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Covering Based Optimistic Multigranular Approximate Rough Equalities and their Properties

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Abstract—Since its inception rough set theory has proved itself to be one of the most important models to capture impreciseness in data. However, it was based upon the notion of equivalence relations, which are relatively rare as far as applicability is concerned. So, the basic rough set model has been extended in many directions. One of these extensions is the covering based rough set notion, where a cover is an extension of the concept of partition; a notion which is equivalent to equivalence relation. From the granular computing point of view, all these rough sets are unigranular in character; i.e. they consider only a singular granular structure on the universe. So, there arose the necessity to define multigranular rough sets and as a consequence two types of multigranular rough sets, called the optimistic multigranular rough sets and pessimistic rough sets have been introduced. Four types of covering based optimistic multigranular rough sets have been introduced and their properties are studied. The notion of equality of sets, which is too stringent for real life applications, was extended by Novotny and Pawlak to define rough equalities. This notion was further extended by Tripathy to define three more types of approximate equalities. The covering based optimistic versions of two of these four approximate equalities have been studied by Nagaraju et al recently. In this article, we study the other two cases and provide a comparative analysis.

Index Terms—Rough Sets, Covering Based Rough Sets, Multigranulations, Covering Based Multigranulations, Approximate Equality.

I. INTRODUCTION

Data in real life are mostly imprecise in nature and so the conventional tools for formal modeling, reasoning and computing, which are crisp, deterministic and precise in characteristics, are inadequate to handle them. This gives rise to the development of several imprecise models, of which rough sets introduced by Pawlak [5, 6] is one of the most efficient one. It is an excellent tool to capture impreciseness in data in a very effective manner. According to Pawlak, the knowledge of human beings depends upon their capability to classify objects of universes. Since equivalence relations on any universe induce classifications through the equivalence classes associated with them, for mathematical reasons equivalence relations were taken as the basic notions in defining the basic rough sets.

A rough set is represented by a pair of crisp sets, called the lower approximation and upper approximation of the set. Lower approximation comprising of elements certainly belong to it and upper approximation comprising of elements certainly or possibly belong to it, with respect to the available information.

This basic rough set has been extended further in many directions. These extensions are actually either based on tolerance relations or any such relations that do not require the stringent restrictions of an equivalence relation.

From the point of view of granular computing, basic rough set theory deals with a single granulation [21]. However, in some application areas we need to handle more than one granulation at a time and this necessitated the development of multi-granular rough sets (MGRS)[7], where at least two equivalence relations are taken for granulation of a universe. This concept is further extended by considering covers and this lead to the development of covering based multi granular rough sets(CBMGRS). Four types of CBMGRS are defined and their properties are established.

The basic notion of equality of two sets is independent of the user or more precisely the user knowledge about the universe dealt with. In an attempt to incorporate the user knowledge about the structure of the universe dealt with in concluding about the equality of two sets the notion of rough equalities were introduced by Novotny and Pawlak. This is an important feature as the sets considered not are equal in the normal sense but they have close features to assume that they are approximately equal. That is, basing upon our knowledge and requirement we can assume that the two sets are indistinguishable. Properties of approximate equalities established by Novotny and Pawlak were analyzed. It was found that the properties failed to hold in their full generalities and mostly parts were found to hold true.

This paper is organized into five sections. First section gives the over view and related literatures. Section two presents various definitions and notions required. Section three introduces rough equalities. Section four specifies multi granular rough equalities, their properties and replacement properties. In this section a real life example is considered to prove few replacement properties as sample. In final section conclusion is written.

II. Definitions and Notations

A. Rough Set

Let U be a universe of discourse and R be an equivalence relation over U. By U/R we denote the family of all equivalence classes of R, referred to as categories or concepts of R and the equivalence class of an element $x \in U$ is denoted by $[x]_R$. By a knowledge base, we understand a relational system $K = (U, P)$, where U is as above and P is a family of equivalence relations over U. For any subset Q ($\neq \phi$) $\subseteq$ P, the intersection of all equivalence relations in Q is denoted by IND(Q) and is called the indiscernibility relation over Q. Given any $X \subseteq U$ and R $\in$ IND (K), we associate two subsets, $\underline{R}X = \bigcup\{Y \in U/R : Y \subseteq X\}$ and $\overline{R}X = \bigcup\{Y \in U/R : Y \cap X \neq \phi\}$, called the R-lower and R-upper approximations of X respectively. The R-boundary of X is denoted by $BN_R\ (X)$ and is given by $BN_R(X) = \overline{R}X - \underline{R}X$. The elements of $\underline{R}$ X are those elements of U, which can certainly be classified as elements of X, and the elements of $\overline{R}$ X are those elements of U, which can possibly be classified as elements of X, employing knowledge of R. We say that X is rough with respect to R if and only if $\underline{R}X \neq \overline{R}X$, equivalently $BN_R(X) \neq \phi$. X is said to be R-definable if and only if $\underline{R}X = \overline{R}X$, or $BN_R(X) = \phi$.

B. Covering based Rough Sets

Basic rough sets introduced by Pawlak have been extended in many ways. One such extension is the notion of covering based rough sets, where the notion of partitions is replaced by the general notion of covers [22, 23]. In this section we introduce the basics of these sets.

Definition 2.2.1: Let U be a universe and C={C_1,C_2,….., C_n} be a family of non-empty subsets of U that are overlapping in nature. If $\bigcup C$ = U, then C is called a covering of U. The pair (U, C) is called covering approximation space. For any X $\subseteq$U, the covering lower and upper approximations of X with respect to C can be defined as follows

(2.2.1) $\underline{C}(X) = \bigcup\{C_i \subset X, i \in 1, 2, ..., n\}$

(2.2.2) $\overline{C}(X) = \bigcup\{C_i \cap X \neq \phi, i \in 1, 2, ..., n\}$

The pair $(\underline{C}(X), \overline{C}(X))$ is called covering based rough set associated with X with respect to cover C if $\underline{C}(X) \neq \overline{C}(X)$, i.e., X is said to be roughly definable with respect to C. Otherwise X is said to be C-definable.

Definition 2.2.2: Given a covering approximation space (U, C) for any x$\in$U, sets $md_c(x)$ and $MD_c(x)$ are respectively called minimal and maximal descriptors of x with respect to C,

(2.2.3) $md_c(x) = \{M \in C \,/\, x \in M \; and \; (\forall N \in C \; such that \; x \in N \; and \; N \subseteq M) => M = N\}$

It is a set of all minimal covers containing x where a minimal cover containing x be one for which no proper sub cover containing x exists.

(2.2.4) $MD_c(x) = \{M \in C \,/\, x \in M \; and \; (\forall N \in C \; such that \; x \in N \; and \; N \supseteq K) => M = N\}$

It is a set of all maximal covers containing x where a maximal cover containing x be one for which no proper super cover containing x exists.

C. Multi Granular Rough Sets

In the view of granular computing (proposed by L. A. Zadeh), an equivalence relation on the universe can be regarded as a granulation, and a partition on the universe can be regarded as a granulation space [5, 6]. For an incomplete information system, similarly, a tolerance relation on the universe can be regard as a granulation, and a cover induced by the relation can be regarded as a granulation space. Several measures in knowledge base closely associated with granular computing, such as knowledge granulation, granulation measure, information entropy and rough entropy. As far as rough set method based on multi-granulations is concerned Qian et al ([7], [10]) proposed two rough set models called the optimistic multigranular rough sets and pessimistic rough set models, which are established by using multi equivalence relations.

Definition 2.3.1: Let K= (U, **R**) be a knowledge base, **R** be a family of equivalence relations, M, N $\in$ **R.** We define the optimistic multi-granular lower approximation and upper approximation of X in U as

(2.3.1) $\underline{M+N}(X) = \bigcup\{x / [x]_M \subseteq X \; or \; [x]_N \subseteq X\}$

and

(2.3.2) $\overline{M+N}(X) = (\underline{M+N}(X^C))^C$

D. Covering based Multi Granular Rough Sets

The notion of Multi-granular rough sets is extended to covering approximation space [1, 2, 4]. They can of two categories, namely, optimistic and pessimistic. By employing minimal and maximal descriptor four types of CBMGRS are possible. The definitions of four types of CBMGRS are given as follows [4].

Let (U, C) be a covering approximation space, C_1 and C_2 be covers in C and X be any subset of U, There are four types of optimistic covering based multi granular rough sets, which are defined as follows.

Definition 2.4.1: The first type CBMGRS lower and upper approximations with respect to C_1 and C_2 are defined as follows

$$(2.4.1)\quad \underline{F_{C_1+C_2}}(X) = \{x \in U / \bigcap md_{c_1}(x) \subseteq X \text{ or } \bigcap md_{c_2}(x) \subseteq X\}$$

And

$$(2.4.2)\quad \overline{F_{C_1+C_2}}(X) = \{x \in U / (\bigcap md_{c_1}(x)) \bigcap X \neq \phi \text{ and } (\bigcap md_{c_1}(x)) \bigcap X \neq \phi\}$$

Definition 2.4.2: The second type CBMGRS lower and upper approximations with respect to C_1 and C_2 are defined as follows

$$(2.4.3)\quad \underline{S_{C_1+C_2}}(X) = \{x \in U / \bigcup md_{C_1}(x) \subseteq X \text{ or } \bigcup md_{C_2}(x) \subseteq X\}$$

And

$$(2.4.4)\quad \overline{S_{C_1+C_2}}(X) = \{x \in U / (\bigcup md_{C_1}(x)) \bigcap X \neq \phi \text{ and } (\bigcup md_{C_2}(x)) \bigcap X \neq \phi\}$$

Definition 2.4.3: The third type CBMGRS lower and upper approximations with respect to C_1 and C_2 are defined as follows

$$(2.4.5)\quad \underline{T_{C_1+C_2}}(X) = \{x \in U / \bigcap MD_{C_1}(x) \subseteq X \text{ or } \bigcap MD_{C_2}(x) \subseteq X\}$$

And

$$(2.4.6)\quad \overline{T_{C_1+C_2}}(X) = \{x \in U / (\bigcap MD_{C_1}(x)) \bigcap X \neq \phi \text{ and } (\bigcap MD_{C_2}(x)) \bigcap X \neq \phi\}$$

Definition 2.4.4: The first type CBMGRS lower and upper approximations with respect to C_1 and C_2 are defined as follows

$$(2.4.7)\quad \underline{L_{C_1+C_2}}(X) = \{x \in U / \bigcup MD_{C_1}(x) \subseteq X \text{ or } \bigcup MD_{C_2}(x) \subseteq X\}$$

And

$$(2.4.8)\quad \overline{L_{C_1+C_2}}(X) = \{x \in U / (\bigcup MD_{C_1}(x)) \bigcap X \neq \phi \text{ and } (\bigcup MD_{C_2}(x)) \bigcap X \neq \phi\}$$

E. Properties of Optimistic Covering based Multi Granulation Rough Sets

The following are the properties of optimistic first type covering based multi granular rough sets. Here 'A' denotes any of the four types first, second, third or fourth of optimistic multigranulation. Let X and Y be any two subsets of U. We omit the proofs of these properties as these are more or less trivial. The proofs can also be found in [4].

$$(2.5.1)\quad X \subseteq Y \Rightarrow \underline{A_{C_1+C_2}}(X) \subseteq \underline{A_{C_1+C_2}}(Y)$$

$$(2.5.2)\quad X \subseteq Y \Rightarrow \overline{A_{C_1+C_2}}(X) \subseteq \overline{A_{C_1+C_2}}(Y)$$

$$(2.5.3)\quad \underline{A_{C_1+C_2}}(\sim X) = \sim \overline{A_{C_1+C_2}}(X)$$

$$(2.5.4)\quad \overline{A_{C_1+C_2}}(\sim X) = \sim \underline{A_{C_1+C_2}}(X)$$

$$(2.5.5)\quad \underline{A_{C_1+C_2}}(X \cup Y) \supseteq \underline{A_{C_1+C_2}}(X) \cup \underline{A_{C_1+C_2}}(Y)$$

$$(2.5.6)\quad \overline{A_{C_1+C_2}}(X \cup Y) \supseteq \overline{A_{C_1+C_2}}(X) \cup \overline{A_{C_1+C_2}}(Y)$$

$$(2.5.7)\quad \underline{A_{C_1+C_2}}(X \cap Y) \supseteq \underline{A_{C_1+C_2}}(X) \cap \underline{A_{C_1+C_2}}(Y)$$

$$(2.5.8)\quad \overline{A_{C_1+C_2}}(X \cap Y) \subseteq \overline{A_{C_1+C_2}}(X) \cap \overline{A_{C_1+C_2}}(Y)$$

III. MAIN RESULTS

A. Approximate Equalities

The equality of sets or domains used in mathematics is too stringent. In most of the real life situations we often consider equality of sets or domains, as approximately equal under the existing circumstances of it. These existing circumstances serve as user knowledge about the set or domain. So, approximate equalities play a significant role in approximate reasoning. Also, one can state that it mostly depends on the knowledge the assessors have about the set of domain under consideration as a whole but not on the knowledge about individuals of the set or domain.

As a step to incorporate user knowledge in considering likely equality of sets, Novotny and Pawlak [xxx] introduced the following rough equalities of two sets X and Y which are subsets of X.

Let K= (U,R) be a knowledge base, $X, Y \subseteq U$ and $R \in IND(K)$.

Definition 3.1: We say that,

(3.1.1) X and Y are bottom rough equal (X *b_R_eq* Y) if and if only $\underline{R}X = \underline{R}Y$.

(3.1.2) X and Y are top rough equal (X *t_R_eq* Y) if and if only $\overline{R}X = \overline{R}Y$.

(3.1.3) X and Y are rough equal (X *R_eq* Y) if and if only $\underline{R}X = \underline{R}Y$ and $\overline{R}X = \overline{R}Y$ i.e., (X *b_R_eq* Y) and (X *t_R_eq* Y).

There are several properties of these approximate equalities established by Novotny and Pawlak in the form of general and replacement properties. The replacement properties are those properties obtained from the general properties by interchanging the top and bottom equalities. As noted by them, all these approximate equalities of sets are relative in character; that is, sets are equal or not equal from our point of view depending on what we have about them. So, in a sense the definition of rough equality incorporates user knowledge about the universe in arriving at likely equality of sets or domains. However, these notions of approximate equalities of sets boil down to equality of sets again. Recently the extension of these approximate equalities to the context of covering based rough sets for the pessimistic case is handled by Tripathy et al [21].

In this paper we shall introduce the concepts of approximate equalities to the context of covering based optimistic multigranulations and prove their properties (both general and replacement). We establish both the direct as well as the replacement properties for both of these notions. In fact four types of covering based multi granular rough sets are found in the literature. But the properties of all these types are similar. So, we shall focus on only the first type of covering based optimistic multigranular rough sets. We shall study the direct properties and establish them. Next, we shall study the replacement properties. We shall consider a real life example to explain the concepts and use it in the proofs to find out counter examples.

B. Optimistic Covering based Multi Granular Approximate Equalities

We now introduce in the following different optimistic covering based multi granular rough equalities for first type of CBOMGRS and study their properties. The definitions for the other types of multigranulations are similar.

Let C_1 and C_2 be two covers on U and $C_1, C_2 \in C$ *and* $X, Y \subseteq U$.

Let F denotes first type of CBOMGRS.

Definition 3.2: We say that,

(3.2.1) X and Y are optimistic bottom rough equal to each other with respect to C_1 and C_2 (X *b_C₁+C₂_eq* Y) if $\underline{F_{C_1+C_2}}(X) = \underline{F_{C_1+C_2}}(Y)$.

(3.2.2) X and Y are optimistic top rough equal to each other with respect to C_1 and C_2 (X *t_C₁+C₂_eq* Y) if $\overline{F_{C_1+C_2}}(X) = \overline{F_{C_1+C_2}}(Y)$.

(3.2.3) X and Y are optimistic total rough equal to each other with respect to C_1 and C_2 (X *r_C₁+C₂_eq* Y) if $\underline{F_{C_1+C_2}}(X) = \underline{F_{C_1+C_2}}(Y)$ and $\overline{F_{C_1+C_2}}(X) = \overline{F_{C_1+C_2}}(Y)$.

C. Properties of First Type of Covering based Optimistic Multi Granular Approximate Equalities

The general properties of first type of covering based rough equalities are stated, proved and substantiated few proofs with examples wherever is necessary.

Let C_1 and C_2 be two covers on U and $C_1, C_2 \in C$ *and* $X, Y \subseteq U$. Let F denotes first type CBOMGRS. Then

(3.3.1) X *b_C₁+C₂_eq* Y if $X \cap Y$ *b_C₁+C₂_eq* X and Y both. But the converse may not be true in general.

Proof:

$$X \cap Y\, b\_C_1 + C_2\_eq\, X \Rightarrow \underline{F_{C_1+C_2}}(X \cap Y) = \underline{F_{C_1+C_2}}(X)$$

and

$$X \cap Y\, b\_C_1 + C_2\_eq\, Y \Rightarrow \underline{F_{C_1+C_2}}(X \cap Y) = \underline{F_{C_1+C_2}}(Y)$$

From the above two expressions we have

$\underline{F_{C_1+C_2}}(X) = \underline{F_{C_1+C_2}}(Y) \Rightarrow$ X *b_C₁+C₂_eq* Y.

For the converse part we see that it depends upon the logical equivalence of the statements $(p \wedge q) \vee (r \wedge s)$ and $(p \vee r) \wedge (q \vee s)$, where p, q, r and s are any four logical statements. However, from their truth values we find that these two statements are not equivalent to each other in the following case.

p	q	r	s
True	True	False	False
True	False	True	False
True	False	False	True
False	True	True	False
False	True	False	True
False	False	True	True

So, examples can be provided which satisfy any of the above cases to show that the converse is not true.

(3.3.2) X *t_C₁+C₂_eq* Y if $X \cup Y$ *t_C₁+C₂_eq* X and Y both. The converse may not be true in general.

Proof:

$$X \bigcup Y\ t\_C_1 + C_2\_eq\ X \Rightarrow \overline{F_{C_1+C_2}}(X \bigcup Y) = \overline{F_{C_1+C_2}}(X)$$

and

$$X \bigcup Y\ \ t\_C_1 + C_2\_eq\ \ Y \Rightarrow \overline{F_{C_1+C_2}}(X \bigcup Y) = \overline{F_{C_1+C_2}}(Y)$$

From the above two expressions we have

$$\overline{F_{C_1+C_2}}(X) = \overline{F_{C_1+C_2}}(Y) \Rightarrow X\ t\_C_1+C_2\_eq\ Y.$$

The converse part is not true as in property 1. We note that the truth of the converse depends upon the logical equivalence of the two statements, $(p \vee q) \wedge (r \vee s)$ and $(p \wedge r) \vee (q \wedge s)$. However, we find the statements quoted are not true in the following cases.

P	q	r	s
False	True	True	False
True	False	False	True

So, examples can be constructed such that the above two cases occur to show that the converse part does not hold.

(3.3.3) X $t\_C_1+C_2\_eq$ X' and Y $t\_C_1+C_2\_eq$ Y' may not imply that $X \bigcup Y$ $t\_C_1+C_2\_eq$ $X' \bigcup Y'$

Proof:

Let us consider the following example to prove the above property.

$$X\ t\_C_1+C_2\_eq\ X' \Rightarrow \overline{F_{C_1+C_2}}(X) = \overline{F_{C_1+C_2}}(X')$$

$$Y\ t\_C_1+C_2\_eq\ Y' \Rightarrow \overline{F_{C_1+C_2}}(Y) = \overline{F_{C_1+C_2}}(Y')$$

From property (2.5.6) we have the following

$$\overline{F_{C_1+C_2}}(X \bigcup Y) \supseteq \overline{F_{C_1+C_2}}(X) \bigcup \overline{F_{C_1+C_2}}(Y)$$
$$\supseteq \overline{F_{C_1+C_2}}(X') \bigcup \overline{F_{C_1+C_2}}(Y') \supseteq \overline{F_{C_1+C_2}}(X' \bigcup Y')$$

$\Rightarrow X \bigcup Y$ is not $t\_C_1+C_2\_eq$ $X' \bigcup Y'$. Hence it is proved

(3.3.4) X $b\_C_1+C_2\_eq$ X' and Y $b\_C_1+C_2\_eq$ Y' may not imply that $X \bigcap Y$ $b\_C_1+C_2\_eq$ $X' \bigcap Y'$

Proof: Let us consider the following example to prove the above property.

Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ and the following are the covers of U induced by C_1 and C_2.

$$U/C_1 = \{\{x_1, x_5\}, \{x_3, x_4, x_5, x_6\}, \{x_2, x_7, x_8\}\}\ and$$
$$U/C_2 = \{\{x_1, x_6\}, \{x_2, x_3, x_4\}, \{x_5, x_6\}, \{x_7, x_8\}\}$$

Table 1

Elements(x)	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$\cap\ md_{C_1}(x)$	$\{x_1, x_5\}$	$\{x_2,x_7,x_8\}$	$\{x_3,x_4,x_5,x_6,\}$	$\{x_3,x_4,x_5,x_6,\}$	$\{x_5\}$	$\{x_3,x_4,x_5,x_6,\}$	$\{x_2,x_7,x_8\}$	$\{x_2, x_7,x_8\}$
$\cap\ md_{C_2}(x)$	$\{x_1, x_6\}$	$\{x_2,x_3,x_4\}$	$\{x_2,x_3,x_4\}$	$\{x_2,x_3,x_4\}$	$\{x_5, x_6\}$	$\{x_6\}$	$\{x_7, x_8\}$	$\{x_7, x_8\}$

Let $X = \{x_3, x_4, x_5, x_6\}; X' = \{x_1, x_3, x_4, x_5, x_6\};$
$Y = \{x_5, x_6, x_7, x_8\}; Y' = \{x_1, x_5, x_6, x_7, x_8\}$

$$\underline{F_{C_1+C_2}}(X) = \{x_3, x_4, x_5, x_6\}\ and\ \underline{F_{C_1+C_2}}(X') = \{x_3, x_4, x_5, x_6\}$$
$$\underline{F_{C_1+C_2}}(Y) = \{x_5, x_6, x_7, x_8\}\ and\ \underline{F_{C_1+C_2}}(Y') = \{x_5, x_6, x_7, x_8\}$$

$$X \bigcap Y = \{x_5, x_6\}\ and\ X' \bigcap Y' = \{x_1, x_5, x_6\}$$

$$\underline{F_{C_1+C_2}}(X \bigcap Y) = \{x_5, x_6\}\ and\ \underline{F_{C_1+C_2}}(X' \bigcap Y') = \{x_1, x_5, x_6\}$$

$$\underline{F_{C_1+C_2}}(X \bigcap Y) \neq \underline{F_{C_1+C_2}}(X' \bigcap Y').$$

Thus $X \bigcap Y$ *not* $b\_C_1 + C_2\_eq$ $X' \bigcap Y'$

(3.3.5) X $t\_C_1+C_2\_eq$ Y $\Rightarrow$ $X \bigcup Y^c$ $t\_C_1+C_2\_eq$ U

Proof: Given X $t\_C_1+C_2\_eq$ Y $\Rightarrow$ $\overline{F_{C_1+C_2}}(X) = \overline{F_{C_1+C_2}}(Y)$. But we know that the following holds

$$\overline{F_{C_1+C_2}}(X \bigcup Y) \supseteq \overline{F_{C_1+C_2}}(X) \bigcup \overline{F_{C_1+C_2}}(Y).$$

Thus we have from this the following

$$\overline{F_{C_1+C_2}}(X \bigcup Y^C) \supseteq \overline{F_{C_1+C_2}}(X) \bigcup \overline{F_{C_1+C_2}}(Y^C)$$
$$= \overline{F_{C_1+C_2}}(Y) \bigcup (\sim \underline{F_{C_1+C_2}}(Y)) =$$
$$\overline{F_{C_1+C_2}}(Y) \bigcup (\sim (\overline{F_{C_1+C_2}} Y - BN_{C_1+C_2} Y))$$
$$= \overline{F_{C_1+C_2}}(Y) \bigcup \sim \overline{F_{C_1+C_2}}(Y) = U \Rightarrow X \bigcup Y^C\ t\_C_1+C_2\_eq\ U.$$

(3.3.6) X $b\_C_1+C_2\_eq$ Y $\Rightarrow$ $X \bigcap Y^C$ $t\_C_1+C_2\_eq$ ϕ

Proof: Given X $b\_C_1+C_2\_eq$ Y $\Rightarrow \underline{F_{C_1+C_2}}(X)=\underline{F_{C_1+C_2}}(Y)$. But we know that the following holds

$$\underline{F_{C_1+C_2}}(X \bigcap Y) \subseteq \underline{F_{C_1+C_2}}(X) \cap \underline{F_{C_1+C_2}}(Y).$$

Thus we have from this the following

$$\underline{F_{C_1+C_2}}(X \bigcap Y^C) \subseteq \underline{F_{C_1+C_2}}(X) \cap \underline{F_{C_1+C_2}}(Y^C)$$

$$= \underline{F_{C_1+C_2}}(X) \cap (\overline{F_{C_1+C_2}}(Y))^C$$

$$= \underline{F_{C_1+C_2}}(X) \bigcap (U - \underline{F_{C_1+C_2}}(Y) - BN_{C_1+C_2}(Y))$$

$$\subseteq \underline{F_{C_1+C_2}}(X) \bigcap (U - \underline{F_{C_1+C_2}}(Y)) \subseteq \phi \bigcap U = \phi.$$

$$\Rightarrow \; X \bigcap Y^C \; t\_C_1+C_2\_eq \; \phi$$

(3.3.7)If $X \subseteq Y$ and Y $t\_C_1+C_2\_eq$ ϕ then X $t\_C_1+C_2\_eq$ ϕ

Proof: Given $X \subseteq Y$ and Y $t\_C_1+C_2\_eq$ ϕ . So we have $\overline{F_{C_1+C_2}}(Y) = \phi$. As $X \subseteq Y \Rightarrow X = \phi$ $\Rightarrow \overline{F_{C_1+C_2}}(X) = \phi \Rightarrow$ X $t\_C_1+C_2\_eq$ ϕ .

(3.3.8) If $X \subseteq Y$ and X $t\_C_1+C_2\_eq$ U then Y $t\_C_1+C_2\_eq$ U

Proof: Given $X \subseteq Y$ and X $t\_C_1+C_2\_eq$ U. So, we have $\overline{F_{C_1+C_2}}(X) = U$ and hence as $X \subseteq Y \Rightarrow \overline{F_{C_1+C_2}}(X) \subseteq \overline{F_{C_1+C_2}}(Y)$, we get $U \subseteq \overline{F_{C_1+C_2}}(Y)$ So, $\overline{F_{C_1+C_2}}(Y) = U$. Hence, Y $t\_C_1+C_2\_eq$ U.

(3.3.9) X $t\_C_1+C_2\_eq$ Y iff ~X $b\_C_1+C_2\_eq$ ~Y

Proof: X $t\_C_1+C_2\_eq$ Y $\Rightarrow \overline{F_{C_1+C_2}}(X) = \overline{F_{C_1+C_2}}(Y)$

But we know that

$$\overline{F_{C_1+C_2}}(X) = (\underline{F_{C_1+C_2}}(X^C))^C \Leftrightarrow (\underline{F_{C_1+C_2}}(X^C))^C$$

$$= (\underline{F_{C_1+C_2}}(Y^C))^C \Leftrightarrow \underline{F_{C_1+C_2}}(X^C) = \underline{F_{C_1+C_2}}(Y^C)$$

$$\Leftrightarrow \text{~X } b\_C_1+C_2\_eq \text{ ~Y}$$

(3.3.10) If X $b\_C_1+C_2\_eq$ ϕ or Y $b\_C_1+C_2\_eq$ ϕ then $X \bigcap Y$ $b\_C_1+C_2\_eq$ ϕ

Proof:

Given X $b\_C_1+C_2\_eq$ ϕ or Y $b\_C_1+C_2\_eq$ ϕ $\Rightarrow$ $\underline{F_{C_1+C_2}}(X) = \phi$ or $\underline{F_{C_1+C_2}}(Y) = \phi$ $\Rightarrow \underline{F_{C_1+C_2}}(X) \cap \underline{F_{C_1+C_2}}(Y) = \phi$. But we know that the following holds

$$\underline{F_{C_1+C_2}}(X \cap Y) \subseteq \underline{F_{C_1+C_2}}(X) \cap \underline{F_{C_1+C_2}}(Y) \Rightarrow$$

$$\underline{F_{C_1+C_2}}(X \cap Y) = \phi \Rightarrow X \bigcap Y \; b\_C_1+C_2\_eq \; \phi.$$

(3.3.11) If X $t\_C_1+C_2\_eq$ U or Y $t\_C_1+C_2\_eq$ U then $X \bigcup Y$ $t\_C_1+C_2\_eq$ U

Proof:

Given X $t\_C_1+C_2\_eq$ U or Y $t\_C_1+C_2\_eq$ U $\Rightarrow \overline{F_{C_1+C_2}}(X) = U$ or $\overline{F_{C_1+C_2}}(Y) = U$ $\Rightarrow \overline{F_{C_1+C_2}}(X) \cup \overline{F_{C_1+C_2}}(Y) = U$. But we know that the following holds

$$\overline{F_{C_1+C_2}}(X \cup Y) \supseteq \overline{F_{C_1+C_2}}(X) \cup \overline{F_{C_1+C_2}}(Y)$$

$$\overline{F_{C_1+C_2}}(X \cup Y) = U \Rightarrow X \bigcup Y \; t\_C_1+C_2\_eq \; U.$$

D. Replacement Properties of Optimistic First Type of Covering Based Multi Granular Approximate Equalities

These properties are also called as interchange properties. We have stated above the observation of Novotny and Pawlak in connection with holding of the properties for rough equalities when the bottom and top equalities are interchanged. They categorically told that the properties do not hold under this change. However, it is shown by Tripathy et al that some of these properties hold under the interchange where as some other hold with some additional conditions which are sufficient but not necessary. They are stated as below along with proofs. We use a real life example as detailed below, which shall be used to illustrate the properties as well as provide counter examples whenever necessary.

E. A Real Life Example

Let us consider that a committee for the school of computing science and engineering (SCSE) to be constituted to carry out continuous assessment test activities such as collecting the question paper bundles and distributing the answer bundles. Assume that there are 8 faculties available for the purpose. Their collection and distribution experiences in years along with their sex are considered. This information is tabulated in the following table.

Table 2.

S.No.	Faculty Name	Collection. Experience (yrs)	Distribution Experience (yrs)	Sex
1	Allen –x_1	1	2	Male
2	Brinda-x_2	2	1	Female
3	Celina-x_3	4	4	Male
4	Danya-x_4	2	3	Female
5	Ershad-x_5	1	2	Male
6	Feroz-x_6	3	2	Male
7	Geeta-x_7	2	3	Male
8	Harsha-x_8	1	2	Male

C_1=Cover obtained based on average collection experience of whole group as 2 years and having male faculty more than or equal to female faculty

$$U/C_1 = \{\{x_1, x_2, x_3\}, \{x_3, x_4\}, \{x_3, x_8\}, \{x_4, x_5\}, \{x_6, x_7\}\}$$

C_2=Cover obtained based on average distribution experience of whole group as 2 years and having male faculty more than or equal to female faculty

$$U/C_2 = \{\{x_1, x_2, x_3\}, \{x_3, x_4\}, \{x_3, x_5\}, \{x_4, x_5\}, \{x_4, x_8\}, \{x_6, x_7\}\}$$

Consider subsets $X, Y \subseteq U$. Then the lower approximation of any set can be interpreted as a group of people who are certainly part of the committee and the upper approximation of any set can be interpreted as a group of people who are either certainly or possibly be part of the committee.

Two sets X and Y are said to be optimistic bottom equivalent to each other with respect to C_1 and C_2 if their lower approximations with respect to C_1+C_2 are the same. That is the set of faculties who are certainly in X with respect to C_1 or with respect to C_2 is same as the set of faculties who are certainly in Y with respect to C_1 or with respect to C_2.

Two sets X and Y are said to be optimistic top equivalent to each other with respect to C_1 and C_2 if their upper approximations with respect to C_1+C_2 are the same. That is the set of faculties who are certainly or possibly be in X with respect to C_1 and with respect to C_2 is same as the set of faculties who are certainly or possibly be in Y with respect to C_1 and with respect to C_2.

Let us consider first type of CBMGRS. Its lower and upper approximations are determined based on minimal descriptors.

The minimal descriptor table for the two covers for the above example is as shown below.

Table 3.

Elements → / Minimum Descriptors ↓	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$\cap\ md_{C_1}(x)$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2, x_3\}$	$\{x_3\}$	$\{x_4\}$	$\{x_4, x_5\}$	$\{x_6, x_7\}$	$\{x_6, x_7\}$	$\{x_3, x_8\}$
$\cap\ md_{C_2}(x)$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2, x_3\}$	$\{x_3\}$	$\{x_4\}$	$\{x_5\}$	$\{x_6, x_7\}$	$\{x_6, x_7\}$	$\{x_3, x_8\}$

(3.4.1) X $t\_C_1+C_2\_eq$ Y if $X \cap Y\ t\_C_1+C_2\_eq$ X and Y both. Converse need not be true.

Proof:

$$X \cap Y\ t\_C_1+C_2\_eq\ X \Rightarrow \overline{F_{C_1+C_2}}(X \cap Y) = \overline{F_{C_1+C_2}}(X)$$

$$X \cap Y\ t\_C_1+C_2\_eq\ Y \Rightarrow \overline{F_{C_1+C_2}}(X \cap Y) = \overline{F_{C_1+C_2}}(Y)$$

From the above two expressions we have, $\overline{F_{C_1+C_2}}(X) = \overline{F_{C_1+C_2}}(Y) \Rightarrow$ $X_{t_}C_1+C_2_eq$Y The following example shows that converse need not be true.

$$Let X = \{x_3, x_6\}, Y = \{x_3, x_7\}\ and$$
$$\overline{F_{C_1+C_2}}(X) = \{x_1, x_2, x_3, x_6, x_7\}, \overline{F_{C_1+C_2}}(Y) = \{x_1, x_2, x_3, x_6, x_7\}$$
$$\overline{F_{C_1+C_2}}(X \cap Y) = \{x_1, x_2, x_3\}$$
$$\overline{F_{C_1+C_2}}(X) = \{x_1, x_2, x_3, x_6, x_7\}, \overline{F_{C_1+C_2}}(Y) = \{x_1, x_2, x_3, x_6, x_7\}$$
$$\overline{F_{C_1+C_2}}(X \cap Y) = \{x_1, x_2, x_3\}$$
$$Thus\ \overline{F_{C_1+C_2}}(X \cap Y) \neq \overline{F_{C_1+C_2}}(X)\ and\ \overline{F_{C_1+C_2}}(X \cap Y) \neq \overline{F_{C_1+C_2}}(Y)$$
$$though\ \overline{F_{C_1+C_2}}(X) = \overline{F_{C_1+C_2}}(Y)$$

The converse part can be interpreted as, though the sets of faculty certainly or possibly be in committee with respect to C_1+C_2 are the same for X and Y, but the set of faculty certainly or possibly be in committee for $X \cap Y$ with respect to C_1+C_2 is not same as that of X and Y. It means that a committee obtained through common people from sets X and Y having same group of people who are either certainly or possibly be in the committee , may not be the same as the committee obtained from X and Y having same group of people who are either certainly or possibly be in the committee.

(3.4.2) X $b\_C_1+C_2\_eq$ Y if $X \cup Y\ b\_C_1+C_2\_eq$ X and Y both. Converse need not be true

Proof:

$$Given\ X \cup Y\ b\_C_1+C_2\_eq\ X \Rightarrow \underline{F_{C_1+C_2}}(X \cup Y) = \underline{F_{C_1+C_2}}(X)$$

$$Given\ X \cup Y\ b\_C_1+C_2\_eq\ X \Rightarrow \underline{F_{C_1+C_2}}(X \cup Y) = \underline{F_{C_1+C_2}}(X)$$

And

$$Given\ X \cup Y\ b\_C_1+C_2\_eq\ Y \Rightarrow \underline{F_{C_1+C_2}}(X \cup Y) = \underline{F_{C_1+C_2}}(Y)$$

From the above two expressions we have

$$\underline{F_{C_1+C_2}}(X) = \underline{F_{C_1+C_2}}(Y) \Rightarrow \text{ X } b\_C_1+C_2\_eq \text{ Y.}$$

The following example shows that converse need not be true.

$$\text{Let} X = \{x_3, x_6\}, Y = \{x_3, x_7\}\ and X \cup Y \{x_3, x_6, x_7\}$$

$\underline{F}_{C_1+C_2}(X)=\{x_3\}$ *and* $\underline{F}_{C_1+C_2}(Y)=\{x_3\}$ $\underline{F}_{C_1+C_2}(X\cup Y)=\{x_3,x_6,x_7\}$

Thus $\underline{F}_{C_1+C_2}(X\cup Y)\neq \underline{F}_{C_1+C_2}(X)$ *and* $\underline{F}_{C_1+C_2}(X\cup Y) \neq \underline{F}_{C_1+C_2}(Y)$

though $\underline{F}_{C_1+C_2}(X)=\underline{F}_{C_1+C_2}(Y)$

The converse part can be interpreted as, though the sets of faculty certainly in committee with respect to C_1+C_2 are the same for X and Y, but the set of faculty certainly in committee for $X\bigcup Y$ with respect to C_1+C_2 is not the same as that of X and Y It means that a committee obtained through all the people from sets X and Y having same group of people who are certainly in the committee , may not be the same as the committee obtained from X and Y having same group of people who are certainly in the committee.

(3.4.3)X $t\_C_1+C_2\_eq$ X' and Y $t\_C_1+C_2\_eq$ Y' may not imply that $X\bigcap Y$ $t\_C_1+C_2\_eq$ $X'\bigcap Y'$

Proof: The following example emphasizes the above proof.

Let X $=\{x_1\}$, X' $=\{x_1,\ x_6\}$, Y $=\{x_4\}$ and Y' $=\{x_4,\ x_6\}$.

$\overline{F}_{C_1+C_2}(X)=\{x_1,x_2,x_3\}$ *and* $\overline{F}_{C_1+C_2}(X')=\{x_1,x_2,x_3\}$

$\overline{F}_{C_1+C_2}(Y)=\{x_1,x_2,x_3,x_6,x_7\}$ *and* $\overline{F}_{C_1+C_2}(Y')=\{x_1,x_2,x_3,x_6,x_7\}$

$\Rightarrow \overline{F}_{C_1+C_2}(X)=\overline{F}_{C_1+C_2}(X')$ *and* $\overline{F}_{C_1+C_2}(Y)=\overline{F}_{C_1+C_2}(Y')$

$X\cap Y=\phi \Rightarrow \overline{F}_{C_1+C_2}(X\cap Y)=\phi$

$X'\cap Y'=\{x_2\}\Rightarrow \overline{F}_{C_1+C_2}(X'\cap Y')=\{x_1,x_2,x_3\}$

Thus $\overline{F}_{C_1+C_2}(X\cap Y)\neq \overline{F}_{C_1+C_2}(X'\cap Y')$

$\Rightarrow X\cap Y$ *not* $t\_C_1+C_2\_eq$ $X'\cap Y'$.

(3.4.4) X t_ C_1+C_2 _eq X' and Y t_ C_1+C_2 _eq Y' may not imply that $X\bigcap Y$ t_ C_1+C_2 _eq $X'\bigcap Y'$.

Proof: It can be seen by constructing a suitable example that the conclusion is true.

(3.4.5) X $b\_C_1+C_2\_eq$ Y => $X\bigcup Y^C$ $b\_C_1+C_2\_eq$ U

Proof: Given X $b\_C_1+C_2\_eq$ Y

=> $\underline{F}_{C_1+C_2}(X)=\underline{F}_{C_1+C_2}(Y)$. But from (2.6.5) we have $\underline{F}_{C_1+C_2}(X\cup Y)\supseteq\underline{F}_{C_1+C_2}(X)\bigcup\underline{F}_{C_1+C_2}(Y)$. Thus we have from this the following

$$\underline{F}_{C_1+C_2}(X\cup Y^C)\supseteq\underline{F}_{C_1+C_2}(X)\bigcup\underline{F}_{C_1+C_2}(Y^C)=$$

$$\underline{F}_{C_1+C_2}(Y)\bigcup(\overline{F}_{C_1+C_2}(Y))^C$$

$$=\ \underline{F}_{C_1+C_2}(Y)\bigcup(U-(\underline{F}_{C_1+C_2}(Y)-BN_{C_1+C_2}Y))$$

$$\subseteq \underline{F}_{C_1+C_2}(Y)\bigcup(U-\underline{F}_{C_1+C_2}(Y))=U \ => X\bigcup Y'\ b\_C_1+C_2\_eq\ U$$

(3.4.6) X $t\_C_1+C_2\_eq$ Y => $X\bigcap Y'$ $b\_C_1+C_2\_eq$ ϕ

Proof: Given X $t\_C_1+C_2\_eq$ Y

$\Rightarrow$ $\overline{F}_{C_1+C_2}(X)=\overline{F}_{C_1+C_2}(Y)$. But we know that the following holds

$$\overline{F}_{C_1+C_2}(X\bigcap Y)\subseteq \overline{F}_{C_1+C_2}(X)\bigcap \overline{F}_{C_1+C_2}(Y).$$

Thus we have from this the following

$$\overline{F}_{C_1+C_2}(X\bigcap Y')\subseteq\overline{F}_{C_1+C_2}(X)\bigcap \overline{F}_{C_1+C_2}(Y')=$$

$$\overline{F}_{C_1+C_2}(X)\bigcap(\underline{F}_{C_1+C_2}(Y))'=$$

$$\overline{F}_{C_1+C_2}(X)\bigcap(U-\underline{F}_{C_1+C_2}(Y))\subseteq BN_{C_1+C_2}(Y)$$

$$\Rightarrow X\bigcap Y' \text{ not } t\_C_1+C_2\_eq\ \phi.$$

(3.4.7) If $X\subseteq Y$ and Y $b\_C_1+C_2\_eq$ ϕ then X $b\_C_1+C_2\_eq$ ϕ

Proof: Given $X\subseteq Y$ and Y $b\_C_1+C_2\_eq$ ϕ . So we have $\underline{F}_{C_1+C_2}(Y)=\phi$ .As $X\subseteq Y \Rightarrow X=\phi \Rightarrow$ $\underline{F}_{C_1+C_2}(X)=\phi$

$\Rightarrow$ X $b\_C_1+C_2\_eq$ ϕ

(3.4.8) If $X\subseteq Y$ and X $b\_C_1+C_2\_eq$ U then Y $b\_C_1+C_2\_eq$ U

Proof: Given $X\subseteq Y$ and Y $b\_C_1+C_2\_eq$ U. So $\underline{F}_{C_1+C_2}(X)=U$ .As $U=X\subseteq Y \Rightarrow Y=U \Rightarrow$ $\underline{F}_{C_1+C_2}(Y)=U$

$\Rightarrow$ Y $b\_C_1+C_2\_eq$ U.

(3.4.9) X $b\_C_1+C_2\_eq$ Y iff X^C $t\_C_1+C_2\_eq$ Y^C

Proof: Given X $b\_C_1+C_2\_eq$ Y

But we know that $\underline{F}_{C_1+C_2}(X)=(\overline{F}_{C_1+C_2}(X^C))^C$

$\Rightarrow(\overline{F}_{C_1+C_2}(X^C))^C=(\overline{F}_{C_1+C_2}(Y^C))^C$

$\Rightarrow \overline{F}_{C_1+C_2}(X^C)=\overline{F}_{C_1+C_2}(Y^C)$

$\underline{F}_{C_1+C_2}(X)=\underline{F}_{C_1+C_2}(Y) \ \Rightarrow\ X^C$ $t\_C_1+C_2\_eq$ Y^C

In a similar way converse can also be proved.

(3.4.10) If X $t\_C_1+C_2\_eq$ ϕ or Y $t\_C_1+C_2\_eq$ ϕ then $X\bigcap Y$ $t\_C_1+C_2\_eq$ ϕ

Proof: Given X $t\_C_1+C_2\_eq$ ϕ or Y $t\_C_1+C_2\_eq$ ϕ

$\Rightarrow$ $\overline{F}_{C_1+C_2}(X)=\phi$ *or* $\overline{F}_{C_1+C_2}(Y)=\phi$

$\Rightarrow$ $\overline{F}_{C_1+C_2}(X)\bigcap\overline{F}_{C_1+C_2}(Y)=\phi$. But from (2.6.8)

$\overline{F}_{C_1+C_2}(X\cap Y)\subseteq \overline{F}_{C_1+C_2}(X)\bigcap\overline{F}_{C_1+C_2}(Y)$

$\Rightarrow \overline{F}_{C_1+C_2}(X\cap Y)=\phi \ \Rightarrow\ X\bigcap Y$ $t\_C_1+C_2\_eq$ ϕ

(3.4.11) If X $b\_C_1+C_2\_eq$ U or Y $b\_C_1+C_2\_eq$ U then $X\bigcup Y$ $b\_C_1+C_2\_eq$ U

Proof: Given X $t\_C_1+C_2\_eq$ U or Y $t\_C_1+C_2\_eq$ U

$\Rightarrow$ $\underline{F}_{C_1+C_2}(X)=U$ *or* $\underline{F}_{C_1+C_2}(Y)=U$

$\Rightarrow$ $\underline{F}_{C_1+C_2}(X)\bigcup\underline{F}_{C_1+C_2}(Y)=U$. But from (2.6.7)

$$\underline{F_{C_1+C_2}}(X \cup Y) \supseteq \underline{F_{C_1+C_2}}(X) \cup \underline{F_{C_1+C_2}}(Y)$$
$$\Rightarrow \underline{F_{C_1+C_2}}(X \cup Y) = U \Rightarrow X \cup Y \; b\_C_1 + C_2\_eq \, \mathrm{U}.$$

IV. CONCLUSIONS

The basic Rough set introduced by Pawlak is unigranular in character from the granular computing point of view. The granularity introduced by attributes through equivalence relations over information systems is mostly multigranular in character. So, in order to handle such situations two types of multigranular approximations were introduced by Qian et al in 2006 1nd 2010. However, the assumption that attributes generate equivalence relations restricts their applicability in real life situations as the relations induced by attributes may not be equivalence relations. So, the covering based rough sets were introduced and following this the covering based Multigranular rough sets were introduced by Lin et al and Liu et al in 2011. Approximate equalities introduced by Novotny and Pawlak in 1985 was motivated to extend the strict notion of equality of sets used in Mathematics and also to make use of user knowledge in deciding the equalities of two sets which are termed as approximate equalities. There are several properties of these approximate equalities. Also, a related set of properties called the replacement properties have also been established. The study of these properties for covering based optimistic multigranular rough sets was done very recently by Tripathy et al. In this paper, we studied the properties of covering based optimistic multigranular equalities and also used examples from real life to illustrate their applicability and prove some negative properties.

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