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CP Violation and Off-Diagonal Neutral Currents

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We discuss the *CP* violation based on the $SU(2)\times U(1)\times SU_F(2)$ gauge model, where the new gauge bosons S_μ^i are introduced in association with the extra subgroup $SU_F(2)$. The effective coupling constant G_S for the interaction mediated by the S_μ^i is found to be $G_S/G_F>5\times10^{-6}$. In this model the gauge bosons S_μ^i are coupled to off-diagonal neutral currents. The ratio $R=[\Gamma(K^\pm\to\pi^\pm e\bar{\mu})+\Gamma(K^\pm\to\pi^\pm\mu\bar{e})]/\Gamma(K^-\to\pi^0\mu\bar{\nu}_\mu)$ is predicted to be $R>2\times10^{-9}$.

§ 1. Introduction

The violation of *CP* invariance is one of the fundamental mysteries in the elementary-particle physics. Many attempts[1)~6)] have been made to describe *CP* violation in the framework of the unified gauge theories. Among them the most interesting one is the model proposed by Kobayashi and Maskawa,[1)] in which they pointed out that the *CP*-violating phases can be introduced into the Weinberg-Salam model[7)] for six quark scheme. Although their model reduces approximately to the superweak theory,[8)] no natural explanation is given for the smallness of *CP* violation. Thus we believe that the complete understanding of *CP* violation is not obtained at the present stage, so it is interesting to explore other approaches.

Phenomenological success[9)] of the superweak theory seems to suggest that *CP* violation is due to a new interaction which is characterized by strangeness-changing neutral currents.[5),6),10)] In recent articles, we proposed a model based on the gauge group $SU(2)\times U_1(1)\times U_2(1)$,[11),12)] where the new gauge boson couples to off-diagonal neutral currents. In this note we discuss *CP* violation in the $SU(2)\times U(1)\times SU_F(2)$ gauge model[13)] which is a natural extension of the previous one and calculate the decay rates of the processes $K^\pm\to\pi^\pm e\bar{\mu}$ and $K^\pm\to\pi^\pm\mu\bar{e}$ induced by off-diagonal neutral currents.

§ 2. The $SU(2)\times U(1)\times SU_F(2)$ gauge model

Assuming the gauge group $SU(2)\times U(1)\times SU_F(2)$, where the subgroup $SU(2)\times U(1)$ corresponds to that of the Weinberg-Salam model,[7)] we take the following multiplets:
for leptons

$$\begin{pmatrix} \nu \\ l \end{pmatrix}_L = \begin{pmatrix} \nu_e & \nu_\mu \\ e & \mu \end{pmatrix}_L, \quad \nu_R = (\nu_e \ \nu_\mu)_R, \quad l_R = (e \ \mu)_R, \tag{1}$$

$$(2, 2, -1) \qquad (1, 2, 0) \qquad (1, 2, -2)$$

and for quarks

$$\begin{pmatrix} u \\ d \end{pmatrix}_L = \begin{pmatrix} p & c \\ n & \lambda \end{pmatrix}_L, \quad u_R = (p \ c)_R, \quad d_R = (n \ \lambda)_R, \tag{2}$$

$$(2, 2, 1/3) \qquad (1, 2, 4/3) \qquad (1, 2, -2/3)$$

where (i, j, k) denotes the eigenvalues of operators $(2I+1, 2I_F+1, Y)$ in subgroups $SU(2)$, $SU_F(2)$ and $U(1)$, respectively. The right-handed neutrinos are introduced to get rid of triangle anomalies.[14] We assume Higgs scalars $\phi_0 = (2, 1, 1)$ and $\chi = (1, 2, 0)$; the vacuum-expectation values of their neutral components, $\langle\phi_0\rangle = a/\sqrt{2}$ and $\langle\chi\rangle = (e, f)/\sqrt{2}$, generate the masses of gauge bosons $W_\mu^{\pm}$, Z_μ and new ones S_μ^i $(i = 1\sim3)$. The extra neutral gauge bosons cause muon-number- and strangeness-changing interactions,[6),11)~13)] whose strength is known to be small.[15] This is realized by the large vacuum-expectation values of the scalar field χ (see Eq. (6)).

We introduce other n Higgs bosons $\phi_i = (2, 3, 1)$ $(i = 1\sim n)$, whose vacuum-expectation values are taken as $\langle\phi_i\rangle = (b_i, c_i, d_i)/\sqrt{2}$. The leptons and the quarks get the diagonal and the off-diagonal masses through their Yukawa interactions with the scalars ϕ_0 and ϕ_i. Then the physical fermion fields ψ_f' are defined by

$$\psi_f' = U_f^\dagger \psi_f, \quad \left(\psi_f = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} e \\ \mu \end{pmatrix}, \begin{pmatrix} p \\ c \end{pmatrix}, \begin{pmatrix} n \\ \lambda \end{pmatrix}\right) \tag{3}$$

where

$$U_f = e^{i\alpha_f} e^{i\beta_f \tau_3} \begin{pmatrix} \cos\theta_f & -\sin\theta_f \\ \sin\theta_f & \cos\theta_f \end{pmatrix} e^{i\gamma_f \tau_3}, \tag{4}$$

and the subscript f stands for ν, l, u and d. Here we have assumed the parity-invariance in the fermion mass matrix M_f, namely, that M_f contains no γ_5 terms, where M_f is defined by $\mathcal{L}^f_{\text{mass}} = -\bar{\psi}_f M_f \psi_f$.*) Furthermore, we assume that the neutral weak boson Z_μ is not mixed with S_μ^i,**) for simplicity.

The effective Hamiltonian for one S_μ^i-exchange is

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*) It may be natural to assume the parity-invariance in M_f, if we take the view of the composite model of leptons and quarks such as the model proposed by Akama and Terazawa (INS-Rep-257 (1976), unpublished). We may replace this assumption by a weaker condition $U_f^L = U_f^R$, where U_f^L and U_f^R are the mixing matrices for the left-handed and the right-handed fermion, respectively.

**) This assumption requires $\mathrm{Im}(\boldsymbol{b}\cdot\boldsymbol{c}^*) = \mathrm{Im}(\boldsymbol{c}\cdot\boldsymbol{d}^*) = \mathrm{Im}(\boldsymbol{d}\cdot\boldsymbol{b}^*) = 0$ (we take the abbreviated notation $\boldsymbol{A}\cdot\boldsymbol{B}$ as $\sum_{i=1}^n A_i B_i$ and $|\boldsymbol{A}|^2$ as $\sum_{i=1}^n |A_i|^2$ hereafter). In this case we need at least three Higgs bosons ϕ_i to introduce CP-violating phases on the assumption of the parity-invariance in M_f.

$$\mathcal{H}_{\text{eff}}^{S}=\frac{G_S}{\sqrt{2}}\sum_{i,j=1}^{3}J_{i\mu}(C^{-1})_{ij}J_j^{\mu}\,, \tag{5}$$

where

$$\frac{G_S}{\sqrt{2}}=\frac{1}{2(|e|^2+|f|^2)}\,, \tag{6}$$

$$J_i^{\mu}=\sum_f \bar{\psi}_f' U_f^{\dagger}\tau_i\gamma^{\mu}U_f\psi_f' \tag{7}$$

and

$$C_{ij}=\begin{pmatrix} 1+4\dfrac{|\boldsymbol{c}|^2+|\boldsymbol{d}|^2}{|e|^2+|f|^2} & -4\dfrac{\boldsymbol{b}\cdot\boldsymbol{c}^*}{|e|^2+|f|^2} & -4\dfrac{\boldsymbol{d}\cdot\boldsymbol{b}^*}{|e|^2+|f|^2} \\ -4\dfrac{\boldsymbol{b}\cdot\boldsymbol{c}^*}{|e|^2+|f|^2} & 1+4\dfrac{|\boldsymbol{d}|^2+|\boldsymbol{b}|^2}{|e|^2+|f|^2} & -4\dfrac{\boldsymbol{c}\cdot\boldsymbol{d}^*}{|e|^2+|f|^2} \\ -4\dfrac{\boldsymbol{d}\cdot\boldsymbol{b}^*}{|e|^2+|f|^2} & -4\dfrac{\boldsymbol{c}\cdot\boldsymbol{d}^*}{|e|^2+|f|^2} & 1+4\dfrac{|\boldsymbol{b}|^2+|\boldsymbol{c}|^2}{|e|^2+|f|^2} \end{pmatrix}. \tag{8}$$

Unless all phases in the Hamiltonian are absorbed into fermion fields, there survive CP-odd terms that have, in general, $|\Delta S|=0$, 1 and 2 parts. In the case $\beta_u=\beta_d\equiv\beta$, the magnitudes of these parts are of the same order*) and the model reduces approximately to the superweak theory[8] which is consistent with the experimental results on CP violation.[9] In this note we restrict ourselves to the case $\beta_u=\beta_d$ for simplicity.

§ 3. *CP* violation in K^0-$\bar{K}^0$ mass matrix**)

Now we discuss CP violation in our model. We assume that the contributions of physical Higgs scalars can be ignored. The Feynman diagram which contributes to the imaginary parts of the K^0-$\bar{K}^0$ mass matrix is illustrated in Fig. 1. The CP-violating effective Hamiltonian is given by (we omit primes for physical fields hereafter)

*) Though the strength of the effective Hamiltonian for the process $\lambda+p\to n+p$ is of order $(G_S/G_F)G_F$, the CP-odd term is suppressed in the case $\beta_u=\beta_d$ and is given by

$$\mathrm{Im}[\mathcal{H}_{\text{eff}}^{\Delta S=1}]=-2\sqrt{2}\delta'(G_S/G_F)^2G_F(\bar{n}\gamma_\mu\lambda)(\bar{p}\gamma^\mu p),$$

where

$$\delta'=\frac{1}{|a|^2+|\boldsymbol{b}|^2+|\boldsymbol{c}|^2+|\boldsymbol{d}|^2}[\{(|\boldsymbol{c}|^2-|\boldsymbol{b}|^2)\sin 4\beta-2\boldsymbol{b}\cdot\boldsymbol{c}^*\cos 4\beta\}\sin 2\theta_d$$
$$+2(\boldsymbol{d}\cdot\boldsymbol{b}^*\sin 2\beta+\boldsymbol{c}\cdot\boldsymbol{d}^*\cos 2\beta)\cos 2\theta_d]+O(G_S/G_F).$$

This is of the same order as $\Delta S=2$ part (see Eq. (9)).

**) We note that the new interaction does not break the parity invariance. Thus the electric dipole moment of a neutron may arise from the fourth order Hamiltonian $[\mathcal{H}^{\text{Weak}}\times\mathcal{H}^{S}]_{\text{eff}}$. The calculation for the electric dipole moment will be given elsewhere.

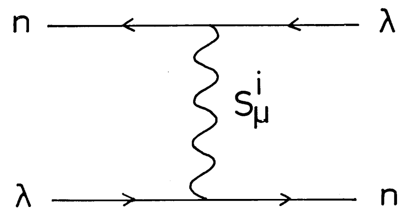

Fig. 1. The diagram which contributes to the $\Delta S=2$ transition.

$$\mathrm{Im}[\mathcal{H}_{\mathrm{eff}}^{\Delta S=2}]=2\sqrt{2}\,\delta\,(G_S/G_F)^2 G_F\,(\bar{n}\gamma_\mu\lambda)\,(\bar{n}\gamma^\mu\lambda), \tag{9}$$

where

$$\delta=\frac{1}{|a|^2+|\boldsymbol{b}|^2+|\boldsymbol{c}|^2+|\boldsymbol{d}|^2}[\{(|\boldsymbol{c}|^2-|\boldsymbol{b}|^2)\sin 4\beta-2\boldsymbol{b}\cdot\boldsymbol{c}^*\cos 4\beta\}\cos 2\theta_d$$
$$-2(\boldsymbol{d}\cdot\boldsymbol{b}^*\sin 2\beta+\boldsymbol{c}\cdot\boldsymbol{d}^*\cos 2\beta)\sin 2\theta_d]+O(G_S/G_F), \tag{10}$$

and the Fermi coupling constant

$$\frac{G_F}{\sqrt{2}}=\frac{1}{2(|a|^2+|\boldsymbol{b}|^2+|\boldsymbol{c}|^2+|\boldsymbol{d}|^2)}. \tag{11}$$

Here the phase factors α_u, α_d, γ_μ and γ_d have been absorbed into quark fields so that no phase factors appear in the weak interaction Hamiltonian.

Though it is difficult to evaluate, from this interaction, the imaginary part of the off-diagonal element of the K^0-$\bar{K}^0$ mass matrix $\mathrm{Im}(m_{12})$, a rough estimation may be given in terms of the nonrelativistic quark model as follows:

$$\mathrm{Im}(m_{12})\simeq 4\sqrt{2}\,\delta\,(G_S/G_F)^2 G_F|f_K(0)|^2. \tag{12}$$

The $f_K(r)$ is the bound-state-wave function[16] and $|f_K(0)|^2\simeq 6\times 10^6(\mathrm{MeV})^3$ which is determined by the decay rate $\Gamma(K^-\to\mu\bar{\nu}_\mu)$. Using the experimental values[15] of mass difference between neutral kaons $\Delta m_K=m_L-m_S\simeq 7\times 10^{-15}m_K$ and of ε-parameter[9] $|\varepsilon|\simeq|\mathrm{Im}(m_{12})|/(\sqrt{2}\Delta m_K)\simeq 2\times 10^{-3}$, we find

$$|\delta|(G_S/G_F)^2 G_F\simeq 3\times 10^{-16}(m_{\mathrm{proton}})^{-2}. \tag{13}$$

Making use of Eq. (10) and Schwarz inequalities for the scalar products among n-dimensional vectors $\boldsymbol{b}$, $\boldsymbol{c}$ and $\boldsymbol{d}$, we obtain the constraint $|\delta|<2/\sqrt{3}$, which leads us to a remarkable result

$$G_S/G_F>5\times 10^{-6}. \tag{14}$$

It should be noted that in the processes, in which only particles of the same $SU_F(2)$-multiplets participate (except for the gauge bosons), the $SU_F(2)$-isospin is almost conserved, for the S_μ^i has the nearly degenerate masses. The smallness of CP violation observed in neutral kaons is attributed not only to the small coupling constant G_S, but also to the suppression due to the approximate $SU_F(2)$ (global) symmetry. Other examples, where such a suppression mechanism works, are the processes $\mu\to 3e$ and $\mu\to e\gamma$.

§ 4. $K^\pm\to\pi^\pm e\bar{\mu}$ and $K^\pm\to\pi^\pm\bar{e}$ decays

There exist some muon-number- and strangeness-changing processes in which the suppression mechanism does not work. Among them the most interesting processes*) which can be tested by high-precision experiments in future would be $K^\pm\to\pi^\pm e\bar{\mu}$ and $K^\pm\to\pi^\pm\mu\bar{e}$. The Feynman diagram for these processes is shown in Fig. 2. The decay rates of these processes are calculated as

$$R_1=\frac{\Gamma(K^\pm\to\pi^\pm e\bar{\mu})}{\Gamma(K^-\to\pi^0\mu\bar{\nu}_\mu)}=2(1\pm\xi)^2\frac{1}{\sin^2\theta_c}\left(\frac{G_S}{G_F}\right)^2, \tag{15}$$

$$R_2=\frac{\Gamma(K^\pm\to\pi^\pm \mu\bar{e})}{\Gamma(K^-\to\pi^0\mu\bar{\nu}_\mu)}=2(1\mp\xi)^2\frac{1}{\sin^2\theta_c}\left(\frac{G_S}{G_F}\right)^2, \tag{16}$$

where θ_c is the Cabibbo angle and

$$\xi=\cos(2\theta_d)\cos(2\theta_l)+\sin(2\theta_d)\sin(2\theta_l)\cos(\beta_d-\beta_l). \tag{17}$$

Although the prediction for each process depends on the unknown parameter ξ, we get an interesting result for the following ratio R:

$$\begin{aligned}R&\equiv R_1+R_2\\&=\frac{4}{\sin^2\theta_c}(1+\xi^2)\left(\frac{G_S}{G_F}\right)^2>\frac{4}{\sin^2\theta_c}\left(\frac{G_S}{G_F}\right)^2.\end{aligned} \tag{18}$$

By substituting the lower bound of (G_S/G_F) given by Eq. (14), the ratio R is predicted to be $R>2\times10^{-9}$. The present experimental upper limit is 4×10^{-7},[15] from which we obtain $(G_S/G_F)<7\times10^{-5}$.

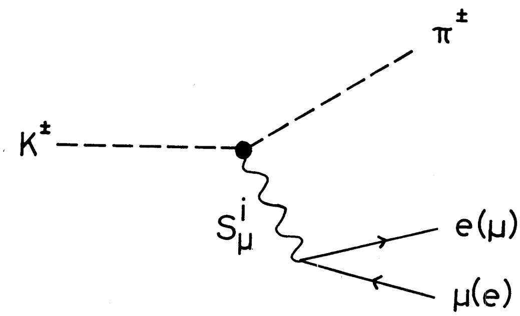

Fig. 2. The diagram for the $K^\pm\to\pi^\pm e\bar{\mu}(\mu\bar{e})$ decay.

*) The $K_L^0\to e\bar{\mu}(\mu\bar{e})$ decay is expected to be so rare, because the process proceeds through the virtual photon-S_μ^i pair.[12]

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