Credit Crises and Liquidity Traps

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1 Introduction

A liquidity trap is a situation where the economy is in a recession, possibly a serious recession, and the nominal interest rate is zero. In this situation, the central bank cannot lower the nominal interest rate to boost private spending, as it would in normal times, and the question arises of what policy tools can be used to achieve this objective.

Hystorically, central banks have found themselves in a liquidity trap, or close to one, in periods characterized by great turbulence in financial markets and in the banking system, the most notable examples being the Great Depression, Japan in the 90s, and the current crisis. Is there a connection between dysfunctional credit markets and the emergence of a liquidity trap? Is it just the size of the recession that tends to make a liquidity trap more likely or is there something special about shocks coming from the credit market?

In this paper, we argue that shocks that affect the private agents' ability to borrow are precisely the type of shocks that can push the economy in a liquidity trap. We show that, when preferences display prudence, these shocks tend to make consumers more cautious, leading both to lower levels of spending and to larger liquidity premia. Larger liquidity premia mean that the required real interest rate on liquid assets like treasuries, tends to drop and can, possibly, go negative. This is what triggers a liquidity trap.

In this context, we compare the effect of various policies. Traditional open market operations are useless, independently of their size. "Quantitative easing" does not help as long as the private sector's expectations about future monetary policy are unchanged. Policies that work are those that change the total amount of "real

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liquidity" in the economy, that is, fiscal transfers that increases the total amount of liquid assets in circulation, including both money *and* liquid bonds, or interventions that help relax the agents' borrowing constraints in the future.

A broader point that comes out of the analysis is that precautionary behavior tends to make traditional monetary policy tools less effective and that credit market interventions, which help agents smooth consumption over time, are a natural complement to standard interest rate policy. That is, the elasticity of private spending to the nominal interest rate is larger when credit constraints are less tight and liquidity premia are lower.

Krugman (1998) and Woodford and Eggertsson (2003) emphasize that the basic problem in a liquidity trap is that the real interest rate that would be required to achieve full employment, i.e., the "natural" real interest rate, is unusually low and negative. If expected inflation is low, in line with the central bank target, or, even worse, if deflation has taken hold, the real interest rate corresponding to a zero nominal interest rate would be too high relative to its natural benchmark and private spending would be stuck at an inefficiently low level. However, the existing literature provides little clues about what shocks would push the economy in this situation. If we think of a recession as a temporary drop in output and consumption, agents should expect an increasing consumption path in the future. This should raise the real interest rate rather than depress it. For this reason, both Krugman (1998) and Woodford and Eggertsson (2003) focus on exogneous preference shocks that depress the marginal utility of consumption today relative to tomorrow. While this is a convenient simplification, it begs the question of what actual shocks are behind this increased preference for saving. The model presented here shows that a shock to credit market access has exactly these features. Moreover, having identified the source of the shock we can discuss analytically the effect of policies that alleviate this precautionary behavior.

In the baseline New Keynesian model there is a liquidity premium between money and bonds, the nominal interest rate, but bonds and all other assets are assumed to be perfect substitutes. The novelty in this paper is that treasuries also carry a liquidity premium, because they can be used to smooth temporary income shocks. In this context, the full-employment real interest rate on treasuries may become negative due to an increase of the liquidity premium, even if the full-employment real interest rate on less liquid assets is still positive.

2 Model

Consider a cash-in-advance model in which consumers have limited credit access. The model has an infinite horizon, but all the action will take place in periods 1 and 2.

There is a continuum of households, with preferences described by the utility function

$$E\left[\sum_{t=1}^{\infty}\beta^{t-1}\left(u\left(c_{j,t}\right)-n_{j,t}\right)\right],$$

where $c_{j,t}$ is consumption and $n_{j,t}$ is labor effort. The assumption of linear disutility of labor effort helps to simplify the analysis.

Each household produces consumption goods using the linear technology:

$$y_{j,t}=\theta_{j,t}n_{j,t},$$

where $\theta_{j,t}$ is an idiosyncratic productivity shock. In period 1,

$$\theta_{j,1}=\begin{cases}0 & \text{with pr. } \alpha\\ 1 & \text{with pr. } 1-\alpha\end{cases},$$

that is, applying a law of large numbers, a fraction α of households receives the productivity shock $\theta_{j,1}=0$ and a fraction $1-\alpha$ the shock $\theta_{j,1}=1$. From period 2 onwards the shock is degenerate, and $\theta_{j,1}=1$ with probability 1 for $t\geq 2$, that is, all households have productivity equal to 1.

Household j enters each period t with cash $m_{j,t-1}$ and nominal bond holdings $b_{j,t-1}$. At the beginning of each period the financial market is open. Households receive the interest rate i_{t-1} on their bond holdings and buy $b_{j,t}$ new bonds. The cash available after trading on the financial market is denoted by

$$\tilde{m}_{j,t}=m_{j,t-1}+\left(1+i_{t-1}\right)b_{j,t-1}-b_{j,t}-\tau_t,$$

where τ_t is a lump sum tax. The initial value of $m_0+\left(1+i_0\right)b_0$ is inherited from the past and is taken as given (the specific values of m_0, b_0 and i_0 are irrelevant).

Then, the good market opens and each household splits into two agents: a consumer who buys the output of other households in exchange for cash and a worker who produces $y_{j,t}$ and sells it. Imagine that consumer and worker of the same household travel to separate islands, where they meet consumers and workers from different households. Also, to simplify matters, assume that the idiosyncratic productivity shocks $\theta_{j,t}$ are realized after the consumer leaves to go shopping and the worker and the consumer cannot communicate when they are in different islands. This implies that each consumer cannot consume the output of his own worker and does not even know his productivity shock. Consumer spending is subject to the cash-in-advance constraint

$$p_t c_{j,t} \leq \tilde{m}_{j,t}. \tag{1}$$

At the end of each period t, when the consumer and the worker meet again, the household's cash balances are

$$m_{j,t} = \tilde{m}_{j,t} - p_t c_{j,t} + p_t y_{j,t}.$$

In addition to the cash-in-advance constraint, households face a borrowing constraint. When they raise cash on the financial market, they are allowed to borrow, i.e., to choose a negative $b_{j,t}$, but their real borrowing position is bounded below by the exogenous limit $f > 0$,

$$b_{j,t}/p_t \geq -f. \tag{2}$$

To close the model we need to specify how we model the government. We assume that the government sets the supply of money and bonds and the interest rate in period 1, m_1, b_1, and i_1 in order to keep y as close as possible to the steady state level. Moreover, he keeps them constant at $\bar{m}$, $\bar{b}$, and $\bar{\imath}$ in periods $2, 3, ...$ Money and bond supply are always assumed to be positive and the nominal interest rate cannot be negative. Finally, the tax τ_t will adjust to ensure budget balance for the government in each period:

$$m_t + b_t = m_{t-1} + (1 + i_{t-1})\, b_{t-1} - \tau_t.$$

3 Equilibrium

An equilibrium is given by sequences of consumption and labor effort decisions $\{y_{j,t}\}$ and $\{n_{j,t}\}$, money and bond holdings $\{b_{j,t}\}$ and $\{m_{j,t}\}$, taxes $\{\tau_t\}$, money supply, bond supply, and interest rate in period 1, m_1, b_1 and i_1, and prices $\{p_t\}$, such that households maximize their utility, the government keep y_1 as close as possible to the steady state level and the government budget is balanced at each time t.

Manipulating the household's optimality conditions for bond holdings gives:

$$u'(c_{j,t}) \geq (1 + i_t)\, \beta E_t \left[\frac{p_t}{p_{t+1}} u'(c_{j,t+1}) \right], \tag{3}$$

where $E_t[.]$ is the expectation formed in period t, without knowing the idyosincratic shock. This is a standard Euler equation which can hold as an inequality only if the borrowing constraint (2) is binding. The optimality condition for consumption can be written as

$$u'(c_{j,t}) \geq \beta E_t \left[\frac{p_t}{p_{t+1}} u'(c_{j,t+1}) \right]. \tag{4}$$

If this condition holds as an inequality the cash-in-advance constraint (1) must be binding. Finally, the optimality condition for labor supply is

$$1 \geq \theta_{j,t} \beta E_t^* \left[\frac{p_t}{p_{t+1}} u'(c_{j,t+1}) \right], \tag{5}$$

where $E_t^*[.]$ is the expectation based on the information available to the worker in the goods market, after the realization of the idiosyncratic shock $\theta_{j,t}$. This condition can be slack if the non-negativity constraint $n_{j,t} \geq 0$ is binding.

3.1 Steady State

Proceeding backwards let us first construct the equilibrium in periods $3, 4, ...$, in which the economy reaches a steady state with constant output. We conjecture that in these periods prices are constant at $\bar{p}$ and all households choose the same level of consumption and labor supply, $\bar{c} = \bar{n} = \bar{y}$. We will check later that households hit by negative productive shocks in period 1 will readjust their bond and money holdings by working more in period 2, so that all households j will enter period 3 with exactly the same financial wealth

$$m_{j,2} + (1 + i_2) b_{j,2} = \bar{m} + (1 + i_2) \bar{b}. \tag{6}$$

Since the representative household needs to hold government bonds, which are in non-negative supply, the borrowing constraint is slack and the Euler equation (3) holds as an equality. Under constant prices and consumption, this requires a constant nominal interest rate equal to $\bar{\imath} = 1 - 1/\beta$.

Combining (3), the labor supply optimality condition (5), and market clearing we obtain

$$1 = \beta u'(\bar{y}),$$

which implicitly defines the steady state output level $\bar{y}$. Notice the presence of a distortion which pushes output below its first best level y^*, characterized by $u'(y^*) = 1$. This distortion, captured by the factor $\beta < 1$, is a monetary distortion due to the fact that monetary policy targets a constant price level. A constant deflation at the rate β, corresponding to the Friedman rule, would be sufficient to eliminate this distortion.

Comparing (3) and (4), shows that the CIA constraint is binding given that $\bar{\imath} > 0$. Hence, the steady state price level is immediately derived as $\bar{p} = \bar{m}/\bar{y}$. For now, let us set the steady state money supply at $\bar{m} = \bar{y}$, so that the steady state price level is $\bar{p} = 1$.

3.2 Periods 1 and 2

Now we can go back and analyze the equilibrium in periods 1 and 2. First, it is useful to derive the budget constraint at the beginning of period 2. At the beginning of period 1, all households choose the same bond and money holdings, $b_{j,1} = b_1$ and $\tilde{m}_{j,1} = m_1$, and the same consumption, $c_{j,1} = y_1 \equiv \alpha y_1^l + (1-\alpha) y_1^h$, since these decisions are made prior to the observation of the idiosyncratic income shock. However, their end-of-period money holdings are different and equal to $m_{j,1} = m_1 + p_1 y_{j,1} - p_1 y_1$. Therefore, the financial wealth of household j at the beginning of period 2 is

$$\begin{aligned} m_{j,1} + (1+i_1) b_{j,1} &= m_1 + (1+i_1) b_1 + p_1 y_{j,1} - p_1 y_1 = \\ &= \bar{m} + \bar{b} + \tau_2 + p_1 y_{j,1} - p_1 y_1, \end{aligned}$$

where the second equality follows from the government budget balance. The budget constraint at the beginning of period 2 can then be written as

$$\tilde{m}_{j,2} + b_{j,2} = \bar{m} + \bar{b} + p_1 y_{j,1} - p_1 y_1. \tag{7}$$

Since $p_3 = 1$, the optimality condition for labor supply in period 2 yields

$$1 = p_2 \beta u'(\bar{y}).$$

We will check later that labor supply is positive for all agents in period 2. Since $\beta u'(\bar{y}) = 1$, the last equation shows that the price level reaches its steady state in period 2, $p_2 = 1$.

To determine the output level in period 1 and complete our equilibrium construction, we need to consider two possibilities. In period 2, either all households are unconstrained or the households hit by the low income shock in period 1 are constrained (since there is a positive supply of bonds we cannot have both types constrained). We analyze these cases separately.

3.2.1 Slack borrowing constraint

Suppose the borrowing constraint in period 2 is slack for all households. Use the superscript h and l to denote households with high and low income in period 1.

Then the Euler equation in period 2 gives

$$u'\left(c_2^l\right) = u'\left(c_2^h\right) = (1 + i_2)\,\beta u'(\bar{y})\,.$$

We then have an equilibrium with a positive interest rate $i_2 = 1/\beta - 1$, a binding cash-in-advance constraint for all households, and all households consuming $c_2^l = c_2^h = \bar{y}$.

To determine the level of output in period 1, it is sufficient to use the Euler equation

$$u'(c_1) = (1 + i_1)\,p_1 \beta u'(\bar{y})\,, \tag{8}$$

and the labor supply condition

$$1 = p_1 \beta u'(\bar{y})\,.$$

We will assume that the central bank has an objective of stabilizing output, that is, to set y_1 as close as possible to the steady state output level $\bar{y}$. If the central bank chooses $m_1 = \bar{m}$ it can then achieve $y_1 = \bar{y}$ with a positive nominal interest rate $i_1 = 1/\beta - 1$. This policy also achieves a stable price level with $p_1 = 1$.

It remains to check three things: that in period 2 the borrowing constraint is indeed not binding, that labor markets clear and that all agents enter period 3 with the same financial wealth. The budget constraint (7), the cash-in-advance constraint $\tilde{m}_{2,j} \geq c_{2,j}$, and the borrowing constraint $b_{2,j} \geq -f$, can be jointly satisfied with $c_2^l = c_2^h = \bar{y}$ if and only if

$$\bar{m} + \bar{b} - 2\bar{y} \geq -f.$$

Define

$$z \equiv \bar{m} + \bar{b} + f,$$

this is a measure of the total liquid resources available to an individual household at time 2, including bonds, cash, and unused debt capacity.

We then have the following.

Proposition 1 *If $z \geq 2\bar{y}$ the central bank can achieve an equilibrium with stable output at $\bar{y}$ where the consumers' borrowing constraint is never binding.*

In other words a liquidity trap does not arise if the liquid resources z are sufficiently abundant. We conjecture that in period 2 the nominal interest rate is positive, so the cash-in-advance constraint is binding and determines an output level equal to $\bar{y} = \bar{m}$.

It is easy to verify our conjecture that low shock households exactly compensate for low earnings in period 1 with higher earnings in period 2, so as to begin period 3 with the same initial financial position.

3.2.2 Binding borrowing constraint

Suppose now that the borrowing constraint is binding for low productivity households. Also assume that $i_2 > 0$ and hence that the cash in advance constraint in period 2 is binding. We will check this later. Now the budget constraint (7), the binding cash-in-advance constraint and the borrowing constraint determine their consumption

$$c_2^l = z - p_1 y_1.$$

Market clearing implies that the consumption of high productivity households is determined by

$$c_2^h = \frac{\bar{y} - \alpha c_2^l}{1-\alpha}.$$

The Euler equations for the two households in period 2 are

$$\begin{aligned} u'\left(c_2^l\right) &\geq (1+i_2)\,\beta u'(\bar{y}), \\ u'\left(c_2^h\right) &= (1+i_2)\,\beta u'(\bar{y}). \end{aligned}$$

These two conditions can only be jointly satisfied if $c_2^l \leq c_2^h$, that is if

$$z \leq \bar{y} + p_1 y_1. \tag{9}$$

We make the simplifying assumption $u'(\bar{y}/(1-\alpha)) > \beta u'(\bar{y})$, so as to ensure $i_2 > 0$ and a binding cash-in-advance constraint in period 2.

Substituting in the Euler equation in period 1 gives

$$u'(y_1) = (1+i_1)\,p_1\beta\left[\alpha u'(z - p_1 y_1) + (1-\alpha)\,u'\left(\frac{\bar{y} - \alpha(z - p_1 y_1)}{1-\alpha}\right)\right]. \tag{10}$$

The labor supply condition only holds as an equality for productive workers and takes the form

$$1 = p_1 \beta u'\left(\frac{\bar{y} - \alpha(z - p_1 y_1)}{1-\alpha}\right). \tag{11}$$

Then, for each value of i_1, we can solve the system of equations (10) and (11) for y_1 and p_1.

In such an equilibrium, the economy is in a liquidity trap, if even at $i_1 = 0$, the equilibrium output level in period 1, y_1, turns out to be smaller than $\bar{y}$, the government target. More formally, the economy is in a liquidity trap if

$$u'(\bar{y}) < p_1\beta\left[\alpha u'(z - p_1 y_1) + (1-\alpha)\,u'\left(\frac{\bar{y} - \alpha(z - p_1 y_1)}{1-\alpha}\right)\right],$$

where p_1 and y_1 solve (10) and (11) with $i_1 = 0$. We can then check wether (9) is satisfied.

4 Precautionary savings and the liquidity trap

In this section, we define formally a liquidity trap and study how for an economy is easier to enter a liquidity trap the lower is the level of liquidity in the economy. In particular, we show two main results: first, that y is monotone decreasing in i and $y < \bar{y}$ if $i = 0$, and second, that y monotone increasing in z, for given i. Recall that (10) and (11) are the two key equations that characterize the equilibrium.

Proposition 2 *If consumer preferences display prudence,* $u''' > 0$*, and elasticity of substitution smaller than* 1*,* $-cu''(c)/u'(c) < 1$*,* y_1 *is monotonically non increasing with* i*. Moreover,* $i_1 = 0$ *yields* $y_1 < \bar{y}$*.*

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