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CP-Conserving Contribution in the Decays $K_L \to \pi^\circ \gamma\gamma$ and $K_L \to \pi^\circ e^+ e^-$

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The *CP*-conserving contribution to the decay $K_L \to \pi^\circ e^+ e^-$ is reexamined. The general form of the decay rates for $K_L \to \pi^\circ \gamma\gamma$ and $K_L \to \pi^\circ \gamma\gamma \to \pi^\circ e^+ e^-$ (the absorptive part) is obtained. Using the vector meson dominance (VMD) model, the branching ratios for $K_L \to \pi^\circ \gamma\gamma$ and $K_L \to \pi^\circ \gamma\gamma \to \pi^\circ e^+ e^-$ (the absorptive part) are shown to be $Br[K_L \to \pi^\circ e^+ e^-]|_{2\gamma} \geq 1.7\times 10^{-11}$, $Br[K_L \to \pi^\circ \gamma\gamma] = 1.4\times 10^{-6}$. Our result suggests that the *CP*-conserving contribution with VMD model may be larger than the *CP*-violating one in the standard model.

§ 1. Introduction

As is well known, the decay $K_L \to \pi^\circ e^+ e^-$ via one photon exchange is a *CP*-violating process.[1] Furthermore in this mode, the standard Kobayashi-Maskawa model yields a contribution from the *direct CP* violation which is larger than that from *state mixing*.[2] Hence if the *CP*-conserving piece is very small compared with the *CP*-violating signal, the direct *CP*-violating amplitude is detectable. In a recent paper[3] Donoghue et al. in the KM model predict the branching ratio

$$\frac{\Gamma[K_L \to \pi^\circ e^+ e^-]|_{1\gamma}}{\Gamma[K_L \to \mathrm{all}]} = 3.7\times 10^{-12}\,. \tag{1}$$

This necessitates considering the *CP*-conserving piece which proceeds via two photon exchange.

In Ref. 3), the authors suggest that the *CP*-conserving amplitude is small compared to the *CP*-violating one, because the *CP*-conserving pieces are regarded as being accompanied by the chirality mismatch of the electron and positron and suppressed by the factor m_e (electron mass). However, in this evaluation the soft pion limit is assumed, which does not reflect the general features of the three body decay. Indeed, the chirality conserving part of the decay amplitude vanishes in this limit, i.e.,

$$\lim_{p_\nu \to 0} \bar{u}(s_2)\gamma_\mu v(s_1) k^\mu = \lim_{p_\nu \to 0} \bar{u}\gamma_\mu v(s_1{}^\mu + s_2{}^\mu + p^\mu) = 0\,, \tag{2}$$

where k^μ, p^μ, $s_1{}^\mu$ and $s_2{}^\mu$ are the four-momenta of K_L, π°, e^+ and e^-, respectively. If the pion momentum is not set to zero, the helicity suppression does not necessarily occur, and the chirality conserving form $k^\mu \bar{u}\gamma_\mu v$ may appear.

In this paper, after reexamining the chirality conserving part in the *CP*-conserving amplitude, we shall demonstrate that the amplitude without suppression does actually occur in the calculation based on the vector meson dominance (VMD)

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model. Furthermore, the decay ratios for $K_L \to \pi^\circ \gamma\gamma$ and $K_L \to \pi^\circ \gamma\gamma \to \pi^\circ e^+ e^-$ (the absorptive part) are calculated. Recently there has been an evaluation[4] of these ratios in chiral perturbation theory, in which there is an unknown coupling constant which is not determined properly. Therefore, it is worth estimating these ratios with the VMD model and comparing the results with those of the chiral perturbation.

This paper is organized as follows: In § 2, the general form of the CP-conserving amplitudes is discussed and the simple formulae for the decay widths of $K_L \to \pi^\circ \gamma\gamma$ and $K_L \to \pi^\circ \gamma\gamma \to \pi^\circ e^+ e^-$ (the absorptive part) are obtained. In § 3, the model calculations for the decay $K_L \to \pi^\circ \gamma\gamma$, $K_L \to \pi^\circ \gamma\gamma \to \pi^\circ e^+ e^-$ are performed, while § 4 is devoted to the conclusion and discussion.

§ 2. The general form of the *CP*-conserving amplitude

The chirality conserving form of the CP-conserving amplitude in the decay $K_2 \to \pi^\circ e^+ e^-$ and its characteristic distribution in the Dalitz plot are first discussed in Ref. 3). In this section, for completeness we first reexamine the CP-conserving amplitude of the decay $K_2 \to \pi^\circ e^+ e^-$. The chirality conserving form of the decay amplitude should be of the form

$$A = \bar{u}(s_2, \sigma_2)\gamma_\mu v(s_1, \sigma_1) k^\mu f(k\cdot s_1, k\cdot s_2)\,, \tag{3}$$

where f is a function of the two independent Lorentz invariants of the external momenta, and σ_1 and σ_2 are the helicity states of e^+ and e^-, respectively. Under the CP transformation, Eq. (3) becomes

$$A^{CP} = \bar{u}(\bar{s}_1, \sigma_1)\gamma_\mu v(\bar{s}_2, \sigma_2)\bar{k}^\mu f(k\cdot s_2, k\cdot s_1)\,, \tag{4}$$

where $\bar{k}^\mu = k_\mu$, etc. In Eq. (4) there is no extra sign change, because the internal CP of K_2 is the same as that of π°. In the center of mass frame of e^+e^-, the time component of $\bar{u}\gamma_\mu v$ is actually zero, because $(s_1+s_2)^\mu \bar{u}\gamma_\mu v = 0$. Moreover in the massless limit of the electron, the chirality conserving nature leads to the following relation in the same frame,

$$\bar{u}(s_2, \sigma_2)\gamma_\mu v(s_1, \sigma_1) = \bar{u}(\bar{s}_1, \sigma_1)\gamma_\mu v(\bar{s}_2, \sigma_2)\,. \tag{5}$$

Therefore, if CP invariance is assumed, in Eq. (3), f must be odd with respect to its two arguments

$$f(k\cdot s_1, k\cdot s_2) = -f(k\cdot s_2, k\cdot s_1)\,. \tag{6}$$

As discussed previously, a CP-conserving contribution to the decay $K_L \to \pi^\circ e^+ e^-$ can arise from a two photon intermediate state. Then in two-photon-exchange, the following quantity is important:

$$T^{\mu\nu}(q_1, q_2, k) = \int e^{iq_1\cdot x_1} e^{iq_2\cdot x_2} \langle \pi_p | T[j^\mu_{em}(x_1) j^\nu_{em}(x_2) H_W(0)] | K_{2k} \rangle d^4x_1 d^4x_2\,, \tag{7}$$

where j^μ_{em} is the hadronic part of the electromagnetic current and H_W is the effective Hamiltonian of the weak interaction. If we assume CP invariance of H_W, $T^{\mu\nu}$ must satisfy the relation,

$$T^{\mu\nu}(q_1, q_2, k) = T_{\mu\nu}(\bar{q}_1, \bar{q}_2, \bar{k}). \tag{8}$$

Further, taking into account Bose symmetry and gauge invariance, we can express $T^{\mu\nu}$ in terms of the four independent form factors. (See also the Appendix in Ref. 4) for the amplitude of the on-mass-shell photons.)

$$\begin{aligned} T^{\mu\nu}(q_1, q_2, k) &= A(k\cdot q_1, k\cdot q_2)(q_{2\mu}q_{1\nu} - q_1\cdot q_2 g_{\mu\nu}) \\ &+ B(k\cdot q_1, k\cdot q_2)\left(\frac{k\cdot q_1 k\cdot q_2}{q_1\cdot q_2}g_{\mu\nu} + k_\mu k_\nu - \frac{k\cdot q_1}{q_1\cdot q_2}q_{2\mu}k_\nu - \frac{k\cdot q_2}{q_1\cdot q_2}q_{1\nu}k_\mu\right) \\ &+ C(k\cdot q_1, k\cdot q_2)\left(\frac{q_1^2 q_2^2}{q_1\cdot q_2}g_{\mu\nu} - \frac{q_2^2}{q_1\cdot q_2}q_{1\mu}q_{1\nu} - \frac{q_1^2}{q_1\cdot q_2}q_{2\mu}q_{2\nu} + q_{1\nu}q_{2\nu}\right) \\ &+ D(k\cdot q_1, k\cdot q_2)\left(q_1^2\frac{k\cdot q_2}{q_1\cdot q_2}g_{\mu\nu} - \frac{k\cdot q_2}{q_1\cdot q_2}q_{1\mu}q_{1\nu} + q_{1\mu}k_\nu - \frac{q_1^2}{q_1\cdot q_2}q_{2\mu}k_\nu\right) \\ &+ D(k\cdot q_2, k\cdot q_1)\left(q_2^2\frac{k\cdot q_1}{q_1\cdot q_2}g_{\mu\nu} - \frac{k\cdot q_1}{q_1\cdot q_2}q_{2\mu}q_{2\nu} + q_{2\nu}k_\mu - \frac{q_2^2}{q_1\cdot q_2}q_{2\nu}k_\mu\right), \end{aligned} \tag{9}$$

where $A(x, y)$, $B(x, y)$ and $C(x, y)$ satisfy the relations,

$$A(x, y) = A(y, x), \quad B(x, y) = B(y, x), \quad C(x, y) = C(y, x). \tag{10}$$

The decay amplitude of $K_2 \to \pi^\circ\gamma\gamma$ is obtained from Eq. (9):

$$A(K_2 \to \pi^\circ\gamma\gamma) = \epsilon_{1\mu}^* \epsilon_{2\nu}^* T^{\mu\nu}(q_1, q_2, k). \tag{11}$$

Then the decay width is

$$\Gamma[K_L \to \pi^\circ\gamma\gamma] = \int dt_1 dt_2 \frac{1}{256\,\pi^3}\frac{1}{m_K^3}\frac{1}{2}\left[(A_s - Bm_K^2)^2 + \left(\frac{B}{s}\right)^2(m_K^2 m_\pi^2 - t_1 t_2)^2\right] \times \frac{1}{2}, \tag{12}$$

where $s = (k-p)^2 = (q_1+q_2)^2$, $t_1 = (k-q_1)^2$, $t_2 = (k-q_2)^2$. Futhermore the amplitude for $K_2 \to \pi^\circ e^+e^-$ via two photons is obtained by contracting the hadronic tensor $T^{\mu\nu}$ in Eq. (9) with the e^+e^- conversion amplitude:

$$\begin{aligned} A(K_2 \to \pi^\circ e^+e^-)|_{2\gamma} &= e^2 \int \frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}(2\pi)^4\delta(q_1+q_2-s_1-s_2)\frac{1}{q_1^2+i\epsilon}\frac{1}{q_2^2+i\epsilon} \\ &\times\left[\bar{u}(s_2)\gamma_\nu\frac{1}{\not{q}_2 - \not{s}_1 - m_e}\gamma_\mu v(s_1) + \bar{u}(s_2)\gamma_\mu\frac{1}{\not{s}_2 - \not{q}_2 - m_e}\gamma_\nu v(s_1)\right] \\ &\times\frac{1}{2}T^{\mu\nu}(q_1, q_2, k). \end{aligned} \tag{13}$$

We can use Eq. (13) to yield the absorptive part of $K_L \to \pi^\circ\gamma\gamma \to \pi^\circ e^+e^-$ amplitude with the Cutkosky rule:

$$\text{Abs. } A(K_L\to\pi^\circ\gamma\gamma\to\pi^\circ e^+e^-)$$
$$=\frac{-\pi^2e^2}{4}\int\frac{d^4q}{(2\pi)^4}\delta(s-2q\cdot(s_1+s_2))\delta(q^2)\theta(q^0)\theta((s_1+s_2)^0-q^0)B$$
$$\times\Big[\Big(\frac{1}{q\cdot s_2}+\frac{1}{q\cdot s_1}\Big)\frac{t_1-t_2}{s}(-4k\cdot q+s+m_K{}^2-m_\pi{}^2)\bar{u}\not{q}v+\Big(\frac{1}{q\cdot s_2}-\frac{1}{q\cdot s_1}\Big)$$
$$\times\frac{1}{s}\{8(k\cdot q)^2-4k\cdot q(s+m_K{}^2-m_\pi{}^2)+(m_K{}^2-m_\pi{}^2)^2+s(s-2m_\pi{}^2)\}\bar{u}\not{q}v$$
$$-\Big(\frac{1}{q\cdot s_2}+\frac{1}{q\cdot s_1}\Big)(t_1-t_2)\bar{u}\not{k}v+\Big(\frac{1}{q\cdot s_2}-\frac{1}{q\cdot s_1}\Big)\{4k\cdot q-(s+m_K{}^2-m_\pi{}^2)\}\bar{u}\not{k}v\Big], \quad (14)$$

where the electron mass has been neglected. Thus, only the term with the form factor B gives a non-vanishing contribution. Note that Eq. (14) shows the general feature of the CP-conserving amplitude, i.e., the r.h.s. is actually an odd function of (t_1-t_2). (See also Eq. (6)). From Eq. (14), the lower bound for $K_L\to\pi^\circ e^+e^-$ via two photons is obtained,

$$\Gamma[K_L\to\pi^\circ e^+e^-]|_{2\gamma}$$
$$\geq\frac{1}{m_K{}^3}\Big(\frac{\alpha}{16}\Big)^2\frac{1}{256\,\pi^3}\int ds\int_{-1}^{1}d\rho\lambda(m_K{}^2,m_\pi{}^2,s)^{5/2}[J(\rho)-J(-\rho)]^2\frac{(1-\rho^2)}{s^2}, \quad (15)$$

where

$$\lambda(a,b,c)=a^2+b^2+c^2-2ab-2bc-2ca,$$
$$J(\rho)=\int_\rho^1 dxB(x_+,x_-)\Big[\frac{1}{1-\rho}\Big(\frac{x^2+1}{2}-\rho x\Big)-1\Big], \quad (16)$$
$$\rho=\frac{t_2-t_1}{\sqrt{\lambda(m_K{}^2,m_\pi{}^2,s)}},$$
$$x_\pm=\frac{m_K{}^2-m_\pi{}^2+s\pm\sqrt{\lambda(m_K{}^2,m_\pi{}^2,s)x}}{4}.$$

In the above expressions, $\arccos(\rho)$ stands for the angle between K_L and the electron in the cm frame of e^+ and e^-, and $\arccos(x)$ for the angle between K_L and the photon in the same frame. The function $J(\rho)$ is free from a singularity at $\rho=1$, which means the collinear singularities are absent in this process as claimed by the authors of Ref. 6). Equations (12) and (15) are the general expressions. They are written on the assumption that the form factors have the following simple kinematical dependence,

$$B(x,y)=F\times\frac{s}{2m_k{}^2}, \quad (17)$$

where F is a constant. Now using the form factor, we calculate Eq. (15),

$$\Gamma[K_L\to\pi^\circ e^+e^-]_{2\gamma}\geq\Big(\frac{F}{2m_K{}^2}\Big)^2\Big(\frac{\alpha}{16}\Big)^2\frac{16}{9}\int ds\int_{-1}^{1}d\rho\frac{\lambda(m_K{}^2,m\pi^2,s^2)^{5/2}}{256\,\pi^3m_K{}^3}(1-\rho^2)\rho^2. \quad (18)$$

Equation (18) coincides with the result of Ref. 6). We shall now proceed in § 3 to examining the *CP*-conserving contribution to the decay $K_2 \to \pi^\circ e^+ e^-$ in a VMD model without the approximation to momentum dependence of the form factor.

§ 3. A vector meson dominance model

The model we use to calculate the form factors (Eq. (9)) is a vector meson dominance model, which has successfully predicted the radiative decays of the neutral pseudoscalar mesons (P) and vector mesons (V). We assume in the calculation that the decay $K_2 \to \pi^\circ \gamma\gamma$ is dominated by the weak transition $K_2 \to P$ followed by the radiative decays, $P \to V\gamma$ and $V \to \pi^\circ \gamma$ (cf. Fig. 1). Other processes in which the weak transition takes place after one or two photons are emitted are neglected, because the mass of the vector mesons, (K^*, ϕ), which join them are large and their contribution is small compared with that of ρ and ω. The same process was considered in Refs. 5) and 6). However the explicit formulae for the differential decay rates are still lacking, then we shall derive them from the following (Eqs. (31) and (33)). First, the weak transition matrix element $\langle \pi^\circ | H_W | K_2 \rangle$ is evaluated by using the effective chiral Lagrangian in which $\Delta I = 1/2$ dominance is assumed:[7]

$$\langle \pi^\circ | H_W | K_2 \rangle = -3.71 \times 10^{-2}\,\mathrm{MeV}^2, \tag{19}$$

where for hadronic states, we use the relativistic invariant normalization. Then, the other matrix elements $\langle P | H_W | K_2 \rangle$ are related to it by the following nonet relation:

$$\langle \pi^\circ | H_W | K_2 \rangle = \sqrt{3} \langle \eta_8 | H_W | K_2 \rangle = \frac{-\sqrt{6}}{4} \langle \eta_1 | H_W | K_2 \rangle, \tag{20}$$

where η_8 and η_1 are the $SU(3)$-octet and -singlet pseudoscalar meson states, respectively. Further, the matrix elements between the physical states of the pseudoscalars and the kaons are easily obtained by using the following relations:

$$\eta = \eta_8 \cos\theta - \eta_1 \sin\theta,$$
$$\eta' = \eta_8 \sin\theta + \eta_1 \cos\theta, \tag{21}$$

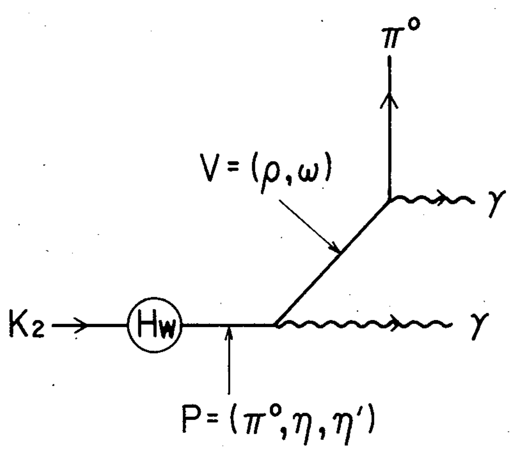

Fig. 1. The Feynman diagram of $K_L \to \pi^\circ \gamma\gamma$ in the vector meson dominance model.

where θ is the mixing angle ($\approx -20°$). Thus the weak transition amplitudes are evaluated, and what remains to be calculated is the $PV\gamma$ vertex. We take the simplest form consistent with the requirements of $SU(3)$ flavor symmetry:

$$\epsilon^{\mu\nu\rho\sigma} \mathrm{tr}[Q\lambda^a\lambda^b] \partial_\mu A_\nu \partial_\rho V_\sigma^a \Pi^b, \tag{22}$$

where V_μ^a's and Π^a's ($a = 0, \cdots, 8$) are the nonets of the vector and pseudoscalar mesons, respectively. A_μ is the photon field. The matrices λ^a ($a = 1, \cdots, 8$) are the Gell-Mann matrices and $\lambda^0 = \sqrt{2/3}\,1$.

Table I. The table of $g_{VP\gamma}/g_{\omega\pi\cdot\gamma}$.

	$\rho=\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	$\omega=\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	$\phi=s\bar{s}$
π°	1/3	1	0
η	$\frac{1}{3}(\sqrt{3}\cos\theta-\sqrt{6}\sin\theta)$	$\frac{1}{9}(\sqrt{3}\cos\theta-\sqrt{6}\sin\theta)$	$\frac{-2}{9}(\sqrt{6}\cos\theta+\sqrt{3}\sin\theta)$
η'	$\frac{1}{3}(\sqrt{3}\sin\theta+\sqrt{6}\cos\theta)$	$\frac{1}{9}(\sqrt{3}\sin\theta+\sqrt{6}\cos\theta)$	$\frac{2}{9}(\sqrt{3}\cos\theta-\sqrt{6}\sin\theta)$

Q is the charge matrix. The magnitude of the coupling constants is determined from $\omega\to\pi^\circ\gamma$ decay. The ratio of the coupling constants are shown in Table I where ideal mixing of the vector mesons is assumed. By calculating the amplitude of Fig. 1, we obtain the following results:

$$A(k\cdot q_1, k\cdot q_2)=-\sum_{V=\omega,\rho} G_V\left[\frac{k\cdot(k-q_2)}{(k-q_2)^2-m_V{}^2}+\frac{k\cdot(k-q_1)}{(k-q_1)^2-m_V{}^2}\right], \tag{23}$$

$$B(k\cdot q_1, k\cdot q_2)=-\sum_{V=\omega,\rho} G_V q_1\cdot q_2\left[\frac{1}{(k-q_2)^2-m_V{}^2}+\frac{1}{(k-q_1)^2-m_V{}^2}\right], \tag{24}$$

$$C(k\cdot q_1, k\cdot q_2)=0\,, \tag{25}$$

$$D(k\cdot q_1, k\cdot q_2)=\sum_{V=\omega,\rho} G_V\frac{q_1\cdot q_2}{(k-q_1)^2-m_V{}^2}, \tag{26}$$

where

$$\begin{aligned}G_\rho=g^2_{\omega\pi\cdot\gamma}\frac{\langle\pi^\circ|H_W|K_2\rangle}{m_K{}^2}\Bigg[&\frac{1}{9}\frac{1}{1-\Delta_\pi{}^2}\\&+\frac{1}{9}(\sqrt{3}\cos\theta-\sqrt{6}\sin\theta)\left(\frac{1}{\sqrt{3}}\cos\theta+\frac{4}{\sqrt{6}}\sin\theta\right)\frac{1}{1-\Delta_\eta{}^2}\\&+\frac{1}{9}(\sqrt{3}\sin\theta+\sqrt{6}\cos\theta)\left(\frac{1}{\sqrt{3}}\sin\theta-\frac{4}{\sqrt{6}}\cos\theta\right)\frac{1}{1-\Delta^2_{\eta'}}\Bigg],\end{aligned} \tag{27}$$

$$\begin{aligned}G_\omega=g^2_{\omega\pi\cdot\gamma}\frac{\langle\pi^\circ|H_W|K_2\rangle}{m_K{}^2}\Bigg[&\frac{1}{1-\Delta_\pi{}^2}+\frac{1}{9}(\sqrt{3}\cos\theta-\sqrt{6}\sin\theta)\left(\frac{1}{\sqrt{3}}\cos\theta+\frac{4}{\sqrt{6}}\sin\theta\right)\frac{1}{1-\Delta_\eta{}^2}\\&+\frac{1}{9}(\sqrt{3}\sin\theta+\sqrt{6}\cos\theta)\left(\frac{1}{\sqrt{3}}\sin\theta-\frac{4}{\sqrt{6}}\cos\theta\right)\frac{1}{1-\Delta^2_{\eta'}}\Bigg],\end{aligned} \tag{28}$$

$$\Delta_P{}^2=\left(\frac{m_P}{m_K}\right)^2. \tag{29}$$

From Eq. (9) and the form factors determined above, it is easy to see that the amplitude is vanishing in soft pion limit. The numerical values of the dimensionless coupling constants $G_V m_K{}^2$ are estimated using the value $g_{\omega\pi\cdot\gamma}=6.84\times10^{-4}\,\mathrm{MeV}^{-1}$ determined from $\omega\to\pi^\circ\gamma$ decay:[8)]

$$G_\rho m_K{}^2=-0.46\times10^{-8}\,,$$

$$G_\omega m_K{}^2=-0.21\times10^{-7}\,, \tag{30)}$$

where we assume that the mixing angle is $\theta=-20°$. With the form factors, Eqs. (23) and (24), we obtain the decay width for $K_L\to\pi°\gamma\gamma$ in the VMD model.

$$\Gamma[K_L\to\pi°\gamma\gamma]_{\rm VMD}=\int dt_1dt_2\frac{1}{256\,\pi^3}\frac{1}{m_K{}^3}\frac{1}{16}\bigg[\bigg(\sum_{V=\omega,\rho}G_V\Big(\frac{t_1}{t_1-m_V{}^2}+\frac{t_2}{t_2-m_V{}^2}\Big)\bigg)^2 s^2$$
$$+\bigg(\sum_{V=\omega,\rho}G_V\Big(\frac{1}{t_1-m_V{}^2}+\frac{1}{t_2-m_V{}^2}\Big)\bigg)^2(m_K{}^2m_\pi{}^2-t_1t_2)^2\bigg]. \qquad (31)$$

We also evaluate the branching ratio,

$$\frac{\Gamma[K_L\to\pi°\gamma\gamma]_{\rm VMD}}{\Gamma[K_L\to{\rm all}]}=1.4\times10^{-6}, \qquad (32)$$

which is significantly below the present experimental upper bound, 2.4×10^{-4}. The calculation of Eq. (15) with the form factor $B(x_+, x_-)$ in Eq. (24) gives a lower bound for the branching ratio of the decay $K_L\to\pi°e^+e^-$ via a two photon intermediate state in this model,

$$\Gamma[K_L\to\pi°e^+e^-]_{2\gamma}$$
$$\geq\Big(\frac{\alpha}{16}\Big)^2\int ds\int_{-1}^{1}d\rho\frac{1}{256\,\pi^3m_K{}^3}\lambda(m_K{}^2, m_\pi{}^2, s)^{3/2}(1-\rho^2)\Big(\sum_{V=\omega,\rho}G_Vf_V\Big)^2, \qquad (33)$$

where

$$f_V=\frac{A_V{}^2+1}{1-\rho^2}\rho\ln\Big(\frac{A_V-1}{A_V+1}\Big)-\Big(\frac{A_V{}^2-1+2\rho^2}{1-\rho^2}\Big)\ln\Big(\frac{A_V-\rho}{A_V+\rho}\Big)-2\rho A_V\ln\Big(\frac{A_V{}^2-1}{A_V{}^2-\rho^2}\Big),$$

$$A_V=\frac{s^2+2m_V{}^2-m_K{}^2-m_\pi{}^2}{\sqrt{\lambda(m_K{}^2, m_\pi{}^2, s)}},$$

$$\frac{\Gamma[K_L\to\pi°e^+e^-]|_{2\gamma}}{\Gamma[K_L\to{\rm all}]}\geq1.7\times10^{-11}. \qquad (34)$$

From Eqs. (32) and (34), the ratio $\Gamma[K_L\to\pi°e^+e^-]|_{2\gamma}/\Gamma[K_L\to\pi°\gamma\gamma]|_{\rm VMD}$ is at least of the order of 10^{-5}

$$\frac{\Gamma[K_L\to\pi°e^+e^-]|_{2\gamma}}{\Gamma[K_L\to\pi°\gamma\gamma]|_{\rm VMD}}\geq1.2\times10^{-5}. \qquad (35)$$

The main results are Eqs. (31) and (33). In the calculation, there are uncertainties in the weak transition matrix element (Eq. (19)) and in the $PV\gamma$ couplings. However the ratio $\Gamma[K_L\to\pi°e^+e^-]|_{2\gamma}/\Gamma[K_L\to\pi°\gamma\gamma]|_{\rm VMD}$ is free from such uncertainties. Therefore in the VMD model the chirality suppression does not occur, which should be compared with the result of Ref. 3).

§ 4. Conclusions and discussion

In this paper we obtain the formulae of the decay rates for $K_L\to\pi°e^+e^-$ and $K_L\to\pi°\gamma\gamma$. Further using these formulae, in the VMD model the branching ratios for these decay modes are also predicted. We have seen that chirality suppression does not always occur in the CP-conserving amplitudes for the decay $K_L \to \pi^\circ e^+ e^-$ with the general, kinematical configuration of the three body decay. The calculation using the VMD model has explicitly shown that the chirality suppression does not occur and the CP-conserving contribution is comparable with or larger than the CP-violating piece. The same conclusions were obtained by the authors of Refs. 5) and 6). The authors of Ref. 4) evaluate the form factors in Eq. (9) with chiral perturbation theory. Their result is that only the form factor A gives a non-vanishing contribution to the lowest non-trivial order in $K_L \to \pi^\circ \gamma\gamma$. Since A does not contribute to the absorptive part of $K_L \to \pi^\circ e^+ e^-$ in the massless limit of the electron and the positron, the chirality suppression does occur in the calculation of the lowest non-trivial order. Furthermore they suggest that the next order contribution to the form factor B is small, nevertheless the chirality suppression does not occur. In contrast to the results of Ref. 4), in the VMD model, even in the tree level evaluation, the form factor B is non-vanishing and gives a large contribution to $K_L \to \pi^\circ e^+ e^-$ as well as to $K_L \to \pi^\circ \gamma\gamma$. Then a detailed study of the coefficient in the form factor B is neccessary in chiral perturbation theory.[4]

The general form of the decay amplitude for $K_L \to \pi^\circ \gamma\gamma$, given in Eq. (12), can be applied to other two-current effects, such as $\eta \to \pi^\circ \gamma\gamma$, i.e., if experiment could give further information on the invariant mass distribution of the two photons, the angular distribution, etc., it could be determined which of the invariant tensors actually describes the data. This kind of analysis is effective to select the appropriate model for the process.

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7) Equation (19) is derived from an effective chiral Lagrangian in which $\Delta I = 1/2$ dominance is assumed,
(See Eq. (6·71) in J. F. Donoghue, E. Golowich and B. Holstein, Phys. Rep. **131** (1986), 319.)

$$L = g_8 \mathrm{Tr}(\lambda_6 \partial_\mu M \partial^\mu M),$$

$$M = \exp\left(i \frac{\pi_a \lambda^a}{F_\pi}\right),$$

$$g_8 = 3.58 \times 10^{-8}\, m_\pi^2,$$

$$F_\pi = 93.3\,\mathrm{MeV}.$$

8) The decay width for $\omega \to \pi^\circ \gamma$ is $\Gamma_{\omega\pi^\circ\gamma} = (g^2_{\omega\pi^\circ\gamma}/96\,\pi) m_\omega^3 (1-(m_\pi^2/m_\omega^2))^3$. We quote the value, $\Gamma_{\omega\pi^\circ\gamma} = 0.68$(MeV).

Note added: There was a mistake about Eq. (15) in our original preprint (KEK-Preprint-88-4). In the previous version of our paper, there remained the collinear singularities. However, as pointed out in Ref. 6), the singularities are cancelled. Then we can neglect the masses of electron and positron from the beginning of the calculation.