{internal symmetries} \tag{3}$$

*) More precisely: in O(n) models the Abelian parameter is $\bar{\varepsilon}_1^i\varepsilon_2^j\varepsilon_{ij}$. In models with only one ε, there is no Abelian parameter.

Suppose that one deals with local supersymmetry, so that $\varepsilon = \varepsilon(x)$. Then the right-hand side describes a translation P_μ over a space-time dependent distance $\xi^\mu(x) = 2\bar{\varepsilon}_2(x)\gamma^\mu\varepsilon_1(x)$, which is the notion of a general co-ordinate transformation. Thus local supersymmetry leads to gravity! The converse holds, too. If one has in curved space a constant ε, then, after a gravitational gauge transformation, ε will have picked up a dependence or a space-time.

What are the gauge fields of local supersymmetry? Quite generally, gauge fields are obtained by adding a four-vector index μ to the parameters. Hence, the supersymmetry gauge field is the vectorial spinor ψ_μ. Its spin content is

$$\text{spin } \psi_\mu = \frac{1}{2} \times (1 + 0) = \frac{3}{2} + \frac{1}{2} + \frac{1}{2} \tag{4}$$

The lower gauge components with spin ½ ($\partial \cdot \psi$ and $\gamma \cdot \psi$) are needed in the action in order that the theory be consistent. One can formulate a theory with only spin 3/2 in the action (and helicity ±3/2 physical states only), but this theory is then not ordinary supergravity but conformal supergravity with higher derivatives (see below).

From a group-theoretical point of view, supergravity can be obtained by gauging the graded Poincaré group, whose generators are $P_\mu, M_{\mu\nu}$ and Q_a, satisfying

$$[Q_a, P_\mu] = 0 \;;\; [Q_a, M_{\mu\nu}] = (\sigma_{\mu\nu})_a{}^b Q_b \;;\; \{Q_a, Q_b\} = (\gamma^\mu C)_{ab} P_\mu \tag{5}$$

where C must be the charge conjugation matrix in order that the Jacobi identities are satisfied. Thus one expects <u>three</u> gauge fields: for P_μ, for $M_{\mu\nu}$ and for Q_a. The gauge field for P_μ is the vierbein field e_μ^a (which is the square root of $g_{\mu\nu}$, $e_\mu^a e_{a\nu} = g_{\mu\nu}$), that of $M_{\mu\nu}$ is the spin connection ω_μ^{ab} and that of Q is ψ_μ. Since e_μ^a and ψ_μ have both two physical states with helicities ±2 and ±3/2, respectively, there are already equal numbers of boson and fermion states and $\omega_{\mu ab}$ must be non-physical. In supergravity based on the graded Poincaré group (Poincaré supergravity, or, just supergravity) this follows from the field equation of ω (Palatini formalism), but in conformal supergravity it follows from a constraint (vanishing of the translational curvature, see below).

3. THE GAUGE ACTION NAIVELY QUANTIZED

The gauge action of Poincaré supergravity is the sum of the Einstein action and the Rarita-Schwinger action in curved space

$$\mathcal{L} = -\frac{e}{2\kappa^2} R - \frac{e}{2} \bar{\psi}_\mu R^\mu \tag{6}$$

$$R = e_b{}^\mu e_a{}^\nu \left(\partial_\mu \omega_\nu{}^{ab} + \omega_\mu{}^{ac} \omega_{\nu c}{}^{b} - \mu \leftrightarrow \nu \right) \tag{7}$$

$$R^\mu = e^{-1} \epsilon^{\mu\nu\rho\sigma} \gamma_5 \gamma_\nu \left(\partial_\rho + \tfrac{1}{2} \omega_{\rho ab} \sigma^{ab} \right) \psi_\sigma \tag{8}$$

It is thus linear in the spin 2 and spin 3/2 curvatures. The field equation for the field $\omega_{\mu ab}$ is non-propagating and can be solved algebraically (an elementary exercise)

$$\omega_{\mu ab} = \omega_{\mu ab}(e) + \frac{\kappa^2}{4} \left(\bar{\psi}_\mu \gamma_a \psi_b - \bar{\psi}_\mu \gamma_b \psi_a + \bar{\psi}_a \gamma_\mu \psi_b \right) \tag{9}$$

where $\omega_{\mu ab}(e)$ follows from the metric postulate

$$\partial_\mu e^a{}_\nu + \omega_\mu{}^{ab}(e) e_{b\nu} - \Gamma^\alpha_{\mu\nu}(g) e^a{}_\alpha = 0 \tag{10}$$

Thus supergravity is a theory with torsion since $\omega_{\mu ab} \neq \omega_{\mu ab}(e)$.

Substituting the solution for $\omega_{\mu ab}$ back into the action, it follows that one never need consider variations of $\omega_{\mu ab}$ since they always are multiplied by zero

$$\frac{\delta I(e,\psi,\omega)}{\delta \omega} \left(\frac{\delta\omega}{\delta e} \delta e + \frac{\delta \omega}{\delta \psi} \delta \psi \right) = 0 \quad \text{since} \quad \left. \frac{\delta I}{\delta \omega} \right|_{e,\psi} = 0 \tag{11}$$

This obvious but invaluable trick treats ω as if it were an independent field (with $\delta\omega = 0$), but one really is working all the time in second order formalism with only e and ψ as independent fields (in order to satisfy $\delta I/\delta\omega = 0$). It combines the simple features of second order formalism[3] (where the transformation rules δe and $\delta\psi$ are simple but a complicated four-fermion term is present) with those of first order formalism[4] (where the complicated four-fermion term is replaced by the independence of the field ω but where $\delta\omega$ is complicated). It has been of incomparable help in constructing new models[5] and has been named 1.5 order formalism[6].

The action is invariant under the following transformation rules

$$\delta e^a{}_\mu = \kappa \bar{\epsilon} \gamma^a \psi_\mu \quad ; \quad \delta\psi_\mu = \frac{2}{\kappa}\left(\partial_\mu + \frac{1}{2}\omega_\mu{}^{ab}\sigma_{ab}\right)\epsilon \tag{12}$$

where, we repeat, $\omega = \omega(e,\psi)$. The invariance is shown as follows. Varying (only 1.5 order!) the vierbein fields in the Einstein action yields the Einstein tensor. Variation of ψ_σ in $D_\rho\psi_\sigma$ yields the commutator $[D_\rho, D_\sigma]$ hence a Riemann curvature. Variation of $\bar{\psi}_\mu$ yields the same result after partial integration and all curvature terms cancel. In the process of partially integrating one picks up terms proportional to $\partial_\mu\gamma_\nu$ and to $\omega_{\mu ab}$, while variation of the vierbein in γ_ν yields a $\psi^3\varepsilon$ term. These last three terms sum up to the torsion field equation in (9) and cancel too.

The gauge action (6) is invariant under (12) and under general co-ordinate and local Lorentz transformations, given by

$$\delta_G \begin{pmatrix} e^a{}_\mu \\ \psi_\mu \end{pmatrix} = \xi^\alpha \partial_\alpha \begin{pmatrix} e^a{}_\mu \\ \psi_\mu \end{pmatrix} + \begin{pmatrix} e^a{}_\alpha \\ \psi_\alpha \end{pmatrix} \partial_\mu \xi^\alpha \tag{13}$$

$$\delta_L e^a{}_\mu = \lambda^a{}_b e^b{}_\mu \quad ; \quad \delta_L \psi_\mu = \frac{1}{2}\lambda_{ab}\sigma^{ab}\psi_\mu \tag{14}$$

If one were to quantize naively, one would add gauge fixing terms; for example

$$\mathcal{L}(fix) = -\frac{1}{4}\left(\partial_\mu \sqrt{g}\, g^{\mu\nu}\right)^2 + \frac{1}{4}\bar{\psi}\cdot\gamma\,\not{\partial}\,\gamma\cdot\psi + \left(e_{a\mu} - e_{\mu a}\right)^2 \tag{15}$$

The ghost actions would then follow as usual, starting with

$$\mathcal{L}(\text{ghost}) = \bar{C}^{\nu} \Box C^{\nu} - \bar{C} \gamma^{\mu} \partial_{\mu} C - \bar{C}^{ab} C_{ab} \tag{16}$$

where the gravitational ghosts C^{ν} and C^{ab} anticommute but the supersymmetry ghost C (a four-component complex spin ½ field) commutes. The necessity of these opposite statistics not only follows from general theoretical arguments, but also from a theoretical "experiment"[7]. Calculating the self-energy of a graviton due to a spin 3/2 loop, one finds that the conservation equation $p_{\mu} D_{\mu\nu,\rho\sigma} p_{\sigma}$ only holds if the spin 3/2 ghost field is commuting (so that no - sign for the loop is added).

At this point propagators and vertices are given and one can calculate loop processes to investigate the renormalizability of supergravity. However, it is clear that there is something unusual going on as far as unitarity is concerned. Usually the ghost fields remove the unphysical modes of the gauge fields from the quantum theory, so that one ends up with two modes for a massless field at the quantum level. But the field ψ_{μ} has 4 (4-component spinors) times 4 (index μ) divided by 2 (since fermions have half as many degrees of freedom as bosons) = 8 components, while C has 4 components (ghosts are complex!). Apparently, there are $8 - 4 = 4$ quantum modes of ψ_{μ}. There is really no problem as we shall see, but new quantization rules are needed[8]. The root of the problem can be traced back to a birth defect of supergravity, the non-closure of the gauge algebra[9].

4. THE GAUGE ALGEBRA

The gauge algebra of supergravity in second order formalism with only e^{a}_{μ} and ψ_{μ} as independent fields, does not close. For the commutator of two local supersymmetry transformations of e^{a}_{μ} one finds[9]

$$\begin{aligned} \left[\delta(\epsilon_1), \delta(\epsilon_2)\right] e^{a}{}_{\mu} &= 2\, \bar{\epsilon}_2 \gamma^{a} D_{\mu} \epsilon_1 - (1 \leftrightarrow 2) = \\ &\left(t^{\lambda} \partial_{\lambda} e^{a}{}_{\mu} + e^{a}{}_{\lambda} \partial_{\mu} t^{\lambda}\right) + \left(t^{\lambda} \omega_{\lambda ab} e^{b}{}_{\mu}\right) + \left(-\tfrac{\kappa^2}{2} t^{\lambda} \bar{\psi}_{\lambda} \gamma_{a} \psi_{\mu}\right) \end{aligned} \tag{17}$$

i.e., a sum of a general co-ordinate transformation G with parameter $t^\lambda \equiv$ $\equiv 2\,\bar{\varepsilon}_2\gamma^\lambda\varepsilon_1$, a local Lorentz transformation L with parameter $t^\lambda\omega_{\lambda ab}$ and a local supersymmetry transformation S with parameter $-\frac{\kappa}{2}t^\lambda\psi_\lambda$. But a tedious calculation for the same commutator acting on ψ_μ yields extra terms

$$\begin{aligned}[\delta(\epsilon_1),\delta(\epsilon_2)]\psi_\mu &= \delta_G(t^\lambda)+\delta_L(t^\lambda\omega_{\lambda ab})+\delta_S\left(-\tfrac{\kappa}{2}t^\lambda\psi_\lambda\right)\\ &+(\bar\epsilon_1\gamma^\alpha\epsilon_2)\left[\tfrac{1}{4}\gamma_\alpha g_{\mu\rho}+\tfrac{1}{2}\epsilon_{\mu\alpha\rho\tau}\gamma_5\gamma_\tau\right]R^\tau\\ &+(\bar\epsilon_1\sigma^{\rho\sigma}\epsilon_2)\left[\tfrac{1}{2}\sigma_{\rho\sigma}g_{\mu\tau}+g_{\mu\rho}g_{\sigma\tau}+\tfrac{e}{2}\epsilon_{\rho\sigma\tau\mu}\gamma_5\right]R^\tau\end{aligned} \tag{18}$$

where R^τ is the spin 3/2 field equation. Thus off-shell $(R^\mu \neq 0)$ the gauge algebra does not close. For all other commutators one finds at the right-hand side a sum of gauge transformations with field-dependent parameters.

In more formal and general notation[10)], we have found the following. If a gauge transformation with parameter ξ^α of a gauge field ϕ^i is given by

$$\delta(\xi)\phi^i \equiv R^i{}_\alpha(\phi)\,\xi^\alpha \tag{19}$$

then the commutator algebra is given by

$$\begin{aligned}[\delta(\eta),\delta(\xi)]\phi^i &= R^i{}_{\alpha,j}R^j{}_\beta\left(\eta^\beta\xi^\alpha-\xi^\alpha\eta^\beta\right)\\ &= R^i{}_\gamma f^\gamma{}_{\alpha\beta}(\phi)\,\xi^\alpha\eta^\beta+\eta^{ij}_{\alpha\beta}\,I_{,j}\,\xi^\alpha\eta^\beta\end{aligned} \tag{25}$$

The parameter of the gauge transformations on the right-hand side is $f^\gamma_{\alpha\beta}(\phi)\xi^\alpha\eta^\beta$ and is, in general, field dependent since the structure constants of the local algebra $f^{\gamma}_{\alpha\beta}$ do in general depend on ϕ^i. The non-closure function $\eta^{ij}_{\alpha\beta}$ is non-zero only if i,j are the indices of a gravitino field and α,β refer to local supersymmetry. Note that we define $I,_j$ by right derivatives; if $\phi^j = \psi_\mu$ then $I,_j = \delta I/\delta\psi_\mu$ and

$$\delta I/\delta\psi_\mu = -D_\rho \bar{\psi}_\sigma \gamma_5 \gamma_\nu \epsilon^{\mu\nu\rho\sigma} = (\mathbb{C} R^\mu)^T \tag{21}$$

where $\mathbb{C}$ is the charge conjugation matrix $(\mathbb{C} = \gamma_0)$ (for a Majorana spinor one has $\lambda = C\bar{\lambda}^T$).

One can repeat the foregoing for supergravity in first order formalism. One finds then that $\eta^{ij}_{\alpha\beta}$ is non-zero for i,j any of the fields e^a_μ, ψ_μ or $\omega_{\mu ab}$ but α,β still referring to local supersymmetry only.

Non-closure of the algebra arises in <u>global</u> supersymmetry if one eliminates auxiliary fields from the algebra by means of their non-propagating field equations which are given by an action which is invariant. Consider, for example, the spin $(1, \frac{1}{2})$ global supersymmetry multiplet

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu V_\nu - \partial_\nu V_\mu)^2 - \tfrac{1}{2}\bar{\lambda}\not{\partial}\lambda + \tfrac{1}{2}D^2$$

$$\delta V_\mu = -\bar{\epsilon}\gamma_\mu\lambda \; ; \; \delta\lambda = F_{\alpha\beta}\sigma^{\alpha\beta}\epsilon + i\gamma_5 D\epsilon \; ; \; \delta D = i\bar{\epsilon}\gamma_5\not{\partial}\lambda \tag{22}$$

With D present, one finds $[\delta(\varepsilon_1), \delta(\varepsilon_2)]\lambda = 2(\bar{\varepsilon}_2\gamma^\mu\varepsilon_1)\partial_\mu\lambda$, but if one first eliminates D from the algebra by inserting its field equations $D = 0$, one finds

$$[\delta(\epsilon_1), \delta(\epsilon_2)]\lambda = 2(\bar{\epsilon}_2\gamma^\mu\epsilon_1)\partial_\mu\lambda + \tfrac{1}{2}(\bar{\epsilon}_1\gamma^\mu\epsilon_2)\gamma_\mu\not{\partial}\lambda + (\bar{\epsilon}_1\sigma^{\rho\sigma}\epsilon_2)\sigma_{\rho\sigma}\not{\partial}\lambda \tag{23}$$

Incidentally, since $\delta(\text{boson}) \approx \bar{\varepsilon}$ fermion, but $\delta(\text{fermion}) \approx \partial_\mu(\text{boson})\varepsilon$, the algebra always closes on boson fields if there are no auxiliary fields[11], since a propagating boson field equation requires two derivatives*). This explains why the gauge algebra of supergravity closes on e^a_μ in second order formalism.

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*) That is assuming that the algebra closes on shell.

Since the beginning of supergravity, there have been speculations that in supergravity the reverse may hold: by adding extra fields to the transformation laws of e_μ^a and ψ_μ, one may obtain a closed algebra. What remains after finding these fields, is to find an action containing these auxiliary, as well as the physical fields, such that upon elimination of the auxiliary fields one regains the supergravity gauge action in second order formalism.

This conjecture has recently been verified[12)]. The auxiliary fields are: a scalar S, a pseudoscalar P and an axial vector A_μ. The gauge transformations for local supersymmetry become

$$\begin{aligned}
\delta e^a{}_\mu &= \kappa \bar{\epsilon}\gamma^a\psi_\mu \\
\delta\psi_\mu &= \frac{2}{\kappa}\left(D_\mu + \frac{i}{2}A_\mu\gamma_5\right)\epsilon + \frac{1}{3}\gamma_\mu N\epsilon \\
\delta A_\mu &= \frac{3i}{2}\bar{\epsilon}\gamma_5\left(\hat{R}_\mu - \frac{1}{3}\gamma_\mu\gamma\cdot\hat{R}\right) + \frac{i\kappa}{2}\bar{\epsilon}N\gamma_5\psi_\mu \\
\delta S &= \frac{1}{2}\bar{\epsilon}\gamma\cdot\hat{R} - \frac{\kappa}{2}\bar{\epsilon}\gamma^\sigma N\psi_\sigma \\
\delta P &= \frac{-i}{2}\bar{\epsilon}\gamma_5\gamma\cdot\hat{R} + \frac{i\kappa}{2}\bar{\epsilon}\gamma_5\gamma^\sigma N\psi_\sigma \\
N &= S - i\gamma_5 P - i\not{A}\gamma_5 \\
e\hat{R}_\mu &= \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\left(D_\rho + \frac{i}{2}A_\rho\gamma_5\right)\psi_\sigma
\end{aligned} \tag{24}$$

The action is even simpler

$$\mathcal{L} = -\frac{e}{2\kappa^2}R - \frac{e}{2}\bar{\psi}_\mu R^\mu + \frac{e}{3}\left(A_\mu A^\mu - S^2 - P^2\right) \tag{25}$$

Note that in the algebra one has chiral minimal coupling (except inside N), but not in the action, since there one finds R^μ instead of $\hat{R}^\mu$ as well as the mass term A_μ^2.

These results reveal an interesting connection between Poincaré and conformal supergravity. In conformal supergravity[13),14)] there are two local supersymmetries associated with $Q \approx \sqrt{P}$ and $S \approx \sqrt{K}$, where K are conformal boosts. There are three gauge fields: e_μ^a, ψ_μ and an axial vector field A_μ! The transformation rules[*)] are

$$\delta_Q e^a_\mu = \bar{\epsilon}\gamma^a\psi_\mu \quad ; \quad \delta_Q \psi_\mu = 2\left(D_\mu + \tfrac{i}{2}A_\mu\gamma_5\right)\epsilon$$
$$\delta_Q A_\mu = \frac{3i}{2}\bar{\epsilon}\gamma_5\left(\hat{R}_\mu - \tfrac{1}{3}\gamma_\mu\gamma\cdot\hat{R}\right) \tag{26}$$

and

$$\delta_S e^a_\mu = 0 \; ; \; \delta_S\psi_\mu = -\gamma_\mu\epsilon \; ; \; \delta_S A_\mu = -\frac{3i}{2}\bar{\epsilon}\gamma_5\psi_\mu \tag{27}$$

Looking now at the Poincaré gauge algebra, we see that δe, $\delta\psi$ and δA are a linear combination of δ_Q and δ_S, with parameters ε and

$$\epsilon_S = -\tfrac{1}{3}\left(S - iP\gamma_5 - i\not{A}\gamma_5\right)\epsilon$$

respectively. The kinetic terms of the conformal action are given by

$$\mathcal{L} = e\kappa^2\left[\frac{\left(R_{\mu\nu}^{\;2} - \frac{1}{3}R^2\right)}{\kappa^2} - \bar{\psi}_\mu\left(\Box g_{\mu\nu} - \partial_\mu\partial_\nu\right)\left(\hat{R}^\nu - \tfrac{1}{3}\gamma^\nu\gamma\cdot\hat{R}\right)\right] - \frac{e}{3}F^2_{\mu\nu}(A) \tag{28}$$

The Q- and S-invariance at the linearized level is obvious.

We now see what has happened. The fields e_μ^a, ψ_μ and A_μ have acquired two extra derivatives, so that A_μ becomes propagating. Thus e_μ^a describes two massless modes (since $\Box^2$ acts as twice $\Box$) and ψ_μ three massless modes (since $\Box\not{\partial}$ is equivalent to three times $\not{\partial}$)[14)]. There are thus an equal number of bosonic and fermionic states (6), and this counting explains why the fields S

*) The normalization is A_μ [Ref. 13)] = $-2/3\, A_\mu$ (here).

and P are absent from the conformal supergravity gauge actions. The Q- and S-supersymmetry is needed to eliminate the $\gamma\cdot\psi$ and $\partial\cdot\psi$ spin $\frac{1}{2}$ parts of ψ_μ so that only three massless modes, each with helicity $\pm 3/2$, remain. It is intriguing that not only the transformation laws of e, ψ, A show a direct connection between Poincaré and conformal supergravity, but that also the fields S and P seem to know about S supersymmetry through N.

Also for various matter couplings, it has been shown that one can add auxiliary fields such that the algebra closes[15]. The auxiliary fields are of two kinds: either S, P, A_μ, or they are the auxiliary fields which are already needed for the matter system in order that the global supersymmetry algebra closes.

We stress that all these results hold only in second order formalism (or in 1.5 order formalism) where ω is not an independent field (but S, P, A_μ are independent).

5. UNITARITY IN SUPERGRAVITY

Suppose a classical action I is invariant under gauge transformation

$$\delta\phi^k = R^k{}_\alpha(\phi)\,\xi^\alpha \tag{29}$$

One quantizes then, according to the Feynman-De Witt-Faddeev-Poppov-'t Hooft-Veltman procedure, by adding a gauge fixing and a ghost term

$$\mathcal{L}^{qu} = \mathcal{L}^{cl} + \tfrac{1}{2}F_\alpha(\phi)\,a^{\alpha\beta}\,F_\beta(\phi) + \bar{C}^\alpha F_{\alpha,j}\,R^j{}_\beta\,C^\beta \tag{30}$$

where $F_{\alpha,j} = \delta F_\alpha(\phi)/\delta\phi^j$ may be field-dependent but $a^{\alpha\beta}$ should not depend on ϕ (otherwise unitarity is not satisfied). The (anti) ghost fields $(\bar{C}^\alpha)C^\beta$ are anticommuting for bosonic gauge invariances, but commute for local supersymmetry.

This is the usual quantization procedure for non-Abelian gauge fields and it is correct in all cases except in supergravity. In supergravity, one needs an extra <u>four-ghost coupling</u>[8),10),16)] in order that the theory be unitary (by which one always means: a unitary <u>and</u> gauge-fixing-term independent S matrix). No six or higher ghosts seagulls are needed for unitarity.

The four-ghost coupling can be expressed in terms of the non-closure function η of the previous section by

$$\mathcal{L}(\text{4-ghost}) = \frac{1}{4}\,\bar{C}^{\gamma} F_{\gamma,j}\,\bar{C}^{\delta} F_{\delta,i}\,\eta^{ij}_{\alpha\beta}\,C^{\alpha} C^{\beta} \tag{31}$$

For the gauge $F_\alpha(\phi) = -(\kappa/2)\gamma\cdot\psi$ one finds from (18)

$$\mathcal{L}(\text{4-ghost}) = \frac{5\kappa^2}{32}\left(\bar{C}\gamma_\alpha \mathcal{C}\,\bar{C}^T\right)\left(C^T \mathcal{C}\,\gamma^\alpha C\right) \tag{32}$$

where we have chosen the normalization of F_α such that the kinetic term of the supersymmetry ghost is the usual $-\bar{C}\gamma^\mu D_\mu C$. Note that one cannot scale F_α, $\bar{C}$ and C such that all ghost terms are a density.

Before explaining these results, we show that these results are consistent. Suppose one would have quantized the gauge action of Poincaré supergravity with the auxiliary fields S, P and A_μ. Since its gauge algebra closes, $\eta^{ij}_{\alpha\beta} = 0$ and

$$\mathcal{L}(\text{ghost}) = -\bar{C}\gamma^\mu\left[D_\mu + \frac{i\kappa}{2}A_\mu\gamma_5 + \frac{1}{6}\gamma_\mu\left(S - i\gamma_5 P - i\not{A}\gamma_5\right)\right]C \tag{33}$$

Eliminating S, P, A_μ from the quantum actions, one finds[12)]

$$A_\mu = \frac{3i\kappa}{4}\left(\bar{C}\gamma_\mu\gamma_5 C\right);\quad S = -\kappa\left(\bar{C}C\right);\quad P = i\kappa\left(\bar{C}\gamma_5 C\right) \tag{34}$$

and, substituting these field equations back into the action, one indeed regains $\mathcal{L}$(4-ghost) of Eq. (32).

These results can be obtained by requiring unitarity[16)]. In order to prove unitarity one needs Ward identities, and in order to prove Ward identities one needs a local or global invariance of the quantum action. Although the classical gauge transformations no longer leave the quantum action invariant (for that purpose F_α was introduced!), there is an extension of the classical transformation rules under which the quantum action is invariant. This is BRS-invariance[17)], and it is a <u>global</u>, non-linear symmetry with a constant anticommuting parameter Λ. The anticommuting nature of Λ has led some authors to conjecture on a deep connection between supersymmetry and BRS-invariance, but the two have really nothing to do with each other: <u>any</u> gauge theory, including supergravity, is BRS invariant at the quantum level. The BRS transformation rules for any theory where the gauge algebra closes on-shell, read

$$\begin{aligned} \delta\phi^i &= \delta\phi^i_{BRS} + \delta\phi^i_{K} = R^i{}_\alpha C^\alpha \Lambda + \tfrac{1}{2}\bar{C}^\gamma F_{\gamma,j}\,\eta^{ji}_{\alpha\beta} C^\alpha C^\beta \Lambda \\ \delta C^\beta &= \tfrac{1}{2} f^\beta{}_{\alpha\gamma} C^\alpha \Lambda C^\gamma \\ \delta\bar{C}^\alpha &= \Lambda F_\beta\, a^{\beta\alpha} \end{aligned} \tag{35}$$

They were found by R. Kallosh in an excellent paper[10] for F_α linear in ϕ *).

The invariance of the action under these extended BRS rules is shown as follows. The classical action is clearly invariant under $\delta\phi^i_{BRS}$. The variation $\delta\bar{C}^\alpha$ in $\mathcal{L}$(2-ghost) cancels the variation $\delta\phi^i_K$ in $\mathcal{L}$(fix). The variation $\delta\bar{C}^\alpha$ in $\mathcal{L}$(4-ghost) cancels the variation $\delta\phi^i_K$ in $\mathcal{L}$(fix). The variation $\delta_{BRS}\phi^i$ in $\mathcal{L}$(2-ghost) yields a commutator on ϕ^j since $\mathcal{L}$(2-ghost) itself is proportional to $\delta_{BRS}\phi^j$. This is thus where the non-closure function η comes in. One has now a large number of remaining terms, all containing η, and, miraculously, they cancel!

The Ward identities follow easily by noting that

$$\delta_{BRS}\left\langle \bar{C}^\alpha(x)\,\phi^k(y)\,\phi^{n_1}(z_1)\cdots\phi^{n_\ell}(z_\ell)\right\rangle^{\mathcal{L}(qu)} = 0 \tag{36}$$

This is because for <u>any</u> $\mathcal{L}$ varying both all external lines and the action from which the n point function is obtained, yields zero <u>identically</u>. But since our $\mathcal{L}$ (qu) is invariant under the extended BRS variations, one only needs to vary the external fields. For the unitarity relation we will only need all lines with on-shell momenta which means that one only needs the linear (pole) parts of $\delta_{BRS}\phi^i$ and $\delta_{BRS}\bar{C}^\alpha$, while $\delta_{BRS}C^\alpha = 0$. Moreover, we will only discuss here the case of one-loop unitarity [for n loop unitarity, see Ref. 16)]. In that case it is sufficient to put all fields $\phi^{n_i}(z_i)$ on-shell with physical momenta and polarizations. They are then invariant under linear BRS transformations, and play in the following the role of inert spectators. We will omit them from here on.

*) For non-linear F_α, see P.K. Townsend and the author, to appear.

Thus one obtains the Ward identity

$$\left\langle \delta^{lin}_{BRS}\, \bar{C}^{\alpha}(x)\, \phi^{k}(y) \right\rangle = -\left\langle \bar{C}^{\alpha}(x)\, \delta^{lin}_{BRS}\, \phi^{k}(y) \right\rangle \tag{37}$$

For a supersymmetry gauge field ψ_μ,

$$\delta^{lin}_{BRS}\, \psi_\mu = \frac{2}{k}\partial_\mu C\, \Lambda$$

and for the supersymmetric antighost field $\bar{C}$ one has

$$\delta^{lin}_{BRS}\, \bar{C} = -\frac{\kappa}{2}\Lambda\, \bar{\Psi}\cdot\gamma$$

The resulting identity is shown in Fig. 1. Note in deriving that result, in (37) all external lines have propagators [for spin 3/2, see Eq. (38)]. It says that saturating a gauge line, whose momentum is physical, at one end with one momentum factor flips the gauge line into a ghost line which at the other end emits the momentum of the ghost line. We have here only considered supersymmetry gauge and ghost fields, but the same holds for the other local symmetries. The other identity we need is the usual conservation relation in Fig. 1, which says that saturating all non-physical lines by one momentum factor yields zero.

We come now to unitarity. Unitarity requires that in an N particle cut the sums of unphysical and of ghost modes cancel each other. For a two-particle cut (one-loop process) this implies Fig. 2.

In the first diagram, the propagators are the renormalizable propagators, obtained from $\mathcal{L}$(cl) + $\mathcal{L}$(fix) by inverting the field equations. For the spin 3/2 field one finds with Eq. (15)

$$P^{3/2}_{\mu\nu} = \langle \psi_\mu \bar{\Psi}_\nu \rangle = \tfrac{1}{2}\gamma_\nu \not{p}\, \gamma_\mu / p^2 \tag{38}$$

It can be decomposed into its two physical modes with helicity ±3/2 plus gauge terms

$$P^{3/2}_{\mu\nu} = P^{phys}_{\mu\nu} + \left(p_\nu \bar{\not{p}}\not{p}\gamma_\mu + \gamma_\nu \not{p}\bar{\not{p}}\, p_\mu + 2p_\nu \bar{\not{p}}\, p_\mu\right)\left(2\, p\cdot\bar{p}\right)^{-1} \tag{39}$$

where $\bar{p} = (\vec{p}, -ip_4)$ is the time-reversed of $p = (\vec{p}, +ip_4)$ and where

$$P^{phys}_{\mu\nu} = \sum_{\lambda=\pm} \left(\epsilon^{\lambda}_{\mu} u^{\lambda}\right)\left(\epsilon^{\lambda *}_{\nu} \bar{u}_{\nu}\right) = \frac{1}{2} \bar{\delta}_{\mu\alpha} P^{3/2}_{\alpha\beta} \bar{\delta}_{\beta\nu} \tag{40}$$

The symbol $\bar{\delta}_{\mu\nu}$ is the transverse photon projection operator

$$\bar{\delta}_{\mu\nu} = \delta_{\mu\nu} - \left(p_{\mu}\bar{p}_{\nu} + p_{\nu}\bar{p}_{\mu}\right)\left(p\cdot\bar{p}\right)^{-1} = \sum_{\lambda=\pm} \left(\epsilon^{\lambda}_{\mu} \epsilon^{\lambda *}_{\nu}\right) \tag{41}$$

The propagators in the last diagram of Fig. 2 are $P^{phys}_{\mu\nu}$.

For definiteness we will consider two intermediate spin 3/2 lines. Consider the term with γ_{ν} in (39). It can be rewritten as

$$p_{\mu} \left(\frac{\bar{p}_{\alpha}}{p\cdot\bar{p}} \right) \left(\frac{1}{2} \gamma_{\nu} \not{p} \gamma_{\alpha} \right) \tag{42}$$

hence as the product of the momentum of the cut line, times a scalar factor which we omit from now on for the time being, times the renormalizable propagator (index α acts on the left, ν on the right). Using the Ward identity once, one has Fig. 3. One can now use k_{ρ} on the right, and cycle once more with the Ward identity, cf. Fig. 4. The original factor $\bar{p}_{\alpha}(p\cdot\bar{p})^{-1}$ contracted with the final factor p_{α} yields one, and since at no time we have added an extra minus sign for this loop, we end up with minus one times the oriented ghost loop (ghosts are complex!). Similarly the term with p_{ν} in (39) cancels the other orientation of the ghost loop. That leaves us with the last term in (39). Cycling p_{μ} once we get the factors k_{ρ} upstairs as before, but now we stop since the factors p_{ν} and k_{ρ} can be used in the conservation equation for the blob on the right to show that this term vanishes.

The conclusion is thus that the unphysical terms in the propagator cancel either with the ghosts, or by themselves, and unitarity holds. This argument has been extended to general N particle cuts (with Ward identities for cut-graphs) with gauge and ghost fields in the intermediate states. Also it has been shown[16)] that starting from the requirement that the Ward identities hold (which is necessary for unitarity), one reconstructs the four-ghost interaction.

The four-ghost term is only effective from the two-loop level on. Thus all explicit one-loop calculations remain valid[18)], while the proofs at the two-loop level[19] (which implicitly assumed that Ward identities on the external lines were satisfied) remain (or, rather, become) valid. One can also prove, using similar techniques, that the S matrix is independent of F_α, provided the Ward identities hold, hence provided the extra ghost term is added to the action.

6. RENORMALIZABILITY

The status of finiteness of the S matrix in pure and O(n) supergravity, supergravity with matter without extra O(n) symmetry, pure Einstein gravity and Einstein gravity plus matter is represented in the following table:

	O(n)	non-O(n)	Einstein	Einstein + matter
one-loop	finite	not finite	finite	not finite
two-loop	finite	not finite	(?)	not finite
three-loop	?	not finite	?	not finite

For a review, see last year's proceedings. New remarks are added.

At the two-loop level there is a proof of two years old by Grisaru, Wu and myself[20] that Einstein gravity is finite. Since however, the proof uses helicity conservation which is only proven at the tree level, a half question mark has been noted down. At the three-loop level, however, no conclusion can be drawn at all for Einstein gravity. For supergravity, a proof of two-loop finiteness was first given[19] by noting that the only possible counterterm which does not vanish on-shell, flips helicities, while supergravity (and Einstein gravity?) conserves helicity. Hence the coefficient of this term is zero. But anomalies might invalidate chirality conservation and hence two-loop finiteness. Also it is possible that supersymmetry anomalies affect the transformation rules under which the two-loop S matrix is invariant, so that proofs that there are no non-vanishing counter terms on-shell which are invariant under the classical rules tell us little about the actual counterterms if there are anomalies. The situation clearly deserves study.

An explicit calculation (see Fig. 5) of photon-photon scattering in the O(8) model[21] at the one-loop level yielded finite results. This was expected, although a proof of one-loop finiteness of the O(8) model is lacking -- but only because it requires some hairy algebra. The finiteness of this process makes it in my opinion almost certain that O(8) is one-loop finite. All divergences are again proportional to the energy momentum tensor of the photon which again is a clear sign that the theory has a combined chirality-duality invariance. In fact, it probably has a global $U(8)$ symmetry. Under each graph in Fig. 5 the coefficient of this divergence is written, and all divergences add up to zero. This work was done with Mark Fischler of Stony Brook[22].

If anomalies do not spoil the two-loop finiteness of supergravity, then the finiteness at the three-loop level is the big question. It has been shown that there exists a possible counter term which is supersymmetric even off-shell[23]. Thus the only hope is that its coefficient vanishes. One possible approach is to consider extra symmetries (which do exist) which forbid this term. An explicit calculation seems impossible.

CONFORMAL SUPERGRAVITY

The conformal group has a special place in general relativity, and it seems interesting to see what Fermi-Bose gauge symmetry does with it. The superconformal group has 24 generators X_A: the 15 bosonic generators P_μ, $M_{\mu\nu}$, K_μ and D of the ordinary conformal group, to which are added one more bosonic generator A for chiral rotations, and the square roots Q_α and S_α satisfying

$$\{Q_\alpha, Q_\beta\} = -(\gamma^\mu C)_{\alpha\beta} P_\mu \quad ; \{S_\alpha, S_\beta\} = (\gamma^\mu C)_{\alpha\beta} K_\mu \tag{43}$$

Recently, the gauge action of the superconformal group was found[13]. It is a Yang-Mills type action because it is quadratic in curvatures $R^A_{\mu\nu}$, defined by

$$R^A_{\mu\nu} = \partial_\nu h^A_\mu - \partial_\mu h^A_\nu + f^A{}_{BC}\, h_\nu{}^B h_\mu{}^C \tag{44}$$

$$[X_A, X_B\} = f_{BA}{}^C X_C \tag{45}$$

The gauge fields h^A_μ assigned to the generators X_A are denoted by

$$\begin{array}{ccccccc} P_a & M_{ab} & K_a & D & Q_\alpha & S_\alpha & A \\ e^a{}_\mu & \omega_\mu{}^{ab} & f^a{}_\mu & b_\mu & \bar\psi^\alpha_\mu & \bar\phi^\alpha_\mu & A_\mu \end{array} \tag{46}$$

The action was found by writing down the most general action bilinear in curvatures, conserving parity and without dimensional constants. It reads

$$I = \int d^4x \Big[\epsilon^{\mu\nu\rho\sigma} \Big\{ R_{\mu\nu}(M)^{ab} R_{\rho\sigma}(M)^{cd} \epsilon_{abcd} - 8 R_{\mu\nu}(Q) \gamma_5 \overline{R(S)}_{\rho\sigma} + 4i R_{\mu\nu}(A) R_{\rho\sigma}(D) \Big\} - 8e R_{\mu\nu}(A) R^{\mu\nu}(A) \Big] \tag{47}$$

It is invariant under all local gauge transformations (replacing P by general co-ordinate transformations)

$$\delta h_\mu{}^A = D_\mu \epsilon^A \equiv \partial_\mu \epsilon^A + f_{BC}{}^A h_\mu{}^B \epsilon^C \tag{48}$$

provided three constraints are imposed. These constraints arise naturally, and the theory is invariant if and only if they hold.

i) Torsion: $R_{\mu\nu}(P) = 0$. This expresses the spin connection field $\omega_{\mu ab}$ in terms of the other fields. In Poincaré supergravity it is a field equation, in conformal supergravity it is not. Hence it is a model-independent constraint. It tells one how a vector rotates under parallel transport: depending on which fields are present, but not on which field equations these fields satisfy.

ii) Self-duality:

$$R_{\mu\nu}(Q) + \frac{e}{2} \gamma_5 \epsilon_{\mu\nu\rho\sigma} R^{\rho\sigma}(Q) = 0$$

Since $Q = \sqrt{P}$ one expects a constraint for Q, too. Pure vanishing, $R_{\mu\nu}(Q) = 0$, is too strong, as it would eliminate 24 field components, while there are only 32 fermion components (ψ_μ and ϕ_μ). Self-duality is the natural choice, and it corresponds in a sense to the square root of true vanishing. It gives 12 constraints, from which one can eliminate $\phi_\mu - \frac{1}{4}\gamma_\mu \gamma\cdot\phi$.

iii) The third constraint:

$$\sigma^{\mu\nu} R_{\mu\nu}(Q) = 0$$

Its geometrical meaning is as yet unclear, but again it seems to have a deep meaning. It allows to eliminate $\gamma\cdot\phi$ and leads to Q supersymmetry.

In addition, one finds that the field equation for the K_μ gauge field f_μ^a is non-propagating, so that it can be eliminated by its field equation, while the

D gauge field b_μ drops from the action altogether. The only remaining fields are: the vierbein, the gravitino and the chiral fields, whose two supersymmetry transformations are given in (26) and (27). For the solution of ϕ_μ in terms of these three fields, one finds

$$
\begin{aligned}
\phi_\mu &= \tfrac{1}{3}\gamma^\nu\left(S_{\mu\nu} + \tfrac{e}{4}\gamma_5\epsilon_{\mu\nu\rho\sigma}S^{\rho\sigma}\right) \\
S_{\mu\nu} &= \left(\partial_\nu + \tfrac{1}{2}\omega_{\nu ab}\sigma^{ab} + \tfrac{1}{2}b_\nu + \tfrac{i}{2}A_\nu\gamma_5\right)\psi_\mu - (\mu\leftrightarrow\nu) \\
\omega_{\mu ab} &= \omega_{\mu ab}(e) + \tfrac{1}{4}\left(\bar\psi_\mu\gamma_a\psi_b - \bar\psi_\mu\gamma_b\psi_a + \bar\psi_a\gamma_\mu\psi_b\right) + \left(e_{a\mu}b_b - e_{b\mu}b_a\right) \\
\omega_{\mu ab}(e) &= \tfrac{1}{2}e_a{}^\nu\left(e_{b\nu,\mu} - e_{b\mu,\nu}\right) + \tfrac{1}{2}e_a{}^\lambda e_b{}^\rho e_{c\lambda,\rho}e^c{}_\mu - (a\leftrightarrow b)
\end{aligned}
\tag{49}
$$

and
$$
\begin{aligned}
\delta_Q\phi_\mu &= \gamma^a\epsilon f_{a\mu} + \tfrac{i}{4}\gamma^\nu\left(\gamma_5 R_{\mu\nu}(A) + \tfrac{e}{2}\epsilon_{\mu\nu\rho\sigma}R^{\rho\sigma}(A)\right)\epsilon \\
\delta_S\phi_\mu &= \left(\partial_\mu + \tfrac{1}{2}\omega_{\mu ab}\sigma^{ab} - \tfrac{i}{2}A_\mu\gamma_5 - \tfrac{1}{2}b_\mu\right)\epsilon
\end{aligned}
\tag{50}
$$

For the proper conformal gauge field one finds

$$
\begin{aligned}
f_{\mu\nu} &= -\tfrac{1}{4}\left(\hat R_{\nu\mu} - \tfrac{1}{6}g_{\mu\nu}\hat R\right) - \tfrac{1}{8}\bar\psi^\lambda\gamma_\nu\overline{R}_{\lambda\mu}(Q) - \tfrac{i}{16}R^{(A)}_{\rho\sigma}\epsilon_{\mu\nu}{}^{\rho\sigma} \\
\hat R_{\nu\mu} &= R_{\nu\mu} - \bar\psi_\nu\sigma_{\lambda\mu}\phi_\lambda + \bar\psi_\lambda\sigma_{\lambda\mu}\phi_\nu \\
\overline{R}_{\lambda\mu}(Q) &= \left(\partial_\mu + \tfrac{1}{2}\omega_{\mu ab}\sigma^{ab} + \tfrac{1}{2}b_\mu + \tfrac{i}{2}A_\mu\gamma_5\right)\psi_\lambda - \gamma_\mu\phi_\lambda - (\lambda\leftrightarrow\mu) \\
R_{\rho\sigma}(A) &= -F_{\rho\sigma}(A) - i\bar\psi_\rho\gamma_5\phi_\sigma + i\bar\psi_\sigma\gamma_5\phi_\rho
\end{aligned}
\tag{51}
$$

The complete action is complicated; for its kinetic part, see (28). Since it is a superconformal theory, there should be no lower gauge components. Indeed, S supersymmetry eliminates the $\gamma\cdot\psi$ spin $\frac{1}{2}$ part of ψ_μ, while Q supersymmetry takes care of the $\partial\cdot\psi$ spin $\frac{1}{2}$ part of ψ_μ. Weyl invariance (= local scale invariance) eliminates the scalar part (trace $g_{\mu\nu}$), while general coordinate invariance eliminates, as usual, the other scalar and the spin 1 part of $g_{\mu\nu}$. Chiral invariance gets rid of the spin 0 part of A_μ. Clearly, we do not need K_μ invariance, and indeed all fields are K-inert.

In some sense, the superconformal action can be viewed as the Poincaré action with two more derivatives on e_μ^a and ψ_μ. Proceeding in this way, one sees that A_μ should be one of the auxiliary fields in Poincaré supergravity because it is <u>here</u> propagating normally. And also one expects that the gauge algebra of superconformal gravity closes, since all non-propagating fields of Poincaré supergravity which are effective in conformal supergravity have now a Dalembertian. It indeed does! Thus, no four-ghost term is needed for conformal supergravity, which does not mean that there are no unitarity problems: it is lore (but only lore) that all higher derivative theories have ghosts at the classical level.

Conformal theories have no intrinsic scale (the fields ψ_μ have dimension $\frac{1}{2}$ instead of 3/2; therefore no explicit κ's are present). They do not reproduce the classical tests of general relativity, and for the time being it seems that their main advantage is that they clarify the structure of Poincaré supergravity. For example, the self-duality and third-constraint in Poincaré supergravity are just the spin 3/2 field equation, so that closure of the Poincaré gauge algebra modulo this field equation can be understood from this more general point of view. However, it is not excluded that future developments have surprises in petto for the superconformal group. For example, spontaneous symmetry breaking might lead to a term $\approx R$ in the action so that the classical tests would be satisfied, and let us not forget that (super)conformal theories are all-loop renormalizable! The parallel with Yang-Mills fields is exciting.

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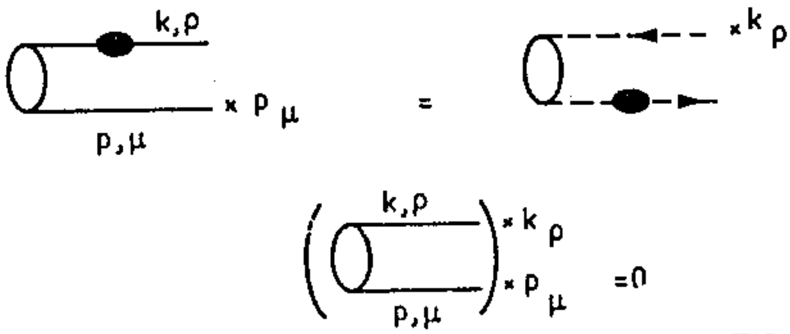


FIG.1

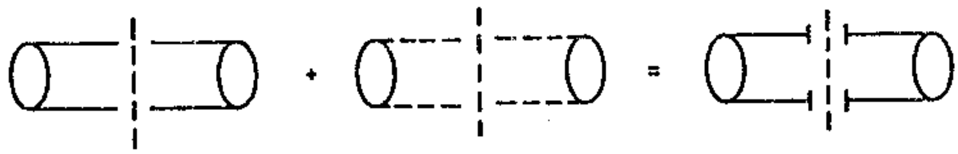


FIG.2

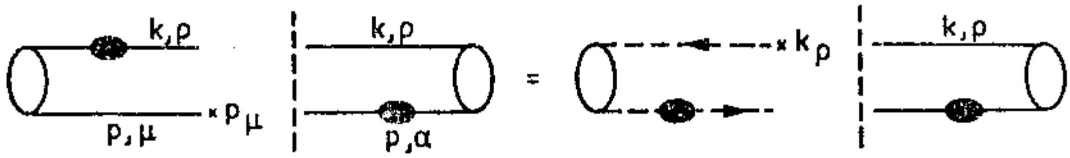


FIG. 3

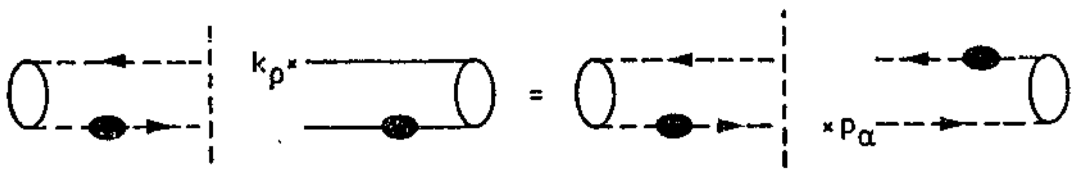
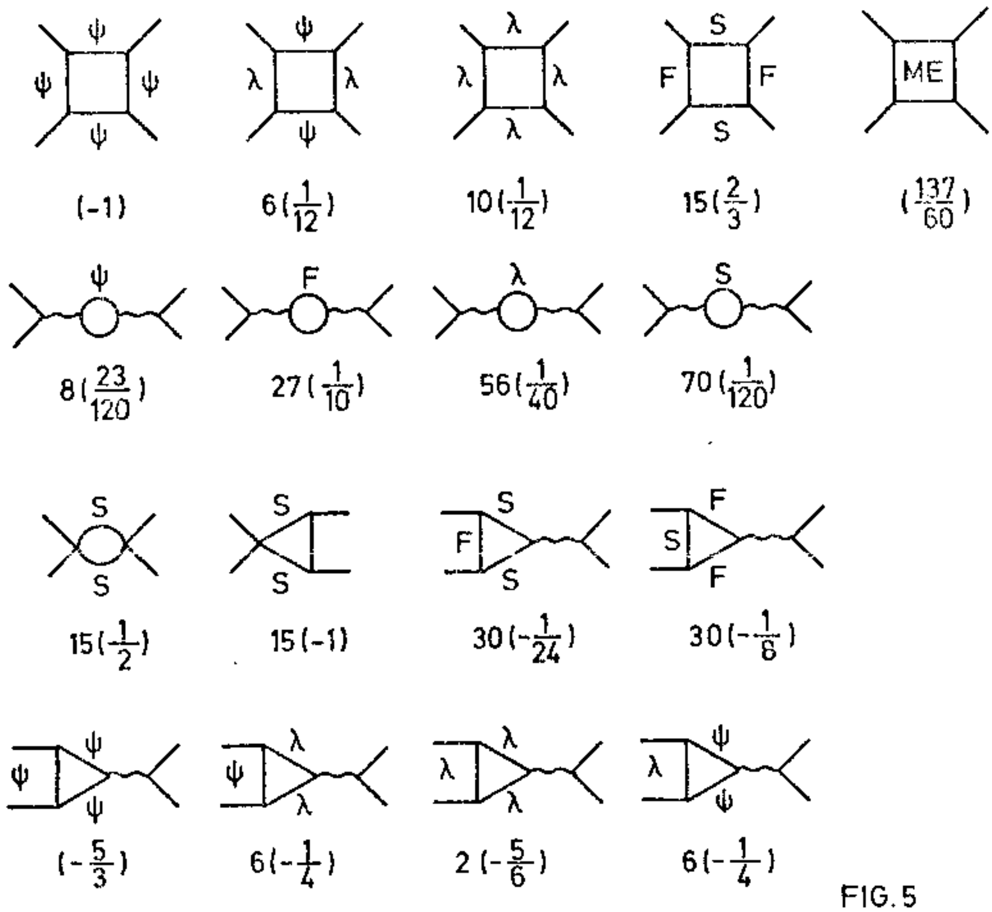


FIG. 4



ψ ψ ψ ψ
ψ λ λ ψ
λ λ λ λ
S F F S
ME
(-1)
6(1/12)
10(1/12)
15(2/3)
(137/60)
ψ F λ S
8(23/120)
27(1/10)
56(1/40)
70(1/120)
S S
S S
S F S
F S F
15(-1/2)
15(-1)
30(-1/24)
30(-1/8)
ψ ψ ψ
ψ λ λ
λ λ λ
λ ψ ψ
(-5/3)
6(-1/4)
2(-5/6)
6(-1/4)

FIG. 5