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CPT-BASED LIQUEFACTION TRIGGERING PROCEDURE

by

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1 **Abstract**

2 A probabilistic cone penetration test (CPT) based liquefaction triggering procedure for
3 cohesionless soils is derived using a maximum likelihood method with an updated case history
4 database. The liquefaction analysis framework includes revised relationships for the magnitude
5 scaling factor (MSF) and for estimating fines contents from CPT data when laboratory test data
6 are not available. The updated case history database and methodology for developing the
7 liquefaction correlation are described. Measurement and estimation uncertainties, the potential
8 effects of false positives and false negatives in the case history database, and the effects of the
9 choice-based sampling bias in the case history database are accounted for. Sensitivity analyses
10 showed that the position of the most likely triggering curve and the magnitude of the total error
11 term are reasonably well constrained by the data. The sensitivity study provides reasonable bounds
12 on the effects of different interpretations, from which probabilistic and deterministic relationships
13 for practice are recommended.

14 *Key Words: Liquefaction, earthquakes, cyclic loads, standard penetration test, probability.*

15 **Introduction**

16 Cone penetration test (CPT) and Standard Penetration Test (SPT) based probabilistic correlations
17 for evaluating liquefaction triggering in cohesionless soils have advanced through the
18 contributions of numerous researchers (e.g., Christian and Swiger 1975, Liao et al. 1988, Liao and
19 Lum 1998, Youd and Nobel 1997, Toprak et al. 1999, Juang et al. 2002, Cetin et al. 2004, Moss
20 et al. 2006, Boulanger and Idriss 2012). Some probabilistic relationships represent the total
21 uncertainty in the evaluation of the case history database; i.e., they include the uncertainty in the
22 triggering relationship (model uncertainty) and the uncertainty in the representative $(N_1)_{60cs}$ or
23 q_{c1Ncs} and $CSR_{M=7.5, \sigma'=1}$ values determined for the case histories (measurement or parameter

24 uncertainty). The approach developed by Cetin et al. (2002) allowed for a separate accounting of
25 the model and measurement uncertainties. For applications, the total uncertainty will include
26 contributions from the liquefaction triggering model and input parameters. The parameter
27 uncertainties in an application are not necessarily the same as the measurement uncertainties in the
28 case history database, and thus it is important to have separately quantified the model uncertainty
29 so that it can be rationally combined with the parameter uncertainties in a probabilistic liquefaction
30 evaluation.

31 The quantity and quality of CPT and SPT case histories has increased with recent earthquake
32 events, including data obtained in the 2010-2011 Canterbury earthquake sequence in New Zealand
33 (e.g., van Ballegooy et al. 2014, Green et al. 2014) and the 2011 $M_w=9.0$ Tohoku earthquake in
34 Japan (e.g., Tokimatsu et al. 2012, Cox et al. 2013). For example, Green et al. (2014) compiled 50
35 case histories representing cases of liquefaction and no liquefaction during the 2010-2011
36 Canterbury earthquake sequence with subsurface profiles for which the critical layer could be
37 identified with relatively high confidence. The inclusion of these and other data provide an
38 opportunity for re-evaluating liquefaction triggering procedures and updating them as warranted.

39 In this paper, a probabilistic CPT-based liquefaction triggering procedure for cohesionless soils
40 is derived using a maximum likelihood method and an updated case history database. The
41 liquefaction analysis framework is described, including revised relationships for magnitude
42 scaling factor (MSF) and for estimating fines contents from CPT data when laboratory test data
43 are not available. The updated case history database and methodology for developing the
44 probabilistic relationships for liquefaction triggering are described. The sensitivity of the
45 maximum likelihood solution to various assumptions regarding measurement uncertainties and the
46 potential influence of false positive or false negative case histories in the database are evaluated.

47 A probabilistic correlation is then proposed and issues regarding its use in practice are discussed.
48 Details for each of these steps are presented in Boulanger and Idriss (2014).

49 **Liquefaction Analysis Framework**

50 The liquefaction analysis framework follows that by Idriss and Boulanger (2008) and incorporates
51 changes to the MSF and procedures for estimating FC from CPT data when laboratory test data
52 are not available. The functional terms provide the means for rationally interpreting case histories
53 and extending the resulting correlation to conditions outside those covered by the case history
54 database. An outcome of the triggering correlation is a FC adjustment relationship, which is
55 presented in this section to facilitate presentation of the case history database in a common format.

56 The earthquake-induced cyclic stress ratio (CSR), at a given depth, z , within the soil profile, is
57 expressed as a representative value equal to 65% of the maximum cyclic shear stress ratio, i.e.:

$$58 \quad CSR_{M, \sigma'_v} = 0.65 \frac{\tau_{\max}}{\sigma'_v} \quad (1)$$

59 where τ_{\max} = maximum earthquake induced shear stress, σ'_v = vertical effective stress, and the
60 subscripts on the CSR indicate that it is computed for a specific earthquake magnitude (moment
61 magnitude, M) and in-situ σ'_v . The value of τ_{\max} can be estimated from dynamic response analyses,
62 but such analyses must include a sufficient number of input acceleration time series and adequate
63 site characterization details to be reasonably adequate. Alternatively, the maximum shear stress
64 can be estimated using the Seed-Idriss Simplified Procedure to arrive at,

$$65 \quad CSR_{M, \sigma'_v} = 0.65 \frac{\sigma_v}{\sigma'_v} \frac{a_{\max}}{g} r_d \quad (2)$$

66 where σ_v = vertical total stress at depth z , a_{\max}/g = maximum horizontal acceleration (as a fraction
67 of gravity) at the ground surface, and r_d = shear stress reduction factor that accounts for dynamic
68 response of the soil profile. The expression for r_d by Idriss (1999), as derived from site response

69 analyses, was used in the present study as,

$$70 \quad r_d = \exp[\alpha(z) + \beta(z) \cdot M] \quad (3a)$$

$$71 \quad \alpha(z) = -1.012 - 1.126 \sin\left(\frac{z}{11.73} + 5.133\right) \quad (3b)$$

$$72 \quad \beta(z) = 0.106 + 0.118 \sin\left(\frac{z}{11.28} + 5.142\right) \quad (3c)$$

73 where z = depth below the ground surface in meters and the arguments inside the sin terms are in
74 radians. Additional details on development of this relationship are in Idriss and Boulanger (2010).

75 CPT penetration resistances are corrected for overburden stress effects as,

$$76 \quad q_{c1N} = C_N q_{cN} = C_N \frac{q_c}{P_a} \quad (4)$$

77 where C_N = overburden correction factor, P_a = atmospheric pressure (101.3 kPa, 1 atm), $q_{cN} = q_c/P_a$,
78 and q_{c1N} is the penetration resistance that would be obtained in the same sand at an overburden
79 stress of 1 atm if all other attributes remain constant (e.g., same relative density, fabric, age, degree
80 of cementation, loading history). Note that q_c should be corrected for pore pressures measured
81 behind the tip whenever such data are available. The C_N relationship by Boulanger (2003) based
82 on calibration chamber test data and numerical modeling of cone penetration was used,

$$83 \quad C_N = \left(\frac{P_a}{\sigma'_v}\right)^m \leq 1.7 \quad (5a)$$

$$84 \quad m = 1.338 - 0.249(q_{c1Ncs})^{0.264} \quad (5b)$$

85 where q_{c1Ncs} = equivalent clean sand penetration resistance (as discussed below). The exponent m
86 can be constrained to its recommend limits of $0.264 \leq m \leq 0.782$ by limiting q_{c1Ncs} values to
87 between 21 and 254 for use in these expressions.

88 The soil's cyclic resistance ratio (CRR) is dependent on the duration of shaking (expressed

89 through the MSF) and σ'_v (expressed through a K_σ factor). The correlation for CRR is therefore
 90 developed by adjusting the case history CSR values to a reference $M = 7.5$ and $\sigma'_v = 1$ atm as,

$$91 \quad CSR_{M=7.5, \sigma'_v=1} = \frac{CSR_{M, \sigma'_v}}{MSF \cdot K_\sigma} \quad (6)$$

92 The soil's CRR is further affected by the presence of sustained static shear stresses, such as may
 93 exist beneath foundations or within slopes. The effect of sustained static shear stresses, expressed
 94 through a K_α factor, is generally small for nearly level ground conditions and is not included herein
 95 because the case history database is dominated by level or nearly level ground conditions.

96 The K_σ relationship by Boulanger (2003), which was based on a compilation of experimental
 97 data interpreted in a critical-state framework, was used as

$$98 \quad K_\sigma = 1 - C_\sigma \ln \left(\frac{\sigma'_v}{P_a} \right) \leq 1.1 \quad (7a)$$

$$99 \quad C_\sigma = \frac{1}{37.3 - 8.27 (q_{c1Ncs})^{0.264}} \leq 0.3 \quad (7b)$$

100 The coefficient C_σ can be limited to its maximum value of 0.3 by restricting q_{c1Ncs} to ≤ 211 in these
 101 expressions. The above relationships have been shown to be in reasonable agreement with an
 102 updated database of laboratory experimental data by Montgomery et al. (2014).

103 The MSF by Boulanger and Idriss (2014, 2015), derived from a compilation of laboratory test
 104 data and analyses of ground motion recordings, includes dependency on soil characteristics as

$$105 \quad MSF = 1 + (MSF_{max} - 1) \left(8.64 \exp \left(\frac{-M}{4} \right) - 1.325 \right) \quad (8)$$

106 where MSF_{max} was related to q_{c1Ncs} values as,

$$107 \quad MSF_{max} = 1.09 + \left(\frac{q_{c1Ncs}}{180} \right)^3 \leq 2.2 \quad (9)$$

108 The resulting MSF relationship for different values of q_{c1Ncs} is shown in Fig. 1. This relationship
 109 produces $MSF_{max} = 1.8$ at $q_{c1Ncs} \approx 160$, which matches the MSF relationship for sand by Idriss
 110 (1999), and $MSF_{max} \approx 1.10$ for $q_{c1Ncs} < 60$, which is consistent with the expected results for very
 111 loose sands or soft low-plasticity silts.

112 The correlation of CRR to q_{c1N} in cohesionless soils is also affected by the soil's FC. For
 113 mathematical convenience, this correlation can also be expressed in terms of an equivalent clean-
 114 sand q_{c1Ncs} values which are obtained using the following expressions:

$$115 \quad q_{c1Ncs} = q_{c1N} + \Delta q_{c1N} \quad (10)$$

$$116 \quad \Delta q_{c1N} = \left(11.9 + \frac{q_{c1N}}{14.6} \right) \exp \left(1.63 - \frac{9.7}{FC + 2} - \left(\frac{15.7}{FC + 2} \right)^2 \right) \quad (11)$$

117 The equivalent clean-sand adjustment $\Delta q_{c1N} = f(FC)$ is derived so that CRR can be expressed as a
 118 function of q_{c1Ncs} alone and thus Δq_{c1N} accounts for the effect that FC has on both CRR and q_{c1N} .
 119 The Δq_{c1N} relationship (Fig. 2) is primarily based on its empirical fit to the liquefaction case history
 120 data, but the sparseness of the case history data for combinations of high q_{c1N} and high FC values
 121 (as described later) provide limited constraint on the shape (form) of the relationship. For this
 122 reason, the selection of its form was also guided by checking consistency of the resulting CPT-
 123 based probabilistic triggering correlation with the SPT-based probabilistic correlation by
 124 Boulanger and Idriss (2012) in terms of implied q_c/N_{60} ratios and relative state parameter indices
 125 for common values of $CRR_{M=7.5, \sigma'=1atm}$ and probability of liquefaction (P_L). The adopted forms for
 126 the $\Delta q_{c1N} = f(FC)$ and $CRR_{M=7.5, \sigma'=1atm} = f(q_{c1Ncs})$ equations were chosen to produce q_c/N_{60} ratios
 127 which are in reasonable agreement with Suzuki et al.'s (1998) empirical data for q_c/N_{60} ratios in
 128 sands and silty sands, including the trends for q_c/N_{60} to decrease with increasing relative density
 129 and increasing FC (details in Boulanger and Idriss 2014). The adjustments begin to plateau for FC

130 values exceeding about 35% because the soil matrix is believed to become fines-dominated for
 131 any FC value greater than about this value. The adjustments are considered appropriate for
 132 nonplastic to low-plasticity silty fines. The adjustments are presented here because they are used
 133 in the following sections for summarizing the case history data and examining their distributions
 134 across a range of conditions.

135 **Soil Classification Estimation using CPT data**

136 The FC and soil classification are often correlated to a soil behavior type index (I_c) which is a
 137 function of the q_c and sleeve friction ratio (f_s). The I_c term by Robertson and Wride (1998) is,

$$138 \quad I_c = \left[(3.47 - \log(Q))^2 + (1.22 + \log(F))^2 \right]^{0.5} \quad (12)$$

139 where Q and F are normalized tip and sleeve friction ratios computed as,

$$140 \quad Q = \left(\frac{q_c - \sigma_{vc}}{P_a} \right) \left(\frac{P_a}{\sigma'_{vc}} \right)^n \quad (13)$$

$$141 \quad F = \left(\frac{f_s}{q_c - \sigma_{vc}} \right) \cdot 100\% \quad (14)$$

142 The exponent n varies from 0.5 in sands to 1.0 in clays (Robertson and Wride 1998).

143 General correlations between FC and I_c or other CPT-based indices exhibit large scatter, such
 144 that site-specific calibration or checking of such correlations is strongly encouraged. The
 145 relationship for estimating FC herein is,

$$146 \quad \begin{aligned} FC &= 80(I_c + C_{FC}) - 137 \\ 0\% &\leq FC \leq 100\% \end{aligned} \quad (15)$$

147 where C_{FC} is a fitting parameter (default value is 0.0). This expression with $C_{FC} = 0.0, -0.29,$ and
 148 0.29 (i.e., \pm an amount equal to the standard deviation in the general correlation) is shown in Fig.

149 3. Site specific calibration of C_{FC} should be for individual geologic strata (common source

150 material, deposition, etc.), such that different C_{FC} values may be obtained for different strata at any
151 one site. For example, setting $C_{FC} = -0.07$ is approximately equal to the relationship developed by
152 Robinson et al. (2013) for liquefiable soils along the Avon River in Christchurch, New Zealand.

153 Ground densification work has been observed to change the $FC-I_c$ correlation at specific sites
154 through its effects on q_c and f_s , with the result that C_{FC} may be different before and after ground
155 densification work (e.g., Nguyen et al. 2014). Similarly, the I_c value used to distinguish clays from
156 sands has often been observed to decrease as a result of densification. The consistency of the
157 inferred soil profile characteristics from before to after ground densification can be used to develop
158 site-specific adjustments in both C_{FC} and the I_c cut-off value.

159 A CPT-based liquefaction triggering evaluation should consider the uncertainty in FC and soil
160 classification estimates when site-specific sampling and lab testing data are not available. For
161 example, liquefaction analyses could be repeated using a range of C_{FC} values to evaluate the
162 sensitivity to FC estimates; e.g., using $C_{FC} = \pm 0.15$ or ± 0.29 would allow for about $\pm 1/2$ or 1 standard
163 deviation in this relationship. Similarly, the I_c cut-off value used to screen out clay-like soils is
164 commonly taken as 2.6 but other values may be justified based on site specific sampling and testing
165 (Robertson and Wride 1998). Liquefaction analyses could be repeated using I_c cut-off values of
166 2.4 and 2.6 to evaluate sensitivity to this parameter. Results of such analyses can be used to
167 evaluate potential benefits of site-specific sampling and testing, while recognizing that some
168 amount of sampling and testing should always be required for high risk/high consequence projects.

169 **CPT-based Case History Database**

170 A database of CPT liquefaction case histories is updated, including adding data from recent
171 earthquake events (e.g., PEER 2000a,b, Sancio 2003, Green et al. 2014, Cox et al. 2013). The
172 individual case histories and key references are summarized in Table S1 in the electronic

173 supplement. We examined the original sources for all cases, as well as interpretations by others
174 (e.g., Moss et al. 2003), to obtain independent interpretations consistent with our current
175 understanding and judgments. For cases where our interpretation was within a few percent of the
176 original investigators, we retained the interpretation of the original investigator.

177 The available information for most case histories with I_c near 2.6 ± 0.2 (i.e., 2.4 to 2.8) in critical
178 strata is insufficient to confidently evaluate whether the soils would be best analyzed with a
179 liquefaction triggering framework (i.e., that for sands) or a cyclic softening framework (i.e., that
180 for clays or silts with high PI). The cases listed in Table S1 are nonetheless limited to cases with
181 $I_c < 2.6$, recognizing this assumes a priori the adequacy of this criterion for identifying the most
182 appropriate analysis framework. Of the 253 cases listed in Table S1, 180 cases had surface
183 evidence of liquefaction, 71 cases had no surface evidence of liquefaction, and 2 cases were
184 described as being at the margin between liquefaction and no liquefaction.

185 Moment magnitudes (M or M_w) are used for all earthquakes (Table S1). The M were obtained
186 from the Next Generation Attenuation NGA-2 project flatfile (Ancheta et al. 2014) and USGS
187 Centennial Earthquake Catalog (Engdahl and Villasenor 2002, and online catalog 2010).

188 Estimates of peak horizontal ground accelerations (PGA or a_{max}) are listed for each site in
189 Table S1. PGA estimates by the original site investigators or from the Moss et al. (2003) database
190 were used in all cases except as noted below. USGS ShakeMaps (Worden et al. 2010) were used
191 to check PGA estimates for a number of sites with no nearby recordings, as described in Boulanger
192 and Idriss (2014). In practice, the use of a geometric mean a_{max} in assessing liquefaction hazards
193 is considered to be a reasonable engineering approach for many geotechnical structures (e.g.,
194 levees, embankment dams) or soil-structure systems (e.g., bridge abutments, pipelines) which
195 often have direction-dependent response characteristics.

196 A number of CPT-based case histories are discussed in Boulanger and Idriss (2014) to illustrate
197 issues important to the interpretation of case histories, including the geologic understanding of the
198 site and methodology used for selecting representative q_{c1Ncs} values from critical strata. In general,
199 the appropriateness of any averaging of q_{cN} values for a specific stratum in case history
200 interpretations or forward analyses depends on the spatial characteristics of the stratum (e.g.,
201 thickness, lateral extent, continuity, variability), the mode of deformation (e.g., reconsolidation
202 settlement, lateral spreading, slope instability), and the spatial dimensions of the potential
203 deformation mechanisms relative to the strata of concern. A familiarity with how representative
204 q_{c1Ncs} values are selected for the database is important for guiding the forward application of these
205 correlations in a manner consistent with their development.

206 Site performance during an earthquake is classified as a "liquefaction", "no liquefaction", or
207 "marginal" case. Cases described as "liquefaction" were generally accompanied with reports of
208 sand boils and/or visible ground surface settlements, cracks, or lateral movements. Cases described
209 as "no liquefaction" were generally accompanied with reports of no visible surface manifestations.
210 Two cases were classified as "marginal" because the available information suggests that conditions
211 at the site are likely at, or near, the boundary of conditions that separate the physical occurrence
212 of liquefaction from non-liquefaction.

213 **Distribution of data**

214 The distributions of the case history data are illustrated in Figs. 4 and 5. Plots of q_{cN} and F versus
215 representative depth of the critical zone (Fig. 4) show the database is limited to average critical
216 depths less than 12 m with few points for average depths greater than about 9 m. Plots of M versus
217 a_{max} (Fig. 5a) show the current database includes few cases for M less than 6 or greater than 7.6.
218 Plots of q_{cN} versus FC (Fig. 5b) show there are relatively few cases with high FC and essentially

219 no cases with both high FC and high q_{cN} values. Explicit statements regarding the plasticity of the
220 fines fraction [i.e., a plasticity index (PI) or statement that the fines are nonplastic] are not provided
221 for most case histories, but the available information and descriptions suggest they correspond
222 primarily to soils with nonplastic or low plasticity silty fines. Additional figures illustrating the
223 distribution of the case history parameters are provided in the electronic Supplement.

224 Liquefaction analyses should evaluate how the conditions of a specific project compare to the
225 conditions covered by the case history database. If project conditions fall outside those constrained
226 by case history data, then the results of liquefaction triggering analyses using different correlations
227 can be strongly dependent on the functional relationships used within those correlations. In such
228 cases, a clear understanding of the bases behind the functional relationships can be important for
229 guiding judgments regarding the applicability of the results so obtained.

230 **Probabilistic Relationship for CPT-based Triggering Procedure**

231 The probabilistic triggering correlation was developed using a maximum likelihood method that
232 utilizes the forms of the limit state and likelihood functions used by Cetin et al. (2002, 2004).
233 Emphasis is placed on developing a reasonable first-order estimate of the total and model
234 uncertainties given that the available case history data are insufficient for quantifying the
235 components of uncertainty on a site-by-site basis. Measurement and estimation uncertainties, the
236 potential effects of false positives and false negatives in the case history database, and the effects
237 of the choice-based sampling bias in the case history database are accounted for. The sensitivity
238 of the maximum likelihood solution to subsets of the database and to a range of estimated
239 measurement uncertainties is evaluated, from which relationships for practice are recommended.

240 ***Limit state function***

241 The limit state function (g) was taken as the difference between the natural logs of $CRR_{M=7.5, \sigma=1atm}$

242 and $CSR_{M=7.5, \sigma'=1atm}$, such that liquefaction is assumed to have occurred if $g \leq 0$ and to have not
 243 occurred if $g > 0$. The $CRR_{M=7.5, \sigma'=1atm}$ value was estimated using the following relationship,

$$244 \quad CRR_{M=7.5, \sigma'=1atm} = \exp\left(\frac{q_{c1Ncs}}{113} + \left(\frac{q_{c1Ncs}}{1000}\right)^2 - \left(\frac{q_{c1Ncs}}{140}\right)^3 + \left(\frac{q_{c1Ncs}}{137}\right)^4 - C_o\right) \quad (16)$$

245 where C_o is an unknown fitting parameter that serves to scale the relationship while maintaining
 246 its shape. This relationship is not tightly constrained by the case history data for low or high values
 247 of q_{c1Ncs} , and thus its shape was also guided by checking its consistency with the SPT-based
 248 correlation by Boulanger and Idriss (2012) in terms of implied q_c/N_{60} ratios and relative state
 249 parameter indices for common values of $CRR_{M=7.5, \sigma'=1atm}$ and P_L . The form of Equation 16 was
 250 shown to produce q_c/N_{60} ratios which are in reasonable agreement with the Suzuki et al.'s (1998)
 251 empirical data for sands including the trend for q_c/N_{60} to decrease with increasing relative density
 252 (Boulanger and Idriss 2014). The limit state function can then be written as,

$$253 \quad \hat{g}(q_{c1Ncs}, C_o, CSR_{M=7.5, \sigma'=1atm}) = \ln(CRR_{M=7.5, \sigma'=1atm}) - \ln(CSR_{M=7.5, \sigma'=1atm}) \quad (17)$$

254 where the hat on g indicates that it is imperfect in its prediction of liquefaction.

255 The uncertainties in the limit state function are represented by three contributors. Measurement
 256 or estimation uncertainties in the case history data points are assumed to be adequately represented
 257 by including uncertainties in the q_{c1Ncs} and $CSR_{M=7.5, \sigma'=1atm}$ values. The uncertainty in q_{c1Ncs} is
 258 assumed to be normally distributed with a constant coefficient of variation (COV) (e.g., Baecher
 259 and Christian 2003). The uncertainty in $CSR_{M=7.5, \sigma'=1atm}$ is assumed to be log-normally distributed,
 260 which is consistent with log-normal distributions for the uncertainty in predictions of peak ground
 261 accelerations (e.g., Abrahamson et al. 2008). Uncertainty in the $CRR_{M=7.5, \sigma'=1atm}$ expression is
 262 represented by inclusion of a random model error term, which is assumed to also be log normally
 263 distributed with mean of zero.

264 The uncertainty in the representative q_{c1Ncs} value assigned to any case history includes
265 contributions from three major sources. One source is the degree to which the available CPT data
266 are representative of the critical strata, which depends on the degree to which the geologic
267 conditions are understood, the heterogeneity of the deposits, the number of soundings, and the
268 placement of the soundings relative to the strata of concern. A second source of uncertainty is the
269 CPT-based estimation of soil types (e.g., FC and fines plasticity), which depends on the availability
270 and quality of site-specific sampling and index testing data. A third source of uncertainty is
271 variability in the CPT equipment and procedures used at different case history sites. The coefficient
272 of variation (COV) of q_{c1N} measurements in sand have been reported to range from 0.20 to 0.60
273 with a mean of about 0.38 (Kulhawy and Trautmann 1996, Phoon and Kulhawy 1999). The large
274 majority of the liquefaction case histories lack sufficient information to justify site-specific
275 estimates of the uncertainty in the representative q_{c1Ncs} values. For this reason, the COV was taken
276 as being the same for all case histories where FC and fines plasticity are based on site-specific
277 sampling and index testing and to be 50% greater when site-specific sampling and index testing
278 data are not available. The 50% increase in the uncertainty for cases without site-specific sampling
279 and index test data is a subjective adjustment based on considering how potential differences in
280 FC adjustments (Δq_{c1N}) would affect estimates of q_{c1Ncs} . Parametric analyses were then used to
281 assess the sensitivity of the solution to the assumed values for the COV.

282 The uncertainty in the $CSR_{M=7.5, \sigma'=1atm}$ values estimated for any case history similarly depends
283 on numerous factors, including the proximity of strong ground motion recordings, potential
284 variability in site responses, availability and quality of indirect measures of shaking levels (e.g.,
285 eye witness reports, damage to structures, disruption of nonstructural contents), variability in the
286 ground motion characteristics (e.g., duration of shaking), and the overburden stress. Ground

287 motion prediction equations (GMPEs) have standard deviations of about 0.45-0.55 in the natural
 288 log of a_{\max} (Abrahamson et al. 2008), which implies similar uncertainty in the CSR if it was
 289 estimated on the basis of a GMPE. The case history estimates of a_{\max} by various researchers are
 290 usually based on several sources of information as discussed above, and likely have smaller
 291 variances than estimates obtained from GMPEs alone. The available data is, however, inadequate
 292 for quantifying the uncertainty in $CSR_{M=7.5, \sigma'=1atm}$ on a case-history specific basis. For this reason,
 293 the standard deviation in $\ln(CSR_{M=7.5, \sigma'=1atm})$ was set to: (1) the small value of 0.05 for the few sites
 294 that had strong ground motion recordings directly at the site, to allow for uncertainty in the MSF,
 295 K_{σ} , and r_d terms even when a_{\max} is known, and (2) a relatively greater value for all other sites. The
 296 sensitivity of the solution to a range of values in this latter parameter is presented in a later section.

297 **Notation**

298 It is convenient to simplify the notation as follows,

299
$$Q = q_{c1Ncs} \tag{18}$$

300
$$S = CSR_{M=7.5, \sigma'=1atm} \tag{19}$$

301
$$R = CRR_{M=7.5, \sigma'=1atm} \tag{20}$$

302 The limit state function can be written using a total error term ε_T , to account for both the inability
 303 of \hat{g} to predict liquefaction perfectly and the uncertainty in the parameters used to compute \hat{g} .

304
$$g(Q, S, C_o, \varepsilon_{\ln(R)}) = \hat{g}(\hat{Q}, \hat{S}, C_o) + \varepsilon_T \tag{21}$$

305 The ε_T is normally distributed with a mean value of zero and includes the effects of uncertainty in
 306 the parameters, which are expressed as,

307
$$Q = \hat{Q} + \varepsilon_Q \tag{22}$$

308
$$\sigma_Q = COV_Q \cdot \hat{Q} \tag{23}$$

309 $\ln(S) = \ln(\hat{S}) + \varepsilon_{\ln(S)}$ (24)

310 $\ln(R) = \ln(\hat{R}) + \varepsilon_{\ln(R)}$ (25)

311 The limit state function and the total error can then be expressed as,

312 $g(Q, S, C_o, \varepsilon_{\ln(R)}) = \hat{g}(\hat{Q}, \hat{S}, C_o) + \left(\frac{1}{113} + \frac{2\hat{Q}}{1000^2} - \frac{3\hat{Q}^2}{140^3} + \frac{4\hat{Q}^3}{137^4} \right) \varepsilon_Q + \varepsilon_{\ln(R)} - \varepsilon_{\ln(S)}$ (26)

313 $\varepsilon_T = \left(\frac{1}{113} + \frac{2\hat{Q}}{1000^2} - \frac{3\hat{Q}^2}{140^3} + \frac{4\hat{Q}^3}{137^4} \right) \varepsilon_Q + \varepsilon_{\ln(R)} - \varepsilon_{\ln(S)}$ (27)

314 The standard deviation in ε_T can be expressed as,

315 $(\sigma_T)^2 = \left(\frac{1}{113} + \frac{2\hat{Q}}{1000^2} - \frac{3\hat{Q}^2}{140^3} + \frac{4\hat{Q}^3}{137^4} \right)^2 (\sigma_Q)^2 + (\sigma_{\ln(R)})^2 + (\sigma_{\ln(S)})^2$ (28)

316 ***Likelihood function***

317 The likelihood function is the product of the probabilities of the individual case history
 318 observations, assuming that the case history observations are statistically independent. For a
 319 liquefaction case ($g \leq 0$), the probability of having observed liquefaction can be expressed as,

320 $P\left[g(Q, S, C_o, \varepsilon_{\ln(R)}) \leq 0\right] = \Phi\left[-\frac{\hat{g}(\hat{Q}, \hat{S}, C_o)}{\sigma_T}\right]$ (29)

321 where Φ is the standard normal cumulative probability function. For example, the probability of
 322 having observed liquefaction is greater than 0.84 if the data point plots more than one σ_T above
 323 the triggering curve. Case history data points for sites without ground motion recordings (almost
 324 all of them) are plotted at the $CSR_{M=7.5, \sigma'=1atm}$ value expected in the absence of liquefaction, and
 325 this $CSR_{M=7.5, \sigma'=1atm}$ value may be greater than the value which was developed if liquefaction was
 326 triggered early in strong shaking. For this reason, the data points that fall well above the triggering

327 curve have probabilities close to unity, and thus have little influence on the overall likelihood
 328 function. The same is true for the no-liquefaction cases that fall well below the triggering curve.

329 The case history database is believed to contain an uneven sampling of liquefaction and no-
 330 liquefaction case histories because researchers more often have chosen to investigate liquefaction
 331 sites. Manski and Leman (1977) suggest that the bias from an uneven choice-based sampling
 332 process can be corrected for by weighting the observations to better represent the actual population.
 333 Cetin et al. (2002) noted that this amounted to writing the likelihood function as,

$$334 \quad L(C_o, \varepsilon_{\ln(R)}) = \prod_{\text{Liquefied sites}} \Phi \left[-\frac{\hat{g}(\hat{Q}, \hat{S}, C_o)}{\sigma_T} \right]^{w_{\text{liquefied}}} \prod_{\text{Nonliquefied sites}} \Phi \left[\frac{\hat{g}(\hat{Q}, \hat{S}, C_o)}{\sigma_T} \right]^{w_{\text{nonliquefied}}} \quad (30)$$

335 where the exponents $w_{\text{liquefied}}$ and $w_{\text{nonliquefied}}$ used to weight the observations are computed as,

$$336 \quad w_{\text{liquefied}} = \frac{Q_{\text{liq,true}}}{Q_{\text{liq,sample}}} \quad (31)$$

$$337 \quad w_{\text{nonliquefied}} = \frac{1 - Q_{\text{liq,true}}}{1 - Q_{\text{liq,sample}}} \quad (32)$$

338 where $Q_{\text{liq,true}}$ is the true proportion of the occurrences of liquefaction in the population, and
 339 $Q_{\text{liq,sample}}$ is the proportion of occurrences of liquefaction in the sample set. Cetin et al. (2002)
 340 adopted weighting values of $w_{\text{liquefied}} = 0.8$ and $w_{\text{nonliquefied}} = 1.2$, producing the ratio
 341 $w_{\text{nonliquefied}}/w_{\text{liquefied}} = 1.5$. Moss et al. (2006) used these same weighting parameters in their
 342 application of this procedure to their CPT-based liquefaction triggering database. These same
 343 values are used herein, except as otherwise noted.

344 The case history database likely contains a number of false negatives and false positives
 345 because the true site performance is either masked or mischaracterized. A scenario of concern for
 346 false negatives is when liquefaction at depth does not produce any visible surface manifestation,

347 such as may occur when a thick crust of non-liquefiable soil overlies a relatively thin zone of
348 liquefaction and there is no significant slope or heavy structure to induce deformations. False
349 positives are not expected to be as common, but it is possible that ground surface cracking or
350 settlement could result from seismic compression of unsaturated loose soils or yielding of soft
351 clays (e.g., bearing failures around buildings), and that such movements could be interpreted as
352 having been caused by liquefaction of a different strata at the site. The potential exists for false
353 positives or false negatives to produce points that fall far from the triggering correlation, which
354 would be incorrectly treated as highly unlikely cases in the maximum likelihood solution. The
355 influence of such outliers was minimized by limiting the probability of any one observation to be
356 no smaller than a specified minimum value, P_{\min} . Sensitivity analyses considered values of $P_{\min} =$
357 0, 0.05, 0.075, and 0.10, as well as an alternative approach where outlier points were omitted.

358 ***Maximum Likelihood Solutions***

359 Maximum likelihood solutions for the triggering correlation were obtained for several subsets of
360 the case history database, including: (1) clean sand ($FC \leq 5\%$) case histories, (2) case histories
361 where the FC is based on laboratory test data alone, (3) all case histories whether the FC was based
362 on laboratory test data or correlation with I_c , and (4) all case histories with σ'_v greater than 40 kPa.
363 For each subset, solutions were obtained using ranges for the estimation parameters: $\sigma_{\ln(S)}$, COV_Q ,
364 $W_{\text{nonliquefied}}/W_{\text{liquefied}}$, and P_{\min} . In addition, the total uncertainty σ_T at high q_{c1Ncs} values was limited
365 to ≤ 0.6 except as otherwise noted. Results of these analyses are in Boulanger and Idriss (2014),
366 from which representative results are used to illustrate the primary observations.

367 Solutions based on the clean sand ($FC \leq 5\%$) case histories are plotted with the clean sand case
368 history data in Fig. 6 for $P_{\min} = 0.0$ (i.e., no allowance for false negatives or false positives) and
369 Fig. 7 for $P_{\min} = 0.05$. Curves for probabilities of liquefaction [PL] equal to 15%, 50%, and 85% in

370 terms of the total uncertainty, which means with inclusion of the estimation errors in
371 $CSR_{M=7.5, \sigma'=1atm}$ and q_{c1Ncs} , are shown in both figures for three scenarios: (a) $\sigma_{ln(S)} = 0.2$, $COV_Q =$
372 0.2 , (b) $\sigma_{ln(S)} = 0.15$, $COV_Q = 0.15$, and (c) $\sigma_{ln(S)} = 0.1$, $COV_Q = 0.1$. The $P_L = 50\%$ curves for the
373 three scenarios are on top of each other in each figure, showing that the expected position of the
374 triggering correlation is insensitive to the estimated values of $\sigma_{ln(S)}$ and COV_Q . The $P_L = 15\%$ and
375 85% curves for the three scenarios in either figure show only small differences, indicating that the
376 total uncertainty in the triggering correlation is also relatively insensitive to the estimated values
377 of $\sigma_{ln(S)}$ and COV_Q . Comparing the solutions in Figs. 6 and 7, the use of $P_{min} = 0.05$ produced a
378 slightly lower median curve (about 5% lower) and smaller total uncertainty terms (e.g., the $P_L =$
379 15% and 85% curves are located closer together). Setting $P_{min} = 0.05$ rather than to 0.0 reduced
380 the influence of the two or three no-liquefaction data points located well above the expected
381 triggering correlation (Figs. 6 or 7), which is why the most likely triggering curve shifted down
382 slightly and the total uncertainty terms were reduced.

383 Solutions based on all the case histories are plotted together with the case history data: (1) in
384 Fig. 8 in terms of the total uncertainty, and (2) in Fig. 9 in terms of model uncertainty alone, which
385 means excluding the estimation errors in $CSR_{M=7.5, \sigma'=1atm}$ and q_{c1Ncs} . Curves for $P_L = 15\%$, 50% ,
386 and 85% are shown in both figures for $P_{min} = 0.075$ and the same $\sigma_{ln(S)}$ and COV_Q scenarios as
387 used above. The $P_L = 15\%$ and 85% curves based on total uncertainty (Fig. 8) are again insensitive
388 to the estimated values of $\sigma_{ln(S)}$ and COV_Q . The total uncertainties for these scenarios, as plotted
389 in Fig. 10, were similar because decreasing the assumed values for $\sigma_{ln(S)}$ and COV_Q was offset by
390 increases in the most likely values for $\sigma_{ln(R)}$; e.g., decreasing $\sigma_{ln(S)}$ and COV_Q from 0.2 to 0.1 caused
391 $\sigma_{ln(R)}$ to increase from 0.05 to 0.24 as listed in the legend of Fig. 10. The $P_L = 15\%$ and 85% curves
392 based on model uncertainty alone (Fig. 9) move progressively closer together with increasing

393 values of $\sigma_{\ln(S)}$ and COV_Q because this results in smaller values for the model uncertainty term
394 $\sigma_{\ln(R)}$. These results illustrate how the maximum likelihood analysis of the case history data
395 provides insight on the total uncertainty, but does not itself tightly constrain partitioning of that
396 uncertainty into the components of Q, S, and R.

397 The solutions based on case histories with FC based on laboratory test data alone (not via
398 correlation to I_c) were not significantly different from those based on all case history data (Figs. 8
399 and 9). The $P_L = 50\%$ curve was about 2% higher and the model uncertainty was slightly smaller.

400 Solutions were also obtained for those case histories with σ'_v greater than 40 kPa, which
401 excludes cases with representative depths less than about 2 m, where the K_σ and C_N relationships
402 are not as well defined and upper limits on their values have been imposed based on judgment and
403 other considerations. The solutions based on cases with σ'_v greater than 40 kPa, compared to those
404 obtained for all case history data, had 12-13% higher $P_L = 50\%$ curves and greater model
405 uncertainties ($\sigma_{\ln(R)}$ of 0.30-0.36 versus 0.05-0.24). This combination resulted in $P_L = 15\%$ curves
406 shifting downward by about 2% whereas the $P_L = 85\%$ curve shifted upward by about 28%.

407 The sensitivity of the solutions to other aspects of the analyses are described in Boulanger and
408 Idriss (2014), including the effects of alternative approaches to handling potential false negatives
409 and false positives, varying the weighting ratio $W_{\text{nonliquefied}}/W_{\text{liquefied}}$, and varying the limits on the
410 total uncertainty σ_T at high q_{c1Ncs} values. The effects of these other factors on the solutions were
411 generally smaller than those examined above.

412 ***Examination of data for potential biases***

413 Distributions of case history data relative to the triggering curves were examined for potential
414 biases with respect to the primary case history parameters. These distributions are illustrated in the
415 electronic Supplement as plots of the case history data across bins of varying FC, M, and σ'_v . These

416 examinations showed no evident biases with regard to these or other case history parameters.

417 The case history distributions, as shown in the Supplement, provide a basis for understanding
418 how various components of the analysis framework may or may not affect the triggering
419 correlation. For example, the C_N and K_σ parameters become less certain at confining stresses less
420 than about 30 or 40 kPa for a number of technical reasons, and thus their expressions include
421 imposed maximum values that are reached in this stress range. If those imposed maximum value
422 limits were increased, then the data points in the bin for $\sigma'_v \leq 0.4$ atm will move downward or to
423 the right. The reverse is true if the maximum limits were decreased. The position of the triggering
424 curve is, however, better constrained by the case history data for σ'_v greater than 0.4 atm and thus
425 these data are given more weight in determining the final correlation. The r_d parameter, on the
426 other hand, becomes more uncertain as the depth increases and thus variations in this parameter
427 only has significant effects on the data points for σ'_v greater than about 0.8 atm. The data for the
428 σ'_v bins of 0.8-1.2 atm and >1.2 atm are relatively limited and scattered, such that changes in the
429 r_d relationship had no significant effect on the final triggering correlation. In contrast, variations
430 in the MSF parameter were found to have a more significant effect on the triggering correlation
431 because it affected data across all bins. The revised MSF relationship improved the fit of the data
432 points across the various bins of M compared to the use of an MSF relationship that did not include
433 dependence on soil properties.

434 ***Recommended relationships***

435 Selecting the most appropriate values for C_o and $\sigma_{ln(R)}$ from these maximum likelihood solutions
436 involves subjective evaluation of the most appropriate partitioning of the total uncertainty in the
437 liquefaction case history database. This evaluation must also consider the limitations of the
438 statistical models and case history database, including uncertainties that are not explicitly

439 accounted for. Of the various analysis scenarios considered, the scenarios with $\sigma_{\ln(S)} = 0.20$, COV_Q
440 $= 0.20$, and $P_{\min} = 0.05-0.075$ are considered most realistic; e.g., $\sigma_{\ln(S)} = 0.10$, $COV_Q = 0.10$ and
441 $P_{\min} = 0.0$ are lower than would be reasonably estimated for these parameters based on available
442 literature as discussed previously. The solutions with larger $\sigma_{\ln(S)}$, COV_Q , and P_{\min} terms, however,
443 often gave model uncertainties that are smaller than seem reasonable. This apparent discrepancy
444 arises from limitations in the case history database, the analysis method, and the ability to define
445 parameter uncertainties accurately. Taking these factors into consideration, the results presented
446 herein are considered reasonable bounds of different interpretations, from which values of $C_o =$
447 2.60 and $\sigma_{\ln(R)} = 0.20$ are recommended as reasonable for use in forward calculations.

448 The liquefaction triggering correlation can then be expressed as,

$$449 \quad CRR_{M=7.5, \sigma'_v=1atm} = \exp\left(\frac{q_{c1Ncs}}{113} + \left(\frac{q_{c1Ncs}}{1000}\right)^2 - \left(\frac{q_{c1Ncs}}{140}\right)^3 + \left(\frac{q_{c1Ncs}}{137}\right)^4 - 2.60 + \varepsilon_{\ln(R)}\right) \quad (33)$$

450 where $\varepsilon_{\ln(R)}$ is normally distributed with a mean of 0.0 and a standard deviation of $\sigma_{\ln(R)} = 0.20$.

451 This expression can also be written as,

$$452 \quad CRR_{M=7.5, \sigma'_v=1atm} = \exp\left(\frac{q_{c1Ncs}}{113} + \left(\frac{q_{c1Ncs}}{1000}\right)^2 - \left(\frac{q_{c1Ncs}}{140}\right)^3 + \left(\frac{q_{c1Ncs}}{137}\right)^4 - 2.60 + \sigma_{\ln(R)} \cdot \Phi^{-1}(P_L)\right) \quad (34)$$

453 where Φ^{-1} is the inverse of the standard cumulative normal distribution, and P_L is the probability

454 of liquefaction. Alternatively, the conditional probability of liquefaction for known values of

455 $CSR_{M=7.5, \sigma'_v=1atm}$ and q_{c1Ncs} can be computed as,

$$456 \quad P_L(q_{c1Ncs}, CSR_{M=7.5, \sigma'_v=1atm}) = \Phi\left[\frac{\frac{q_{c1Ncs}}{113} + \left(\frac{q_{c1Ncs}}{1000}\right)^2 - \left(\frac{q_{c1Ncs}}{140}\right)^3 + \left(\frac{q_{c1Ncs}}{137}\right)^4 - 2.60 - \ln(CSR_{M=7.5, \sigma'_v=1atm})}{\sigma_{\ln(R)}}\right] \quad (35)$$

457 The recommended triggering curves for P_L equal to 15%, 50%, and 85% with model uncertainty

458 alone [i.e., conditional on known values of $CSR_{M=7.5, \sigma'=1atm}$ and q_{c1Ncs}] are plotted together with the
459 clean sand ($FC \leq 5\%$) case history data in Fig. 11a and the full case history database in Fig. 11b.
460 For deterministic analyses, it is recommended that the $P_L = 15\%$ curve be used (i.e., approximately
461 one standard deviation below the mean; Equation 34 with $\varepsilon_{ln(R)} = -0.20$).

462 The probabilistic triggering relationship expressed in Equations 33-35 is conditional on known
463 values for $CSR_{M=7.5, \sigma'=1atm}$ and q_{c1Ncs} values. Therefore, to assess the probability of liquefaction in
464 a hazard evaluation, the conditional probability of liquefaction provided by these equations needs
465 to be combined with the probabilities of the $CSR_{M=7.5, \sigma'=1atm}$ and q_{c1Ncs} values; i.e., the parameter
466 uncertainties. The uncertainties in estimating the latter parameters are often greater than the
467 uncertainty in the triggering model, such that the formal treatment of uncertainties in the seismic
468 hazard analysis and a detailed site characterization effort are generally more important to a
469 probabilistic liquefaction analysis than the uncertainty in the liquefaction triggering model.

470 For example, a probabilistic liquefaction hazard analysis can be structured to branch through
471 a range of seismic hazards (accounting for the majority of uncertainty in $CSR_{M=7.5, \sigma'=1atm}$ values)
472 and a range of site characterizations (accounting for the majority of the uncertainty in the q_{c1Ncs}
473 values) before it gets to the liquefaction triggering analysis. In that scenario, it may be reasonable
474 to only include model uncertainty in the liquefaction triggering analysis because the parameter
475 uncertainties were already accounted for in the previous branches of the analysis.

476 **Summary**

477 A probabilistic CPT-based liquefaction triggering correlation was developed using an updated case
478 history database and a maximum likelihood approach. The liquefaction analysis framework
479 followed that by Idriss and Boulanger (2008) and incorporated changes to the MSF relationship
480 and the procedures for estimating FC from the I_c index when site specific sampling and lab testing

481 data are not available. The revised correlation was shown to exhibit no apparent trends or biases
482 relative to the case history data with respect to FC, M, or σ'_v .

483 For analyses in the absence of site-specific lab testing data, it is suggested that liquefaction
484 analyses be repeated using a range of C_{FC} values (e.g., ± 0.15 or ± 0.29) to evaluate the sensitivity
485 to FC estimates and a range of I_c cut-off values for identifying clay-like soils (e.g., 2.4 versus 2.6)
486 to evaluate sensitivity to soil classification estimates. The results can be used to evaluate the
487 potential benefits of site-specific sampling and testing for a given project, while recognizing that
488 some amount of sampling and testing should be required for high risk/high consequence projects.

489 Measurement and estimation uncertainties in CSR and q_{c1Ncs} , the potential effects of false
490 positives and false negatives in the case history database, and the effects of the choice-based
491 sampling bias in the case history database were accounted for. The results of sensitivity analyses
492 showed that the position of the most likely triggering curve and the magnitude of the total error
493 term were well constrained by the data. The most likely value for the standard deviation of the
494 error term in the triggering correlation was, however, found to be dependent on the uncertainties
495 assigned to CSR and q_{c1Ncs} and the potential presence of false negatives and false positives in the
496 case history database. Despite this and other limitations, the results of the sensitivity study appear
497 to provide reasonable bounds on the effects of different interpretations. The probabilistic
498 relationship for liquefaction triggering proposed herein is considered a reasonable approximation
499 in view of these various findings.

500 Probabilistic liquefaction hazard analyses should consider the uncertainties in the seismic
501 hazard, the site characterization, and the liquefaction triggering model. The uncertainty in the
502 liquefaction triggering model is smaller than the uncertainty in the seismic hazard, and will often
503 be smaller than the uncertainty in the site characterization. For this reason, the seismic hazard

504 analysis and the site characterization efforts are often the more important components of any
505 probabilistic assessment of liquefaction hazards.

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518 **REFERENCES**

- 519 Abrahamson, N., Atkinson, G., Boore, D., Bozorgnia, Y., Campbell, K., Chiou, B., Idriss, I. M.,
520 Silva, W., and Youngs, R. (2008). "Comparisons of the NGA ground-motion relations."
521 *Earthquake Spectra*, 24(1), 45-66.
- 522 Ancheta, T. D., Darragh, R. B., Stewart, J. P., Seyhan, E., Silva, W. J., Chiou, B. S. J., Wooddell,
523 K. E., Graves, R. W., Kottke, A. R., Boore, D. M., Kishida, T., and Donahue, J. L. (2014).
524 "NGA-West 2 database." *Earthquake Spectra*, EERI, [dx.doi.org/10.1193/070913EQS197M](https://doi.org/10.1193/070913EQS197M).
- 525 Baecher, G. B., and Christian, J. T. (2003). *Reliability and Statistics in Geotechnical Engineering*.
526 John Wiley and Sons, Ltd., 619 pp.

527 Boulanger, R. W. (2003). High overburden stress effects in liquefaction analyses, *J. Geotechnical*
528 *and Geoenvironmental Eng.*, ASCE **129**(12), 1071–082.

529 Boulanger, R. W., and Idriss, I. M. (2012). "Probabilistic SPT-based liquefaction triggering
530 procedure." *J. Geotechnical and Geoenvironmental Eng.*, ASCE, 138(10), 1185-1195.

531 Boulanger, R. W., and Idriss, I. M. (2014). "CPT and SPT based liquefaction triggering
532 procedures." Report No. UCD/CGM-14/01, University of California, Davis, CA, 134 pp.

533 Boulanger, R. W., and Idriss, I. M. (2015). "Magnitude scaling factors in liquefaction triggering
534 procedures." *Soil Dynamics and Earthquake Engineering*, 10.1016/j.soildyn.2015.01.004.

535 Cetin, K. O., Der Kiureghian, A., Seed, R. B. (2002). "Probabilistic models for the initiation of
536 seismic soil liquefaction." *Structural Safety*, 24: 67-82.

537 Cetin, K. O., Seed, R., Der Kiureghian, A., Tokimatsu, K., Harder, L., Kayen, R., Moss, R. E. S.
538 (2004). Standard penetration test-based probabilistic and deterministic assessment of seismic
539 soil liquefaction potential, *J. Geotechnical and Geoenviron. Eng.*, ASCE **130**(12), 1314–340.

540 Christian, J. T., and Swiger, W. F. (1975). "Statistics of liquefaction and SPT results." *Journal of*
541 *Geotechnical Engineering Div.*, ASCE, 101(GT11), 1135-1150.

542 Cox, B. R., et al. (2013). "Liquefaction at strong motion stations and in Urayasu City during the
543 2011 Tohoku-Oki earthquake." *Earthquake Spectra*, EERI, 29(S1), S55-S80.

544 Engdahl, E. R., and Villasenor, A. (2002). "Global seismicity: 1900-1999." *Intl Handbook of*
545 *Earthquake and Engineering Seismology*, Committee on Education, Vol. 81A, 665-690.

546 Green, R. A., Cubrinovski, M., Cox, B., Wood, C., Wotherspoon, L., Bradley, B., and Mauer, B.
547 (2014). "Select liquefaction case histories from the 2010-2011 Canterbury earthquake
548 sequence." *Earthquake Spectra*, EERI, <http://dx.doi.org/10.1193/030713EQS066M>.

549 Idriss, I. M. (1999). An update to the Seed-Idriss simplified procedure for evaluating liquefaction

550 potential, *Workshop on New Approaches to Liquefaction*, Publication No. FHWA-RD-99-165.

551 Idriss, I. M., and Boulanger, R. W. (2004). Semi-empirical procedures for evaluating liquefaction
552 potential during earthquakes, *11th Intl Conf Soil Dynamics and Earthquake Engrg, and 3rd Intl*
553 *Conf Earthquake Geotechnical Engrg*, D. Doolin et al., eds., Stallion Press, Vol. 1, pp. 32–56.

554 Idriss, I. M., and Boulanger, R. W. (2008). *Soil liquefaction during earthquakes*. Monograph
555 MNO-12, Earthquake Engineering Research Institute, Oakland, CA, 261 pp.

556 Idriss, I. M., and Boulanger, R. W. (2010). "SPT-based liquefaction triggering procedures." Report
557 UCD/CGM-10/02, Center for Geotechnical Modeling, Univ. of California, Davis, CA, 259 pp.

558 Juang, C. H., Jiang, T., and Andrus, R. D. (2002). "Assessing probability-based methods for
559 liquefaction potential evaluation." *J. Geotechnical and Geoenv. Eng.*, ASCE, 128(7), 580-589.

560 Kulhawy, F. H. and Trautmann, C. H. (1996). "Estimation of in-situ test uncertainty." *Uncertainty*
561 *in the Geologic Environment*, Madison, WI, ASCE: 269–286.

562 Liao, S. S. C., and Lum, K. Y. (1998). "Statistical analysis and application of the magnitude scaling
563 factor in liquefaction analysis." *Geotech. Earthquake Eng. and Soil Dyn. III*, ASCE, 1:410–21.

564 Liao, S. S. C., Veneziano, D., and Whitman, R. V. (1988). "Regression models for evaluating
565 liquefaction probability." *J. Geotech. Eng. Div.*, ASCE, 114(4), 389–411.

566 Manski, C. F., and Lerman, S. R. (1977). "The estimation of choice probabilities from choice-
567 based samples." *Econometrica*, 45(8): 1977-1998.

568 Mitchell, J. K., and Tseng, D. J. (1990). "Assessment of liquefaction potential by cone penetration
569 resistance", *Proc., H. Bolton Seed Memorial Symposium*, BiTech Publishers, Vol. 2, 335-350.

570 Montgomery, J., Boulanger, R. W., and Harder, L. F., Jr. (2014). "Examination of the K_{σ}
571 overburden correction factor on liquefaction resistance." *J. Geotechnical and*
572 *Geoenvironmental Eng.*, ASCE, dx.doi.org/10.1061/(ASCE)GT.1943-5606.0001172.

573 Moss, R. E. S., Seed, R. B., Kayen, R. E., Stewart, J. P., Youd, T. L., and Tokimatsu, K. (2003).
574 "Field case histories for CPT-based in situ liquefaction potential evaluation." *Geoengineering*
575 *Research Rep. UCB/GE-2003/04*, University of California, Berkeley.

576 Moss, R. E. S., Seed, R. B., Kayen, R. E., Stewart, J. P., Der Kiureghian, A., and Cetin, K. O.
577 (2006). CPT-based probabilistic and deterministic assessment of in situ seismic soil
578 liquefaction potential, *J. Geotechnical and Geoenvironmental Eng.*, ASCE **132**(8), 1032–051.

579 Nguyen, T., Shao, L., Gingery, J., Robertson, P. (2014). "Proposed modification to CPT-based
580 liquefaction method for post-vibratory ground improvement." *Geo-Congress 2014*, 1120-1132.

581 PEER (2000a). "Documenting incidents of ground failure resulting from the Aug. 17, 1999,
582 Kocaeli, Turkey Earthquake." peer.berkeley.edu/publications/turkey/adapazari/ (9/2013).

583 PEER (2000b). "Documentation of soil conditions at liquefaction sites from 1999 Chi-Chi, Taiwan
584 Earthquake." peer.berkeley.edu/lifelines/research_projects/3A02/ (last accessed 9/2013).

585 Phoon, K. K. and Kulhawy, F. H. (1999). "Characterization of geotechnical variability." *Canadian*
586 *Geotechnical Journal* 36(4): 612–624.

587 Robertson, P. K., and Wride, C. E. (1998). "Evaluating cyclic liquefaction potential using the cone
588 penetration test." *Canadian Geotechnical J.* 35(3), 442–59.

589 Robinson, K., Cubrinovski, M., and Bradley, B.A. (2013). "Comparison of actual and predicted
590 measurements of liquefaction-induced lateral displacements from the 2010 Darfield and 2011
591 Christchurch Earthquakes." *Proc. 2013 Conference of the New Zealand Society for Earthquake*
592 *Engineering (NZSEE 2013)*, Wellington, New Zealand, 26-28 April.

593 Sancio, R. B. (2003). *Ground failure and building performance in Adapazari, Turkey*. Ph.D. thesis,
594 University of California, Berkeley, 790 pp.

595 Suzuki, Y., Tokimatsu, K., Taya, Y., and Kubota, Y. (1995). "Correlation between CPT data and

596 dynamic properties of in situ frozen samples." Proc., 3rd International Conference on Recent
597 Advances in Geotechnical Earthquake Engineering and Soil Dynamics, Vol. I, St. Louis, MO.
598 Suzuki Y., Sanematsu, T., and Tokimatsu, K. (1998). "Correlation between SPT and seismic CPT."
599 Proc., Conference on Geotechnical Site Characterization, Balkema, Rotterdam, pp. 1375–380.
600 Tokimatsu, K., Tamura, S., Suzuki, H., and Katsumata, K. (2012). "Building damage associated
601 with geotechnical problems in the 2011 Tohoku Pacific Earthquake." Soils and Foundations,
602 Japanese Geotechnical Society, 52(5): 956-974.
603 Toprak, S., Holzer, T. L., Bennett, M. J., Tinsley, J. C. (1999). "CPT- and SPT-based probabilistic
604 assessment of liquefaction potential." Report MCEER-99-0019, MCEER, NY.
605 Van Ballegooy, S., Malan, P., Lacrosse, V., Jacka, M. W., Cubrinovski, M., Bray, J. D., O'Rourke,
606 T. D., Crawford, S. A., and Cowan, H. (2014). "Assessment of liquefaction-induced land
607 damage for residential Christchurch." Earthquake Spectra, EERI, in-press.
608 Worden, C. B., Wald, D. J., Allen, T. I., Lin, K., and Cua, G. (2010). "Integration of macroseismic
609 and strong-motion earthquake data in ShakeMap for real-time and historic earthquake analysis."
610 USGS web site, <http://earthquake.usgs.gov/earthquakes/shakemap/>.
611 Youd, T. L, and Noble, S. K. (1997). "Liquefaction criteria based on statistical and probabilistic
612 analyses." Report NCEER-97-022, NCEER, 201–205.

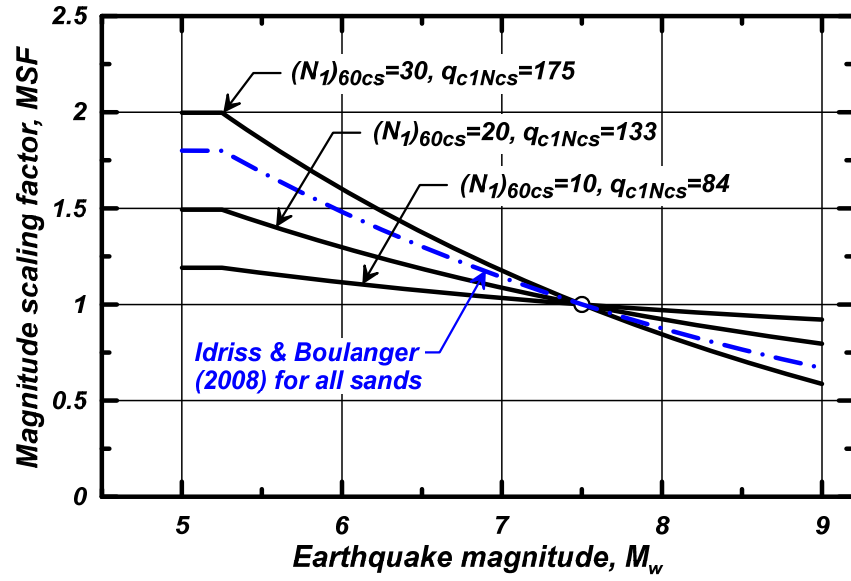


Fig. 1. Variation in the MSF relationship with q_{c1Ncs} and with $(N_1)_{60cs}$ for cohesionless soils

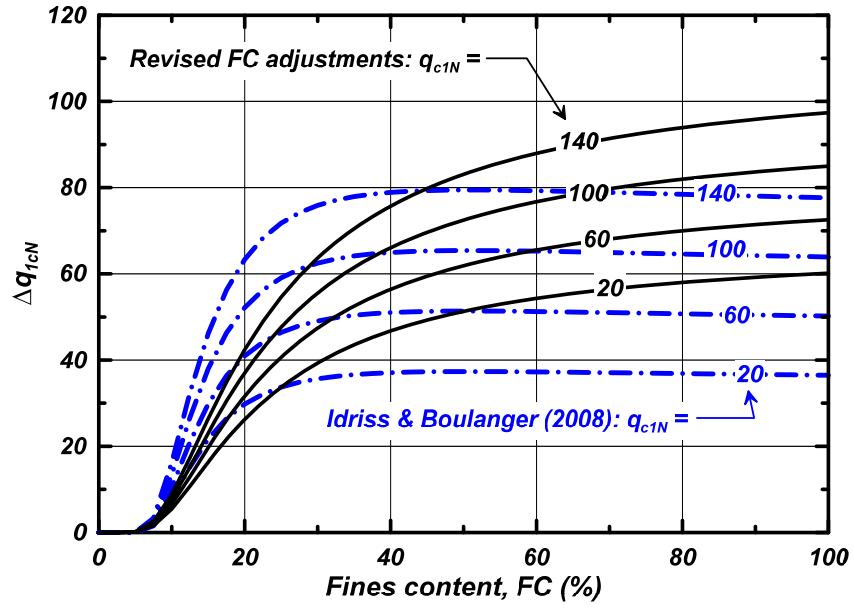


Fig. 2. Equivalent clean sand adjustments for CPT-based liquefaction triggering procedures

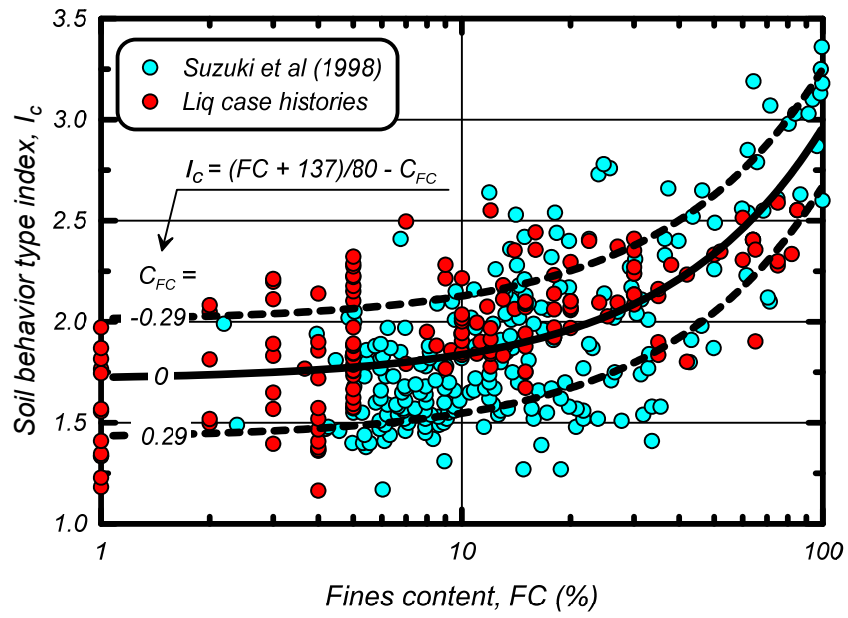


Fig. 3. Recommended correlation between I_c and FC with plus or minus one standard deviation against the dataset by Suzuki et al. (1998) and the liquefaction database

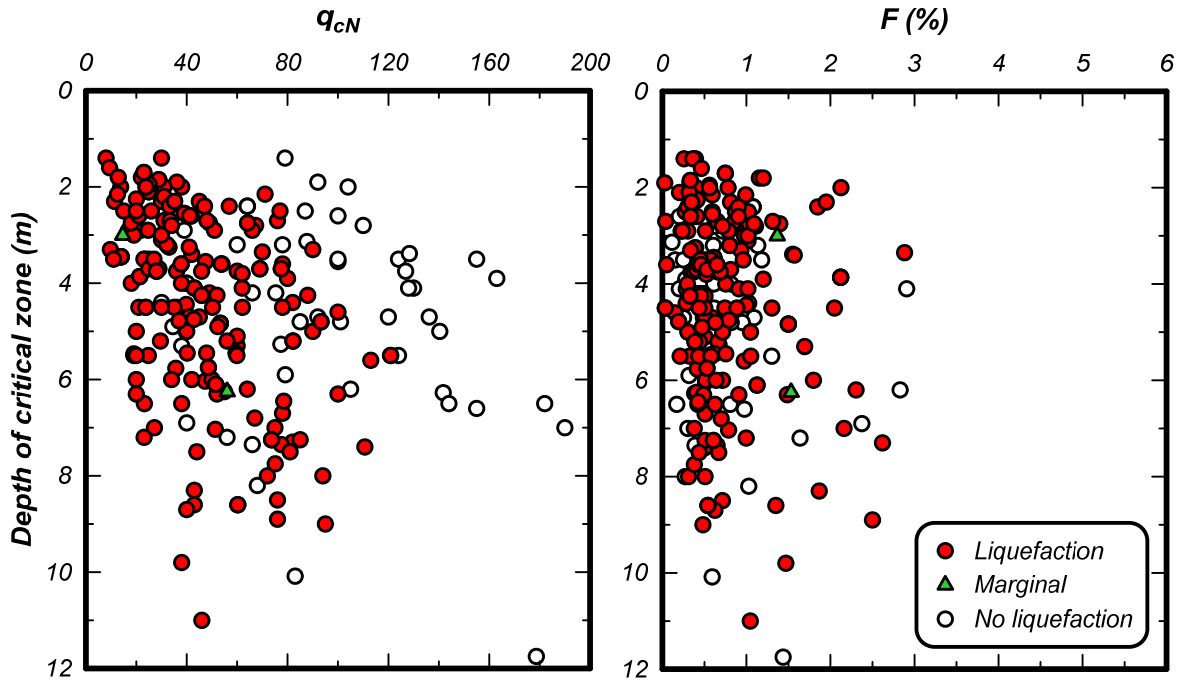


Fig. 4. Distributions of q_{cN} and F versus the representative depth of the critical zone

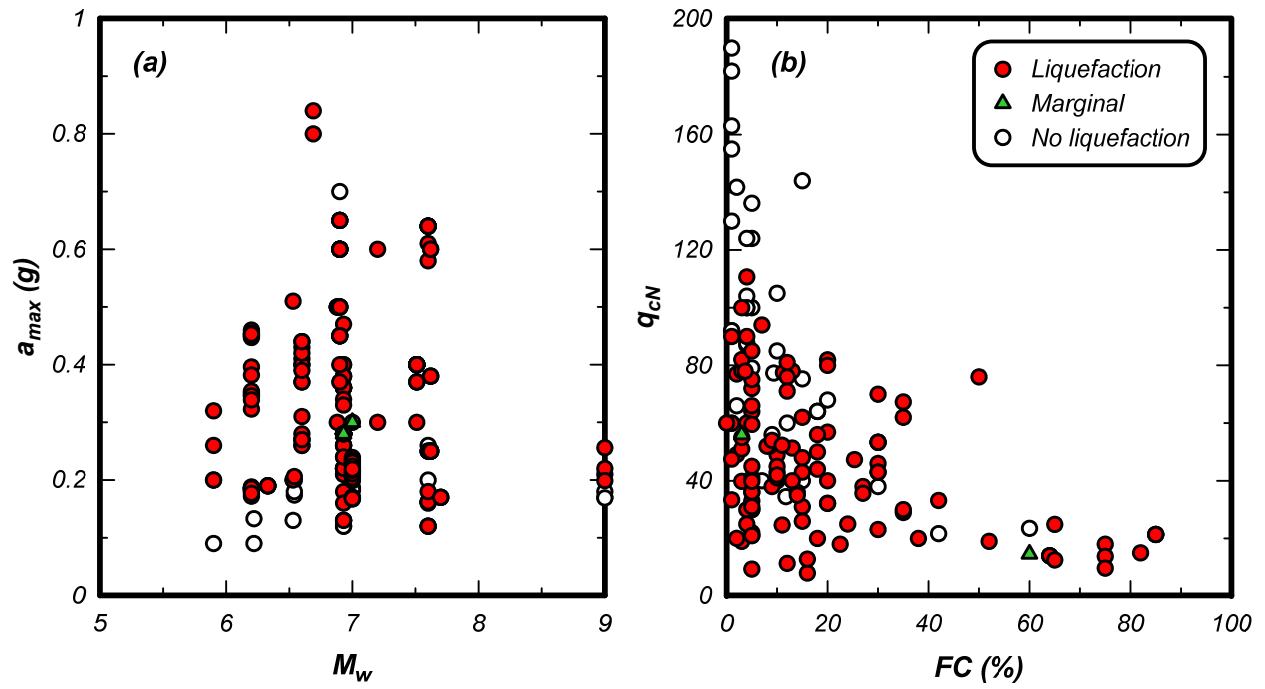


Fig. 5. Distributions of: (a) a_{max} versus M and (b) q_{cN} versus FC for cases with FC determined by laboratory testing

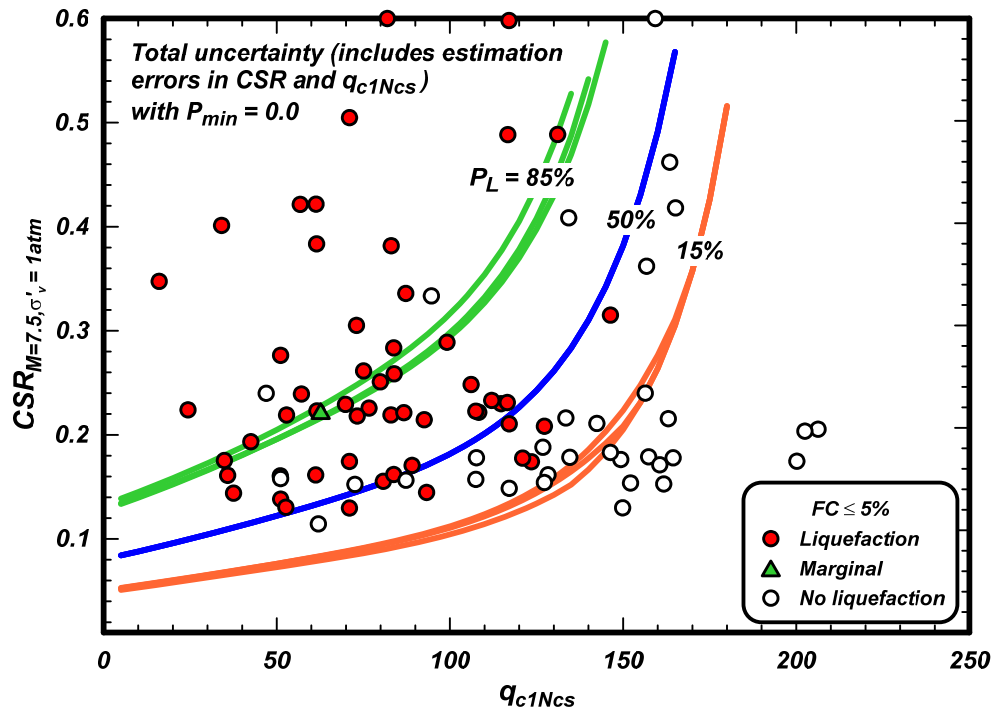


Fig. 6. $CRR_{M=7.5, \sigma'_v=1atm}$ versus q_{c1Ncs} for $PL = 15, 50,$ and 85% in clean sands with inclusion of estimation errors in $CSR_{M=7.5, \sigma'_v=1atm}$ and q_{c1Ncs} and using $P_{min} = 0.0$

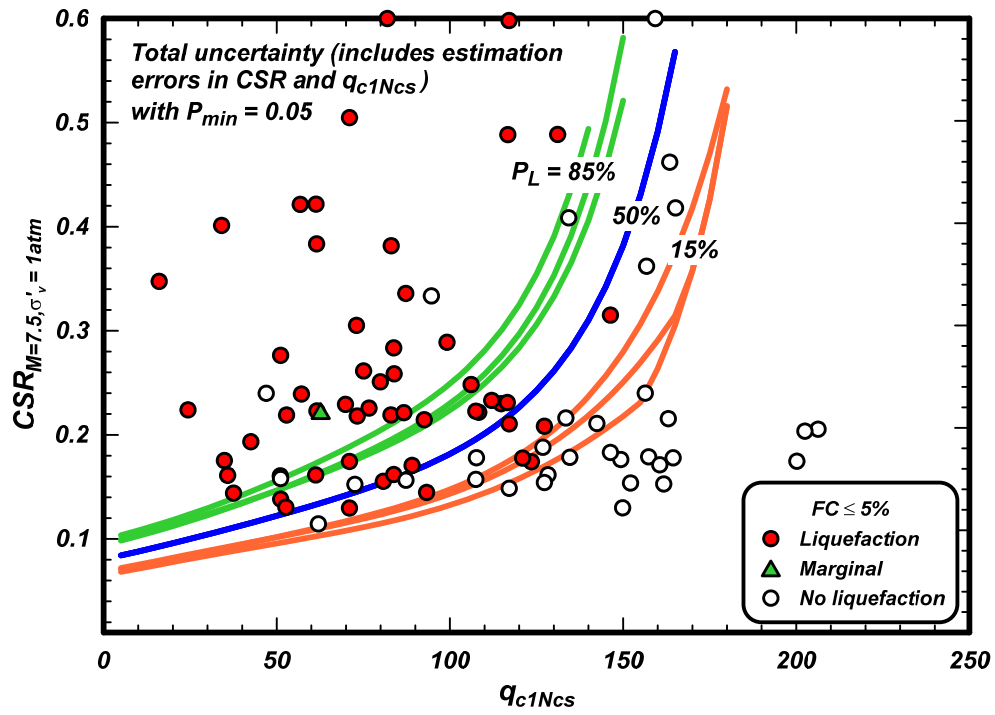


Fig. 7. $CRR_{M=7.5, \sigma'_v=1atm}$ versus q_{c1Ncs} for $P_L = 15, 50,$ and 85% in clean sands with inclusion of estimation errors in $CSR_{M=7.5, \sigma'_v=1atm}$ and q_{c1Ncs} and using $P_{min} = 0.05$

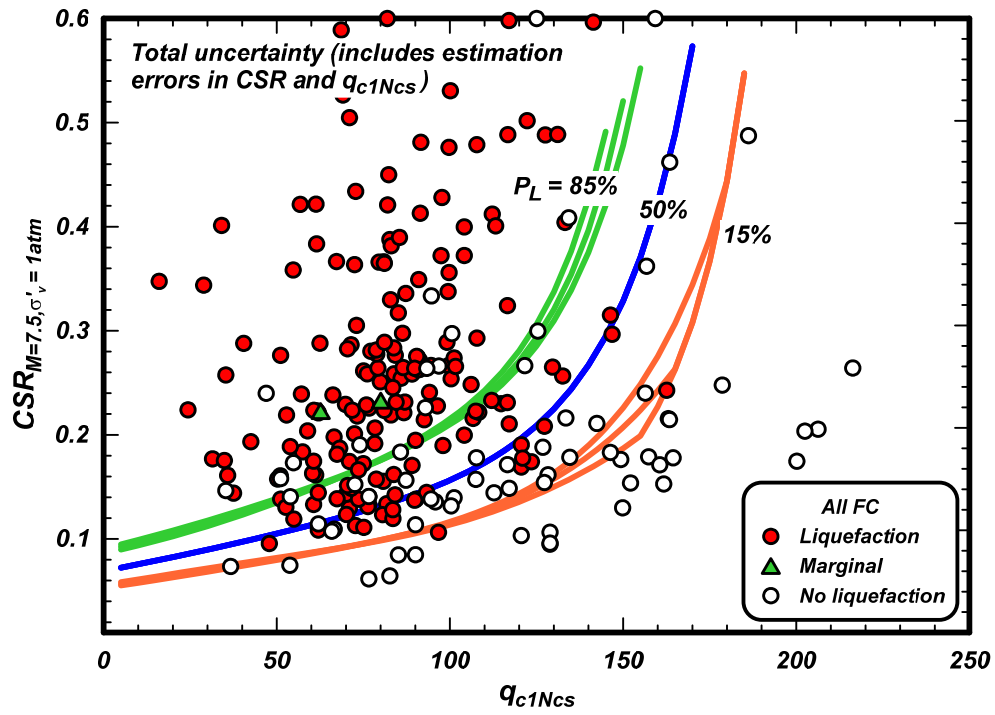


Fig. 8. $CRR_{M=7.5, \sigma'_v=1atm}$ versus q_{c1Ncs} for $P_L = 15, 50,$ and 85% for all sands with inclusion of estimation errors in $CSR_{M=7.5, \sigma'_v=1atm}$ and q_{c1Ncs} with $P_{min} = 0.075$

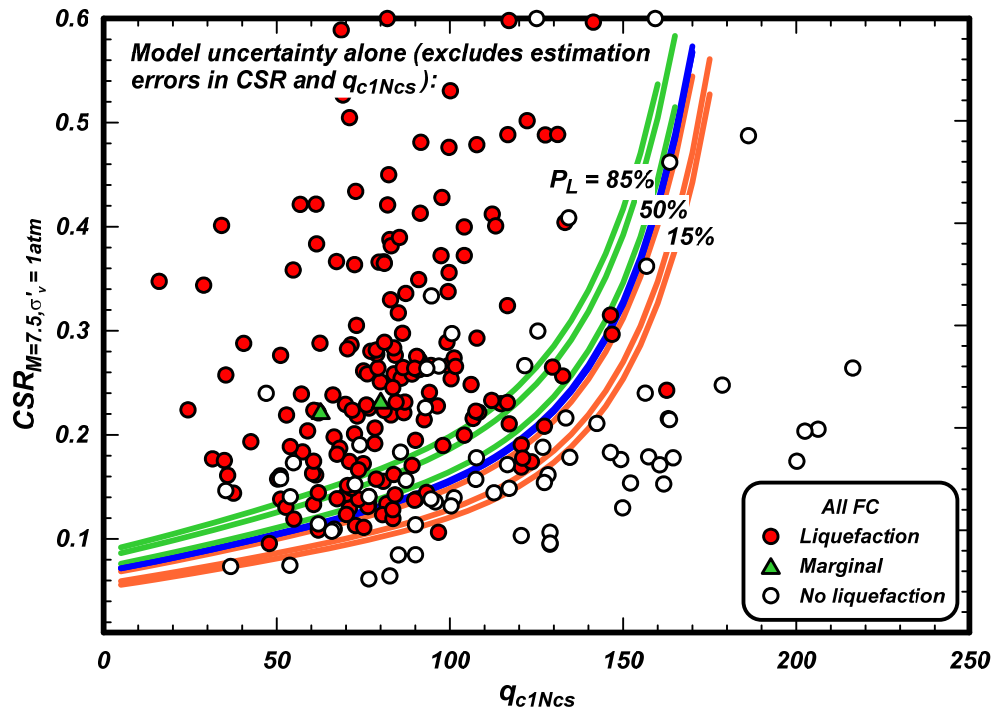


Fig. 9. $CRR_{M=7.5, \sigma'_v=1atm}$ versus q_{c1Ncs} for $P_L = 15, 50,$ and 85% for all sands with model uncertainty alone (excluding estimation errors in $CSR_{M=7.5, \sigma'_v=1atm}$ and q_{c1Ncs}) with $P_{min} = 0.075$

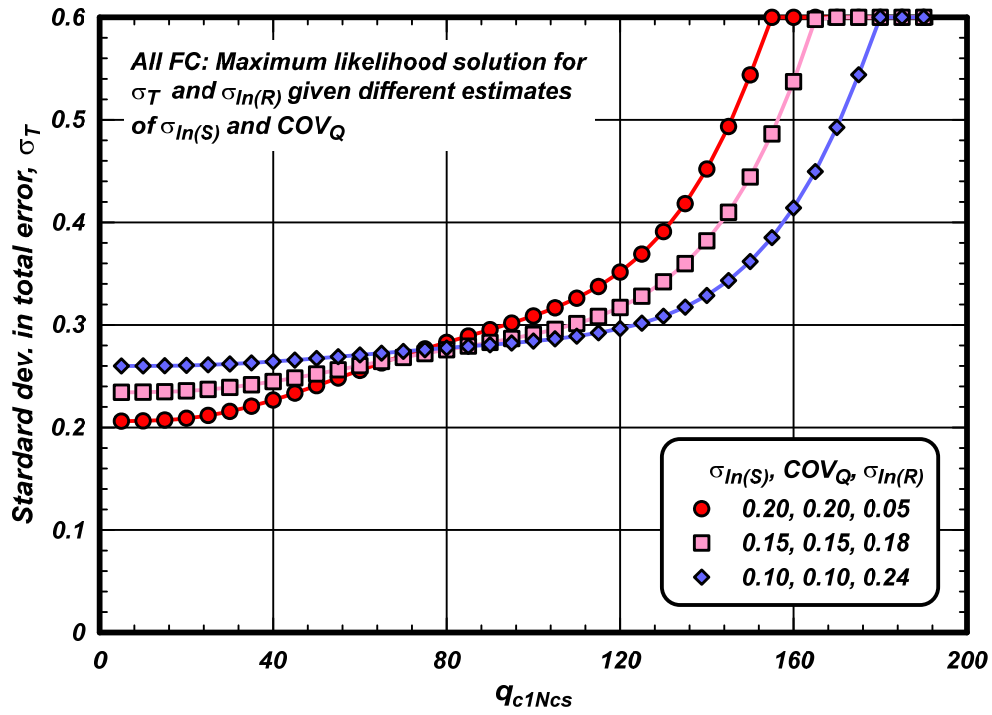
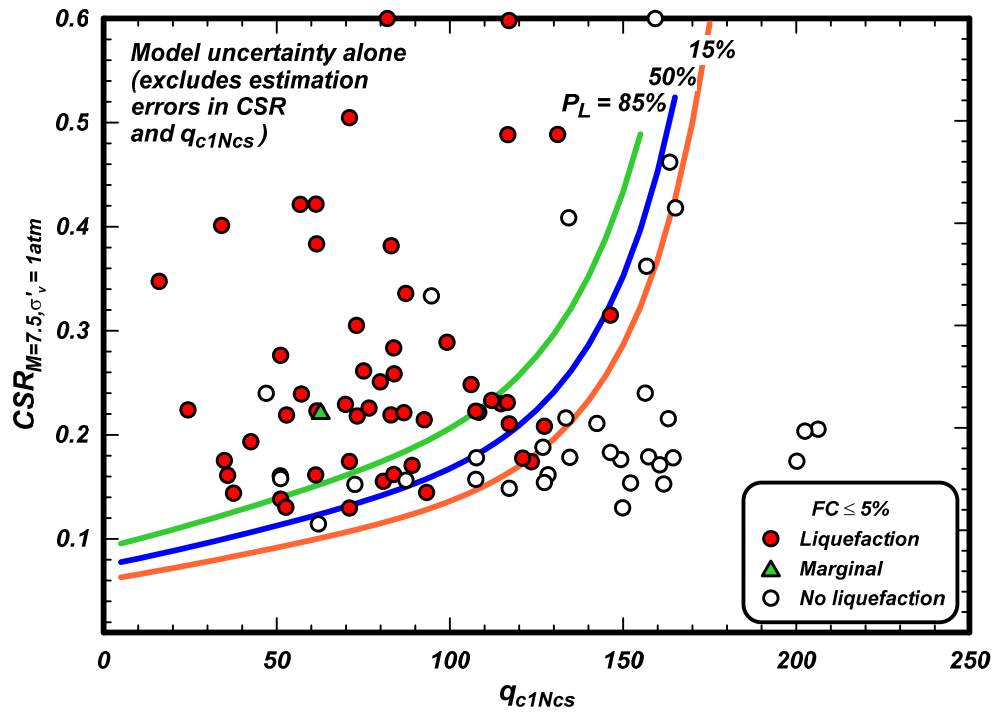
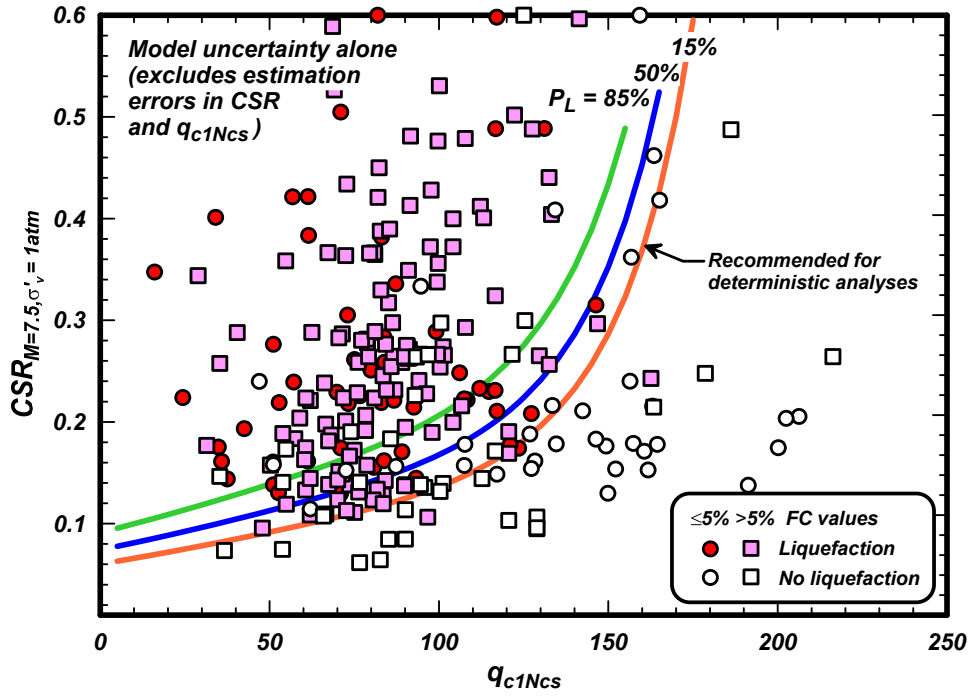


Fig. 10. Standard deviation in the total error term (σ_T) and CRR relationship ($\sigma_{ln(R)}$) for different estimates of $\sigma_{ln(S)}$ and COV_Q in any FC sand with $P_{min} = 0.075$



(a)

Fig. 11. Curves of $CRR_{M=7.5, \sigma'_v=1atm}$ versus q_{c1Ncs} for probabilities of liquefaction of 15%, 50%, and 85%: (a) clean sands, (b) all sands



(b)

Fig. 11. Curves of $CRR_{M=7.5, \sigma'_v=1atm}$ versus q_{c1Ncs} for probabilities of liquefaction of 15%, 50%, and 85%: (a) clean sands, (b) all sands