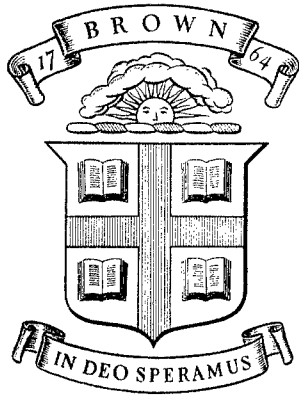


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**CREEP OF POLYURETHANE  
UNDER VARYING TEMPERATURE FOR  
NONLINEAR UNIAXIAL STRESS**

**J. S. Y. LAI and W. N. FINDLEY**

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FOR NONLINEAR UNIAXIAL STRESS

by

J. S. Y. Lai\* and W. N. Findley\*\*

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Summary

Two methods are described to account for varying temperature during creep. Both employ the modified superposition principle. One uses a reduced time involving a shift factor which is a function of both stress and temperature history. The other considers the strain to be a function of the current values of stress and temperature.

Experiments on polyurethane include constant stress creep and recovery at several temperatures in the nonlinear range and an experiment in which the stress was held constant while the temperature increased at a constant rate, then the stress was removed and the temperature decreased at a constant rate. The strain in this experiment was predicted by means of the theories from the results of the constant temperature creep tests.

The strains in the constant temperature creep and recovery tests were described by means of the multiple integral representation and the modified superposition principle. Most of the nonlinearity and temperature effect were found in the coefficient of the time-dependent term.

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## Introduction

In a recent paper [1]<sup>\*</sup> the authors described the results of creep experiments in the nonlinear range on polyurethane in torsion at various temperatures. The results showed that the time dependence could be described adequately by a power function whose exponent was independent of stress and temperature. In the present paper tension creep experiments in the nonlinear range performed on the same specimen of polyurethane as part of the same series of experiments are reported for several temperatures. Constitutive equations describing these results are employed to predict the behavior under a constant rate of temperature rise (and fall) during creep (and recovery) at constant stress.

Previous investigations of nonlinear viscoelasticity have mostly been confined to room temperature. However, Bernstein, Kearsly and Zapas [2] and Lianis [3] have extended the concept of a thermorheologically simple viscoelastic material to the nonlinear range. Earlier investigations by the authors [4 to 9] on poly (vinyl chloride) and polyurethane at room temperature under combined tension and torsion showed that the creep strain could be separated into time-dependent and time-independent parts. The latter could be expressed as a power function whose exponent was independent of stress state. The time-independent term was nearly linear, but the time-dependent coefficient was strongly nonlinear. It was found that the nonlinear behavior could be adequately described by the first three terms of a multiple integral series. For varying loads either the modified superposition method [5,10] for the mixed time parameters in the kernel functions or the product form [6,10] gave satisfactory results.

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\* Numbers in brackets identify references listed at the end of this paper.

### Material and Specimen

The material used in the experiments was the same as employed in previous work [8,9,11,12]. It was a full density solid polyurethane having a specific gravity of 1.25. This material, identified as XR6-77, was produced by CPR Division, the Upjohn Company, and Supplied by the Lawrence Radiation Laboratory. The material was prepared from the prepolymer and combined with toluene diisocyanate. The polyester resin was derived from a dicarboxylic acid and a triol. The carbon content was 59 per cent, hydrogen 6.5 per cent, nitrogen 7.5 per cent, chlorine less than 0.3 per cent, ash 0.05 per cent, and the balance oxygen. The material was cured at a temperature of 250°F for two hours.

The specimen was a tubular type with enlarged threaded ends machined from a solid rod. The average outside diameter was 0.9971 in., with a maximum deviation of 0.0003 in., the average wall thickness was 0.05928 in., with a maximum deviation of 0.0005 in., and the gage length was 4.00 in. All experiments were performed on the same specimen.

### Experimental Apparatus

The testing machine was designed to test tubular specimens under combined tension, torsion, and internal pressure [13]. It has been used to investigate creep and stress relaxation behavior under various loading conditions for plastics [4-12] and metals [14-16] and plasticity [17].

The loading and measuring devices are as follows. Tension and torsion were produced by dead weights or servo control acting through levers and pulleys. Dead weights were used in the present work. The tensile strain measurement was accomplished by using pairs of rods attached to the upper and lower gage points of the test specimen to transmit the motion of the gage points below the furnace. The lower ends of one pair of the rods were attached to the core and the ends

of the other pair were attached to the coil of a differential transformer. Relative motion of the upper and lower gage points during the test caused an output of the differential transformer which was directly related to the strain. The sensitivity of the strain measurement was  $2 \times 10^{-6}$  in./in.

In order to perform creep tests of polyurethane above room temperature, the following heating devices were added. The specimen was heated in an electric split-tube furnace which was maintained at constant temperature by a C. N. S. Instruments' Sirect Mark III proportional temperature controller. Due to the poor conductivity of polyurethane a second heating source inside the specimen tube was used to achieve a more uniform temperature distribution. Two small resistor heating cores were attached at both ends of a copper bar and inserted inside the specimen. It was found that the smaller the spacing between the copper bar and the inner surface of the specimen the better the temperature was controlled. One thermocouple was embedded in each end of the copper bar. These served as the heat sensing device in the control system. Temperature in the copper bar was controlled by two Research Incorporated Thermac Model TC5192 temperature controllers operating on the two heaters.

During each of the tests, temperature was measured at six positions by means of chromel-alumel thermocouples with fused hot junctions. Two of these thermocouples were cemented on the inner surface of the specimen at positions slightly beyond the gage length. Four thermocouples were attached with spring clips on the outer surface at equal distances within the gage length of the specimen. It was observed that the temperature could be maintained constant throughout the testing period within  $\pm 1/2^{\circ}\text{F}$ . The temperature variation along the gage length was uniform at  $75^{\circ}\text{F}$  and  $102^{\circ}\text{F}$ , but increased to  $2^{\circ}\text{F}$  on the outside and  $7^{\circ}\text{F}$  on the inside of the specimen at  $150^{\circ}\text{F}$ .

### Experimental Procedure and Results

The rate of heating the specimen was controlled to about 2°F per minute. After reaching the test temperature the specimen was kept at this temperature for one hour before the load was applied. Pilot tests indicated that a soaking period from 0.5 hr. to 5.0 hr. at the test temperature (75°F to 160°F) did not show a significant difference in the creep response under the same loading conditions. Except for one experiment the temperature was maintained at a constant value throughout the testing period. Some adjustments of the set points of the controllers were required, however.

In performing the constant stress creep tests, the specimen was loaded for a one-hour period for each test. After unloading at the end of one hour, the specimen was left to recover at zero stress at the same test temperature for about one hour. After that, heat was gradually turned down until it reached room temperature, and the specimen was left to recover at room temperature until the strain had returned to less than  $40 \times 10^{-6}$  in./in. or appeared to have stabilized before proceeding to the next test. Tests 69; 46; 48; 84; 91; and 97 followed tests whose recovery stabilized at  $\epsilon_{12} = 102$  ;  $\epsilon_{12} = 78$  ;  $\epsilon_{11} = 95$  ;  $\epsilon_{11} = 120$  and  $\epsilon_{12} = 90$  ;  $\epsilon_{11} = -84$  ; and  $\epsilon_{11} = 63 \times 10^{-6}$  in./in., respectively. The zero strain for each test was taken to be the strain reading at temperature just prior to loading.

Thermal expansion has two effects on tensile strain: it shows up as an apparent creep strain during temperature changes, and it alters the gage length of the strain measuring instrument. The former was taken into account in the constant rate of temperature rise test, but the latter was ignored in all tests. The gage length was fixed at 75°F and no correction was made for change in gage length with temperature. The thermal expansion was found to be  $35.2 \times 10^{-6}$  in./in./°F. Thus at 160°F the gage length increased about 0.3 per cent.

The test program of the present investigation is shown in Table I. The tensile strain versus time curves from these tests are shown in Fig. 1 and 2. A constant stress creep test was also conducted in which the temperature increased at a nearly constant rate, then decreased at a nearly constant rate while the stress was removed when the temperature started to decrease. The strain versus time, together with the programmed and measured temperature, are shown in Fig. 3.

In previous work on the same material at room [8,9,11] and elevated [1] temperature, it was found possible to describe the results of creep strain versus time under constant stress by a power law with a constant exponent  $n$  ,

$$\epsilon_{11} = \epsilon_{11}^{\circ} + \epsilon_{11}^{+} t^n , \quad (1)$$

where  $\epsilon_{11}^{\circ}$  and  $\epsilon_{11}^{+}$  are functions of stress and  $n$  is a constant. From the results shown in Fig. 1 and 2, it seems that (1)\* can be used to describe the strain versus time relationship of the present tests also. The effect of temperature on  $\epsilon_{11}^{\circ}$  ,  $\epsilon_{11}^{+}$  ,  $n$  in (1) was determined by first obtaining the best fit of each data set to (1) by a least squares method. As in the companion tests in torsion it was found that there was considerable scatter in  $n$  but no definite trend as a function of either stress or temperature.  $\epsilon_{11}^{\circ}$  showed scatter and about the same modest temperature effect observed in torsion [1];  $\epsilon_{11}^{\circ}/\sigma$  increased from about 0.19 to 0.23 from 75°F to 160°F. However,  $\epsilon_{11}^{+}$  showed a strong effect; it more than doubled over the same temperature range, the same as in torsion and for the same reason [1].

In view of the scatter in  $n$  resulting from the individual determinations the value  $n = 0.143$  used in earlier work [8,11] with this material at room temperature and in the companion work in torsion at elevated temperature [1] was chosen as a common value for all stresses and temperatures.

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\* Numbers in parentheses identify equations.

Also consistent with the work on torsion [1]  $\epsilon_{11}^0$  was taken to be independent of temperature in view of the much smaller apparent effect of temperature on  $\epsilon_{11}^0$  than on  $\epsilon_{11}^+$ . Employing these restrictions values of  $\epsilon_{11}^0$  and  $\epsilon_{11}^+$  were recalculated and are shown in Table I. In the analysis in following sections equation (1) with the values of  $\epsilon_{11}^0$ ,  $\epsilon_{11}^+$  and  $n$  in Table I were employed rather than the experimental data.

#### Discussion of Results

Several experiments were repeated one or more times at intervals during the series of 118 creep tests under tension, torsion and combined tension and torsion. These repeated experiments were used to indicate how much the creep behavior was changing as a result of the history of stress and temperature on the single specimen employed. As described in [1] the effect of history was greater in torsion than tension. The results of the repeated tests in tension are summarized in Table II. Examination of all the data showed that there were changes which generally resulted in increased strains. The greatest changes occurred during the early tests and again when the largest stresses were applied. Possible causes were discussed in [1]. There was, however, an extensive period in which changes were small. Compare, for example, tests 39 and 76 and tests 41 and 67 in Table II.

In the analysis of the effect of temperature the data from the first 20 experiments were ignored because of the changes which were occurring. Also some of the last tests were omitted and a few others whose strains were so out of line as to suggest errors of observation. The experiments employed in the analysis were undoubtedly influenced to some extent by the prior history of stress and temperature. These history effects were ignored in the analysis.



Analysis

Nonlinear Constitutive Equation

A nonlinear constitutive equation in the multiple integral form has been described by several investigators; see for example [4]. For uniaxial stress  $\sigma_{11} = \sigma$  the following equation can be obtained for the creep strain  $\epsilon_{11} = \epsilon$ ,

$$\begin{aligned} \epsilon = & \int_0^t R(t-\xi_1; T) \dot{\sigma}(\xi_1) d\xi_1 \\ & + \int_0^t \int_0^t M(t-\xi_1, t-\xi_2; T) \dot{\sigma}(\xi_1) \dot{\sigma}(\xi_2) d\xi_1 d\xi_2 \\ & + \int_0^t \int_0^t \int_0^t N(t-\xi_1, t-\xi_2, t-\xi_3; T) \dot{\sigma}(\xi_1) \dot{\sigma}(\xi_2) \dot{\sigma}(\xi_3) d\xi_1 d\xi_2 d\xi_3, \end{aligned} \quad (2)$$

where  $t$  is time,  $T$  is temperature  $\xi$  an arbitrary time prior to  $t$  and  $\dot{\sigma}(\xi) = \partial\sigma(\xi)/\partial\xi$ . In this equation only terms from the first three orders have been retained. The kernel functions  $R, M, N$  of (2) depend on temperature (under different isothermal conditions) and time arguments as indicated. Under constant stress  $\sigma(t) = \sigma H(t)^*$ , where  $H(t)$  has the value 1 when  $t \geq 0$  and 0 when  $t < 0$ . Thus under isothermal conditions and constant stress (2) becomes

$$\epsilon(t, T) = R(t, T)\sigma + M(t, T)\sigma^2 + N(t, T)\sigma^3. \quad (3)$$

Determination of Kernel Functions

Comparison of (3) with (1), representing the test data, suggests that the kernel functions  $R, M, N$  may have the following forms

\*  $H(t)$  is the Heaviside unit function.

$$\begin{aligned} R(t,T) &= a_1 + b_1(T)t^n \\ M(t,T) &= a_2 + b_2(T)t^n \\ N(t,T) &= a_3 + b_3(T)t^n \end{aligned} \quad (4)$$

where  $n = 0.143$  ,  $a_1$  ,  $a_2$  ,  $a_3$  are independent of temperature, and  $b_1$  ,  $b_2$  ,  $b_3$  are functions of temperature as suggested by the test results. Inserting (4) into (3) and comparing with (1) results in

$$\epsilon^o = a_1\sigma + a_2\sigma^2 + a_3\sigma^3 \quad (5a)$$

$$\epsilon^+ = b_1(T)\sigma + b_2(T)\sigma^2 + b_3(T)\sigma^3 \quad (5b)$$

From the results in Table I, the time-independent and the time-dependent coefficients (5a) and (5b), respectively, were determined as follows.

The data of  $\epsilon_o$  for each stress as given in Table I shows that  $\epsilon_o$  can be approximated as a linear function of stress. Therefore

$$\begin{aligned} \epsilon_o &= a_1\sigma \quad , \\ a_1 &= 0.195 \text{ per cent/ksi.} \quad , \quad a_2 = a_3 = 0 \quad . \end{aligned} \quad (6)$$

The time-dependent part of the strain  $\epsilon^+$  from Table I versus temperature  $T$  was plotted for each stress level as shown in Fig. 4. Then  $\epsilon^+/\sigma$  versus  $\sigma$  for each of several temperatures (75°F, 100°F, 120°F and 140°F) was plotted as shown in Fig. 5. In Fig. 5 values of  $\epsilon^+$  for temperatures 100°F, 120°F and 140°F at all stress levels were obtained from the lines of Fig. 4.

From (5b)

$$\frac{\epsilon^+(\sigma, T_1)}{\sigma} = b_1(T_1) + b_2(T_1)\sigma + b_3(T_1)\sigma^2 \quad , \quad (7a)$$

where  $T_1 = T - 75^\circ\text{F}$  . The reference temperature was chosen as 75°F because

$T = 75^{\circ}\text{F}$  was the lowest temperature used in this study. In Fig. 5, curves of  $\epsilon^+/\sigma$  versus  $\sigma$  at different temperatures are parallel straight lines. Thus the curve for different temperatures can be obtained by a vertical shift of the  $\epsilon^+/\sigma$  versus  $\sigma$  curve for  $T_1 = 0$ . This implies that only  $b_1$  depends on temperature,  $b_2$  and  $b_3$  are independent of temperature. Therefore (7a) can be rewritten as

$$\frac{\epsilon^+(\sigma, T_1)}{\sigma} = b_1 + b_2\sigma + b_3\sigma^2 + F(T_1) \quad , \quad (7b)$$

where  $b_1, b_2, b_3$  are independent of temperature,  $F(T_1)$  is the vertical shift, and  $F(0) = 0$ .  $b_1, b_2, b_3$  were determined graphically from  $\epsilon^+/\sigma$  versus  $\sigma$ , in Fig. 5, at  $T_1 = 0$  with the result

$$\begin{aligned} b_1 &= 0.017, \text{ per cent/ksi} \\ b_2 &= 0.004, \text{ per cent}/(\text{ksi})^2 \\ b_3 &= 0 \quad . \end{aligned} \quad (7c)$$

$F(T_1)$  was determined from the amount of vertical shift of  $\epsilon^+/\sigma$  versus temperature as shown in Fig. 6. Thus  $F(T_1)$  was approximated by

$$F(T_1) = \theta_1 T_1 + \theta_2 T_1^2 \quad , \quad (7d)$$

where  $\theta_1 = 0.00038$  per cent/ksi  $^{\circ}\text{F}$  and  $\theta_2 = 0.000,0045$  per cent/ksi( $^{\circ}\text{F}$ )<sup>2</sup>.

From (7a), (7b), (7c), (7d) the time-dependent part of the strain can be represented by

$$\epsilon^+(\sigma, T) = (b_1 + \theta_1 T_1 + \theta_2 T_1^2)\sigma + b_2\sigma^2 \quad . \quad (8)$$

Thus the constitutive relation of creep strain under constant stress, constant temperature as characterized in this section may be summarized as follows by introducing (4) through (8) into (3)

$$\epsilon = a_1\sigma + [(b_1 + \theta_1 T_1 + \theta_2 T_1^2)\sigma + b_2\sigma^2]t^n ,$$

or

$$\epsilon = 0.195\sigma + [(0.017 + 0.00038T_1 + 0.000,0045T_1^2)\sigma + 0.004\sigma^2]t^{0.143} , \quad (9)$$

where  $\epsilon$  is in per cent,  $\sigma$  in ksi, and  $t$  in hours. Creep strains were calculated from (9) for each stress and temperature and are shown in Fig. 1 and 2.

#### Recovery at Constant Temperature

The modified superposition method [5] for multi-step load histories can be expressed as follows for tensile loading  $\sigma$

$$\epsilon_{11}(t) = \sum_{i=0}^p [f(\sigma_i, t-t_i) - f(\sigma_{i-1}, t-t_i)] , \quad t > t_p \quad (10)$$

where for the present experiments

$$f(\sigma, t, T) = R(t, T)\sigma + M(t, T)\sigma^2 + N(t, T)\sigma^3 , \quad (11)$$

which becomes (9) when numerical values are supplied. Thus the recovery at zero stress after a period  $t_1 = 1$  hr. at constant stress  $\sigma_0$  is from (10) for isothermal conditions

$$\epsilon_{11}(t) = f(\sigma_0, t) - f(\sigma_0, t-t_1) , \quad (12)$$

where  $f$  has the form of (9) and the  $a_1\sigma$  term vanishes.

The recovery strains following constant stress creep were predicted from (9) by means of (12). The results for some of the tests are shown in Fig. 7. During recovery the contribution of the nonlinear terms is very small. Agreement between the theory and test data for tension (Fig. 7) is not as good as for torsion [1]. With the exception of one test the actual strains during recovery were smaller than the strains predicted from the tension tests.

Creep under Continuously Varying Temperature

Method A

In the analysis of the creep behavior of linear viscoelastic material at different but constant temperatures, the effects of temperature and time for certain classes of material can be combined into a single parameter through the "time-temperature superposition principle" which may be stated as follows:

$$J(T, \zeta) = J(T_0, t) \quad , \quad (13)$$

$$\zeta = t/a_T(T) \quad , \quad (14)$$

where  $J$  is the creep compliance,  $\zeta$  is the reduced time,  $a_T$  is the temperature shift factor,  $T_0$  is the reference temperature and  $t$  is the real time. Thus, the determination of  $a_T$  as a function of temperature  $T$  will provide the necessary information for determination of the reduced time. For tests involving varying temperature during loading, the definition given by Morland and Lee [18] for reduced time is

$$\zeta = \int_0^t \frac{d\xi}{a_T(\xi)} \quad , \quad (15)$$

where  $t$  is the current time,  $\xi$  is any prior time, and  $t = 0$  corresponds to the first application of stress.

Aside from the work reported in [2] and [3], no similar method is available to describe nonlinear viscoelastic behavior under varying temperature during loading. It is intended in this section to explore a possible way of describing the effect of nonlinear creep behavior under varying temperature by generalizing the constitutive relation obtained through creep tests under different but constant temperatures.

Consider a general constitutive relation under constant stresses and temperatures, as follows:

$$\varepsilon = J_0 \sigma + J_t \sigma \quad , \quad (16)$$

where

$$J_0 = F_1(T) + F_2(T)\sigma + F_3(T)\sigma^2 \quad (16a)$$

$$J_t = [F_1'(T) + F_2'(T)\sigma + F_3'(T)\sigma^2]G(t) \quad . \quad (16b)$$

In (16) it is assumed that the total creep strain can be separated into a time-independent part  $J_0 \sigma$  and a time-dependent part  $J_t \sigma$ . Equation (9) can be considered as a special case of (16). Rearrange (16b) such that

$$J_t = [\alpha_1 + \alpha_2 \sigma + \alpha_3 \sigma^2 + \beta_1(T_1) + \beta_2(T_1)\sigma + \beta_3(T_1)\sigma^2]G(t) \quad . \quad (17)$$

Rearranging by factoring out the temperature-independent terms yields

$$J_t = [\alpha_1 + \alpha_2 \sigma + \alpha_3 \sigma^2] f(T_1, \sigma) G(t) \quad , \quad (18)$$

where

$$f(T_1, \sigma) \equiv \left[ 1 + \frac{\beta_1(T_1) + \beta_2(T_1)\sigma + \beta_3(T_1)\sigma^2}{(\alpha_1 + \alpha_2 \sigma + \alpha_3 \sigma^2)} \right]$$

and  $\alpha_1, \alpha_2, \alpha_3$  are independent of temperature, while  $\beta_1, \beta_2, \beta_3$  are functions of temperature  $T_1 = T - T_0$ , and  $\beta_1 = \beta_2 = \beta_3 = 0$  at  $T = T_0$  or  $T_1 = 0$ .

$f(T_1, \sigma)G(t)$  in (18) can be combined into  $G(\zeta)$  by defining a reduced time  $\zeta$ ,

$$\zeta = \frac{t}{a_T(T_1, \sigma)} \quad , \quad (19)$$

where  $a_T(T_1, \sigma)$  is a shift factor depending on both temperature and stress.

Therefore,

$$\varepsilon = J_0 \sigma + [\alpha_1 \sigma + \alpha_2 \sigma^2 + \alpha_3 \sigma^3]G(\zeta) \quad . \quad (20)$$

Equation (20) can be interpreted as a time-temperature superposition principle

[20] provided that the shift factor  $a_T$  depends on both temperature and stress.

For continuously varying temperature and/or stress, (18) and (19) can be used to redefine the reduced time  $\zeta$  as follows:

$$\zeta = \int_0^t \frac{ds}{a_T[T_1(s), \sigma(t)]} \quad , \quad (21)$$

where  $s$  is time prior to the current time  $t$  and  $t = 0$  at the time stress is first applied.

For describing creep behavior under an arbitrary stress history, it has been found that the concept of a "modified superposition principle," proposed by Findley and Lai [5] in summation form and employed in integral form by Pipkin and Rogers [19], yields satisfactory agreement with experiment.

The modified superposition method considers that the strain following a change in stress at time  $t_1$  is the sum of: (a) the strain that would have occurred had the stress not been changed, (b) minus the strain that would have occurred had the prior stress been applied at the time  $t_1$  to a previously unstressed material, (c) plus the strain that would have occurred had the current stress been applied at time  $t_1$  to the previously unstressed material.

In integral form the modified superposition principle can be written as follows

$$\epsilon(t) = \int_0^t \frac{\partial Q[\sigma(\xi), t-\xi, T]}{\partial \sigma(\xi)} \frac{d\sigma(\xi)}{d\xi} d\xi \quad . \quad (22)$$

Introducing (20) into (22) for  $Q[\sigma(\xi), t-\xi, T]$  provides a means of accounting for creep behavior under continuously varying stress and temperature as follows:

$$\epsilon(t) = J_0 \sigma(t) + \int_0^\xi \frac{\partial}{\partial \sigma} \{ [\alpha_1 \sigma(\zeta') + \alpha_2 \sigma(\zeta')^2 + \alpha_3 \sigma(\zeta')^3] G(\zeta - \zeta') \} \frac{\partial \sigma(\zeta')}{\partial \zeta'} d\zeta' \quad , \quad (23)$$

where

$$\zeta = \int_0^t \frac{ds}{a_T[T_1(s), \sigma(t)]} \quad , \quad (24)$$

$$\zeta' = \int_0^\xi \frac{ds}{a_T[T_1(s), \sigma(\xi)]} \quad . \quad (25)$$

For  $T = T_0$  , a constant, (23) reduces to the form originally proposed for the modified superposition method as follows:

$$\epsilon(t) = J_0 \sigma(t) + \int_0^t \frac{\partial}{\partial \sigma} \{ [\alpha_1 \sigma(\xi) + \alpha_2 \sigma^2(\xi) + \alpha_3 \sigma^3(\xi)] G(t-\xi) \} \frac{\partial \sigma(\xi)}{\partial \xi} d\xi \quad , \quad (26)$$

since as defined in (18) for  $T = T_0$  ,  $f(T, \sigma) = 1$  ,  $\zeta = t$  , and  $\zeta' = \xi$  .

For  $T > T_0$  but constant, and for varying stress  $\sigma = \sigma(t)$  , the strain may be described by (23) provided that

$$\zeta = \int_0^t \frac{ds}{a_T[T_1, \sigma(t)]} = \frac{t}{a_T[T_1, \sigma(t)]} \quad , \quad (27)$$

$$\zeta' = \int_0^\xi \frac{ds}{a_T[T_1, \sigma(\xi)]} = \frac{\xi}{a_T[T_1, \sigma(\xi)]} \quad . \quad (28)$$

For varying temperature but constant stress  $\sigma = \sigma_0 H(t)^*$  . Thus (23) reduces to

$$\epsilon(t) = J_0 \sigma_0 + [\alpha_1 \sigma_0 + \alpha_2 \sigma_0^2 + \alpha_3 \sigma_0^3] G(\zeta) \quad , \quad (29)$$

where

$$\zeta = \int_0^t \frac{ds}{a_T[T_1(s), \sigma_0]} \quad (30)$$

and  $\zeta'$  is not involved.

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\*  $H(t)$  is the Heaviside function. It has the value one when  $t \geq 0$  and zero when  $t < 0$  .



Equation (30) was used to predict the creep strain (total strain minus strain due to thermal expansion) for the test program shown in Fig. 3, as follows. In this program the stress was constant  $\sigma = 4000$  psi, and the temperature  $T_1(t) = T - 75^\circ\text{F} = 60t$  where  $t$  is time in hours.

Equation (9) may be expressed in the form of (18) as follows

$$\epsilon(t) = a_1\sigma + (b_1\sigma + b_2\sigma^2) \left[ 1 + \frac{\theta_1 T_1 + \theta_2 T_1^2}{b_1 + b_2\sigma} \right] t^n \quad (31)$$

Comparing (31) with (18) shows that

$$G(t) = t^n, \quad f(T_1, \sigma) = \left[ 1 + \frac{\theta_1 T_1 + \theta_2 T_1^2}{b_1 + b_2\sigma} \right]$$

Therefore

$$G(\zeta) = \zeta^n = \left( \frac{t}{a_T} \right)^n = f(T_1, \sigma) G(t) = \left[ 1 + \frac{\theta_1 T_1 + \theta_2 T_1^2}{b_1 + b_2\sigma} \right] t^n \quad (32)$$

Hence (31) becomes

$$\epsilon(t) = a_1\sigma + (b_1\sigma + b_2\sigma^2) \zeta^n \quad (33)$$

(32) implies that

$$\left( \frac{1}{a_T} \right)^n = \left[ 1 + \frac{\theta_1 T_1 + \theta_2 T_1^2}{b_1 + b_2\sigma} \right],$$

or

$$\frac{1}{a_T[T_1, \sigma]} = \left[ 1 + \frac{\theta_1 T_1 + \theta_2 T_1^2}{b_1 + b_2\sigma} \right]^{\frac{1}{n}} \quad (34)$$

For varying temperature in which  $T_1 = 60t$ , and  $\sigma = \sigma_0 = 4$  ksi the reduced time  $\zeta$  is defined from (34) in (30) as

$$\zeta = \int_0^t \left[ 1 + \frac{\theta_1(60s) + \theta_2(60s)^2}{b_1 + b_2(4)} \right]^{\frac{1}{n}} ds \quad (35)$$

Equation (35) was evaluated numerically by using Simpson's rule. Then the creep strain was calculated by using (35) in (33) with values of  $a_1$ ,  $b_1$ ,  $b_2$ ,  $\theta_1$ ,  $\theta_2$ ,  $n$  given in (9). The total strain  $\epsilon_T(t)$  is

$$\epsilon_T(t) = \alpha T_1 + \epsilon(t) \quad (36)$$

where  $\alpha = 35.2 \times 10^{-6}$  in./in./°F is the observed value of the coefficient of thermal expansion determined by measuring the thermally induced strain at zero load.

In (36) it was assumed that the thermal expansion coefficient was independent of the stress.

The values of  $\epsilon(t)$  and  $\epsilon_T(t)$  predicted from (33,35) were calculated and are shown in Fig. 3. The strains were also evaluated numerically from the actual temperature instead of  $T_1 = 60t$  as shown in Fig. 3. For this calculation the temperature change was approximated as steps.

#### Method B

A more direct means of describing the effect of temperature (both constant temperature and varying temperature) on the time-dependent part of the strains (16b) is as follows. Consider that temperature only affects the coefficients  $F_1'(T)$ ,  $F_2'(T)$  and  $F_3'(T)$  of (16b). Employing the modified superposition method the creep behavior under varying load and temperature can be described from (16) and the modified superposition principle (22) as follows

$$\epsilon(t) = J_0 \sigma(t) + \int_0^t \frac{\partial}{\partial \sigma} \{J_t[(t-\xi), \sigma(\xi), T(t)] \sigma(\xi)\} \frac{\partial \sigma(\xi)}{\partial \xi} d\xi \quad (37)$$

where

$$J_t[(t-\xi), \sigma(\xi), T(t)] = [\alpha_1 + \alpha_2 \sigma + \alpha_3 \sigma^2] f(T_1(t), \sigma) G(t-\xi) \quad .$$

For the present material the temperature and stress function in  $J_t$  is given by the function in square brackets in (9). In the test program shown in Fig. 3  $\sigma = 4000$  psi, and  $T - 75 = T_1 = 60t$ . Since the stress was constant, integration of (37) by means of the Heaviside function and making use of (9) yielded the following prediction of strains during this part of the experiment:

$$\epsilon(t) = a_1\sigma + \{[b_1 + \theta_1(60t) + \theta_2(60t)^2]\sigma + b_2\sigma^2\}t^n \quad (38)$$

The corresponding total strain is

$$\epsilon_T(t) = \alpha(60t) + \epsilon(t) \quad (39)$$

The results calculated from (38,39) using the values of  $a_1$ ,  $b_1$ ,  $b_2$ ,  $\theta_1$ ,  $\theta_2$ ,  $n$  given in (9) are also plotted on Fig. 3.

Also shown in Fig. 3 are the strains calculated from (38) and (39) using the actual temperature instead of  $T_1 = 60t$ .

Figure 3 shows that (38) and (39) (Method B) are closer to the experimental data than (33-36) (Method A). Also shown by square points in Fig. 3 are the strains at the indicated time from creep tests at a constant stress of 4000 psi and the constant temperature indicated. Comparing these with the data for constant stress but increasing temperature it is observed that the four points nearly coincide with Method B, as would be expected from the nature of Method B, but they lie below and nearly parallel to the increasing temperature test instead of above as would be expected. This fact may result from differences in temperature distribution between the two types of tests. The prediction of Method A lies still farther below the test data for the increasing temperature test.

#### Recovery under Continuously Varying Temperature

After the rising temperature reached the programmed value of 135°F at one hour the temperature was caused to decrease gradually. Also, the load was removed at one hour. The temperature and strain during this period of recovery at

decreasing temperature were as shown in Fig. 8. The strain corresponding to the recovery at decreasing temperature was computed for both Methods A and B by using the modified superposition principle to account for the abrupt removal of the stress at one hour. That is, the actual history was considered to be equivalent to (a) creep at constant stress through the entire period but at the actual variation in temperature, minus (b) the creep at the same stress applied at one hour with the decreasing temperature that actually occurred from one hour on. Since the temperature variation did not occur at a constant rate, especially during falling temperature, the temperature variation was approximated by a series of appropriate steps of constant temperature. This method accounted for the small variations from constant rate of temperature rise shown in Fig. 3. The results for the rising portion by the step method were nearly the same as for  $T_1 = 60t$  as shown in Fig. 3.

Computation of the recovery strain by Method A using the modified superposition principle was as follows:

$$\epsilon(t) = (b_1\sigma_o + b_2\sigma_o^2)[G(\zeta) - G(\zeta - \zeta_1)] \quad , \quad (40)$$

$$t > 1 \text{ hr.}$$

where  $\sigma_o = 4 \text{ ksi}$  ,  $G(\zeta) = \zeta^n$  and  $G(\zeta - \zeta_1) = (\zeta - \zeta_1)^n$  , and

$$\zeta = \int_0^t \left[ 1 + \frac{\theta_1 T_1(s) + \theta_2 T_1^2(s)}{b_1 + b_2\sigma_o} \right]^{\frac{1}{n}} ds \quad , \quad (41)$$

$$\zeta_1 = \int_0^{t_1} \left[ 1 + \frac{\theta_1 T_1(s) + \theta_2 T_1^2(s)}{b_1 + b_2\sigma_o} \right]^{\frac{1}{n}} ds \quad , \quad (42)$$

where  $t = 0$  was taken to be the time at which the first stress was applied, and (41,42) were evaluated by Simpson's rule.

The calculated values of strain by Methods A and B for the recovery at decreasing temperature are shown in Fig. 8. Figure 8 shows that in the early

stage of recovery the test data lie between predictions of the two methods but closest to Method A. However, the shape is predicted more accurately by Method B.

#### Conclusions

The following conclusions may be drawn from the experiments and analyses presented in this paper.

(1) Results of the constant stress creep tests at different temperatures indicate that creep strains are separable into time-independent and time-dependent parts. The time-independent part of the strain was found to be independent of temperature and linearly proportional to the stress. The time-dependent part of the strain was represented by a power law of time with a constant exponent independent of temperature. It was found that the time-dependent strain was a strongly nonlinear function of stress and temperature.

(2) Two methods were considered to account for varying temperature by using the constitutive relation derived from different constant temperatures. One method employed a reduced time involving a shift factor which was a function of both stress and temperature history. The other considered the strain to be a function only of the current temperature not its past history. Both employ the modified superposition principle. The latter method agrees somewhat better than the former with the trend of experiments on varying temperature and recovery. It is also the easier to employ.

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TABLE I  
Tension Creep Experiments

TEST	TENSILE STRESS, ksi	TEMPERATURE, °F	$\epsilon = \epsilon^o + \epsilon^+ t^n, n = 0.143$	
			$\epsilon^o, \%$	$\epsilon^+, \%$
38	1.0	75	0.195	0.017
40	1.0	100	"	0.028
51	1.0	130	"	0.053
52	1.0	140	"	0.062
53	1.0	150	"	0.068
54	1.0	160	"	0.092
63	2.0	75	0.390	0.052
65	2.0	102	"	0.081
41	2.0	115	"	0.100
46	2.0	130	"	0.118
72	2.0	135	"	0.125
48	2.0	140	"	0.142
77	3.0	75	0.585	0.083
47	3.0	130	"	0.185
50	3.0	140	"	0.267
84	4.0	75	0.780	0.133
92	4.0	102	"	0.178
91	4.0	115	"	0.216
78	4.0	126	"	0.268
81	4.0	135	"	0.298
97	4.0	145	"	0.328



TABLE II  
Repeated Experiments in Tension

TEST	STRESS, psi	TEMPERATURE, °F	STRAIN AT 1 hr., Per Cent
39	2000	75	0.4341
63	"	"	0.4419 <sup>+</sup>
76	"	"	0.4368
41	"	115	0.4902 <sup>+</sup>
67	"	"	0.4925
49	"	150	0.6368
114	"	"	0.5894
2	3000	75	0.6439
77	"	"	0.6672 <sup>+</sup>
87	4000	115	1.0843
91	"	"	0.9957 <sup>+</sup>

+ Used in Fig. 1, 2.

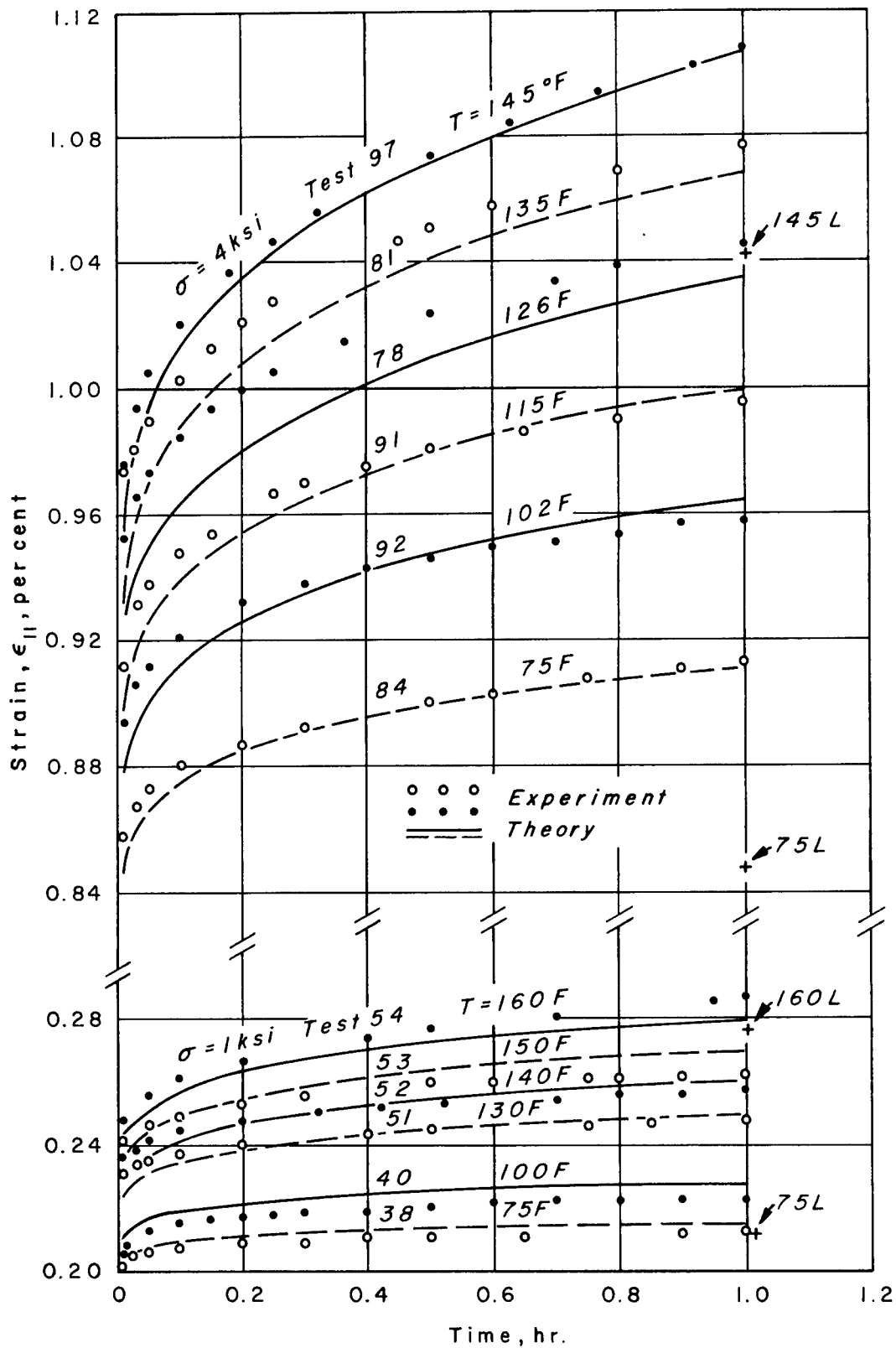


Fig. 1. Creep curves for 1 and 4 ksi stress at different constant temperatures

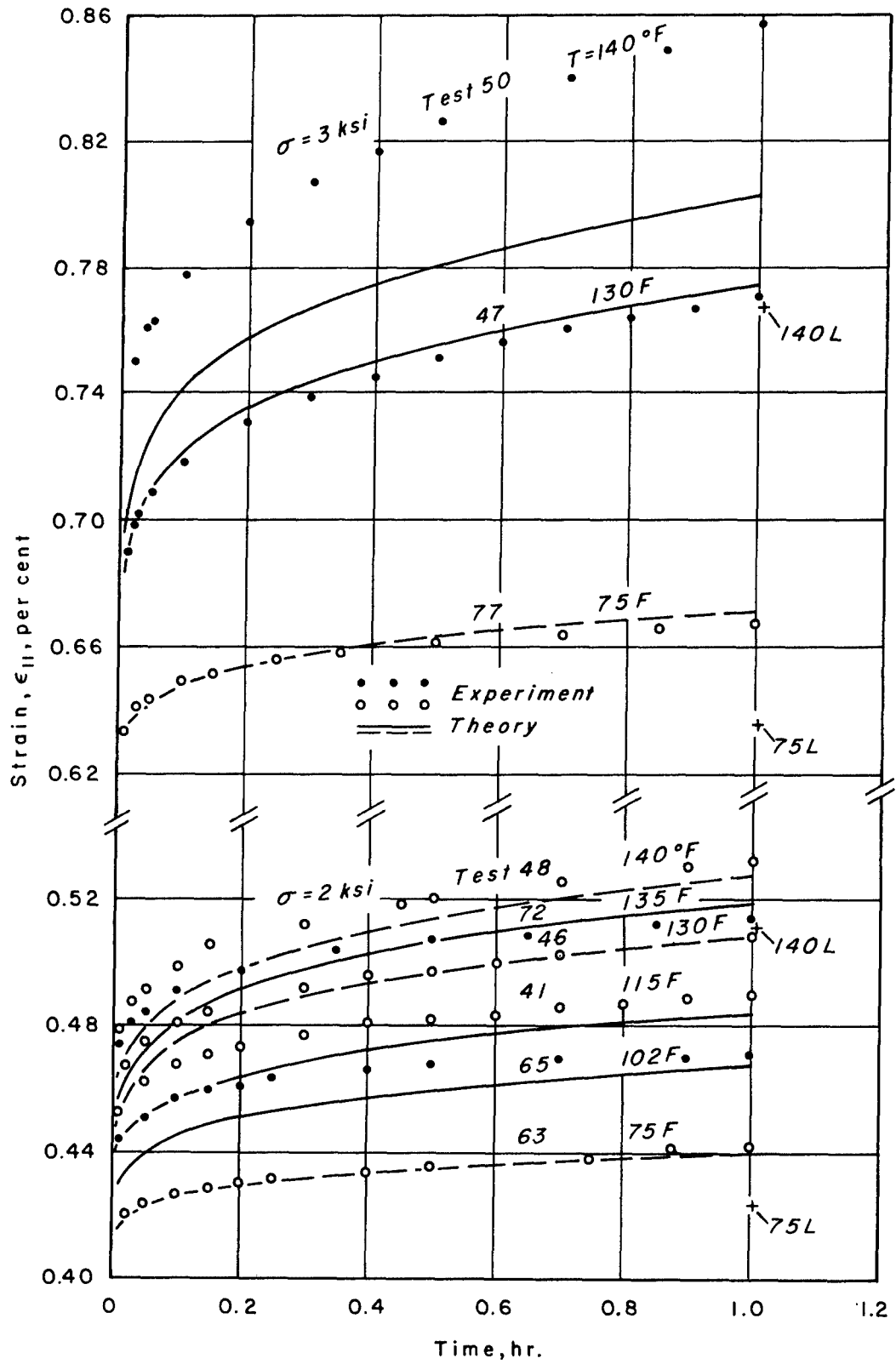


Fig. 2. Creep curves for 2 and 3 ksi stress at different constant temperatures

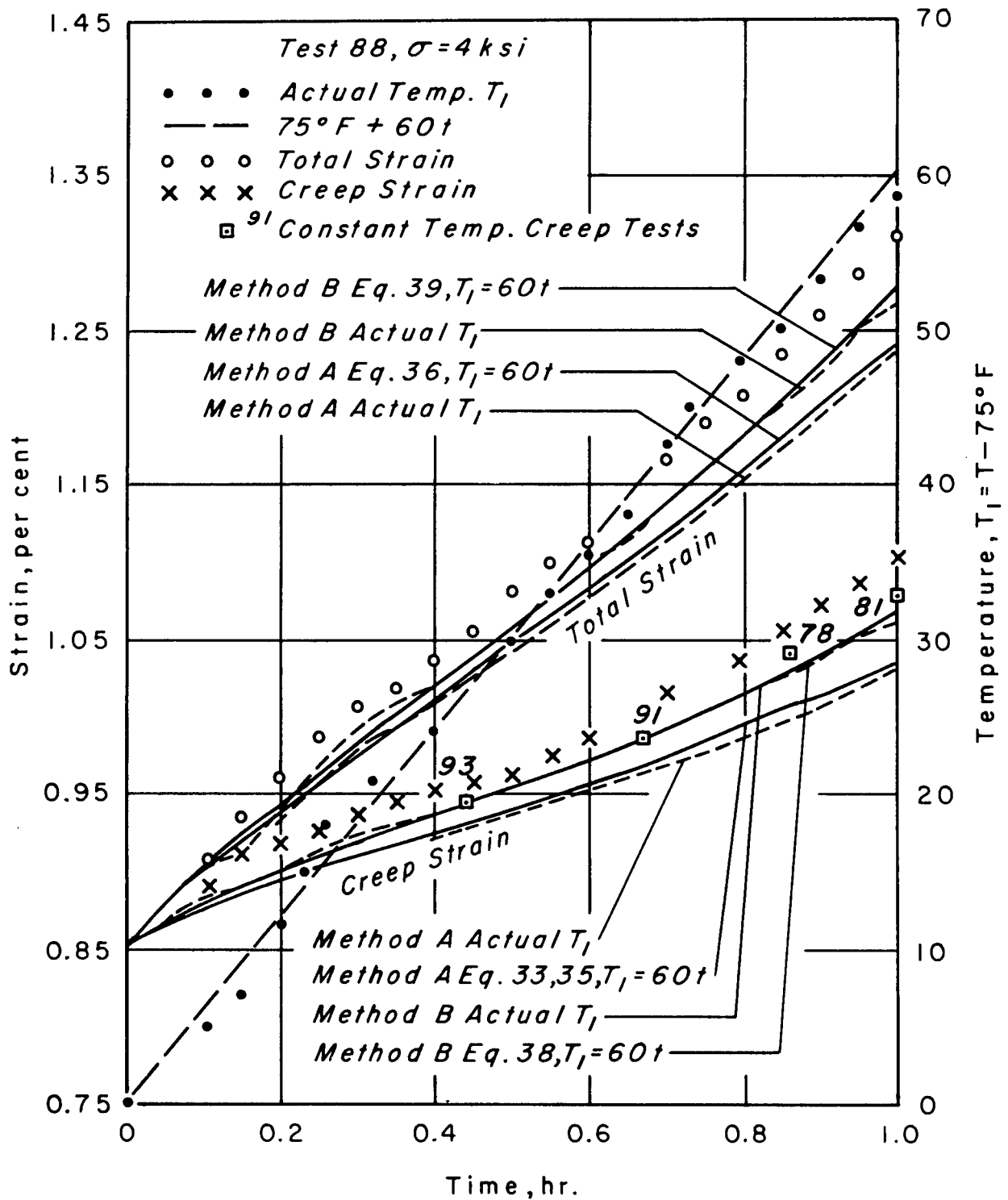


Fig. 3. Creep at a constant stress of 4 ksi and increasing temperature

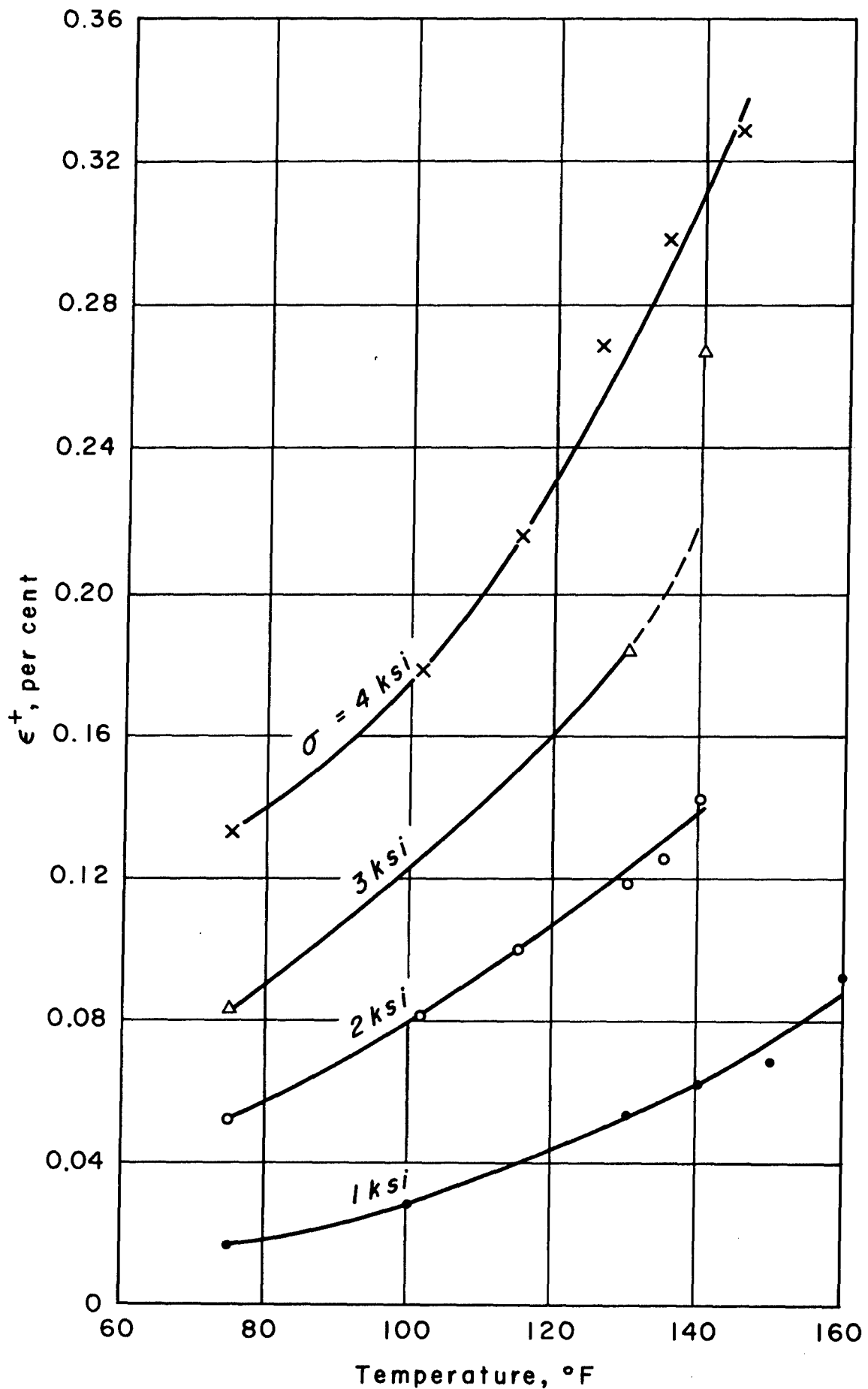


Fig. 4. Coefficient of time-dependent strain versus temperature for several stresses

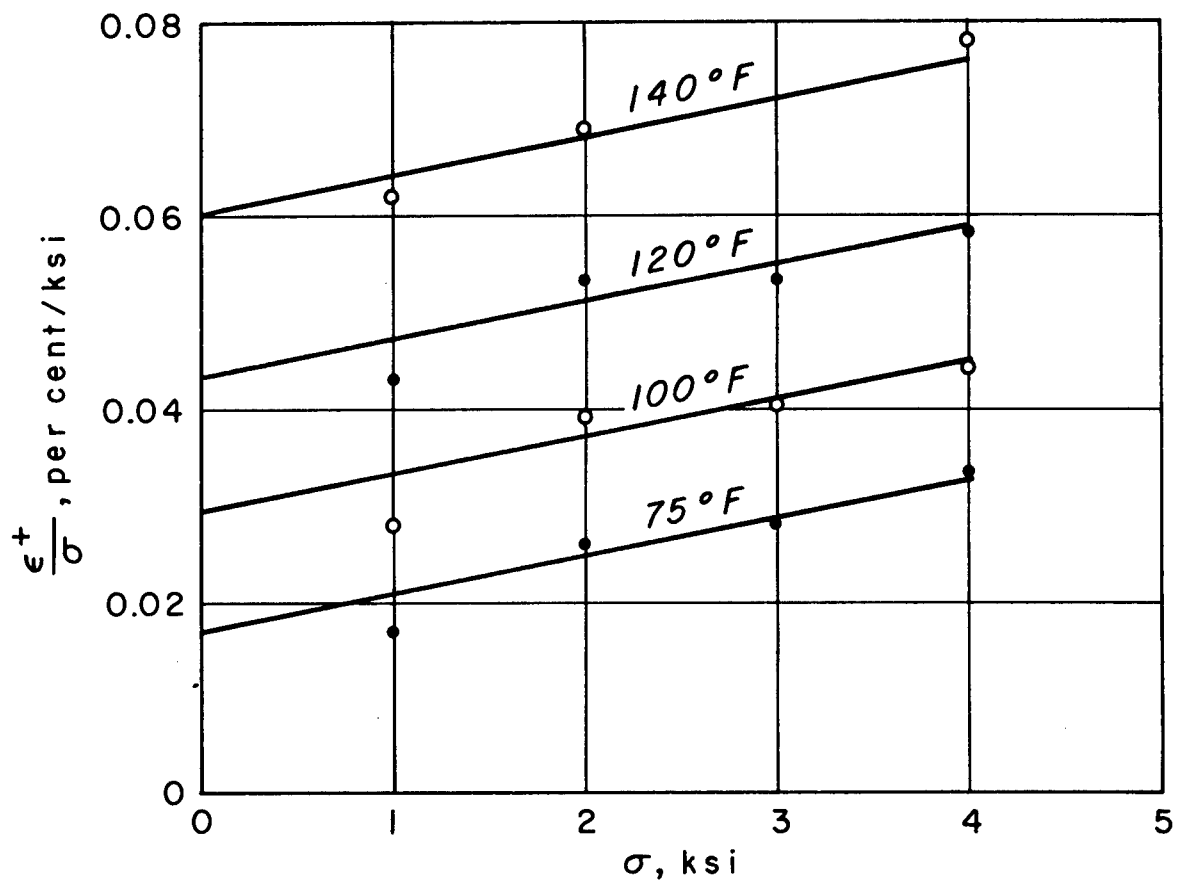


Fig. 5.  $\epsilon^+/\sigma$  versus  $\sigma$  (see Eq. 7a)

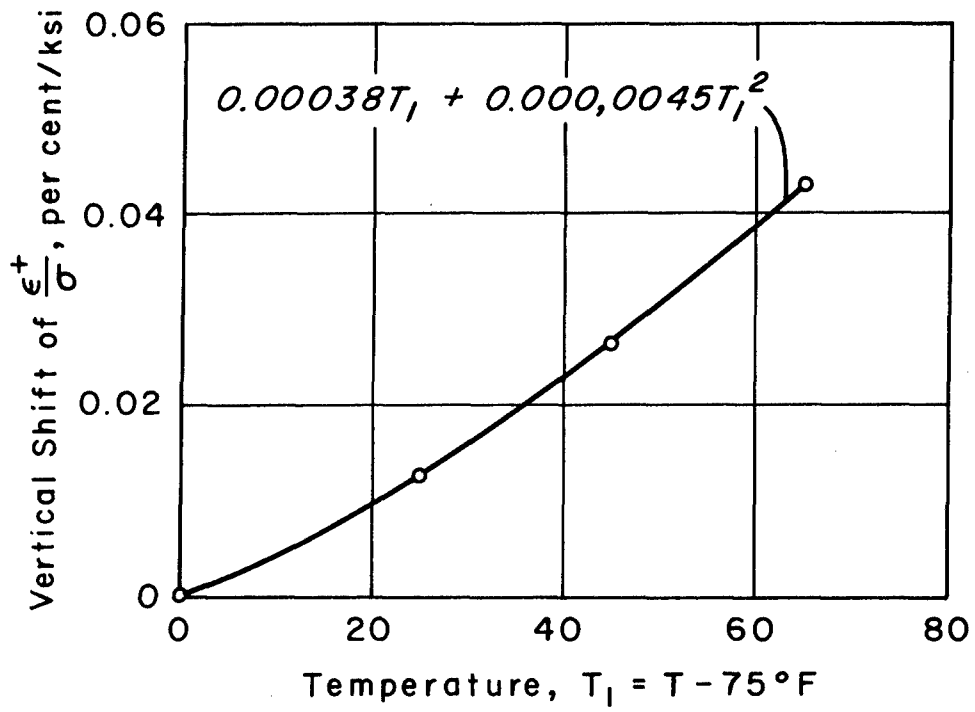


Fig. 6. Vertical shift of  $\epsilon^+/\sigma$  with temperature (see Eq. 7d)

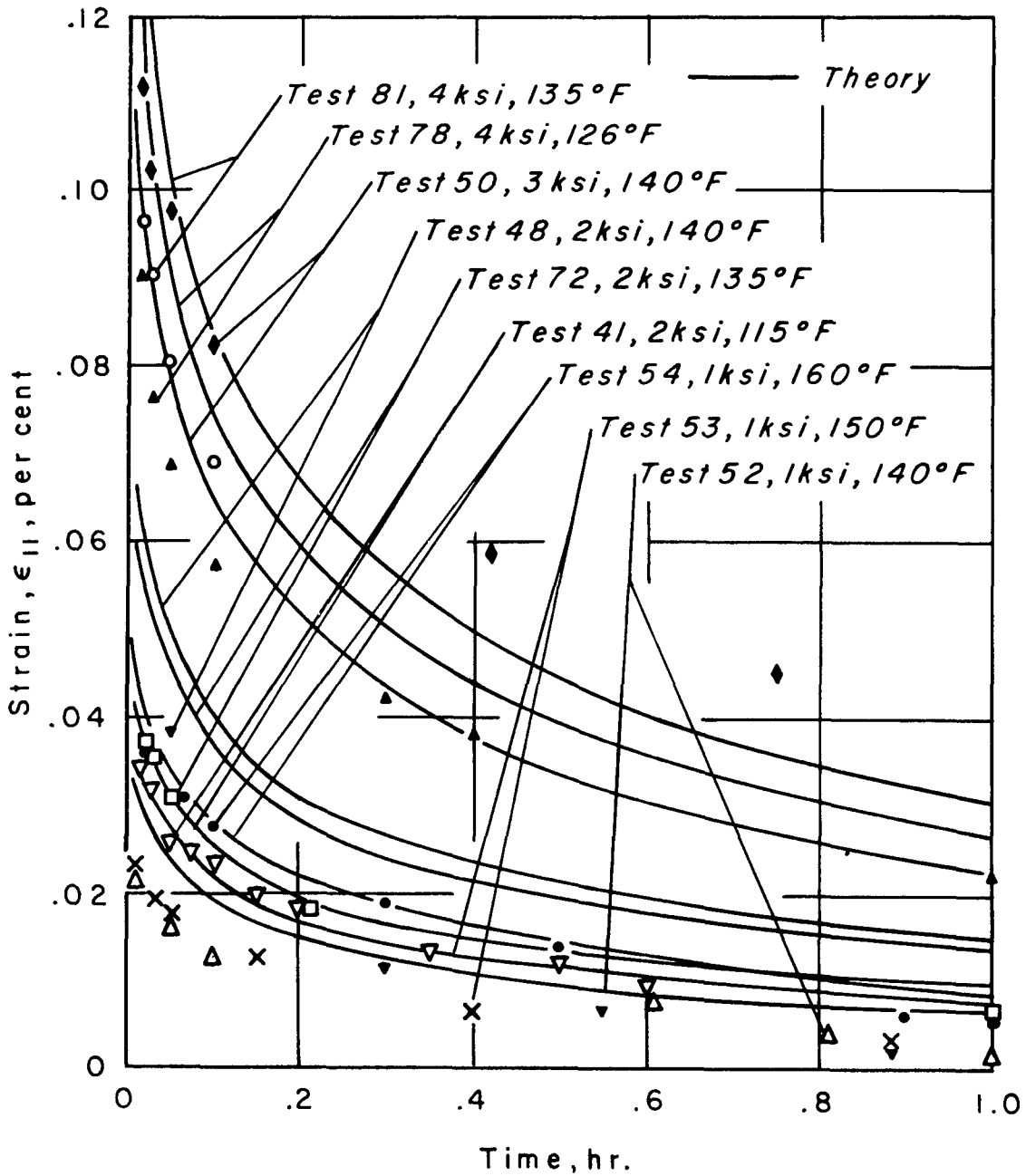


Fig. 7. Recovery at zero stress and at the indicated temperature following creep for one hour at the same temperature and at the stress shown



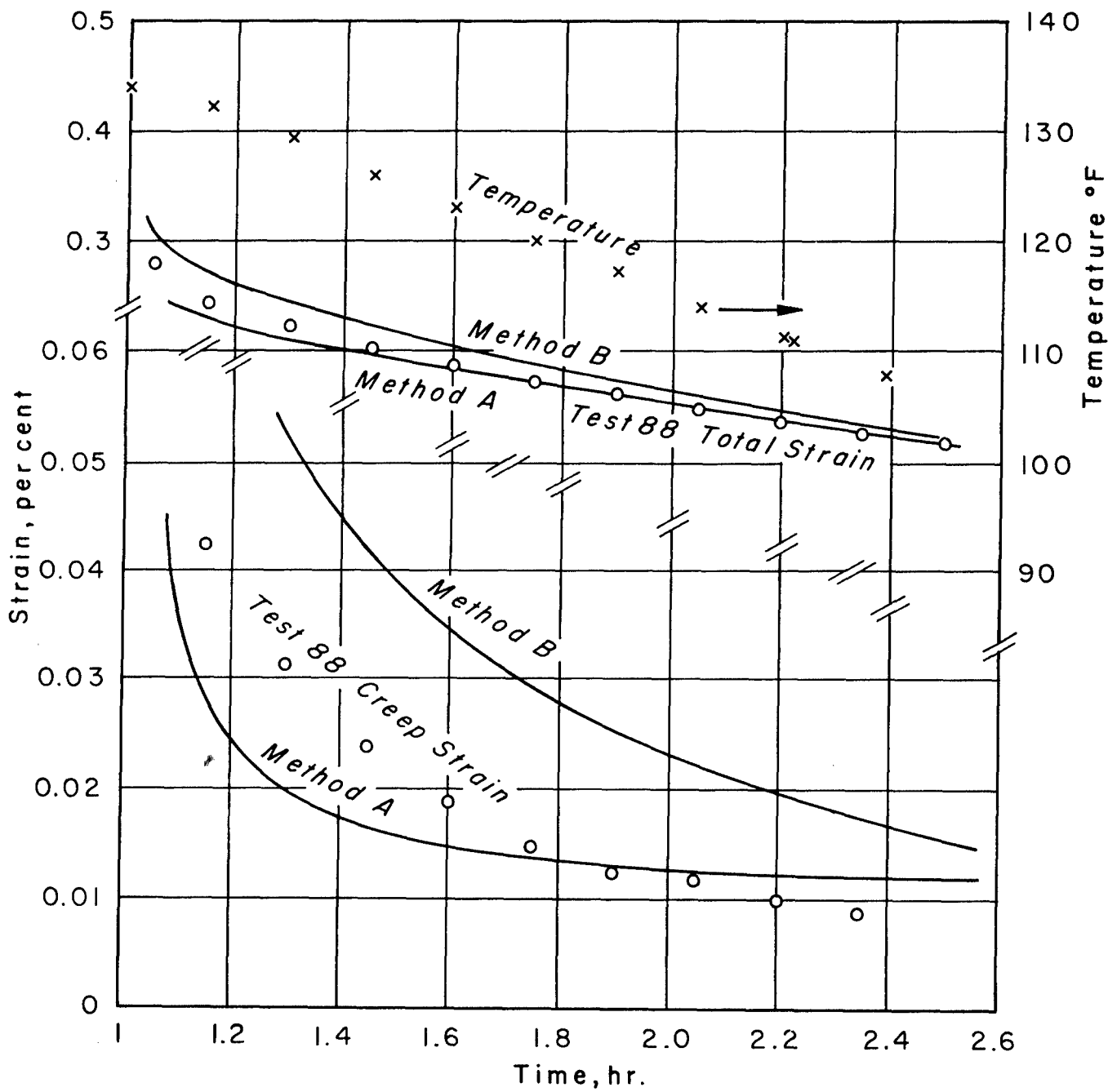


Fig. 8. Recovery at zero stress during decreasing temperature following creep at a constant stress of 4 ksi during increasing temperature (see Fig. 3)