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CREDIT AND BUSINESS CYCLES*

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This paper presents two dynamic models of the economy in which credit constraints arise because creditors cannot force debtors to repay debts unless the debts are secured by collateral. The credit system becomes a powerful propagation mechanism by which the effects of shocks persist and amplify through the interaction between collateral values, borrowers' net worth and credit limits. In particular, when fixed assets serve as collateral, I show that relatively small, temporary shocks to technology or wealth distribution can generate large, persistent fluctuations in output and asset prices.
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1. Introduction

In this paper I will explain why I believe that theories of credit are useful for understanding the mechanism of business cycles. In the 1980s and 1990s, real business cycle theory has emerged as a focal point in the business cycle debate. The standard real business cycle (RBC) model is a competitive economy whose equilibrium corresponds to the solution of the social planner's problem: the planner chooses an allocation of goods and labour to maximize the expected discounted utility of the representative agent subject to the resource constraint. The strength of the RBC approach has been to show that such a simple, yet fully coherent, dynamic general equilibrium model can be calibrated to match a surprisingly large number of business cycle observations, especially aggregate quantities. The RBC model, however, has been much less successful in explaining price movements, either relative or nominal. Indeed, the RBC theory often neglects the problems of money and credit altogether, by using the representative agent model.

Moreover, the RBC model needs large, persistent and exogenous aggregate productivity shocks as a mainspring of fluctuations. And I find it difficult to identify such productivity shocks as exogenous events. A majority of the shocks appear to be either shocks on particular sectors of the economy or shocks on distribution, rather than shocks on the aggregate productivity itself. For example, the oil shocks appear to be shocks on distribution between oil producers and oil consumers, and monetary shocks appear to be shocks mainly on distribution between debtors and creditors. Also, many shocks do not appear to be large compared with the size of the aggregate economy. I think that what is missing in the RBC models is a powerful propagation mechanism by which the effects of small shocks amplify, persist and spread across sectors.

In this paper I wish to study how, in theory, the credit system may act as such a

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propagation mechanism. In particular, when the credit limits are endogenously determined, I wish to examine how relatively small and temporary shocks on technology or wealth distribution may generate large, persistent fluctuations in aggregate productivity, output and asset prices.

For this purpose, I shall construct two dynamic models of the economy in which credit constraints arise because creditors cannot force debtors to repay debts unless the debts are secured by collateral. At each date, there are two groups of agents: productive agents and unproductive agents. Both have the technology to invest goods in the present period to obtain returns in the following period and productive agents have the technology to achieve a higher rate of return. Over time, some of the present productive agents become unproductive, and some of the unproductive agents become productive in the subsequent periods. We will examine questions such as:

(1) To what extent does the credit market transfer the purchasing power from unproductive to productive agents at each date, when credit contracts are difficult to enforce?
(2) How does the distribution of wealth between productive and unproductive agents interact with the aggregate productivity, output and the value of assets over time?
(3) How does a small, unanticipated temporary shock on the aggregate productivity or wealth distribution generate large and persistent effects on aggregate output and the value of assets?

In the basic model of Section 2, the collateral is a proportion of the future returns from present investment. In equilibrium, productive agents borrow up to the credit limit and use their own net worth to finance the gap between the amounts invested and borrowed. The transmission mechanism works as follows. Suppose that, at some date t, all agents experience a temporary productivity shock which reduces their net worth. Because productive agents have debt obligations from previous periods, their net worth falls more severely than does that of unproductive agents. Thus, productive agents cut back more investment than the decrease of aggregate saving, and the average productivity of investment falls together with the share of investment of productive agents. After date t, it takes time for the share of net worth of productive agents and the aggregate productivity to recover through saving and investment. Thus, the temporary productivity shock leads to persistent decreases in the share of net worth of productive agents, the aggregate productivity and the growth rate of the economy.

In the model of Section 3 a fixed asset, such as land, is introduced. When it is difficult to ensure that debtors repay their debts, the fixed asset serves as collateral for loans, in addition to being a factor of production. The credit limits of productive agents are determined by the value of collateralized fixed assets. At the same time, the asset price is affected by the credit limit. The dynamic interaction between the credit limit and the asset price turns out to be a powerful propagation mechanism. When the forward-looking agents expect that the temporary productivity shock will persistently reduce the aggregate output, investment and marginal product of the fixed asset in future, the present land price will fall significantly. Because land is a major asset in the balance sheet, the balance sheet worsens with the fall of the land price, especially for productive agents who have outstanding debt obligations. Thus, the share of investment of productive agents, aggregate productivity and aggregate investment fall even further, and it takes time for them to recover. Through the value of the fixed asset, therefore, persistence and amplification reinforce each other.[1)]

2. Basic model: persistence

Consider a discrete-time economy with a single homogeneous good and a continuum of agents. Everyone lives for ever and has the same preferences; i.e.,

$$E_t\left(\sum_{\tau=0}^{\infty}\beta^{\tau}\ln c_{t+\tau}\right), \tag{1}$$

where $c_{t+\tau}$ is consumption at date $t+\tau$, $\ln x$ is natural log of x, $\beta \in (0, 1)$ is discount factor for future utility, and E_t is expectations formed at date t. At each date t, there is a competitive one-period credit market, in which one unit of goods at date t is exchanged for a claim to r_t units of goods at date $t+1$.

At each date, some agents are productive and the others are unproductive. The productive agents have a constant-returns-to-scale production technology:

$$y_{t+1} = \alpha x_t, \tag{2}$$

where x_t is investment of goods at date t and y_{t+1} is output of goods at date $t+1$. The unproductive agents have a similar constant-returns-to-scale production technology with lower productivity:

$$y_{t+1} = \gamma x_t, \qquad \text{where } 1 < \gamma < \alpha. \tag{3}$$

Each agent shifts stochastically between productive and unproductive states according to a Markov process. Specifically, each agent who is productive in this period may become unproductive in the next period with probability δ, and each unproductive agent may become productive with probability $n\delta$. The shifts of the productivity of individuals are exogenous, and are independent across agents and over time. Assuming that the initial ratio of population of productive agents to unproductive agents is n:1, the ratio is constant over time.

We assume that the probability of the productivity shifts is not too large:

$$\delta + n\delta < 1. \tag{A1}$$

Assumption (A1) is equivalent to the condition that the productivity of each individual agent is positively correlated between the present period and the next period. We introduce these recurrent shifts in productivity of an individual agent in order to analyse how the credit system affects the dynamic interaction between distribution of wealth and productivity.

The production technology is specific to each producer. Once a producer has invested goods at date t, only he has the necessary skill to obtain the full returns described by the production function at date $t+1$. Without the skill of the producer who initiated the investment, other people can obtain only a fraction θ of the full

1) The model of the credit-constrained economy with fixed assets is based on Kiyotaki and Moore (1997a). See also Bernanke and Gertler (1989), Chen (1997), Kiyotaki and Moore (1997b), Scheinkman and Weiss (1986) and Shleifer and Vishny (1992). Gertler (1988) and Bernanke *et al.* (1997) are excellent surveys on the interaction between credit and business cycles.

returns. On the other hand, each producer is free to walk away from the production and from any debt obligations between the dates of investment and harvest with some fraction of the returns. As a consequence, if a producer owes a lot of debt, he may be able to renegotiate with the creditor for a smaller debt before harvesting time. Assuming that the debtor–producer has strong bargaining power, he can reduce his debt repayment to a fraction θ of the total returns.[2] Since the creditor can obtain a fraction θ of the total returns without the help of debtor–producer, this fraction can be thought of as the collateral value of the investment. Anticipating the possibility of the default between dates t and $t+1$, the creditor limits the amount of credit at date t, so that the debt repayment of the debtor–producer in the next period b_{t+1} will not exceed the value of the collateral:

$$b_{t+1} \leqslant \theta y_{t+1}. \tag{4}$$

Because the productivity of each producer between dates t and $t+1$ is known to the public at date t, people have perfect foresight about both debt repayment and output returns in future (aside from an unanticipated shock).

We assume that the rate of return on investment of productive agents without their specific skill is lower than the return on investment of unproductive agents:

$$\theta\alpha < \gamma. \tag{A2}$$

Assumption (A2) implies that the collateralized return on unit investment is smaller than the debt repayment on unit borrowing, so that productive agents cannot borrow unlimited amounts, when the real interest rate is at least as high as the rate of return on the investment of unproductive agents.

Each individual chooses a sequence of consumption, investment, output and debt from present to future $\{c_t, x_t, y_{t+1}, b_{t+1}\}$ to maximize the discounted expected utility (1), subject to the technological constraints (2) and (3), the borrowing constraint (4) and the flow of funds constraint:

$$c_t + x_t = y_t + b_{t+1}/r_t - b_t, \tag{5}$$

taking the initial output and debt as given. Equation (5) says that the expenditures on consumption and investment are financed by the returns from previous investment and new debt after repaying the old debt.

The market equilibrium implies that the aggregate consumption and investment of productive and unproductive agents (C_t, C'_t, X_t and X'_t) are equal to the aggregate output of productive and unproductive agents (Y_t and Y'_t):

$$C_t + C'_t + X_t + X'_t = Y_t + Y'_t. \tag{6}$$

By Walras's law, the goods market equilibrium (6) imples that the aggregate value of debt of productive agents, B_t, is equal to the aggregate credit of unproductive agents.

Before characterizing the equilibrium of our economy, it is helpful to think about what the economy would look like, if there were *no default problem* so that there were no borrowing constraint. Then the productive agent would borrow an unlimited amount as long as the rate of return on investment exceeded the real interest rate,

2) Here there is no issue of reputation, because the producer who walks away from production and debt can start a new life with a clear record. See Hart (1995) and Hart and Moore (1994, 1997) for more analysis of default and renegotiation.

$\alpha > r_t$. Nobody would borrow if the rate of returns were less than the real interest rate, $\alpha < r_t$. Thus, the equilibrium interest rate would be equal to the rate of return on investment of productive agents:

$$r_t = \alpha. \tag{7}$$

Then no unproductive agent would invest, and only productive agents would invest. The aggregate investment of productive agents would be equal to the aggregate saving of the economy, which turns out to be equal to a fraction β of aggregate wealth of the economy under log utility function of (1):[3)]

$$X_t = \beta W_t = \beta Y_t = \beta \alpha X_{t-1}. \tag{8}$$

Here, the aggregate wealth of the economy is simply the output from the previous investment of productive agents.

The important feature of the economy without credit constraint is that aggregate output and investment do not depend upon the distribution of wealth between productive and unproductive agents. Given that everyone has the same homothetic preference for present and future goods, aggregate output, consumption and investment are at the point on the efficient production frontier that is independent of wealth distribution. The growth rate of aggregate wealth is also independent of wealth distribution:

$$G_t \equiv W_{t+1}/W_t = \alpha\beta. \tag{9}$$

Now let us examine our economy *with* the borrowing constraint (4). In order to highlight the importance of the borrowing constraint, let us assume that the probability of a present productive agent becoming unproductive in the next period (δ) is large, and that the ratio of population of productive to unproductive agents (n) is small:

$$\delta > \theta \frac{\alpha - \gamma}{\gamma} \frac{\gamma - \theta\alpha}{\gamma - \theta\alpha - n\theta\gamma}. \tag{A3}$$

The first two terms on the right-hand side of (A3) are the fraction of collateralized returns and the proportion of productivity gap between productive and unproductive agents. The right-hand side is less than one for a small enough n, by (A2). Under (A3), we can show that the equilibrium real interest rate is equal to the rate of return on investment of unproductive agents,

$$r_t = \gamma, \tag{10}$$

in the neighbourhood of the steady state. (We shall verify (10) after we describe the credit constrained equilibrium.)

Productive agents invest by borrowing up to the credit limit, because the rate of return on their investment exceeds the real interest rate. The investment of the productive agent becomes:

$$x_t = \frac{y_t - b_t - c_t}{1 - (\theta\alpha/r_t)}. \tag{11}$$

3) From the first-order condition of consumption-saving choice, we have $1/c_t = \beta r_t / c_{t+1}$. Together with the flow of funds constraint, $a_{t+1} = r_t(a_t - c_t)$, where a_t is net worth ($= y_t - b_t$), we find that c_t is a fraction $1 - \beta$ of the net worth.

Since $(\theta\alpha/r_t)$ is the present value of collateralized returns from unit investment, the numerator is the required down payment for unit investment. Equation (11) implies that the productive agent uses the net worth minus consumption, $y_t - b_t - c_t$, to finance the required down payment. Equation (11) captures important features of investment under the borrowing constraint: the investment of productive agents is an increasing function of their net worth and productivity α, and is a decreasing function of the real interest rate r_t. From (10), (11) and (4) with equality, the flow-of-funds constraint can be written as:

$$y_{t+1} - b_{t+1} = (1-\theta)\alpha x_t = \alpha^+(y_t - b_t - c_t), \tag{12}$$

where $\alpha^+ \equiv [(1-\theta)\alpha]/[1-(\theta\alpha/\gamma)] > \alpha$ is the rate of return on saving for productive agents, taking account of the leverage effect of debt. Because of the log utility, the saving of productive agents is a fraction β of the net worth.

Unproductive agents are indifferent between lending and investing by themselves, because the real interest rate is the same as the rate of return on their investment. Their saving is also a fraction β of their net worth. Then the aggregate lending and investment of unproductive agents are determined by the market-clearing condition (6). Since consumption, debt and investment are linear functions of the net worth, we can aggregate across agents to find the equations of motion of the aggregate wealth (W_t) and the aggregate net worth of productive agents at the beginning of date t (A_t):

$$\begin{aligned} W_{t+1} = Y_{t+1} + Y'_{t+1} &= \alpha\frac{\beta A_t}{1-(\theta\alpha/\gamma)} + \gamma\left(\beta W_t - \frac{\beta A_t}{1-(\theta\alpha/\gamma)}\right) \\ &= \gamma\beta W_t + (\alpha-\gamma)\frac{1}{1-(\theta\alpha/\gamma)}(A_t/W_t)\beta W_t, \end{aligned} \tag{13}$$

$$\begin{aligned} A_{t+1} &= (1-\delta)(Y_{t+1} - B_{t+1}) + n\delta(Y'_{t+1} + B_{t+1}) \\ &= (1-\delta)\alpha^+\beta A_t + n\delta\gamma\beta(W_t - A_t). \end{aligned} \tag{14}$$

Equation (13) says that the aggregate wealth is the sum of returns on investment of productive agents and unproductive agents. The investment of productive agents is equal to their saving times the leverage of debt, while the investment of unproductive agents is the difference between aggregate saving and the investment of productive agents. Equation (14) implies that the aggregate net worth of productive agents is the sum of the net worth of those who continue to be productive from the previous period and the net worth of those who switch from being unproductive to being productive. The important difference from the previous economy of no credit constraint is that, for a given present aggregate wealth, the aggregate wealth of the next period is an increasing function of the share of net worth of productive agents, $s_t \equiv A_t/W_t$. Intuitively, with the credit constraint, the larger the share of net worth of productive agents is, the larger is the share of investment of productive agents, and the larger is the aggregate productivity of the economy.

The growth rate of aggregate wealth is also an increasing function of the share of net worth of productive agents:

$$G_t \equiv W_{t+1}/W_t = \beta\left(\gamma + (\alpha-\gamma)\frac{1}{1-(\theta\alpha/\gamma)}s_t\right). \tag{15}$$

The growth rate is lower in the economy with the borrowing constraint than in the economy without the borrowing constraint (equation (9)). From (13) and (14), we find that the share of net worth of productive agents evolves according to:

$$s_{t+1} = \frac{(1-\delta)\alpha^{+} s_t + n\delta\gamma(1-s_t)}{\alpha^{+} s_t + \gamma(1-s_t)} \equiv f(s_t). \quad (16)$$

Equation (16) implies that the share of net worth of productive agents monotonically converges to a unique steady-state s^* from any initial value $s_0 \in [0, 1]$. The steady-state share of net worth of productive agents s^* solves $s^* = f(s^*)$, and the value lies in between $n\delta$ and $1-\delta$ (see Figure 1).

In order to verify that (10) holds in equilibrium, we only need to check that unproductive agents invest positive amounts of goods:

$$X'_t = Y_t + Y'_t - C_t - C'_t - X_t = \beta W_t - \frac{1}{1-(\theta\alpha/\gamma)}\beta s_t W_t > 0, \quad (17)$$

because the interest rate is equal to the rate of return on investment of unproductive agents, if they invest positive amounts. Using (16), we find that (17) holds in the neighbourhood of the steady state if, and only if, assumption (A3) holds. Intuitively, if

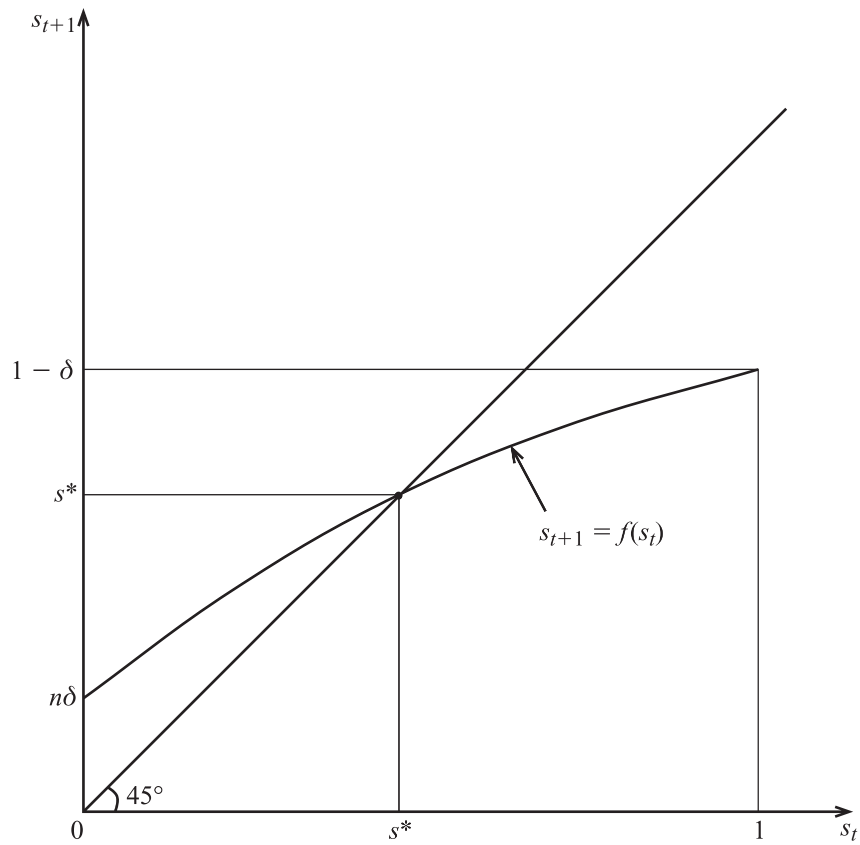

FIGURE 1.

the turnover rate from the productive state to the unproductive state is large and the population of productive agents is small, then the share of the net worth of productive agents is small in the steady state; then, given that the fraction of the collateralized returns is not too large, aggregate saving is larger than the investment of productive agents, and unproductive agents end up investing, using their inferior technology.

To understand the dynamics of the economy, it is helpful to consider the impulse response to an unexpected shock. Suppose that at date $t-1$ the economy is in the steady state: $s_{t-1} = s^*$ and $G_{t-1} = G^*$. There is then an unexpected shock to the productivity of every agent; both productive and unproductive agents find that their returns at the beginning of date t are $(1+\Delta)$ times their expectations. For example, let us assume that Δ is negative. The productivity shock, however, is temporary. The productivity of date t investment and thereafter returns to the normal as in (2) and (3). We assume that the unanticipated temporary productivity shock occurs after the agents have input their labour, so that it is too late for the debtor–producer to renegotiate a smaller debt. Then the aggregate net worth of productive agents at date t is:

$$\begin{aligned} A_t &= (1-\delta)[1+\Delta)\alpha X_{t-1} - B_t] + n\delta[(1+\Delta)\gamma X'_{t-1} + B_t] \\ &= (1+\Delta)[(1-\delta)\alpha X_{t-1} + n\delta\gamma X'_{t-1}] - (1-\delta-n\delta)\theta\alpha X_{t-1}. \end{aligned} \tag{18}$$

Since productive agents have a net debt in the aggregate (even with the turnover under assumption (A1)), the net worth of productive agents decreases proportionately more than the aggregate productivity as a result of the leverage effect of the debt. Because the aggregate wealth decreases in the same proportion as the aggregate productivity, the share of net worth of productive agents s_t decreases at date t. Then the growth rate of the economy is lower than the steady state between date t and $t+1$. Moreover, since the recovery of the share of the net worth of productive agents takes time, according to (16), the growth rate also takes time to recover after the productivity shock at date t.

In contrast, if there were *no* borrowing constraint, then the growth rate would go back to the steady-state level immediately after date t (see Figure 2). Intuitively, we can see that the temporary productivity shock worsens the wealth distribution of productive agents who have debt obligations, and this redistribution lowers the aggregate productivity and the growth rate persistently with credit constraint.

Since our framework does not have money, we cannot analyse the effect of monetary policy *per se*. However, one possible impact of the monetary policy may be considered as the unanticipated redistribution of wealth between debtors and creditors. For example, if the debt is nominal and is not indexed, the unanticipatedly lower inflation redistributes wealth from debtors to creditors. Then the share of net worth of productive agents decreases and the growth rate will decrease persistently.[4)]

I will add a few remarks concerning the case in which the turnover rate of productive agents is not high enough to satisfy assumption (A3). Then the share of net worth of productive agents is so large that the borrowing constraint is no longer binding in the steady state. The steady state is exactly the same as in the economy without the borrowing constraint. If the negative temporary shock reduces the share of net worth of productive agents, the growth rate after the date of the shock is unchanged as long as the shock is not too large. However, if the negative shock is

4) Fisher (1933) and Tobin (1980) emphasize the monetary transmission mechanism through the redistribution of wealth between creditors and debtors.

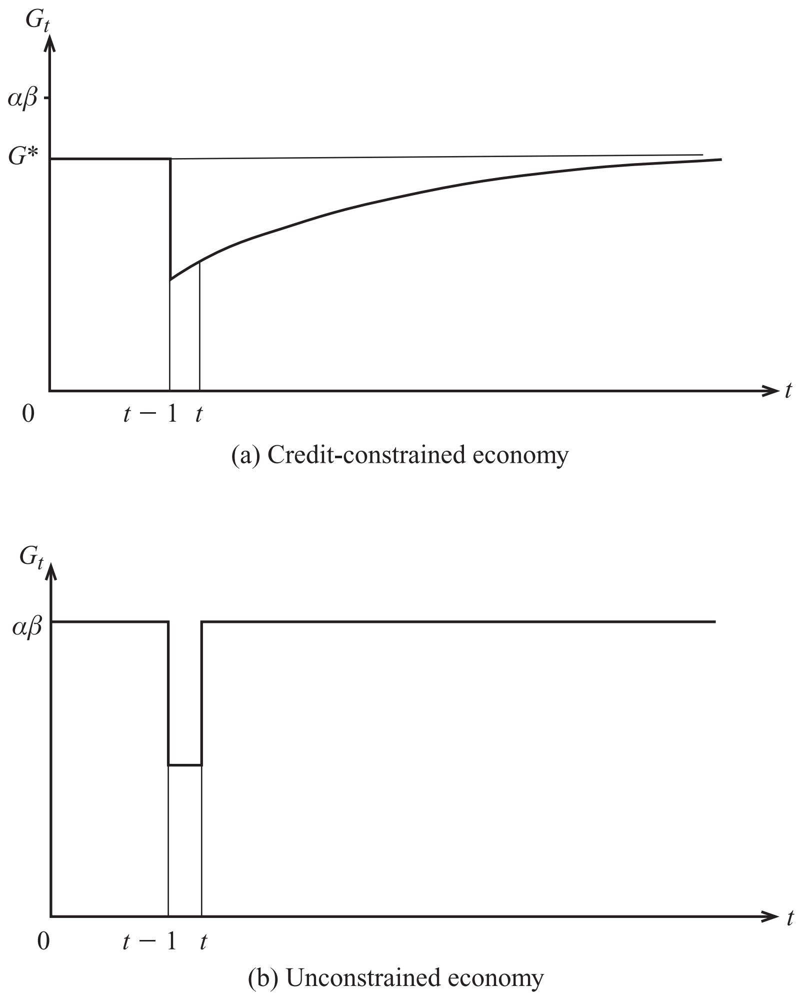

FIGURE 2.

large enough to make productive agents' borrowing constrained and to make unproductive agents invest at date t, then the growth rate will be lower than the steady state, until productive agents accumulate enough net worth so that unproductive agents no longer invest in their less productive technology.

3. Model with fixed asset: propagation and persistence

In the basic model of Section 2, there was only one homogeneous good with no fixed asset, and all the returns from present investment were realized in the following period. However, one of the variables that fluctuates noticeably over the business cycle is the value of fixed assets, such as land, buildings and machinery. Moreover, when lenders find it difficult to force debtors to repay debts, these fixed assets not only are factors of production but also serve as collateral for loans. In this section, I introduce the fixed asset in order to analyse the interaction between the value of the fixed asset, credit, and production over the business cycle.

There are two substantive modifications from the basic model. First, in addition to the homogeneous goods, we have a fixed asset, called land. The land does not depreciate and has a fixed supply, which is normalized to be one. Productive agents and unproductive agents use land and investment in goods as inputs to produce the homogeneous goods; i.e.,

$$y_{t+1} = \alpha \left(\frac{k_t}{\sigma}\right)^{\sigma} \left(\frac{x_t}{1-\sigma}\right)^{1-\sigma}, \tag{19}$$

$$y_{t+1} = \gamma \left(\frac{k_t}{\sigma}\right)^{\sigma} \left(\frac{x_t}{1-\sigma}\right)^{1-\sigma}, \tag{20}$$

where k_t is land, x_t is investment of goods, and $0 < \gamma < \alpha$; parameter $\sigma \in (0, 1)$ is the share of land in costs of input. The productivity of an individual agent follows the same Markov process as before. Beside the credit market, there is a competitive spot market for land, in which one unit of land is exchanged for q_t units of goods.

The second substantive modification concerns the borrowing constraint. We assume that, if the agent who has invested at date t with land k_t withdraws his labour between dates t and $t+1$, there would be no output at date $t+1$: there would be only land, k_t. At the same time, each producer is able to walk away from the production and the debt obligation with some fraction of goods in process between dates t and $t+1$. Thus, the value of collateral is the value of land, and the creditor limits the credit so that the debt repayment of the debtor–producer does not exceed the value of collateral:

$$b_{t+1} \leqslant q_{t+1} k_t. \tag{21}$$

The fraction of collateralized returns $\theta_{t+1} = q_{t+1} k_t / (y_{t+1} + q_{t+1} k_t)$ is no longer constant here, but fluctuates with the value of land.[5)]

Each agent chooses a path of consumption, investment, land holding, output and debt $\{c_t, x_t, k_t, y_{t+1}, b_{t+1} | t = 0, 1, 2, \ldots\}$ to maximize the expected utility, subject to the technological constraints (19) and (20), the borrowing constraint (21) and the flow-of-funds constraint:

$$c_t + x_t + q_t(k_t - k_{t-1}) = y_t + b_{t+1}/r_t - b_t, \tag{22}$$

taking the initial y_0, b_0 and k_{-1} as given. Equation (22) implies that the expenditure on consumption, investment and net purchase of land is financed by internal returns from the previous investment and outside finance by new debt net of repayment of the old debt. The market-clearing conditions are given by the goods market-clearing (equation (6)) and the land market-clearing:

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5) Here, we assume that the producer buys land rather than renting it. Because the producer can buy as much land as he can rent under the borrowing constraint (21), the producer prefers to buy land in order to avoid being held up by landlords after he has invested goods on land. (Renegotiation would generate more complications, if debtor–producer, creditor and landlord were all involved.) The perfect-foresight equilibrium path is the same for buying and renting, except for the impulse response to unanticipated shocks.

$$K_t + K'_t = 1. \tag{23}$$

Equation (23) says that the sum of the aggregate land holdings of productive and unproductive agents (K_t and K'_t) is equal to the total land supply.

In order to describe the competitive equilibrium, it is again helpful to describe first the economy *without* the borrowing constraint. Without the borrowing constraint (21), only productive agents invest in goods and use land. Thus, $K_t = 1$, and the competitive equilibrium corresponds to an efficient allocation, which maximizes the weighted average of utility of productive and unproductive agents, subject to the resource constraint:

$$C_t + C'_t + X_t = Y_t = \frac{\alpha}{\sigma^{\sigma}}\left(\frac{X_{t-1}}{1-\sigma}\right)^{1-\sigma}. \tag{24}$$

This is a deterministic version of the Brock and Mirman (1972) model. Investment becomes proportional to output, and the economy is going to converge to the steady state; i.e.,

$$X_t = (1-\sigma)\beta Y_t, \tag{25}$$

$$Y_{t+1} = \frac{\alpha}{\sigma^{\sigma}}(\beta Y_t)^{1-\sigma}. \tag{26}$$

The land price is the present value of the future marginal product of land, which turns out to be proportional to the present aggregate output and the present aggregate wealth owing to the log utility function:

$$q_t = \frac{\beta}{1-\beta}\sigma Y_t = \frac{\beta\sigma}{1-\beta+\beta\sigma} W_t, \tag{27}$$

where $W_t = Y_t + q_t$ is the aggregate wealth at date t. Without the borrowing constraint, aggregate output, investment and land price do not depend upon the distribution of wealth.

Let us now analyse our economy *with* borrowing constraint (21). We continue to assume that the turnover rate of productive agents is relatively high:

$$\delta > \frac{\alpha-\gamma}{\gamma}. \tag{A4}$$

Under assumption (A4), we can show that, in the neighbourhood of the steady state with a small enough ratio of population of productive to unproductive agents, the real interest rate is equal to the rate of return on investment of unproductive agents:

$$r_t = \gamma u_t^{-\sigma}. \tag{29}$$

$u_t \equiv q_t - q_{t+1}/r_t$ is the opportunity cost, or user cost, of holding land from date t to date $t+1$. I shall explain later why the right-hand side of (29) is the rate of return on investment of unproductive agents, and shall verify (29) after describing the equilibrium.

Productive agents borrow up to the credit limit, because their rate of return on investment exceeds the real interest rate. The investment of goods and land holding of the productive agent becomes:

$$x_t = (1 - \sigma)(y_t + q_t k_{t-1} - b_t - c_t), \tag{30}$$

$$k_t = \frac{\sigma(y_t + q_t k_{t-1} - b_t - c_t)}{q_t - (q_{t+1}/r_t)}. \tag{31}$$

Equation (30) says that the productive agent spends $(1 - \sigma)$ fraction of his net worth after consumption on investment of goods, when he maximizes the return from saving with Cobb–Douglas production function (19). Equation (31) says that the productive agent spends σ fraction of his saving to finance the difference between the land value, $q_t k_t$, and the amount he can borrow against land, $q_{t+1} k_t / r_t$. The difference $q_t - q_{t+1}/r_t$ is thought of as the down payment required to purchase one unit of land, which happens to be equal to the user cost of land, u_t. With the binding borrowing constraint, the flow-of-funds constraint of the productive agent is now:

$$y_{t+1} + q_{t+1} k_t - b_{t+1} = y_{t+1} = \alpha u_t^{-\sigma}(y_t + q_t k_{t-1} - b_t - c_t), \tag{32}$$

where $\alpha u_t^{-\sigma}$ is the return on saving for the productive agent.

Unproductive agents have a similar production function as productive agents, except that the productivity is low. Thus, when the unproductive agent maximizes the return from investment, he uses goods and land at the same ratio with the productive agent, $x_t{:}k_t = 1 - \sigma{:}\sigma/u_t$, and the rate of return on investment of the unproductive agent becomes $\gamma u_t^{-\sigma}$, using land as collateral. Therefore, when (29) holds, the unproductive agent is indifferent between investing and lending. The aggregate land holding of unproductive agents is determined by the market-clearing condition for land, (23).

Both productive and unproductive agents consume $1 - \beta$ fraction of their net worth as a result of their preferences. Thus, the market-clearing condition for goods (6) can be written as:

$$W_t - q_t = (1 - \beta) W_t + (1 - \sigma) u_t / \sigma, \qquad \text{where } W_t = Y_t + Y'_t + q_t. \tag{33}$$

The left-hand side is the aggregate output, and the first term in the right-hand side is the aggregate consumption, which is equal to a fraction $1 - \beta$ of the aggregate wealth. The second term is the aggregate investment of goods, because the ratio of investment of goods to usage of land is $1 - \sigma{:}\ \sigma/u_t$ for both productive and unproductive agents, and because the aggregate land holding is equal to one. Along the perfect-foresight equilibrium path, the aggregate wealth and the share of net worth of productive agents follow:

$$W_{t+1} = (\alpha u_t^{-\sigma} s_t + \gamma u_t^{-\sigma}(1 - s_t))\beta W_t, \tag{34}$$

$$s_{t+1} = \frac{(1 - \delta)\alpha s_t + n\delta\gamma(1 - s_t)}{\alpha s_t + \gamma(1 - s_t)} \equiv g(s_t). \tag{35}$$

Equation (34) says that, along the perfect-foresight equilibrium path, given the aggregate saving of the economy, βW_t, the fraction of the net worth of productive agents earns a rate of return $\alpha u_t^{-\sigma}$, while the fraction of unproductive agents earns a rate of return $r_t = \gamma u_t^{-\sigma}$. The law of motion for the share of net worth of productive agents in (35) is very similar to that of the basic model (equation (16)), except that the ratio of the rates of return on saving by productive and unproductive agents is $\alpha{:}\gamma$ here rather than $\alpha^{+}{:}\gamma$.[6)]

From the definition of the user cost of land and equation (29), the land price should satisfy the dynamic equation:

$$q_t = u_t + q_{t+1}u_t^{\sigma}/\gamma. \tag{36}$$

The perfect-foresight equilibrium is described by a sequence of $\{q_t,\ W_t,\ s_t,\ u_t | t = 0, 1, 2, \ldots\}$, satisfying (33), (34), (35), (36) and the initial conditions:

$$W_0 = Y_0 + Y_0' + q_0, \tag{37}$$

$$s_0 = [(1-\delta)(Y_0 + q_0 K_{-1} - B_0) + n\delta(Y_0' + q_0 K_{-1}' + B_0)]/W_0, \tag{38}$$

for a given initial value of Y_0, Y_0', K_{-1}, K_{-1}' and B_0. The value in brackets in (38) is the aggregate net worth of productive agents, which is the sum of the net worth of those who continue to be productive and those who newly become productive. The initial values of aggregate wealth and the share of net worth of productive agents are both functions of the initial land price, for given values of historically predetermined variables. We also rule out the exploding bubbles in the land price:

$$\lim_{t\to\infty} E_0(q_t/(r_0 \cdot r_1 \ldots r_{t-1})) = 0. \tag{A5}$$

From (33), (34), (35) and (36), there is a unique steady state, $(q^*,\ W^*,\ s^*,\ u^*)$. In particular, from (35), we know that the steady-state share of net worth of productive agents s^* satisfies $g(s^*) = s^* \in (n\delta,\ 1-\delta)$, as in the basic model. Unlike the basic model, however, the economy does not grow in the steady state, because the land is the fixed factor of production. In order to verify (29), we need only show that aggregate land holding of unproductive agents is positive:

$$K_t' = 1 - K_t = 1 - \sigma\beta s_t W_t/u_t > 0. \tag{39}$$

Using the steady-state conditions, inequality (39) holds under assumption (A4) for a small enough n in the neighbourhood of the steady state.

To examine the dynamics, we solve (33) for u_t, and substitute the expression of u_t in (34), (35) and (36) to obtain the dynamical system for $\{q_t,\ W_t,\ s_t\}$. Then we take linear approximation of the dynamical system around the steady state. It can be shown that there are two stable eigenvalues and one unstable one:

$$\left(\lambda,\ 1-\sigma,\ \frac{\gamma}{\beta(1-\sigma)[\alpha s^* + \gamma(1-s^*)]}\right), \qquad \text{where } \lambda \equiv \frac{(1-\delta)\alpha - n\delta\gamma - (\alpha-\gamma)s^*}{\alpha s^* + \gamma(1-s^*)}. \tag{40}$$

The eigenvalue $\lambda \in (0,\ 1)$ is the eigenvalue of the linearized system of (35). The eigenvalue $1-\sigma$ is the same as the eigenvalue of the linearized economy without the credit constraint. The last eigenvalue is larger than one, and corresponds to explosive paths.

We take the land price to be a jump variable, so that $\{q_t,\ W_t,\ s_t\}$ lie on a two-

6) Of gross returns on investment of date t, $y_{t+1} + q_{t+1}k_t$, for the output alone, the ratio of the productivities between productive and unproductive agents is $\alpha{:}\gamma$ in the economy with land. The ratio of the returns on saving between productive and unproductive agents becomes $\alpha{:}\gamma$ through the leverage effect of the debt under the borrowing constraint (21). In the economy without land, the ratio of the rates of return on investment is already $\alpha{:}\gamma$, and the ratio of the rates of return on saving is enlarged to $\alpha^{+}{:}\ \gamma$ through leverage.

dimensional stable manifold, in order to satisfy the non-exploding condition (A5). For the linear approximation, this stable manifold, expressed in terms of the deviations from the steady state, is a plane:

$$\hat{q}_t = \hat{W}_t + \mu \hat{s}_t, \qquad \text{where } \mu \equiv \frac{(\alpha - \gamma)s^*}{(\alpha - \gamma)s^* - (1-\delta)\alpha + n\delta\gamma + \gamma/\beta(1-\sigma)}, \tag{41}$$

and $\hat{X}_t \equiv (X_t - X^*)/X^*$. Since μ is positive, the land price should move proportionately more than the aggregate wealth on the stable manifold, if the share of net worth of productive agents moves in the same direction. Intuitively, if, for example, the share of net worth of productive agents falls with the aggregate wealth, then the recovery of the aggregate wealth is expected to be slow because of the lower aggregate productivity resulting from a lower share of investment of productive agents during the transition. Anticipating a slow recovery of the aggregate wealth and the user cost of land, the present land price falls proportionately more than the present aggregate wealth. In contrast, in the economy without a credit constraint, the land price is proportional to aggregate wealth and does not depend on the wealth distribution in (27), because the aggregate productivity is independent of the wealth distribution.

Consider the impact of a small, unanticipated, temporary productivity shock $\Delta < 0$ at date t. At date $t-1$, the economy was at the steady state. Using (37), (38) and (41), we can solve simultaneously for $\hat{q}_t$, $\hat{W}_t$ and $\hat{s}_t$ to obtain:

$$\hat{q}_t = \frac{1}{d}\left[1 + \mu \frac{q^*}{W^* - q^*}\left(1 - \frac{n\delta}{s^*}\right)\right]\Delta, \tag{42}$$

$$\hat{W}_t = \frac{1}{d}\left[1 + \mu \frac{q^*}{W^* - q^*}\left(1 - \frac{n\delta}{s^*}\right) - \mu(1 - \delta - n\delta)\frac{q^* K^*}{s^* W^*}\right]\Delta, \tag{43}$$

$$\hat{s}_t = \frac{1}{d}(1 - \delta - n\delta)\frac{q^* K^*}{s^* W^*}\Delta, \tag{44}$$

where

$$d \equiv 1 + \mu \frac{q^*}{W^* - q^*}\left[1 - \frac{n\delta}{s^*} - (1 - \delta - n\delta)\frac{K^*}{s^*}\right],$$

and K^* is the aggregate land holding of productive agents in the steady state. Equation (42) implies that the land price falls proportionately more than the temporary productivity shock itself at date t. Without the credit constraint, the land value would decrease only in the same proportion as the productivity shock and the aggregate output in (27).[7] Equation (43) says the aggregate wealth (which is the sum of aggregate output

7) The land price decreases as much as the temporary aggregate productivity shock without the credit constraint in (27) owing to the log utility function and the Cobb–Douglas production function. From (24), (25) and (26), the marginal product of land at date t is proportional to $Y_t^{1-\sigma}$, while the real interest rate is proportional to $Y_t^{-\sigma}$, and thus the present value of the marginal product of land at date t is proportional to Y_t. Similarly, the marginal product of land at date $t+1$ is proportional to $Y_{t+1}^{1-\sigma}$, whose present value at date t is proportional to $Y_{t+1} \cdot Y_t^{\sigma}$ (which in turn is proportional to Y_t). Thus, the present value of the marginal product of land at dates t, $t+1$, $t+2$, ... are all proportional to Y_t, and therefore the land price becomes proportional to Y_t. If the real interest rate is fixed, say because of perfect capital mobility in the small open economy, then the effect of the temporary productivity shock on the land price would be much smaller.

and land value) also decreases more than the productivity shock in proportion. Equation (44) implies that the share of net worth of productive agents decreases with negative productivity shock.

Figure 3 explains diagrammatically the dynamic effect of the temporary negative productivity shock. The plane *OABC* is the stable manifold (41), and the point E^* on this plane is the steady-state equilibrium. When the negative productivity shock hits at date t, the aggregate net worth decreases, and the net worth of productive agents falls proportionately more than the aggregate wealth owing to the leverage effect of debt. Thus, as the direct impact of the productivity shock, the aggregate wealth and the

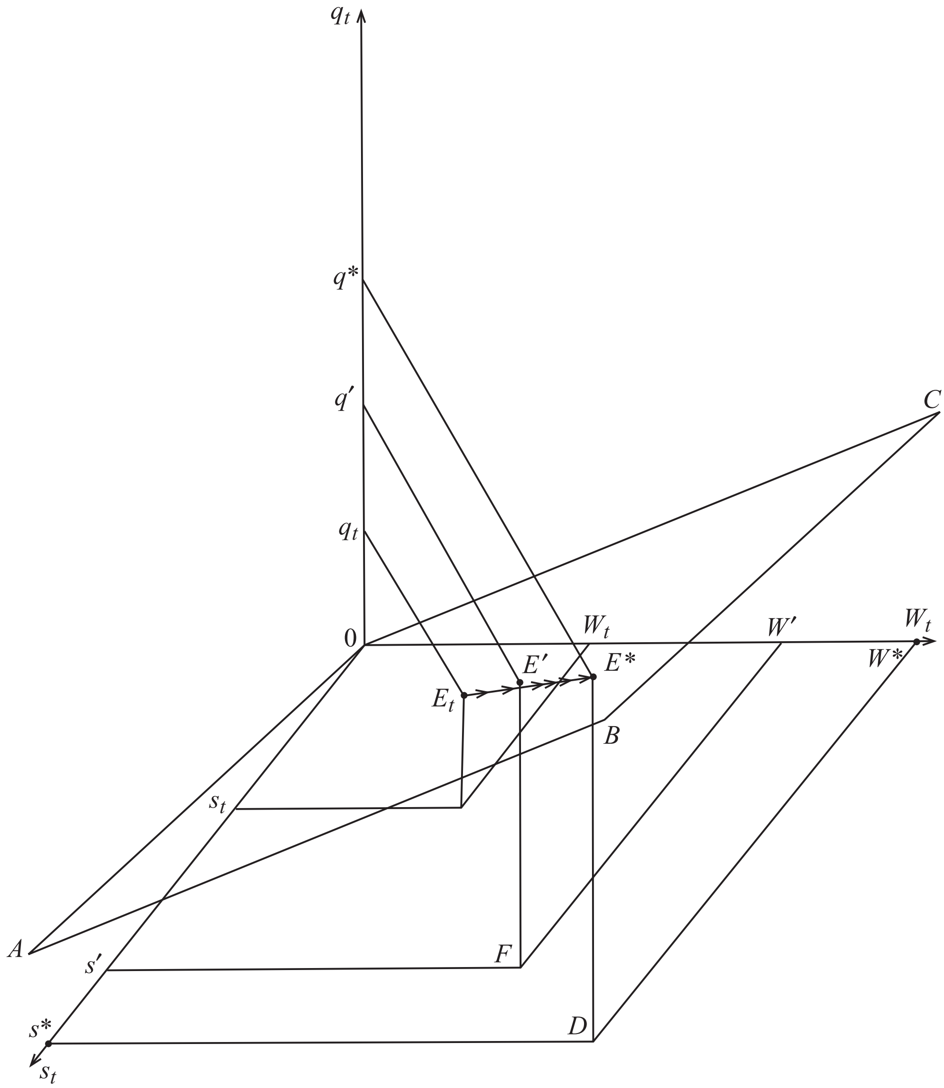

FIGURE 3.

share of net worth of productive agents fall from point $D = (W^*, s^*)$ to point $F = (W', s')$. After date t, it takes time for the aggregate wealth and the balance sheet of productive agents to recover through saving and investment. Then the user cost of land is expected to continue to be low in dates t, $t+1$, $t+2, \ldots$.

This anticipated, persistent decline in the user cost in future dates is reflected by a significant fall in the land price at date t. The land price falls from q^* to q', so that the point $E' = (W', s', q')$ is on the stable manifold. However, the effect does not stop here. Land is a major asset. Thus, the fall in land price at date t further reduces the aggregate wealth, and particularly the net worth of productive agents who have outstanding debts. The decrease in the aggregate wealth and the share of net worth of productive agents in turn further reduces the land price. Therefore, the economy at date t settles at point $E_t = (W_t, s_t, q_t)$ on the stable manifold, in which the aggregate wealth, the share of net worth of productive agents and the land price are all significantly lower than at the steady state. After date t, the economy will gradually recover to converge to the steady state along the path $E_t E^*$.

The basic model of Section 2 was a simple framework to highlight how the shock has more *persistent* effects on the growth rate in the credit-constrained economy than in the unconstrained economy. However, since the dynamical system that characterized the equilibrium path was not forward-looking, this persistent effect in the future did not feed back to the present condition.[8] In fact, both the aggregate wealth and the share of net worth of productive agents at the initial date were predetermined by past investment, except for the direct consequence of the productivity shock. In this section, I introduce the fixed asset in order to highlight the interaction between *persistence* and *amplification*. The introduction of the fixed asset makes the equilibrium system both history-dependent and forward-looking. When the productivity shock is expected to have a persistent impact on the future user cost of the fixed asset, the forward-looking agents change the valuation of the fixed asset significantly at present. Moreover, since the fixed asset is a major component of the balance sheet (particularly for productive agents who have outstanding debt obligations), the aggregate wealth and the balance sheet of productive agents change significantly, which amplifies the effects of the shock today. Intuitively, the persistence and the amplification reinforce each other.[9] I hope that this interaction between the value of the fixed asset, credit constraint, balance sheets and investment may shed light on recent business cycles of the Japanese economy.

8) In general, the consumption-saving choice at present may be affected by how persistent the effects of the shock are. But the effect on consumption is generally ambiguous for each level of present wealth, and is absent here because of the log utility function.

9) It is difficult to compare the size of the propagation between the basic model and the model with the fixed asset, because the former has endogenous growth and the latter does not. Perhaps a better model to compare with the model with the fixed asset is the basic model with decreasing returns to scale rather than constant returns. However, in such a model the aggregation is no longer simple, and we have to keep track of the entire distribution of wealth and productivity in order to describe the equilibrium. See Ortalo-Magné (1996) for an overlapping-generations model with decreasing-returns-to-scale investment technology with fixed assets, in which the younger generations are credit-constrained with smaller wealth, and the older generations are not constrained with larger wealth.

4. Conclusion

In this paper I have discussed how theories of credit may be useful for business cycle studies. In conclusion, I wish to add a few remarks about how theories of money, credit and banking may be useful for understanding the working of a decentralized economy.

Ever since I first became interested in economics, I have been fascinated by the coordination of the market economy, which Adam Smith called "the invisible hand of God". Each of us plays a very specialized role in production and consumption, and relies critically on the production and consumption of many other people. Yet when each person decides his or her production and consumption, each person is relatively selfish and has limited knowledge about other people's activities. How is this enormously interdependent production and consumption of many people coordinated in the market economy, when the individuals who choose their production and consumption are selfish and short-sighted?

The standard answer to this question is the competitive general equilibrium theory. Roughly, the argument goes as follows. Even if individual producers and consumers choose their production and consumption plans selfishly with limited knowledge about the others, as long as everyone is looking at the same market prices and the auctioneer adjusts the market prices to equate the demand and supply of all goods, all the individual plans become mutually consistent and feasible. Moreover, if all the scarce resources are privately owned and privately consumed, the allocation is efficient in the competitive general equilibrium.

Although I think that the competitive general equilibrium theory provides a decent first-cut answer, I also think that the standard general equilibrium theory, along the line of Arrow–Debreu, does not capture some important aspects of the decentralized economy, especially as applied to the dynamic economy. For example, how is the exchange of present goods and of future goods enforced? Suppose that A exchanges present goods for future goods with B. After A provides present goods to B, how can A make sure that B will provide goods in future? Arrow–Debreu assumed that the auctioneer has the power to impose an infinite penalty on B if B does not deliver, and thus can ensure that B will provide the promised goods to the market and then the market supplies goods to A. But when I start thinking about the problem of money, credit and banking, I begin to believe that this problem of enforcing intertemporal exchange is at the heart of the problem. Since the Arrow–Debreu model does not explain the roles of money, credit and banking, in order to analyse these problems, we obviously need to think about an economy that is more decentralized, an economy without an auctioneer with overall authority to enforce all the contracts. At the same time, I now believe that, by developing theories of money, credit and banking, we can understand better how mutually dependent production and consumption of numerous selfish people are coordinated in the decentralized economy.

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