Journal of Solid Mechanics and Materials Engineering

Vol.1, No.7, 2007

Crack Identification in Beam Using Genetic Algorithm and Three Dimensional p-FEM*

Tadashi HORIBE** and Kuniaki TAKAHASHI***

** Department of Mechanical Engineering, Ibaraki University
4–12–1, Nakanarusawa-cho, Hitachi-shi, Ibaraki 316–8511, Japan
E-mail: horibe@mx.ibaraki.ac.jp

*** Fuji Heavy Industries Ltd.
1–1, Subaru-cho, Ohta-shi, Gunma 373–8555, Japan

Abstract

In this paper, a method for identification of a crack in a beam is demonstrated by the use of the genetic algorithm (GA) based on changes in natural frequencies. To calculate the natural frequencies of cracked beams, p-FEM code, which is based on a parametric three dimensional finite element, is developed because the accuracy of the forward analysis is important. In the analysis, an edge crack model is considered. To identify the crack location and the depth from frequency measurements, crack parameters of the beam are coded into a fixed-length binary digit string. By using GA, the square sum of residuals between the measured data and the calculated data is minimized in the identification process and thus the crack is identified. To avoid a high calculation cost, a response surface method (RSM) is also adopted in the minimizing process. The combination of GA and RSM have made the identification more effective and robust. The availability of the proposed method is confirmed by the results of numerical simulation.

Key words : Computational Mechanics, Inverse Problem, Crack Identification, Numerical Analysis, p-FEM, Structural Analysis

1. Introduction

The static or dynamic behaviour of cracked structures has been the subject of much research works because it may cause unexpected failures or accidents. Therefore the preemptive detection of cracks in structures is crucially important for maintaining their safety.

Some of techniques to detect a crack by measuring dynamic quantities, eg. natural frequency, mode shape, have been developed by many researchers. Rizos et al.(1) proposed a crack detection method through measurements of the dynamic response in the bending of a beam. Narkis(2) developed an identification method for a simply supported beam using axial and bending frequencies. Hasen(3) applied the Narkis's technique to a beam on an elastic foundation. Nikolakopoulos et al.(5) used a procedure with natural frequencies to find the crack location and the depth in the frame structure. Horibe et al.(6)(7)(8) presented a method to identify a crack using genetic algorithm (GA). Kim et al.(10) discussed a frequency-based method and a mode-shape-based method in detection of the damage. Douka et al.(11) employed the wavelet analysis to detect a crack in a plate. All these studies mainly dealt with a crack in a beam or a plate structure, whereas few studies have addressed a crack in a three dimensional structure. In this study, we present an inverse analysis method for a cracked beam using the three dimensional parametric finite element method (p-FEM)(12). The identification method for the beam follows the previously proposed method(6)(7)(8). First, an element stiffness matrix and a consistent mass matrix for three dimensional p-FEM are derived, and a finite element analysis program is developed. Then, the inverse analysis of a crack in a beam is performed using both three dimensional p-FEM program and GA. In the process of the inverse analysis, the natural frequency data generated by p-FEM program are used as search data. Finally, sev-

*Received 6 Mar., 2007 (No. 07-0126)
[DOI: 10.1299/jmmp.1.886]

Vol.1, No.7, 2007

eral examples of numerical analysis are shown and an applicability of the present method to a three dimensional cracked beam is discussed.

2. Vibration Analysis Using FEM

2.1. Three Dimensional p-FEM

Present research is based on the shifts in natural frequencies of a cracked beam. The approach requires precise measurement and hence we need to employ a more accurate calculation method to evaluate the natural frequencies in a forward analysis. Using p-FEM, we can obtain accurate results without increasing the DOFs (degrees of freedom) of the problem considered.

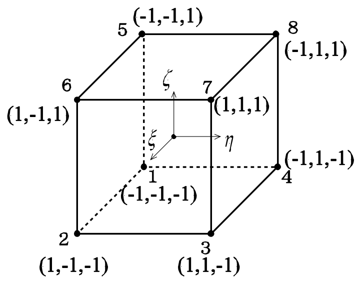

Fig. 1 Eight-node hexahedral element

As shown in Fig.1, we use an eight-node solid hexahedral element for a cracked beam. In this element, each node is assumed to have three degrees of freedom: displacements shown as u_i, v_i, w_i ($i = 1 \sim 8$) in the x, y and z directions.

Using these values of nodal displacements and the idea of p-FEM(12), the displacements within the element are given as

$$\begin{aligned}
u &= \sum_{i=1}^{8} N_i u_i + \alpha_1(1-\xi^2) + \alpha_2(1-\eta^2) + \alpha_3(1-\zeta^2),\\
v &= \sum_{i=1}^{8} N_i v_i + \beta_1(1-\xi^2) + \beta_2(1-\eta^2) + \beta_3(1-\zeta^2),\\
w &= \sum_{i=1}^{8} N_i w_i + \gamma_1(1-\xi^2) + \gamma_2(1-\eta^2) + \gamma_3(1-\zeta^2),
\end{aligned} \tag{1}$$

where, N_i ($i = 1 \sim 8$) are interpolation functions and they are expressed as

$$\begin{aligned}
N_1 &= \frac{1}{8}(1-\xi)(1-\eta)(1-\zeta), & N_2 &= \frac{1}{8}(1+\xi)(1-\eta)(1-\zeta),\\
N_3 &= \frac{1}{8}(1+\xi)(1+\eta)(1-\zeta), & N_4 &= \frac{1}{8}(1-\xi)(1+\eta)(1-\zeta),\\
N_5 &= \frac{1}{8}(1-\xi)(1-\eta)(1+\zeta), & N_6 &= \frac{1}{8}(1+\xi)(1-\eta)(1+\zeta),\\
N_7 &= \frac{1}{8}(1+\xi)(1+\eta)(1+\zeta), & N_8 &= \frac{1}{8}(1-\xi)(1+\eta)(1+\zeta).
\end{aligned} \tag{2}$$

Additional terms in Eq.(1), which include $\alpha_1 \sim \alpha_3$, $\beta_1 \sim \beta_3$ and $\gamma_1 \sim \gamma_3$, are slave degrees of freedom, whereas nodal displacements u_i, v_i, w_i are master degrees of freedom. The number of physical degrees of freedom of the element can be reduced by the following static condensation method, which was proposed by Guyan(13). The implementation of p-FEM not only reduces the time and the memory required, but also gives more accurate results.

Partitioning the element stiffness matrix into master and slave DOFs, one can obtain:

$$\begin{bmatrix} [\boldsymbol{K}]_{aa} & [\boldsymbol{K}]_{ab} \\ [\boldsymbol{K}]_{ba} & [\boldsymbol{K}]_{bb} \end{bmatrix} \begin{Bmatrix} \{\boldsymbol{u}\} \\ \{\alpha\} \end{Bmatrix} = \begin{Bmatrix} \{\boldsymbol{f}\} \\ \{\boldsymbol{0}\} \end{Bmatrix}, \tag{3}$$

where:

$\{\boldsymbol{u}\}$: vector of the displacement of the analysis set is to be retained,
$\{\boldsymbol{\alpha}\}$: vector of the displacement of the omitted set is to be eliminated,
$\{\boldsymbol{f}\}$: vector of the nodal force of the analysis set is to be retained,
$[K]$: stiffness matrices.

The partitioned stiffness matrices are represented by the following matrix product.

$$[\boldsymbol{K}_{aa}] = \iiint_V [\boldsymbol{B}]^t[\boldsymbol{D}][\boldsymbol{B}]dV,\ \ [\boldsymbol{K}_{ab}] = \iiint_V [\boldsymbol{B}]^t[\boldsymbol{D}][\bar{\boldsymbol{B}}]dV,$$
$$[\boldsymbol{K}_{ba}] = \iiint_V [\bar{\boldsymbol{B}}]^t[\boldsymbol{D}][\boldsymbol{B}]dV,\ \ [\boldsymbol{K}_{bb}] = \iiint_V [\bar{\boldsymbol{B}}]^t[\boldsymbol{D}][\bar{\boldsymbol{B}}]dV, \tag{4}$$

where, $[\boldsymbol{B}]$ is the usual strain-displacement matrix, $[\bar{\boldsymbol{B}}]$ is the strain-displacement matrix originating from the additional terms for the displacements and $[\boldsymbol{D}]$ is the three dimensional elasticity matrix.

Elimination of the slave parameters $\{\boldsymbol{\alpha}\}$ in Eq.(3) leads to

$$[\bar{\boldsymbol{K}}]\{\boldsymbol{u}\} = \{\boldsymbol{f}\}, \tag{5}$$

where

$$[\bar{\boldsymbol{K}}] = [\boldsymbol{K}_{aa}] - [\boldsymbol{K}_{ab}][\boldsymbol{K}_{bb}]^{-1}[\boldsymbol{K}_{ab}]. \tag{6}$$

The reduced element stiffness matrix expressed by Eq.(6) has 24 DOFs, which is the same as that of the usual hexahedral element.

Using the interpolation function $\lfloor N_i \rfloor$ in Eq.(2), a consistent mass matrix $[M]$ can be deduced as

$$[M_e] = \rho \iiint_V \lfloor N_i \rfloor^t \lfloor N_i \rfloor dxdydz = \rho \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \lfloor N_i \rfloor^t \lfloor N_i \rfloor |J| d\xi d\eta d\zeta, \tag{7}$$

where ρ is the density of the material and J is Jacobian[14]. Therefore, in order to calculate the natural frequency ω of the cracked beam, we must solve the following conventional characteristic equation, which is constructed by superposing individual element stiffness and consistent mass matrices:

$$([K] - \omega^2[M])\{\delta_i\} = 0, \tag{8}$$

where $[K]$, $[M]$ and $\{\delta_i\}$ represent the global stiffness matrix, the global mass matrix and the global nodal displacement vector, respectively. In this study, employing the subspace iteration method for the eigenvalue problem expressed by Eq.(8), we developed a three dimensional p-FEM code, written in Visual C++.

2.2. Automatic Mesh and Crack Generation

The inverse analysis method proposed herein makes use of natural frequencies of a beam computed by p-FEM program. As detailed in the following section 3.3, several values of the natural frequencies need to be determined under various crack parameters (a crack position and a crack depth) when we employ GA. Therefore, numerous natural frequency data on three dimensional grid points that represent any crack parameters need to be analyzed in advance.

In order to execute the finite element analysis of a beam having a crack with arbitrary size at an arbitrary position, an automatic hexahedral mesh generator was made by piling the hexahedral blocks on top of another one. An example of hexahedral mesh of a beam, which is generated by the mesh generator, is shown in Fig. 2.

An edge crack was generated using a procedure in which new nodes are created near the original nodes so as to make a small gap between the neighboring elements. Figure 3 shows an edge crack, which was generated in accordance with this procedure. Using input data generated in this way, natural frequencies of a cracked beam were analyzed based on p-FEM as described in section 2.1.

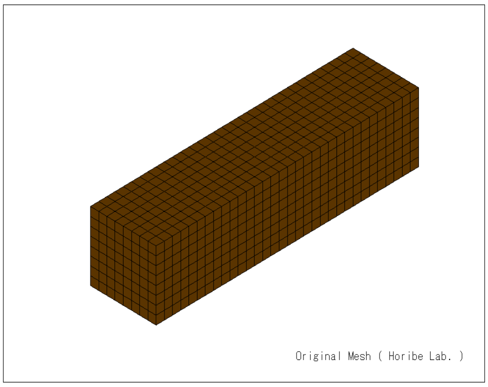

Fig. 2 An Example of hexahedral mesh of a beam

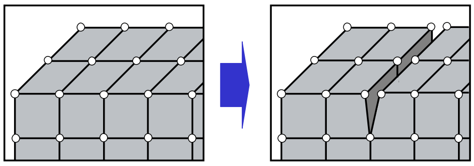

Fig. 3 Crack generation method in hexahedral element

3. Crack Identification Using Genetic Algorithm and Response Surface Method

The inverse problem considered in this paper is to estimate the crack parameters based on the measurement of various order natural frequencies. Consecutive order of natural frequencies of a cracked beam were adopted as observation data for the crack estimation in the present analysis. GA, which requires function values only and has the advantage of being able to deal with multiple peaks, was adopted as an estimation technique based on the results of our previous study [(6)(7)]. For the sake of simplicity, natural frequency values obtained by a forward analysis based on the p-FEM were used as the measured values $\bar{f}_i$ (in Eq.(9) described later) for the natural frequencies of the cracked beam.

3.1. Crack Search Procedure Using GA

In this section, we describe our crack search procedure. As shown in Fig.4, only edge cracks oriented parallel to the y-axis, which have a larger effect on bending vibration, are considered in this study.

GA is a randomized algorithm based on Darwin's theory of survival of the fittest. GA, which is different from optimization algorithms based on gradient information, requires only functional value. This advantage is extremely useful for the present study because it is difficult to calculate the differential of the objective function, which is shown later by Eq.(10).

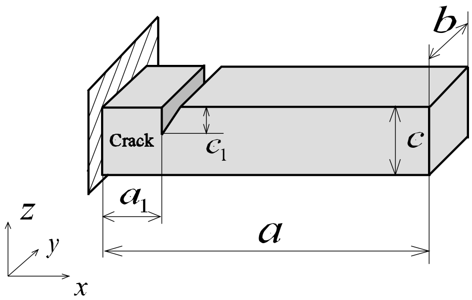

Fig. 4 Schematic diagram of the cracked Beam

First, crack position a_1/a and crack depth c_1/c are set as unknown parameters for crack search (see Fig.5). Each parameter is expressed as an 8-bit binary string of "1s" and "0s", and a 16-bit string is provided to each individual as genetic information.

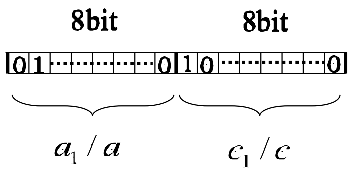

Fig. 5 Binary string representation of unknown parameters $a_1/a, c_1/c$

This means that each individual has crack parameters. Then, various operations based on the GA, such as crossover, selection and mutation, are repeatedly performed among these individuals over a certain number of generations. In this study, an algorithm that gradually decreases its mutation rate with increasing number of generations(15) was employed. The initial mutation rate was set at 0.9 and then gradually decreased. When the elapsed number of generations was half of a given number of generations, the mutation rate had declined to 0.01. The mutation rate was subsequently held constant. This technique enabled us to narrow down a solution with diversity to a true solution.

3.2. Error Evaluation Function and Fitness Value

In this study, we must minimize the difference between the exact and the approximate natural frequencies. By introducing following notation,

f_i: Natural frequency of i-th order on a estimated crack position,

$\bar{f}_i$: Natural frequency of i-th order on a true crack position,

n : Number of natural frequency to detect a crack position and a crack size,

error evaluation function E_n is constracted as Eq.(9):

$$E_n = \frac{1}{n}\sqrt{\sum_{i=1}^{n}\left(\frac{f_i}{\bar{f}_i} - 1\right)^2}. \tag{9}$$

Therefore, the error evaluation function E_n will reach its minimum value at the true crack position and size. The reciprocal of Eq.(9) was defined as fitness:

$$fitness = \frac{1}{\frac{1}{n}\sqrt{\sum_{i=1}^{n}\left(\frac{f_i}{\bar{f}_i} - 1\right)^2}}. \tag{10}$$

The fitness function will reach its maximum value at the true crack position and size.

3.3. Approximation of Natural Frequency

To estimate the crack parameters by applying GA described above, we must now compute the natural frequencies using p-FEM. However, this method requires eigenvalue analysis by p-FEM for each individual and if we employ it in this way, it would require an enormous time of calculation.

In our study we focus on the fact that natural frequency is a function of two variables: crack position x $(= c_1/c)$ and crack size y $(= a_1/a)$. Based on these natural frequency values, the natural frequency at any point is evaluated at given parameters using the following spline interpolation function.

First, we calculate the i-th natural frequencies $(i = 1, 2, , \cdots)$ on rectangular grid points for the cracked beam using p-FEM. Then we form a spline interpolating surface of i-th natural frequencies using a set of frequency data, given on a rectangular grid of the (xy) plane.

Therefore, i-th natural frequency $f_i(x, y)$ at any point (x, y) can be derived using the spline interpolation function.

Without using p-FEM, natural frequencies at all crack parameters are calculated employing the above interpolation function, and then we can evaluate the fitness values defined by Eq.(10). A schematic illustration of the proposed method is shown in Fig.6.

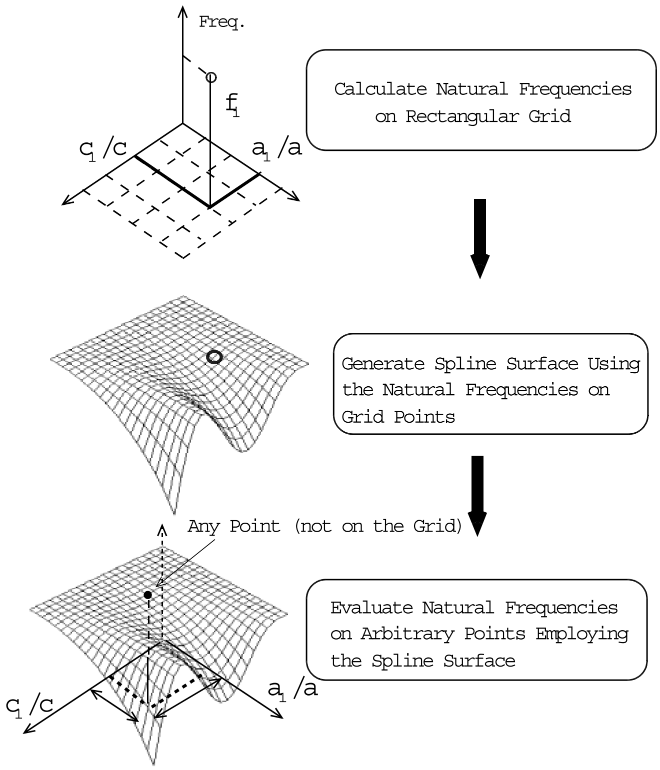

Fig. 6 Evaluation of natural frequency on spline surface

This technique can significantly reduce the time required to calculate the natural frequencies and helps in rapid identification. Except that unknown parameters are derived by a spline interpolation function, the above technique is the same as the conventional response surface methodology (RSM), where unknown parameters are evaluated by using a polynomial.

4. Inverse Analysis

4.1. Natural Frequency Analysis

Prior to the inverse analysis, accuracy of natural frequency by p-FEM are verified. To discuss the accuracy of p-FEM, we consider natural frequencies of a cantilever beam as shown in Table 1. The beam has three types of frequencies, i.e., bending, twisting and stretching mode of frequencies. To compare the results of p-FEM with those of the beam theory, we consider only bending natural frequencies.

Table 1 Configuration and physical values of the beam

Length of beam	$a = 0.25$[m] or $a = 0.1$[m]
Height of beam	$c = 0.01$[m]
Width of beam	$b = 0.01$[m]
Young's modulus	$E = 206$[GPa]
Density	$\rho = 7850$[kg/m^3]
Poisson's ratio	$\nu = 0.3$

Figure 7 shows first, second and third natural frequencies of the cantilever uncracked beam in relation to the total number of nodes for p-FEM, multi-purposes package software ANSYS and h-FEM. The numerical results of h-FEM (hierarchical finite element method) were based on the FEM code using the standard hexahedral solid element with eight nodes, coded by the authors. The figure shows that p-FEM converges considerably earlier than h-FEM. Next, natural frequencies of the cantilever beam with a crack as shown in Fig.4 is addressed. The dimensions and physical parameters of the beam are the same as that shown in Table 1. The positions of the crack are set to be $a_1/a = 0.1$ and the crack is assumed to be oriented parallel to the y-axis.

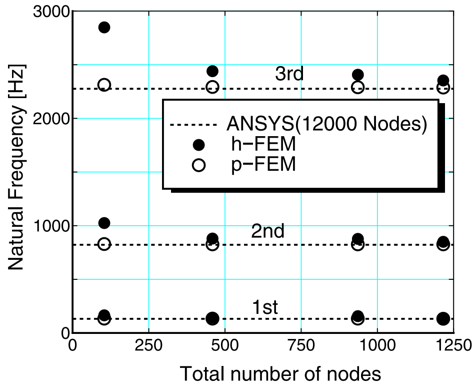

Fig. 7 Variation of natural frequencies with total number of nodes for the cantilever uncracked beam

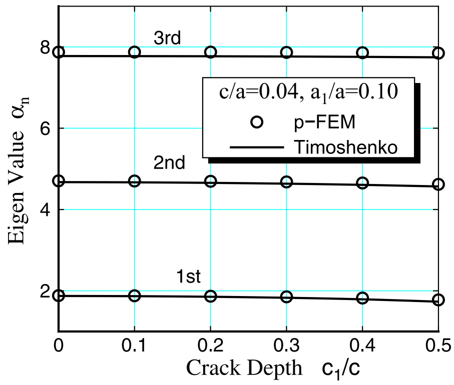

Fig. 8 Comparison of natural frequencies of the cantilever cracked beam

Figure 8 presents the relationship between a crack depth and eigenvalue α_n for the cantilever cracked beam. Eigenvalues α_n $(n = 1, 2, 3)$ of p-FEM are deduced by

$$\alpha_n = \sqrt{\frac{2\pi a^2 f_n}{\sqrt{(EI/\rho A)}}} \tag{11}$$

where, f_n is the bending natural frequency obtained by p-FEM, $A(=bc)$, $I\ (=bc^3/12)$ are cross-sectional area, moment of inertia of the cross-section of the beam, respectively. In the figure, the results of p-FEM were compared with those obtained by Horibe based on the Timoshenko beam theory(7).

The figure indicates that the results of p-FEM are fully in agreement with the those of the Timoshenko beam theory. Taking the above numerical results into consideration, the beam is divided into 750 hexahedral elements (3969 nodes in total) in the subsequent calculation and the resulting natural frequency values are employed in the inverse analysis.

4.2. Results of Crack Identification

Based on a preliminary calculation, the parameters of GA were set as Table 2. Identified solutions were deduced from the crack parameters of the individual that has the maximum fitness value during the calculation period up to the last generation.

Table 2 GA parameters for the cantilever beam with a crack

Population size	$p = 100$
Number of generations	$g = 100$
Initial mutation	$m = 0.9$
Rate of mutation decrement	$mc = 0.95$
Selection scheme	Roulette selection
Crossover scheme	Uniform crossover
Length of chromosome	16bit(=8bit×2)

Before we identify the crack parameters using the technique shown in section 3.3, we need to analyze the natural frequency values at grid points in advance. The values of various-order natural frequencies were calculated for sixty different types of cracks, which are created by dividing the position of crack $x(= a_1/a)$ into 10 at intervals of 0.1 (i.e., at $a_1/a = 0.1 \sim 0.9$ (step 0.1)) and the crack size at each position $y(= c_1/c)$ into 6 at intervals of 0.1 (i.e., $c_1/c = 0.05 \sim 0.55$). Based on this grid point data, the values of consecutive order of natural frequencies at any crack parameter were calculated using the spline surfaces as described in section 3.2. After calculating the fitness values using Eq.(10) in the process of GA , the crack parameters are determined from the best genes. The number of bending natural frequencies used for the identification was set at 2 or 3: the first to second or third bending natural frequencies.

Figures 9 and 10 show the results of the inverse analysis of the cracked cantilever beam with a height of $c/a = 0.04$ $(a = 0.25[\mathrm{m}])$. First two or three bending natural frequencies

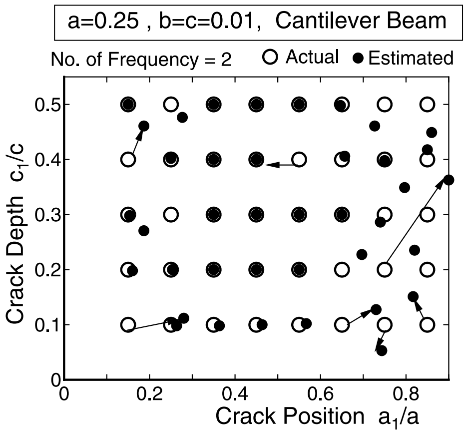

Fig. 9 Identified results for the cracked cantilever beam with c/a = 0.04 (No. of frequency is 2)

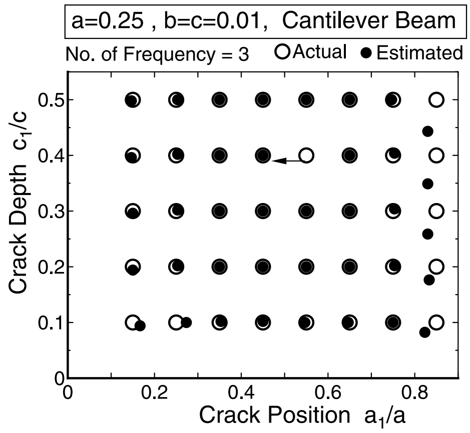

Fig. 10 Identified resuts for the cracked cantilevered beam with c/a = 0.04 (No. of frequency is 3)

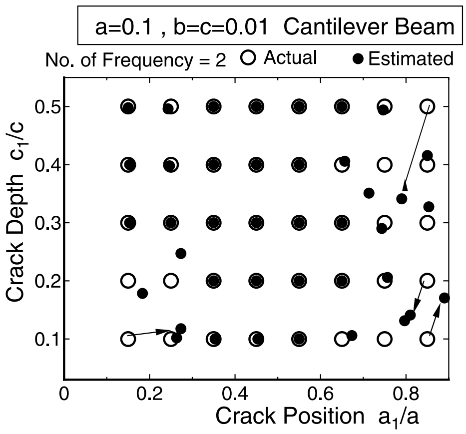

Fig. 11 Identified results for the cracked cantilevered beam with c/a = 0.1 (No. of frequency is 2)

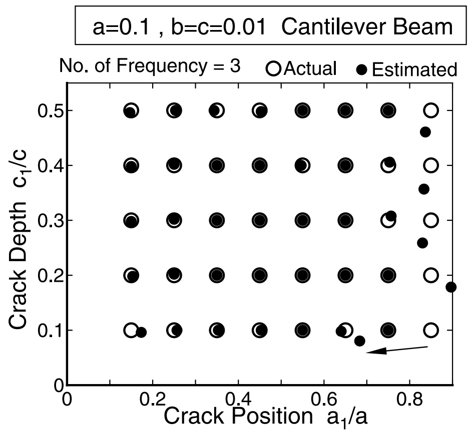

Fig. 12 Identified results for the cracked cantilevered beam with c/a = 0.04 (No. of frequency is 3)

were employed in Fig.9 and Fig.10, respectively. Shear effects on the natural frequency is negligible in this case. The blank circles in the figures represent the actual crack positions and sizes, and the solid circles represent the identified crack positions and sizes. When the crack is at a position where the modal shape of the virgin beam has a small curvature or is located at a region near free end of the beam, it produces very small effects on the natural frequency. In the figures, it was found that identification fails at those positions. However, sufficient identification is obtained at any position except for the free end when we employ the first three natural frequencies.

Figures 11 and 12 give the results of the inverse analysis of the cracked cantilever beam with a height of c/a = 0.1 (a = 0.1[m]), which is a rather thick beam. When a beam becomes shorter, shear effects becomes dominant, especially for higher modes. In our inverse analysis, transverse shear effects on the frequency was automatically taken into consideration since we employed the three dimensional p-FEM. The identified results are almost the same as that of the slender beam.

5. Conclusions

This paper describes a crack identification method based on both p-FEM and genetic algorithm. The proposed method has been applied to find a crack position and a crack depth for a cantilever beam. From numerical examples it is illustrated that the crack parameters can be successfully obtained by the present method.

In this study we treated only a simple beam problem, but we can extend the same methodlogy to identify crack parameters in complex structures.

Acknowledgements

The authors would like to thank A. Saitou of Hitachi Ltd. for her helps in this study.

References

(1) Rizos P. F., and Aspragathos N., Identification of crack location and magnitude in a cantilever beam from the vibration modes, J. Sound and Vib., **138**-3(1990), pp.381-388.
(2) Narkis Y., Identification of crack location in vibrating simply supported beams, J. Sound and Vib., **172**-4(1994), pp.549-558.
(3) Hasen W. M., Crack detection from the vibration of the eigenfrequencies of a beam on elastic foundation, Engng. Fract. Mech., **52**-3(1995), pp.409-421.
(4) Narkis Y. and Elmalah E., Crack identification of a cantilever beam under uncertain end conditions, Int. J. Mech. Sci., **38**-5(1996), pp.499-507.
(5) Nikolakopoulos P. G., Katsareas D. E. and Papadopoulos C. A., Crack identification in frame structures, Comput. and Struct., **64**-4(1997), pp.389-406.
(6) Horibe T., Asano N. and Okamura H., Inverse analysis of cracked frame structures by genetic algorithm, Trans. of the JSME, Ser. A, **65**-637(1999), pp.1962-1968 (in Japanese).
(7) Horibe T., Crack identification in a Timoshenko beam from frequency change using genetic algorithm, Inverse Problems in Engineering Mechanics (M. Tanaka ed.), Elsevier Science(2003), pp.221-228.
(8) Horibe T. and Watanabe K., Crack identification of plates using genetic algorithm, JSME International, Ser. A, Vol.**49**, No.3 (2006), pp.403-410.
(9) Murigendrappa S. M., Maiti S. K. and Srirangarajan H. R., Experimental and theoretical study on crack detection in pipes filled with fluid, J. of Sound and Vib., Vol.**270** (2004), pp1013-1032.
(10) Kim J. T., Ryu Y. S., Cho H. M. and Stubbs N., Damage identification in beam-type structure: frequency-based method vs mode-shape-based method, Engng. Struct., Vol.**25** (2003), pp.57-67.
(11) Douka E., Loutridis S. and Trochidis A., Crack identification in plates using wavelet analysis, J. of Sound and Vib., **270** (2004), pp.279-295.
(12) Taylor R. L., Beresford P. J. and Wilson E. L., A non-conforming element for stress analysis, Int. J. for Num. Methods for Engng., **10** (1976), pp.1211-1220.
(13) Guyan, R., Reduction of mass and stiffness matrices, AIAA Journal, **3**-2 (1965), pp. 380-386.
(14) Washizu K., Miyamoto H., Yamada Y., Yamamoto Y. and Kawai T., Finite element method handbook I, (1982), p.257, Baifukan (in Japanese).
(15) Tomita K. and Tosaka N., Optimum design of truss structure by genetic algorithm with variable mutation rate, Trans. of the JSME, Ser. C, **61**-585(1995), pp.1096-1101 (in Japanese).