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Coverage Reduction: A Mathematical Model

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This paper deals with a mathematical model for reduction of the lack of coverage (LC) involving multiple coverage in presence of partial covering. The model proposes a new structure of assignment of facilities in a facility location system to cover in greater proportion of the demand territory, avoiding assignment of several facilities in the same space of the territory. A comparison between the engendered solution and its representation is carried out through known indicators to measure the improvement of the solution. The results of our proposed model are contrast and better compared to defined referred models in order to evaluate the reduction of LC.

Keywords: Integer programming; facility location; lack of coverage reduction; multiple coverage.

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1. Introduction

Available facilities in location cover the largest number of customers or users who in turn will decide whether to use the facility or not. In a real environment, coverage may be divided into the capacity of several facilities located at strategic points, leading to not only a fully covered market but also partially, and thus it maximizes the coverage. This type of problem seeks to increase the population covered within a desired service distance by locating a fixed number of facilities to operate.[8] In this model, some variations in which contemplated situations are not considered in the base modeling, e.g. the location of maximum coverage by hierarchies. It is addressed by Moore[19] under the consideration of two levels of service related to the interaction of two types of facilities where the goal is to minimize the unmet demand of facilities located at all levels. Daskin[10] reformulates these types of model by incorporating stochastic elements related to the availability of the facilities taking into account the possibility that the facilities located to cover the demands are not available for a certain node as they are attending another demand node. This enables a better approach to the features of a real environment where a certain facility can be arranged to meet several demand nodes. But, at some point, some of them are not being available. The concept of availability of facilities are continued being further investigated and improved by including specific aspects of reality. The certainty of each demand node having at least one facility can meet coverage at some time[21] which is incorporated to the traditional maximal covering location problem (MCLP).

Another approach to the problem of maximum coverage is introduced in later years when continuous points for the location are considered because any point in the plane could be taken as a possible spot to place the facility.[9] This is called the maximal covering problem in the plane which establishes that there is at least one optimal solution. All the facilities are to be located inside the circle that intersects the demand points. Such problem is more efficient as covering levels are improved. In this way, an acceptable service is maintained for the same distance and the same number of facilities available for placement. Likewise, applying the maximal covering model in the plane, fewer facilities need to be located to obtain the same level of coverage than Church's[9] traditional model of maximal covering. In fact, it is taken as a reference to generate a new formulation for the MCLP with rectilinear distance measures where placement is not limited to establish the places. Facility location points form parallelogram-shaped structures in which the extent of coverage is governed by the localized facility such that the length between the facility and the point of demand is less than the rectilinear distance where covering is not provided.[34]

The capacity of the facility plays a key role in the system's coverage. For this reason, a model with coverage of two facilities within an established radius is proposed. If one of them could not meet the required demand of the nearest node, the other facility has the opportunity to provide partial or full coverage.[13] This model is called double coverage. It includes the simple model covering limitations and ensures that all demand is covered by the larger radius of coverage where at least the facility

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installed within the smaller radius is assured. This notion is taken as reference by other researchers to continue studying the problem of double coverage. This is the basis for an ambulance relocation problem proposal[7] in which each relocated facility is penalized and location points constantly are changed. In 2009, the time variable is first incorporated into the model when demand coverage is maximized with mobile facilities that could, at some point in time, cover a certain point of demand to enhance double coverage for multiple periods.[2] This approach is addressed later by introducing the MCLP dynamically.[35]

A new MCLP generalization model is shown later since it is assumed that each demand node could have a set of multiple-coverage levels with specified covering radii. Consequently, this variable can take several values depending on the proximity of each facility to the nearest center. Hence, the level of coverage is a temporary function. In this way, covering levels and radius for each demand node[3] are defined. This model is useful for locating facilities where the level of coverage can adopt intermediate values. Berman *et al.*[4] analyzes the widespread MCLP model based on the concept of gradual coverage in order to maximize the demand covered where each demand node is fully, partially or not covered at all by the inclusion of a function relating to the covered area. Another application of coverage functions in location problems[17] is revealed where MCLP is taken as reference to include bias in coverage, a sigmoid coverage function related to logistics. Later, Berman *et al.*,[6] based on previous research findings, build a new modeling which includes the concept of gradual coverage and varying demands so as to minimize LC that can be produced when placing at certain known point is occurred.

In all the above models, coverage radii are constituted as parameters for each demand node. In some cases, the radii are the same for each node. Sometimes, this is far from reality as there are systems where several demand points could be covered within different coverage radii which are dependent on the objectives set for the type of covering. This is studied by Berman[5] in order to develop a problem where node coverage radius is a variable generated in terms of minimizing installation costs of a given facility to provide coverage to a determined demand point. The variables uncertainty is also considered using the Hurwicz criterion based on an intermediate stance between pessimistic and optimistic results taking into account different probability distributions.[11] To continue with the consideration of various levels of coverage, a model sought to maximize the total demand covered by several facilities located at a certain level[16] is formulated in order to provide a minimum number of facilities those should be assigned within the service covering radius. Literature provides a lot of information about node demand coverage where demand is concentrated in node points. There are cases like those of mobile telephony networks and Internet where demand is not only concentrated in nodes but also along paths because the signal must stay on these roads or areas. Addressing this situation, nonlinear problem modeling arises to find out an optimal set of facilities that meet the requirements for maximum demand nodes and roads.[12] Shah and Soni[28] analyze a periodic review inventory system with service level constraint. Guerra-Olivares *et al.*[15] suggest a heuristic method to optimize container space assignment problem in port terminals.

Consistent with the previous modeling, a model is adjusted to a Wi-Fi network that aims to maximizing the population covered by router facilities providing the Wi-Fi service. This model proposes Maximal Covering with reliable connectivity of network services and relates the location of facilities with the uncommon roads between established facilities.[18] Another maximum coverage model is set forth by Alexandris and Giannikos.[1] Two location models for problems with this type of approach are presented and better results are obtained compared to the traditional MCLP model. The benefits of these models revolve around considering demand points as areas represented by polygons with known centroids rather than as discrete points which makes the extent of coverage closer to the real environment. Sarkar and Majumder[23] develop various dimensional facility location models in different types of transportation modes. The localization problems represent a decision-making process which can be easily applied to different fields.[20] Their mathematical model is developed based on integer linear programming which integrally optimizes the location of the urban distribution center(s) in one city where its clients visit the routes. Valipour[32] compares the non-linear autoregressive neural network (NARNN), the nonlinear input–output (NIO) and the NARNN with exogenous input (NARNNX) in case of the annual precipitation forecasting which has a great importance to assess fresh water and management related to land use, agriculture and hydrology, the risk reduction of flood and drought. Valipour[29–31,33] study irrigation management problems using noteworthy optimization techniques and software such as SIRMOD software, time series analysis methods and neural network techniques.[14]

It is noted that coverage can be studied in different ways considering aspects such as the variability in its components, facility multiplicity, decision variables, and the level of coverage, among others. The proposed model considers any type of fixed facilities such as an ATM, bank, warehouse, school, stadium, airport, among others which in the short term are not mobilized due to their natures. Thus, this paper assesses a modeling proposal in regards to a concept of LC that incorporates multiple coverage within the location system caused by the coverage generated by several facilities on the same area of demand which directly affects the quantification of LC within a system. A comparison between the engendered solution and its representation is carried out through established indicators to measure the quality of the solution obtained, and the results of the proposed model are contrasted to defined reference models so as to evaluate the reduction of LC. Through the research presented in this paper is shown a new structure of assignation of facilities in a system so that their location allows covering in greater proportion of the demand territory, avoiding the several facilities which cover the same space of territory. Thus our model seeks to improve the distribution of the available facilities in order to cover uniformly the territory of demand. To reduce the lack of coverage, facilities should be located so that the coverage generated over all areas of demand is equal to be the difference between the coverage in the areas of demand and the coverage in the intersection spaces between the coverage areas of the facilities. The multiple coverage, generated in the intersection zones or spaces between the coverage areas of several facilities, are eliminated and thus facilities in intersection zones are considered once at a time.

2. Notation

The following notations are used to formulate the proposed model.

2.1. *Sets*

I: Set of areas of demand
J: Set of candidate locations
$M(j)$: Set of locations that intersect with other coverage areas
$W(i)$: Set of locations that have partial coverage over the area of demand i of at least percentage b but less than 100%

2.2. *Indices*

i: Index of areas of demand
j,l: Index of candidate locations

2.3. *Parameters*

D_t: Total population of the territory
S: Number of facilities available for location
A_{jli}: Area of intersection between the coverage area of the facility located in j, the coverage area of the facility located in l and the area of demand i
t_{ij}: Area of intersection between area of demand i and the coverage area of the facility located in j
D_i: Population of area of demand i
b: Minimum percentage of acceptable coverage in the range [0,100]
b_i: Minimum number of facilities with partial coverage that must assist the area of demand i for full coverage
G_i: Value of area of demand i
R: Coverage radius of each facility

$$e_{ij}\left\{\begin{array}{ll} 1, & \text{if the facility located at } j \text{ provides total coverage} \\ & \text{to the demand area } i \\ 0, & \text{otherwise} \end{array}\right\}$$

$$v_{jl}\left\{\begin{array}{ll} 1, & \text{if the coverage area from facility located at } j \text{ intercepts} \\ & \text{coverage area from facility located at } l \\ 0, & \text{otherwise considering } j \neq 1 \end{array}\right\}$$

$$n_{jli}\begin{Bmatrix} 1, & \text{if the coverage area from facility located at } j \text{ intercepts coverage} \\ & \text{area from facility located at } l \text{ and the demand area } i \\ 0, & \text{otherwise} \end{Bmatrix}$$

$PortCov(i,j)$: Coverage proportion of demand area i produced by facility located at j, which is calculated as folows:

$PortCov(i,j) = \left(\frac{t_{ij}}{G_i}\right)$ for every $i \in I$

$Cov(i,j)$: Coverage of demand area i produced from facility located at j, which is calculated as follows:
$Cov(i,j)$:$D(i) * PortCov(i,j)$

2.4. *Variables*

$$x_j\begin{Bmatrix} 1, & \text{if the facility is located at } j \\ 0, & \text{otherwise} \end{Bmatrix}$$

$$h_{jl}\begin{Bmatrix} 1, & \text{if the facilities } j \text{ and } l \text{ are located} \\ 0, & \text{otherwise} \end{Bmatrix}$$

$$y_i\begin{Bmatrix} 1, & \text{if demand area } i \text{ is covered for at least one facility} \\ 0, & \text{otherwise} \end{Bmatrix}$$

$$f_i\begin{Bmatrix} 1, & \text{if demand area } i \text{ is covered partially for at least } b_i \text{ times} \\ 0, & \text{otherwise} \end{Bmatrix}$$

$$p_{ij}\begin{Bmatrix} 1, & \text{if demand area } i \text{ is covered by facility located at } j \\ 0, & \text{otherwise} \end{Bmatrix}$$

3. Methodology

To determine an optimal point of location, an assessment on whether there is an amount of demand still not covered within a given market should be performed allowing subsequent placement decision of a facility that can provide the products or services required by it. In order to decide where to place facilities so as to try increasing the demand coverage presented by the market. Where demand is concentrated, its quantification is required so that the benefit of covering a specific area or space can be revealed. The distance between location points and the area where demand is concentrated is quantified. In this connection, possible facility locations considering the limitations regarding the availability of space, cost and safety, the availability of the facilities; and the ability to supply a certain number of customers are considered. After these quantification, the concepts of coverage established in two models[1,4] are examined in order to assess them. The proposed model incorporates a new concept of LC with the realistic assumptions of the problem.

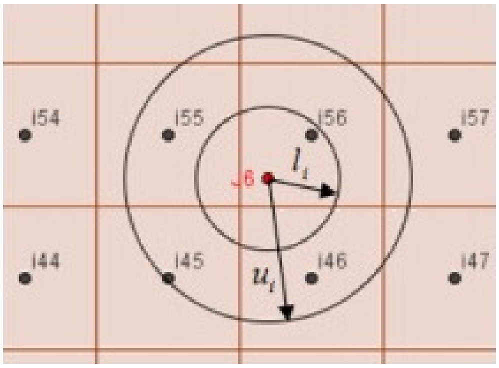

Fig. 1. Coverage concept according to Berman's model.

The first model of Berman *et al.*[4] seeks to locate a known number of facilities so that the total demand is maximized by incorporating a concept of coverage determined by a function of gradual coverage between a couple of coverage radii assigned according to the capacity of each facility. Concepts of full and partial coverage are established through this function. So, an area or node is completely covered if its centroid is located within the radius of full coverage l_i and without coverage if it is out of the non-coverage radius u_i, and partially covered if it is in between the two coverage radii. This can be understood more clearly in Fig. 1.

According to Fig. 1, based on the concept expressed by this author, demand areas *i57* and *i47* are not covered by the facility in *j6* because its centroid is outside the non-coverage radius u_i. The *i56* demand area is completely covered since the centroid is within the range of full coverage l_i. This model seeks to maximize the total proportion demand that is covered by all facilities located.

While the central idea of the second model[1] is the integration of the concept of partial coverage. It is assumed that, if an area is partially covered by a sufficient number of servers, it is assumed that this area is completely covered. Given that each facility is limited in terms of its resources, a minimum distance at which each facility can serve a certain area of demand is determined. This model defines the locations j so that the maximum demand is met. The concept of coverage can be understood more clearly in Fig. 2.

Consequently, according to the concept of coverage of this author, *i70* demand area is completely covered by being attended by 2 facilities with an established minimum coverage of 50%. Hence, the purpose of this model is to maximize the total population covered by establishing a benefit which is equal to the population of the area of demand.

To evaluate the models, 30 test instances of 15, 20, 30, 40, 50, 60, 80, and 100 candidate locations are built. From the set of 30 instances, a subset of 10 instances and another of 20 are considered to assess coverage of placed facilities on 100 and 200 demand areas of equal size, respectively. Once instances are created, LC value is

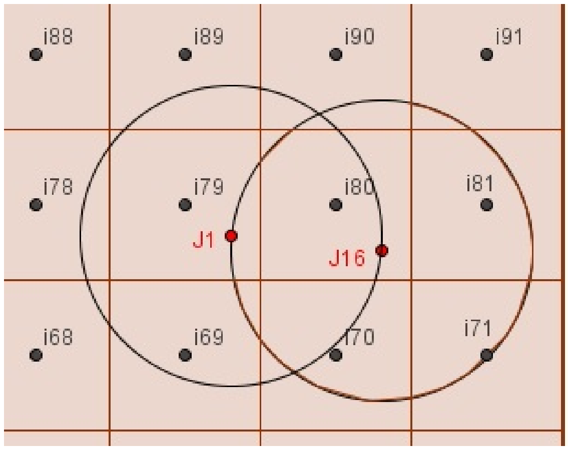

Fig. 2. Coverage concept according to Alexandris' model.

calculated by proposed model and the reference models.[1,4] After that, those values are compared. In the case of the first model[4]; LC value is attained through the difference between the total population of the territory D_t and the demand covered. This can be understood with Eq. (1), where $c_i(x)$ is the proportion of the demand weight in node i that is covered by the facility located at the point x, and $y_i(x)$ is the variable that is determined when the facility located at point x provides coverage in node i. Then, the objective function is

$$\text{Min } Z(x) = D_t - \sum_{i\in N}\sum_{x\in X} c_i(x)y_i(x). \tag{1}$$

In the case of the second model,[1] LC is obtained through the difference between the total population of the territory D_t and the demand covered. This can be understood through Eq. (2), where w_i is the benefit of fully covering the demand area, y_i is the variable that is determined while the area of demand is covered by at least one facility, α is the section of the partial coverage considered as full coverage, and v_i is the variable that is determined if the area of demand is partially covered by at least a minimum number of partial coverage facilities. In this case, the objective function is

$$\text{Min } Z(x) = D_t - \sum_{i\in I} w_i(y_i + \alpha v_i). \tag{2}$$

As the value of the objective function is a measure of the result obtained by each model of integer linear programming (ILP), the reduction of LC (ΔLC), the amount of real covering per test instance, and the amount of false covering per test instance are used. This allows establishing or recognizing the quality of the result obtained for assessment of each model. Here, the formula of ΔLC is

$$\Delta LC = \frac{LC_{\text{Reference model}} - LC_{\text{Proposed model}}}{LC_{\text{Reference model}}} \times 100. \tag{3}$$

3.1. *Mathematical model with multiple coverage*

The main objective of the proposed model is to calculate the LC generated by locations from a new indicator which uses as reference of the coverage of facilities those are located throughout the space where demand and coverage are generated in the intersection areas between various facilities. Coverage generated in the areas of demand originates from the coverage of demand areas where the percentage of coverage is total or partial and from the coverage of the different intersection areas since the latter should be considered in order not to estimate it multiply in the calculations of real coverage. This coverage concept can be understood more clearly in Fig. 3.

Figure 3 shows that LC as the white area within the territory of 4 areas of demand (objects of squared area) is obtained after subtracting the population of the space covered by the L, H, and N facilities and the intersection zones of their coverage areas (intersection between L and N coverage area, intersection between L and H coverage area, and the intersection between N and H coverage area) from the total population of the territory. This model determines which locations j to be located and the coverage relationship between located facilities and coverage areas such that LC of demand is minimized. The unmet demand or LC is understood as the demand not served in areas where facilities cannot cover it entirely or largely. The proposed model aims to minimize LC that can be incurred by locating at certain points,

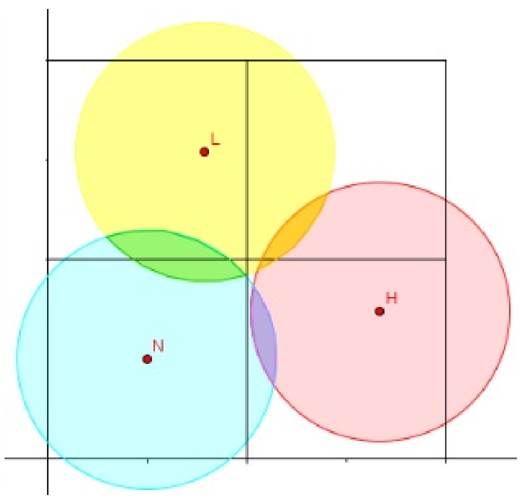

Fig. 3. Coverage concept suggested.

providing an optimal solution with localization points of the available facilities: In this case, the new objective function of LC is formulated as follows:

$$\text{Min } Z = LC = D_t - \underbrace{\sum_{i \in I} \sum_{j \in J} [\text{Cov}(i,j)] * x(j)}_{\substack{\text{Converage of facilities located at} \\ \text{demand areas}}} + \underbrace{\sum_{j \in M(j)} \left[\frac{\sum_l^J v_{jl} * h_{jl} \left[\sum_i^I \left[n_{jli} * \left(\frac{A_{jli}}{G_i} \right) * D_i \right] \right]}{2} \right]}_{\substack{\text{Covering produced in the intersection} \\ \text{zones of different facilities}}} \quad \text{where } l \neq j. \tag{4}$$

Subject to:

$$\sum_{j \in J} x_j = S, \tag{5}$$

$$\sum_{j \in J} e_{ij} x_j \geq y_i \quad \forall i \in I, \tag{6}$$

$$y_i + f_i \leq 1 \quad \forall i \in I, \tag{7}$$

$$\sum_{j \in W(i)} x_j \geq b_i f_i \quad \forall i \in I, \tag{8}$$

$$\sum_{j \in J} p_{ij} \geq y_i \quad \forall i \in I, \tag{9}$$

$$\sum_{j \in J} p_{ij} \geq b_i f_i \quad \forall i \in I, \tag{10}$$

$$x_j \in \{0,1\} \quad \forall j \in J, \tag{11}$$

$$y_i f_i \in \{0,1\} \quad \forall i \in I, \tag{12}$$

$$p_{ij} \in \{0,1\} \quad \forall i \in I \quad \text{and} \quad \forall j \in J. \tag{13}$$

The number of facilities available for location is limited by constraint (5). The second constraint ensures that, when $y_i = 1$, the demand for the area i is completely covered by at least one facility. The distinction between full and partial coverage is expressed through constraint (7). Constraint (8) states that a minimum number of facilities with partial coverage should be located in order to complete full coverage if partial coverage is considered on the demand for the area i. Constraint (9) establishes that at least one facility should provide coverage to the area of demand i. Constraint (10) states that at least b_i from located facility should provide partial coverage to the area of demand i. Constraints (11)–(13) define the model's binary variables.

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4. Results and Discussion

The three models are assessed to obtain the LC regarding the concept introduced in each modeling. It is found that, in 93% of the assessed instances, the proposed model minimizes LC in contrast to the reference models since lower values for this indicator are gathered. This shows a more accurate location decision when comparing the results from the reference models. Likewise, for the same percentage of instances, LC is significantly reduced (Table 1) because, only in two instances, one of the reference models improved. But, the presence of over coverage is an unreal situation that causes redundancy in the calculation of the indicator since the value of coverage in which multiple facilities can have on the same space of a specific area of demand is contemplated.

Analyzing the values mentioned above, it is observed that LC by proposed model is reduced by an average of 36% compared to the second model[1] and 25% compared

Table 1. Benchmarking for indicator ΔLC.

	Size		ΔLC	ΔLC
Instance	No. of i	No. of j	Proposed model vs. second model	Proposed model vs. first model
1	100	20	26%	13%
2	100	30	33%	16%
3	100	15	47%	41%
4	100	15	89%	55%
5	100	15	34%	No improvement
6	100	20	61%	46%
7	100	30	37%	19%
8	100	40	No improvement	No improvement
9	100	50	27%	4%
10	100	50	39%	24%
11	200	15	31%	29%
12	200	15	26%	22%
13	200	20	26%	24%
14	200	20	13%	8%
15	200	30	20%	12%
16	200	30	20%	11%
17	200	40	14%	6%
18	200	40	51%	46%
19	200	50	35%	22%
20	200	50	55%	41%
21	200	30	49%	41%
22	200	30	53%	45%
23	200	40	30%	18%
24	200	40	36%	22%
25	200	15	17%	15%
26	200	20	9%	3%
27	200	20	32%	25%
28	200	30	8%	3%
29	200	40	18%	7%
30	200	50	70%	85%

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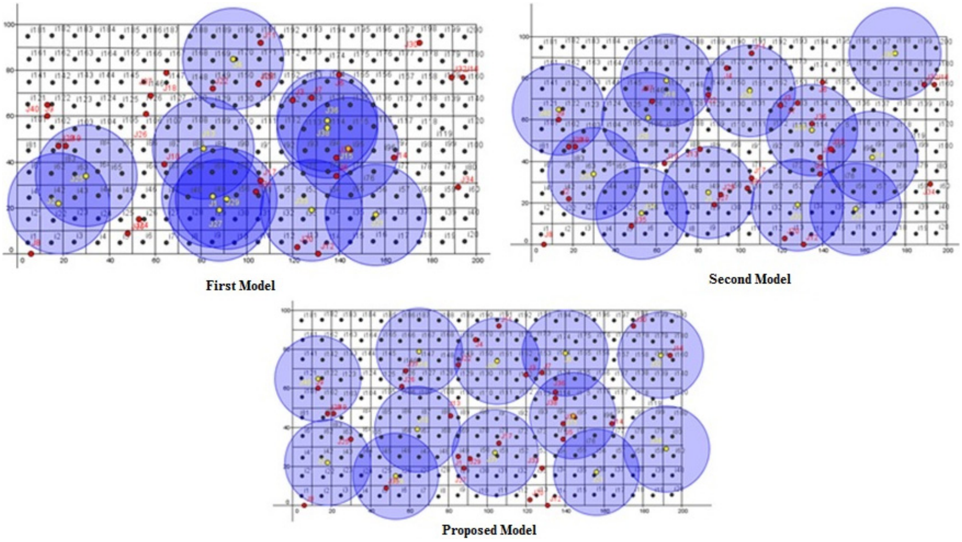

Fig. 4. Comparison of results of assessed models.

to the first model.[4] This clearly shows that the proposed model achieves greater reduction in LC compared to the reference models. Consistent with the results summarized in the table mentioned above, Fig. 4 is introduced to present a visual comparison of the results of instance 18 for the models. Figure 4 shows that the placement of facilities in the candidate locations is performed more effectively since facilities available are distributed more equitably.

According to the results and the plotting, the proposed model considers all the provided covering for the calculation of the objective function (LC), whereas the results of all test instances obtained from the reference models show that a high number of real and false coverage of facilities on specific demand areas are not considered.

5. Conclusions and Future Work

This research puts forward the assessment of three location models by means of the concept of coverage set in each which is a determining factor in the placement of facilities with a limited set of resources. These models are evaluated through 30 different instances and results are plotted. It is demonstrated that, when coverage in terms of the area covered and the intersection between coverage areas of located facilities are considered, the LC can be reduced and the plot is consistent with the actual situation.

By evaluating the validated models, it is observed that, when coverage is considered as a gradual function dependent on the distance between the centroid of the demand area and the location point of the given facility, false coverage values are generated and there is an absence of real coverage values which are not taken into account. Similarly, having a minimum number of facilities with acceptable coverage for a certain area of demand does not ensure the coverage of the entire territory. It is real fact that plotted results are not consistent with it. On the other hand, establishing coverage as part of the space where demand is covered by specific facilities places taking into account intersection zones among them), location outcome is more accurate and consistent with plotting because the real space partially or completely covered is contemplated and thus space covered by two or more facilities simultaneously is not considered multiple times.

As a result, the development of the proposed model shows that this work can encompass several aspects and presents great opportunities for improvement. Unlike the previous literature, many features such as the assessment of irregular areas in scenarios where demand is not uniform, the presence of locations established by the competition, capacity for providing different services through available facilities, intermittent availability of the facilities, decision criteria different from the distance of the facility, personalized search for the model in terms of the number of locations required to have full coverage of the territory, producing the ideal scenario rather than exploring for it, and constraint of location points in terms of area boundaries may be included in the proposed model. The model helps to a management to eliminate the over-coverage understood as a non-real situation that occurs when several installations provide coverage in more than 100% to a demand area. The over-coverage causes redundancy in the calculation of the indicator because it contemplates the coverage value of several facilities over the same space of a demand area. In this way, the space covered by two or more installations simultaneously is not considered in a double way and therefore results are obtained more accurate and consistent with reality.

Moreover, it would be interesting to engage the measurement indicators to evaluate the quality of the results in the proposed model in future and thus have a multi-objective mathematical model. This model can be extended further applying management tools of supply chain[22,24–27] for better facilities distributions.

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