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COVERAGE CONTROL FOR MOBILE SENSING NETWORKS: VARIATIONS ON A THEME

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Abstract

This paper presents control and coordination algorithms for networks of autonomous vehicles. We focus on groups of vehicles performing distributed sensing tasks where each vehicle plays the role of a mobile tunable sensor. We design distributed gradient descent algorithms for a class of utility functions which encodes optimal coverage and sensing policies. These utility functions are studied in geographical optimization, vector quantization, and sensor allocation contexts. The algorithms exploit the computational geometry of spatial structures such as Voronoi diagrams.

1 Introduction

Motivation: The objective of this paper is the design of control and coordination algorithms for groups of vehicles. We focus on vehicles that perform distributed sensing tasks and refer to them as active sensor networks. Such systems are being developed for applications in remote autonomous surveillance, exploration, information gathering, and automatic monitoring of transportation systems. For active sensor networks, we envision the need for a distributed control and coordination architecture: a wireless network provides the vehicles with the ability to share some information, but no overall leader might be present to coordinate the group. As the vehicle network evolves in time, the ad-hoc communication graph and neighborhood relationships change. It is interesting therefore to design distributed algorithms for ad-hoc multi-vehicle networks.

References: The technical approach proposed in this paper relies on methods from computational geometry [1], facility location [2], and distributed algorithms [3]. We exploit a formulation of a vector quantization problem, whose solution is closely related to the computational geometric notion of centroidal Voronoi partition [4]. This problem and its variations are also related to the p-median and p-center problem in facility location [2, 5].

More generally, this problem is related to a number of technological areas including data compression in image processing (vector quantization), optimal quadrature rules (integration), grid generation for finite differences methods (PDE discretization), clustering analysis, optimal resource placement, facility location and combinatorial optimization, mesh optimization methods (mesh relaxation, Laplacian smoothing), and statistical pattern recognition (learning vector quantization).

Contribution and paper organization: We consider a coverage optimization problem for an active sensor network. To characterize the quality of service provided by a spatially distributed active sensor network, we introduce a notion of coverage based on locational optimization. With this motivation, Section 2 reviews certain locational optimization problems and their solutions as centroidal Voronoi tessellations. In Section 3, we provide a continuous-time version of the classic Lloyd algorithm from vector quantization and apply it to the setting of multi-vehicle networks. We consider an interesting worst-case setting, referred to as the p-center problem in facility location, and design a similar coverage control law.

In Section 4, we describe in some detail a distributed version of the Lloyd algorithm and discuss a supporting infrastructure required for its implementation in an ad-hoc network. Next, in Section 5, we design density functions that lead the multi-vehicle network to predetermined geometric patterns. In this sense, the proposed coverage control scheme can be regarded as a formation control algorithm. We then consider the setting of time-varying density functions and investigate target tracking problems. Finally, we extend the proposed coverage control scheme to the vehicle models with higher order dynamics than a simple integrator. Section 6 presents control designs for second-order dynamics, and mobile wheeled dynamics. We present our conclusions and directions for future research in Section 7.

2 From location optimization to centroidal Voronoi partitions

Locational optimization & facility location

In this section we describe a collection of known facts about a meaningful optimization problem. References include the theory and applications of centroidal Voronoi partitions, see [4], and the discipline of facility location, see [2].

Let Q be a convex polygon in $\mathbb{R}^2$; we shall also consider the extension to convex polyhedra in $\mathbb{R}^N$. We call a map $\phi : Q \to \mathbb{R}_+$ a *distribution density function* if it represents a measure of information or probability that some event take place over Q. In equivalent words, we can consider Q to be the bounded support of the function ϕ. Let $P = (p_1, \ldots, p_n)$ be the *location of n sensors* moving in the space Q. Because of noise and loss of resolution, the *sensing performance* at point q taken from ith sensor at the position p_i degrades with the Euclidean distance $\|q - p_i\|$ between q and p_i; we describe this degradation with a non-decreasing function $f : \mathbb{R}_+ \to \mathbb{R}_+$. Accordingly, $f(\|q - p_i\|)$ provides a quantitative assessment of how poor the sensing performance is. This assumption on the sensing performance is well-suited for various electromagnetic and sound sensors that have signal-to-noise ratios inversely proportional to distance.

We consider the task of minimizing the location optimization function

$$\mathcal{H}(P, \mathcal{W}) = \sum_{i=1}^{n} \int_{W_i} f(\|q - p_i\|) d\phi(q), \qquad (1)$$

where we let $\mathcal{W} = \{W_1, \ldots, W_n\}$ be a partition of Q, and we assume that the ith sensor is responsible for measurements over its "dominance region" W_i. Note that the function $\mathcal{H}$ is to be minimized with respect to both (1) the sensor location P, and (2) the assignment of the dominance regions $\mathcal{W}$. This problem is referred to as a facility location problem and in particular as a continuous p-median problem in [2].

Voronoi partitions

One can easily see that, at fixed sensors location, the optimal partition of Q is the *Voronoi partition* $\mathcal{V}(P) = \{V_1, \ldots, V_n\}$ generated by the points $(p_1, \ldots, p_n)$:

$$V_i = \{q \in Q | \ \|q - p_i\| \leq \|q - p_j\| \ \forall j \neq i\}.$$

We refer to [1, 5] for comprehensive treatments on Voronoi diagrams, and briefly present some relevant concepts. Since Q is a convex polyhedron in a finite dimensional Euclidean space, the boundary of each V_i is a convex polygon. When the two Voronoi regions V_i and V_j are adjacent, p_i is called a *(Voronoi) neighbor* of p_j (and vice-versa). It is known that (i) the nearest vehicle p_j to p_i is a neighbor, (ii) the average number of neighbors on $Q \subset \mathbb{R}^2$ is six. In what follows, we shall write

$$\mathcal{H}_{\mathcal{V}} = \mathcal{H}(P, \mathcal{V}(P)).$$

Remarkably, one can show [4, 6] that

$$\begin{aligned} \frac{\partial \mathcal{H}_{\mathcal{V}}}{\partial p_i}(P) &= \frac{\partial \mathcal{H}}{\partial p_i}(P, \mathcal{V}(P)) \\ &= \int_{V_i} \frac{\partial}{\partial p_i} f(\|q - p_i\|) \, d\phi(q), \end{aligned} \qquad (2)$$

and deduce some smoothness properties of $\mathcal{H}_{\mathcal{V}}$. Since the Voronoi partition $\mathcal{V}$ depends at least continuously on $P = (p_1, \ldots, p_n)$, and assuming f is a continuous function, the function $\mathcal{H}_{\mathcal{V}}$ is at least continuously differentiable.

Centroidal Voronoi partitions

Let us recall some basic quantities associated to a region $V \subset \mathbb{R}^N$ and a mass density function ρ. The (generalized) mass, centroid (or center of mass), and polar moment of inertia are defined as

$$M_V = \int_V \rho(q)\, dq, \quad C_V = \frac{1}{M_V} \int_V q\, \rho(q)\, dq,$$
$$J_{V,p} = \int_V \|q-p\|^2\, \rho(q)\, dq.$$

We refer to [7] for closed form expressions for area, centroid, and polar moment of inertia for uniform densities over $\mathbb{R}^N$; see also [6] for expressions in the $\mathbb{R}^2$ setting.

Let us consider again the locational optimization problem (1), and suppose now we are strictly interested in the setting

$$\mathcal{H}(P, \mathcal{W}) = \sum_{i=1}^{n} \int_{W_i} \|q-p_i\|^2 d\phi(q), \qquad (3)$$

that is, we consider the setting $f(\|q-p_i\|) = \|q-p_i\|^2$. Under this assumption, an application of the parallel axis theorem leads to simplifications for both the function $\mathcal{H}_{\mathcal{V}}$ and its partial derivative:

$$\mathcal{H}_{\mathcal{V}}(P) = \sum_{i=1}^{n} J_{V_i, C_{V_i}} + \sum_{i=1}^{n} M_{V_i} \|p_i - C_{V_i}\|^2$$
$$\frac{\partial \mathcal{H}_{\mathcal{V}}}{\partial p_i}(P) = 2 M_{V_i}(p_i - C_{V_i}).$$

It is useful to write $\mathcal{H}_{\mathcal{V}}$ as the sum of two terms and compute their respective partials as

$$\mathcal{H}_{\mathcal{V},1} = \sum_{i=1}^{n} J_{V_i, C_{V_i}}, \quad \frac{\partial \mathcal{H}_{\mathcal{V},1}}{\partial p_i} = 0,$$

and

$$\mathcal{H}_{\mathcal{V},2} = \sum_{i=1}^{n} M_{V_i} \|p_i - C_{V_i}\|^2,$$
$$\frac{\partial \mathcal{H}_{\mathcal{V},2}}{\partial p_i} = 2 M_{V_i}(p_i - C_{V_i}).$$

Therefore, the (not necessarily unique) local minimum points for the location optimization problem are described as follows. The critical points for $\mathcal{H}_{\mathcal{V}}(P)$ are *centroids*, i.e., the point p_i satisfies two properties simultaneously, it is the generator for the Voronoi cell V_i and it is its centroid. In other words

$$C_{V_i} = \operatorname{argmin}_{p_i} \mathcal{H}_{\mathcal{V}}(P)$$
$$\mathcal{H}_{\mathcal{V},1} = \min_{(p_1,\dots,p_n)} \mathcal{H}_{\mathcal{V}}(P).$$

Accordingly, the critical partitions and points for $\mathcal{H}$ are *centroidal Voronoi partitions*; see [4]. This discussion provides a proof alternative to the one given in [4] for the necessity of centroidal Voronoi partitions as solutions to the continuous p-median location problem.

3 Coverage control: a continuous-time Lloyd descent

In this section, we describe algorithms to compute location of sensors that minimize the cost $\mathcal{H}$. We propose a continuous-time version of the classic Lloyd algorithm; see [8] for a reprint of the original report, [9] for a historical overview, and [4, 5] for numerous applications in other technological areas. In our setting, both the positions and partitions evolve in continuous time, whereas Lloyd algorithm for vector quantization is usually designed in discrete time. Similarly to the original Lloyd's scheme, the proposed algorithm is a *gradient descent flow*.

Assume the sensors location obeys a first order dynamical behavior described by

$$\dot{p}_i = u_i.$$

Consider $\mathcal{H}_{\mathcal{V}}$ a cost function to be minimized and impose that the location p_i follows a gradient descent. In equivalent control theoretical terms, consider $\mathcal{H}_{\mathcal{V}}$ a Lyapunov function and stabilize the multi-robot system to one of its local minima via a dissipative $L_g V$ control. Formally, we set

$$u_i = -k(p_i - C_{V_i}), \qquad (4)$$

where k is a positive gain, and where we assume that the partition $\mathcal{V}(P) = \{V_1, \dots, V_n\}$ is continuously updated.

Lemma 3.1 (Continuous-time Lloyd descent)**.** *For the closed loop induced by equation* (4)*, the sensors location* $P = (p_1, \dots, p_n)$ *converges asymptotically to a critical point of the cost function* $\mathcal{H}_{\mathcal{V}}$*. The cost function* $\mathcal{H}_{\mathcal{V}}$ *converges to a critical value* $\mathcal{H}_{\mathcal{V},1}$ *with exponential convergence rate* $2k$*.*

Proof. Since $\mathcal{H}_{\mathcal{V}}(P) = \mathcal{H}_{\mathcal{V},1} + \mathcal{H}_{\mathcal{V},2}(P)$, the closed loop is a gradient flow for the cost function $\mathcal{H}_{\mathcal{V},2}(P)$. We have

$$\frac{d}{dt}\mathcal{H}_{\mathcal{V},2}(P) = \sum_{i=1}^{n} \frac{\partial \mathcal{H}_{\mathcal{V},2}}{\partial p_i}\, \dot{p}_i$$
$$= -2k \sum_{i=1}^{n} M_{V_i} \|p_i - C_{V_i}\|^2 = -2k\mathcal{H}_{\mathcal{V},2}.$$

□

Note that this gradient descent is not guaranteed to find the global minimum. For example, in the vector quantization and signal processing literature [9], it is known that for bimodal distribution density functions, the solution to the gradient flow reaches local minima where the number of generators allocated to the two region of maxima are not optimally partitioned.

Despite the difficulty in obtaining global minima of $\mathcal{H}_{\mathcal{V}}$, we regard the continuous-time Lloyd descent as at least an interesting heuristic. To study the performance of this heuristic, we implemented it in `Mathematica`. The algorithm is implemented as a single centralized program; it computes the bounded Voronoi diagram using the Mathematica package `ComputationalGeometry`, and computes mass, centroid, and polar moment of inertia of polygon via the numerical integration routine `NIntegrate`. Careful attention was paid to numerical accuracy issues in the computation of the Voronoi diagram and in the integration. We illustrate the performance of the closed loop in Figure 1.

Generalized settings, worst case design, and the p-center problem

Different performance functions f in equation (1) and different distance powers in equation (3) correspond to different optimization problems. However, the Voronoi partition computed with respect to the Euclidean metric remains the optimal partition. In general, it is not possible anymore to decompose $\mathcal{H}_{\mathcal{V}}$ into the sum of terms similar to $\mathcal{H}_{\mathcal{V},1}$ and $\mathcal{H}_{\mathcal{V},2}$. Nevertheless, it is still possible to implement the gradient flow via the expression for the partial derivative (2).

More generally, various distance notions can be used to define performance functions and accordingly compute the optimal partition. We refer to [10, 11] for a discussion on locational optimization via weighted Voronoi partitions. According to the definition of performance function, one can then define various notions of "center of a region" (any notion of geometric center, mean, or average is an interesting candidate). These can then be adopted in designing coverage algorithms.

Choosing between these possible avenues of investigation, let us here focus on an interesting variation on the original problem. The location optimization problem for the function in equation (1) can be stated as

$$\min_{p_1,\dots,p_n} E\left[\min_{i\in\{1,\dots,n\}} \|q - p_i\|^2\right],$$

where the expected value is computed with respect to ϕ regarded as a probability density function. As mentioned above, the facility location literature [2, 12] refers to this optimization problem as the continuous p-median problem.

In addition to the p-median problem, it is instructive to consider the worst case problem

$$\min_{p_1,\dots,p_n}\left[\max_{q\in Q}\left[\min_{i\in\{1,\dots,n\}} \|q - p_i\|^2\right]\right].$$

This optimization is referred to as the p-center problem in [12, 13]. It corresponds to a version of the sphere packing problem: how to cover a region with (possibly overlapping) disks of minimum radius disks. If $D_1 \subset \mathbb{R}^2$ is unit disk, the problem reads:

$$\min_{\cup_i(RD_1+p_i)\supseteq Q} R.$$

It is immediate to derive an heuristic for the p-center problem exactly similar to the Lloyd algorithm: each vehicle moves toward the center of the minimum spanning circle containing its own Voronoi polygon, i.e., the center of the circle of minimum radius enclosing the polygon. Note that the center of the minimum spanning circle for a given convex polygon can be computed via a convex problem [14], or via the closed form algorithm in [15]. It is interesting to note that no convergence proof appears to be available for this heuristic; see [13].

In what follows, we shall restrict our attention to the p-median problem and on centroidal Voronoi tessellations.

4 A distributed implementation

In this section we show how the Lloyd algorithm can be implemented in a distributed fashion. By distributed

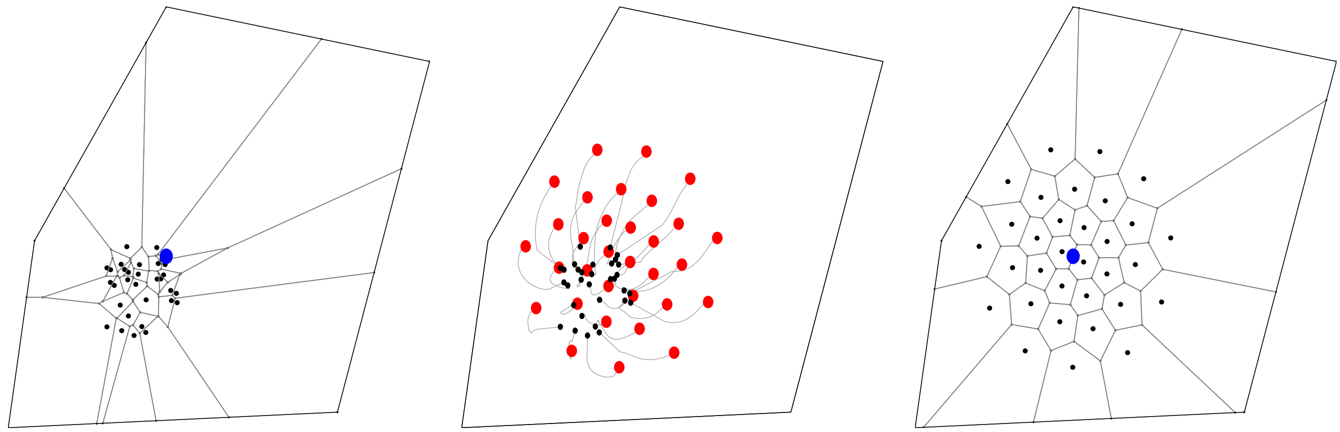

Figure 1: Lloyd continuous-time algorithm on a convex polygonal environment, with Gaussian density $\phi = \exp(5.(x^2 - y^2))$ centered about the gray point in figure. The left (respectively, right) figure illustrates the initial (respectively, final) locations and Voronoi partition. The central figure illustrates the gradient descent flow.

we mean that the algorithm is run on a group of agents performing the same sequence of instructions and sharing information in a predetermined way. We refer to [3] for a comprehensive treatment on distributed algorithms.

In its distributed form, the coverage algorithm is a feedback mechanism in the sense that it allows the network to adapt to changes in the number of nodes due to agents departures, arrivals or failures. It is also a natural way of obtaining scalability with respect to the number of agents. Note that we consider for simplicity the setting of synchronous networks, and leave the extensions to asynchronous networks to future works.

Distributed algorithms for groups or networks of mobile agents can be regarded as local interaction rules; e.g., see [16] and references therein. Behavioral rules are then studied in terms of emerging behaviors they induce for the overall network. By casting the coverage control laws in distributed fashion, we show how a coverage behavior emerges from a local Lloyd interaction rule.

Because the communication and computation in a distributed network typically take place over discrete time, we assume the agents follow a first order dynamics in discrete time with bounded input:

$$p_i(t+1) = p_i(t) + u_i(t), \qquad \|u_i\| \leq 1.$$

A distributed version of Lloyd algorithm for the solution of the optimization problem (1) is as follows:

Name:	Coverage behavior
Goal:	distributed optimal agent location
Assumes:	$p_i(t+1) = p_i(t) + u_i$, $\|u_i\| \leq 1$
Requires:	(i) own Voronoi cell computation, (ii) centroid computation

For all i, agent i performs:

1: determine own Voronoi cell V_i
2: determine centroid C_{V_i} of V_i
3: set $u_i = (C_{V_i} - p_i)/(1 + \|C_{V_i} - p_i\|)$

A key requirement of this implementation of Lloyd algorithm is that each agent must be able to compute its own Voronoi cell. To do so, each agent needs to know the relative location (distance and bearing) of each Voronoi neighbor. Therefore, this implementation of the coverage algorithm is distributed only to the extent that Voronoi neighbors can be computed in a distributed fashion.

Let us therefore sketch a distributed algorithm to compute the Voronoi cell of an agent; we do so following the lines of [17]. The algorithm is based on basic properties of Voronoi diagrams; e.g., see [1]. We assume that each vehicle has the ability to detect the relative location of other vehicles within a certain distance. The objective is to determine the smallest distance R_i for vehicle i which provides sufficient information to compute the Voronoi cell V_i. We start by noting that V_i is a subset of the convex polygon

$$W(p_i, R_i) = Q \cap \big(\cap_{j:\|p_i - p_j\| \leq R_i} S_{ij} \big), \qquad (5)$$

where the half planes S_{ij} are

$$\{q \in \mathbb{R}^N : 2q \cdot (p_i - p_j) \geq (p_i + p_j) \cdot (p_i - p_j)\}.$$

The equality $V_i = W(p_i, R_i)$ holds when all Voronoi neighbors of p_i are within distance R_i from p_i. This is guaranteed to happen provided R_i is twice as large as the maximum distance between p_i and the vertices of $W(p_i, R_i)$. The minimum adequate sensing radius is therefore $R_{i,\min} = 2 \max_{q \in W(p_i, R_{i,\min})} \|p_i - q\|$.

Remark 4.1. The ability of locating neighbors plays a central role in numerous (distributed) algorithms for localization, media access, routing, and power control in ad-hoc wireless communication networks; e.g., see [18, 19, 20] and references therein. Therefore, any motion control scheme might be able to obtain this information from the underlying communication layer.

Instead of assuming that each vehicle can observe the relative location of neighbors within a certain radius, one might assume that the agents can query an ad-hoc communication network for this type of information. For example, the work in [18] provides a synchronous distributed algorithms based on a 1-hop communication exchange for a wireless network.

5 Density function design

In this section, we investigate interesting ways of designing density functions and solving problems apparently unrelated to coverage.

Geometric patterns and formation control

Here we suggest the use of decentralized coverage algorithms as formation control algorithms, and we present various density functions that lead the multi-vehicle network to predetermined geometric patterns. In particular, we present simple density functions that lead to segments, ellipses, polygons, or uniform distributions inside convex environments.

Consider a planar environment, let k be a large positive gain, and denote $q = (x, y) \in Q \subset \mathbb{R}^2$. Let a, b, c be real numbers, consider the line $ax + by + c = 0$, and define the density function

$$\phi_1(q) = \exp(-k(ax + by + c)^2).$$

Similarly, let (x_c, y_c) be a reference point in $\mathbb{R}^2$, let a, b, r be positive scalars, consider the ellipse $a(x - x_c)^2 + b(y - y_c)^2 = r^2$, and define the density function

$$\phi_2(q) = \exp\big(-k(a(x - x_c)^2 + b(y - y_c)^2 - r^2)^2\big).$$

We illustrate this density function in Figure 2.

Finally, define the smooth ramp function $\mathrm{SR}_k(x) = x(\arctan(kx)/\pi + (1/2))$, and the density function

$$\phi_3(q) = \exp(-k\,\mathrm{SR}_k(a(x - x_c)^2 + b(y - y_c)^2 - r^2)).$$

This density function leads the multi-vehicle network to obtain a uniform distribution inside the ellipsoidal disk $a(x - x_c)^2 + b(y - y_c)^2 \leq r^2$. We illustrate this density function in Figure 3.

It appears straightforward to generalize these types of density functions to the setting of arbitrary curves or shapes. The proposed algorithms are to be contrasted with the classic approach to formation control based on rigidly encoding the desired geometric pattern. We refer to [21] for previous work on algorithms for geometric patterns, and to [22, 23] for formation control algorithms.

Tracking in time-varying environments

Next, we consider environments in which the density function is allowed to depend on time. The time-dependence might model an example situation where a target of interest enters the environment under observation. In this case, we would define $(x_{\text{target}}(t), y_{\text{target}}(t))$ as the target location and define

$$\phi(q, t) = \exp\left(-k(x - x_{\text{target}}(t))^2 - k(y - y_{\text{target}}(t))^2\right).$$

Given a time-varying distribution density function $\phi(q, t)$, we define a time-varying locational optimization function

$$\mathcal{H}_{\mathcal{V}}(P, t) = \sum_{i=1}^{n} \int_{V_i} \|q - p_i\|^2 \phi(q, t) dq.$$

We can compute its time derivative as

$$\begin{aligned}\frac{d}{dt}\mathcal{H}_{\mathcal{V}}(P, t) &= \frac{d}{dt}\mathcal{H}_{\mathcal{V},1}(t) + \frac{d}{dt}\mathcal{H}_{\mathcal{V},2}(P, t) \\ &= \sum_i \Big(\frac{d}{dt} J_{V_i, C_{V_i(t)}}(t) + \dot{M}_{V_i} \|p_i - C_{V_i}\|^2 \\ &\qquad + M_{V_i}(p_i - C_{V_i})'(\dot{p}_i - \dot{C}_{V_i}) \Big),\end{aligned}$$

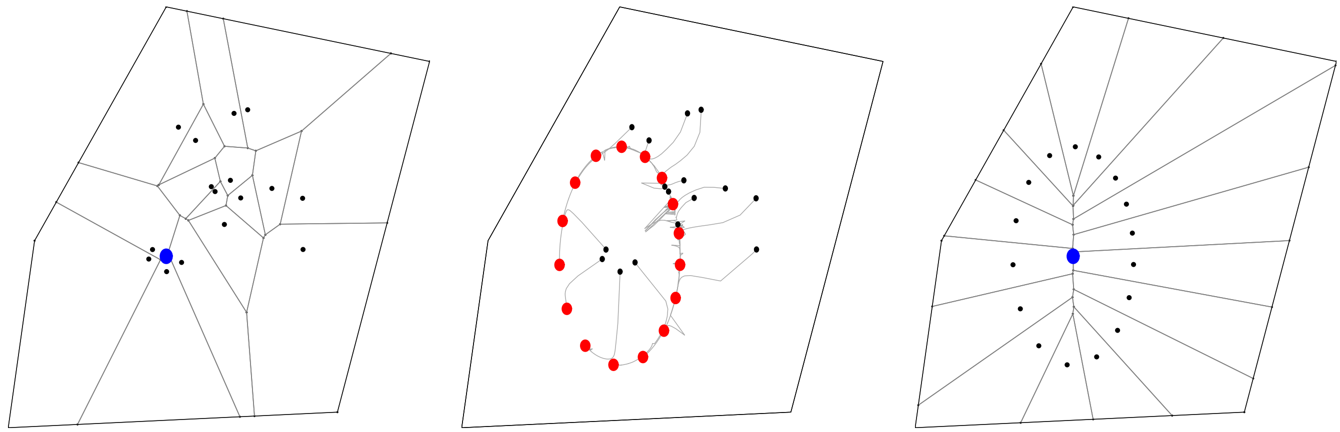

Figure 2: Coverage control with "circular" density function ϕ_2. The parameter values are: $k = 500$, $a = 1.4$, $b = .6$, $x_c = y_c = 0$, $r^2 = .3$.

where, at fixed V_i, we compute

$$\dot{M}_{V_i} = \int_{V_i} \dot{\phi}(q,t)dq,$$

$$\dot{C}_{V_i} = \frac{1}{M_{V_i}} \left(\int_{V_i} q\dot{\phi}(q,t)dq - \dot{M}_{V_i} C_{V_i} \right).$$

Considering a first order dynamics, we design a feedback plus feedforward control law as

$$\dot{p}_i = \dot{C}_{V_i} - \left(k + \frac{\dot{M}_{V_i}}{M_{V_i}} \right) (p - C_{V_i}), \qquad (6)$$

to obtain the closed loop behavior:

$$\frac{d}{dt}\mathcal{H}_{\mathcal{V}} = \frac{d}{dt}\mathcal{H}_{\mathcal{V},1} - k\mathcal{H}_{\mathcal{V},2}.$$

Now, assume the time-varying density function is characterized by a constant optimal value for the locational optimization function, i.e., assume that $\mathcal{H}_{\mathcal{V},1}$ is constant and that $\frac{d}{dt}\mathcal{H}_{\mathcal{V},1} = 0$. Under this assumption, the control law in equation (6) obtains perfect tracking in the following sense: the vehicles asymptotically converge to form a (moving) centroidal Voronoi tessellation or, if starting from one such tessellation, their configuration remains optimal at all time.

6 Variations in vehicle dynamics

In this section we consider vehicles systems described by more general linear and nonlinear dynamical models.

Second order dynamics

We start by considering second order systems described by an equation of motion of the form $\ddot{p}_i = u_i$. For such systems, we devise a proportional derivative (PD) control via,

$$u_i = -2k_{\text{prop}} M_{V_i}(p_i - C_{V_i}) - k_{\text{deriv}}\dot{p}_i,$$

where k_{prop} and k_{deriv} are scalar positive gains. The closed loop induced by this control law can be analyzed with the Lyapunov function

$$\mathcal{E} = k_{\text{prop}}\mathcal{H}_{\mathcal{V}} + \frac{1}{2}\sum_{i=1}^{n} \dot{p}_i^2,$$

and its derivative along the closed loop: $\dot{\mathcal{E}} = -k_{\text{deriv}} \sum_{i=1}^{n} \dot{p}_i^2$. Convergence to a centroidal Voronoi tessellation is obtained invoking the classic LaSalle's invariance principle.

Mobile wheeled dynamics

Next, we consider a classic model of mobile wheeled dynamics and we propose a feedback law based on the design in [24]. Assume the ith vehicle has configuration $(\theta_i, x_i, y_i) \in \mathrm{SE}(2)$ evolving according to

$$\begin{aligned} \dot{\theta}_i &= \omega_i \\ \dot{x}_i &= v_i \cos\theta_i \\ \dot{y}_i &= v_i \sin\theta_i, \end{aligned}$$

where (ω_i, v_i) are the control inputs for vehicle i.

Note that the definition of (θ_i, v_i) is unique up to the discrete action $(\theta_i, v_i) \mapsto (\theta_i + \pi, -v_i)$. We use this symmetry to require the equality $(\cos\theta_i,\ \sin\theta_i) \cdot (p_i - C_{V_i}) \leq 0$ for all time t. Should the equality be violated at some time $t = t_0$, we shall redefine $\theta_i(t_0^+) = \theta_i(t_0^-) + \pi$ and v_i as $-v_i$ from time $t = t_0$ onwards.

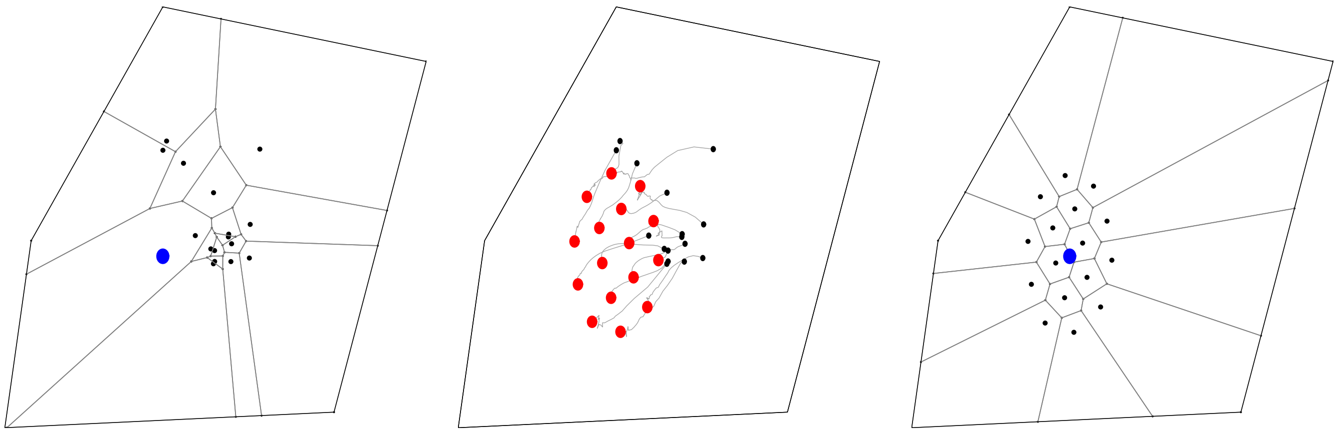

Figure 3: Coverage control to a ellipsoidal disk. The density function parameters are as in Figure 2.

We consider the control law

$$\omega_i = 2k_{\text{prop}} \arctan \frac{(-\sin\theta_i,\ \cos\theta_i)\cdot(p_i - C_{V_i})}{(\cos\theta_i,\ \sin\theta_i)\cdot(p_i - C_{V_i})}$$
$$v_i = -k_{\text{prop}}(\cos\theta_i,\ \sin\theta_i)\cdot(p_i - C_{V_i}),$$

where k_{prop} is a positive gain. This law differs from the original stabilizing strategy in [24] only in the fact that no final angular position is preferred. We illustrate the performance of this control law in Figure 4. Stability of the multi-vehicle network is guaranteed since, for each vehicle, the proposed control law leads to decreasing error $\|p_i - C_{V_i}\|^2$ for all time.

7 Conclusions

We have presented a novel approach to coordination algorithms for multi-vehicle networks. The scheme can be thought of as an interaction law between agents and as such it is implementable in a distributed fashion.

Numerous extensions appear worth pursuing. We plan to investigate the setting of non-convex environments and non-isotropic sensors. Furthermore, we plan to consider more general sensing tasks, such as target identification, and uncontrollable vehicle dynamics, such as aircraft.

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References

[1] M. de Berg, M. van Kreveld, M. Overmars, and O. Schwarzkopf, *Computational Geometry: Algorithms and Applications*, Springer Verlag, New York, 2 edition, 2000.

[2] Z. Drezner, Ed., *Facility Location: A Survey of Applications and Methods*, Springer Series in Operations Research. Springer Verlag, New York, 1995.

[3] N. A. Lynch, *Distributed Algorithms*, Morgan Kaufmann Publishers, San Mateo, CA, 1997.

[4] Q. Du, V. Faber, and M. Gunzburger, "Centroidal Voronoi tessellations: Applications and algorithms," *SIAM Review*, vol. 41, no. 4, pp. 637–676, 1999.

[5] A. Okabe, B. Boots, K. Sugihara, and S. N. Chiu, *Spatial Tessellations: Concepts and Applications of Voronoi Diagrams*, Wiley Series in Probability and Statistics. John Wiley, New York, 2 edition, 2000.

[6] J. Cortés, S. Martínez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," in *IEEE Int. Conf. on Robotics and Automation*, Arlington, VA, May 2002, pp. 1327–1332.

[7] C. Cattani and A. Paoluzzi, "Boundary integration over linear polyhedra," *Computer-Aided Design*, vol. 22, no. 2, pp. 130–5, 1990.

[8] S. P. Lloyd, "Least squares quantization in PCM," *IEEE Transactions on Information Theory*, vol. 28, no. 2, pp. 129–137, 1982, Presented as Bell Laboratory Technical Memorandum at a 1957 Institute for Mathematical Statistics meeting.

[9] R. M. Gray and D. L. Neuhoff, "Quantization," *IEEE Transactions on Information Theory*, vol. 44, no. 6, pp. 2325–2383, 1998, Commemorative Issue 1948-1998.

[10] A. Okabe, B. Boots, and K. Sugihara, "Nearest neighbourhood operations with generalized Voronoi diagrams: a review," *International Journal of Geographical Information Systems*, vol. 8, no. 1, pp. 43–71, 1994.

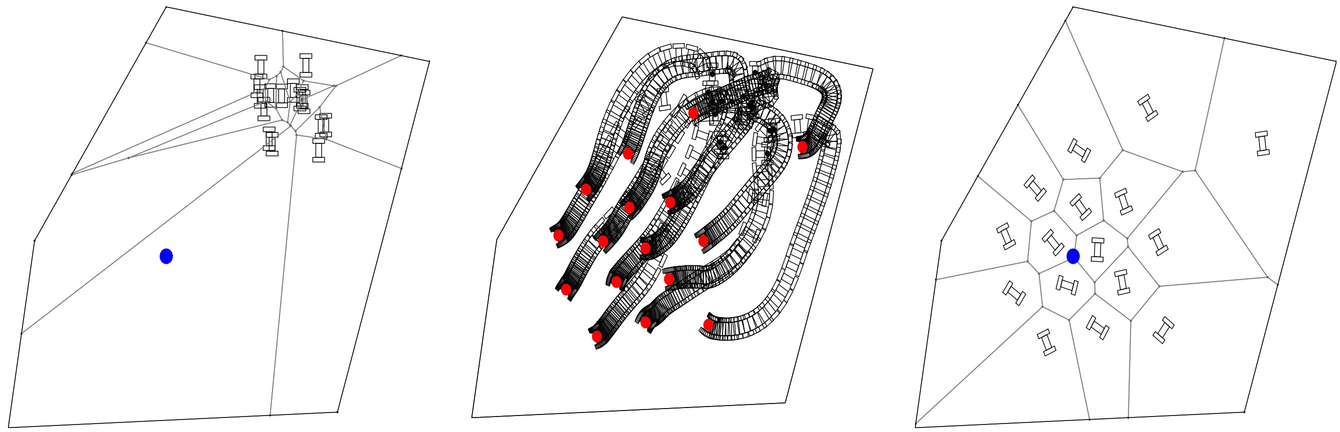

Figure 4: Coverage control for vehicles with mobile wheeled dynamics. The environment and Gaussian density function are as in Figure 1.

[11] A. Okabe and A. Suzuki, "Locational optimization problems solved through Voronoi diagrams," *European Journal of Operational Research*, vol. 98, no. 3, pp. 445–56, 1997.

[12] A. Suzuki and A. Okabe, "Using Voronoi diagrams," In Drezner [2], pp. 103–118.

[13] A. Suzuki and Z. Drezner, "The *p*-center location problem in an area," *Location Science*, vol. 4, no. 1/2, pp. 69–82, 1996.

[14] S. Boyd and L. Vandenberghe, "Convex optimization," Preprint, Dec. 2001.

[15] S. Skyum, "A simple algorithm for computing the smallest enclosing circle," *Information Processing Letters*, vol. 37, no. 3, pp. 121–125, 1991.

[16] L. E. Parker, "Current state of the art in distributed robotic systems," in *Distributed Autonomous Robotic Systems 4*, L. E. Parker, G. Bekey, and J. Barhen, Eds., pp. 3–12. Springer Verlag, 2000.

[17] M. Cao and C. Hadjicostis, "Distributed algorithms for Voronoi diagrams and application in ad-hoc networks," Preprint, Oct. 2002.

[18] J. Gao, L. J. Guibas, J. Hershberger, L. Zhang, and A. Zhu, "Geometric spanner for routing in mobile networks," in *ACM International Symposium on Mobile Ad-Hoc Networking & Computing (MobiHoc)*, Long Beach, CA, Oct. 2001, pp. 45–55.

[19] X.-Y. Li and P.-J. Wan, "Constructing minimum energy mobile wireless networks," *ACM Journal of Mobile Computing and Communication Survey*, vol. 5, no. 4, pp. 283–286, 2001.

[20] S. Meguerdichian, S. Slijepcevic, V. Karayan, and M. Potkinjak, "Localized algorithms in wireless ad-hoc networks: Location discovery and sensor exposure," in *ACM International Symposium on Mobile Ad-Hoc Networking & Computing (MobiHoc)*, Long Beach, CA, Oct. 2001, pp. 106–116.

[21] K. Sugihara and I. Suzuki, "Distributed algorithms for formation of geometric patterns with many mobile robots," *Journal of Robotic Systems*, vol. 13, no. 3, pp. 127–39, 1996.

[22] T. Balch and R. Arkin, "Behavior-based formation control for multirobot systems," *IEEE Transactions on Robotics and Automation*, vol. 14, no. 6, pp. 926–939, 1998.

[23] J. P. Desai, J. P. Ostrowski, and V. Kumar, "Modeling and control of formations of nonholonomic mobile robots," *IEEE Transactions on Robotics and Automation*, vol. 17, no. 6, pp. 905–908, 2001.

[24] A. Astolfi, "Exponential stabilization of a wheeled mobile robot via discontinuous control," *ASME Journal on Dynamic Systems, Measurement, and Control*, vol. 121, no. 1, pp. 121–127, 1999.